MODELS FOR ANALYSIS OF ANNIHILATIONS

C. GHESQUIERE

Laboratoire de Physique Nucléaire, Collège de France - PARIS

I - INTRODUCTION

Mainly aimed at opening a discussion on the models we have tried to use in our analysis of antiproton annihilations, this talk will briefly review the main trends of the analysis in the last years. But, I will raise questions about what looks to me open to criticism, hoping that together we might reach some conclusion about the future.

Looking at what we have searched for in the annihilation processes, I see more or less three periods in the game. The first, extending from 1956 to the early 60 's when statistics were scarce, is oriented towards global processes, concentrating on multiplicities, angular correlations...

Soon after opened the wide field of boson resonances, and the annihilations in π and K appeared as a very valuable tool to look for bosons in the absence of baryon fields. Most of the effort came in the study of resonant amplitudes : branching ratios for production, quantum numbers and decay properties. This research appeared very fruitful, it being needless to remind you of the many discoveries : C,D,F_1,E... The model in favour for reaching this goal was the Final-State-Interaction model.

Trying to refine on the use of Final-State-Interactions, including higher terms and a more complete approach, some attemps have been made to solve the three-body problem, through the use of Faddeev type equations, but difficulties showed up with the problem of introducing the effect of initial states.

Very fortunately, as at the time this model was going into such intricacy as to become useless, there arrived a very attractive and clear way to deal with all crossed channels involved in the annihilation : I mean duality and the Veneziano formulation of duality. Its success in finding a simple explanation to the puzzling problem of $\bar{p}\, n \longrightarrow \pi^+ \pi^- \pi^-$ at rest gave a boost that brought hope to experimentalists.

These models being well know, it would be a tedious job to go through them in detail. I will rather, from an experimentalist 's point of view, go through some definite applications and try to pinpoint what problems we are confronted with when applying the models.

As a remark I would also mention that I shall not talk about the treatment of angular momenta and spins. As the kinematical part of the matrix element will be the subject of the next talk, I shall stick to the dynamics.

II- Model of Final State Interaction (F.S.I.)

In a paper in 1952 (1) Watson formulated his model of F.S.I., which was later developped by many others -Blackenbecler for instance(2)- Essentially, the idea is that if several particles - π and K- in our case - are produced within a rather small interaction volume ($4/3\ \pi r_0^3$) at low relative momentum q, if the relation

$$q r_0 < \hbar$$

is fulfilled, they interact strongly between themselves. At the limit we could forget about the initial production mechanism, and assume that the interaction between final particles dominates. The elementary cross-section in then proportional to σ_q between two particles.

$$d\sigma \simeq \sigma_q\, d^3q$$

If the two-body interaction in a state of angular momentum L saturates the unitarity, we can make the approximation

$$d\sigma \simeq \frac{\sin^2 \delta_L}{q^{2L}}\, dq$$

This condition is well fulfilled when a resonance is very strong in the L channel (ρ, K^* for instance) and we can make the realistic approximation

$$\mathrm{tg}\, \delta_L = \frac{\frac{1}{2}\Gamma_L}{E_L - E}$$

and all calculations being done

$$\sigma = \frac{\pi(2L+1)}{k^2}\ \frac{\Gamma_L \Gamma_a}{(E-E_L)^2 + \frac{1}{4}\Gamma_L^2}$$

This formula describes the two-body interaction. We now have to sum over all possible pairs of particle and integrate it over the available phase space.

In fact, in most of the analysis of $N\bar{N}$ states, all waves have been approximated either by $\sin \delta_L = c^{st}$ or resonant.

The model has in fact been reduced in its use to the isobaric model, but we could as well use the phase shift expression extracted from $\pi\pi$ or $K\pi$ scattering obtained by some other method (3).

At this point I would make three remarks :

1) First is that we have no proof that the condition $r_0 q < \hbar$ defined by Watson holds.

We know almost nothing about the interaction range. In the first attempts of the early experiments in the late 50's (4,5,6,7,8) trying to explain the mean multiplicities in terms of a statitical model, in order to reproduce the data they had to play with the only available parameter, the interaction volume, and they currently found of the order of IO times the elementary volume defined from the pion compton wavalengh

$$V \;\#\; 10 \left[\frac{4}{3} \pi \left(\frac{\hbar}{m_\pi c} \right)^3 \right]$$

If this were true the condition would be very far from being filled.

Many theoricists have tried to explain an increase of multiplicity avoiding such a large interaction volume. I may quote Gatto (9), Sudarshan (IO), even Pomeranchuk (II) Koba and Takeda (I2).

The Ball and Chew (I3) and the Briyan andPhillips (I4) models for low energy $\bar{p}p$ interaction lead to the conclusion

$$r \leqslant 1 \text{ Fermi}$$

The best answer to these divergent opinions may be to say from an experimental point of view, that since we did apply this model with some success, there must be something realistic in it.

2) The next remark is relative to the parametrization of the phase - shift. Trying to go beyondthe usual expression for the phase-shift, R. Bizzari et AL (I5) have tried to include in their analysis of the channel

$$\bar{p}\,p \rightarrow \omega^\circ \pi^+ \pi^- \quad \text{at rest}$$

a more sophisticated expression for δ_0°, in order to explain a rather peculiar behaviour of the $\pi^+\pi^-$ effective mass distribution (Fig. I) and extended it later to the analysis of the channel

$$\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^- \quad (16)$$

Although their result is inconclusive between a type of formula with a C.D.D. pole

$$\frac{q}{m} \operatorname{cotg} \delta_0^\circ = a \frac{(b-m^2)(c-m^2)}{(d-m^2)}$$

or a phase shift à la Baton (3) (Fig.2) with a strong absorption opening around 960 MeV/c, it shows that we can probably do better than the rough isobar approximation -even with some correction for variable width.

3) The last remark is about the symmetrization of the matrix element. It is an essential feature, though not the only one if we want to reproduce the so called Goldhaber effect (I7) between the angular distribution of like and unlike pions.

In I962, Bouchiat and Flamand (I8) have shown how the angular distribution for the ρ decay could be influenced by the proper symmetrization of the amplitudes.

Nevertheless the effect of symmetrization is much less marked on the effective mass distribution, and the relative amount of resonance is not much effected by the model used.

Take for example the result shown in table 1 (extracted from (I9)) showing the percentage of resonant states in $\bar{p}p$

Table I

Gev/c	0.	1.2	2.5	3.28	5.7
$\bar{p}p \to 3\pi\rho^0$	0.28±0.04	0.34±0.04	0.51±0.09	0.29±0.07	0.26±0.03
$3\pi\rho^\pm$	0.34±0.04	0.41±0.04	0.49±0.09	0.21±0.07	0.45±0.07
$3\pi f^0$		0.06±0.01	0.16±0.05	0.13±0.06	0.09±0.03
$3\pi g$			0.026±0.01		
$2\pi\omega$	0.35±0.03	0.29±0.02	0.18±0.03	0.10±0.03	0.09±0.02

The partial percentages seems to have a rather smooth behaviour, although they have been obtained with widely different methods (20).

- At rest : smooth polynomial background + resonances, χ^2 fit over scatter plot.
- I,2 and 2,5 GeV/c : Incoherent or coherent addition of amplitudes gives comparable amount of resonance. Incoherent gives better agreement with experiment. Maximum likehood fit.
- 3,28 GeV/c : Phase space + Breit Wigner (or gaussian) resonances. χ^2 fit over histograms.
- 5,7 GeV/c : Phase space + Resonant symmetrized amplitudes χ^2 over histograms.

The F.S.I. model such as it has been used is nevertheless a first approximation model for the particles produced and neglects all successives reinteractions. Some attempts have been done to introduce these effects into the analysis.

III- The three-body problem

The three-body channels have proven in $\bar{N}N$ annililations, as well as in other reactions, one of the most valuable and easy to study. It is why around 1964, following the work of Faddeev (21), some Theorists (22,23) tried to adapt to the high energy three-body problem the method of resolution derived from the non-relativistic one. Lovelace on one side (24), Basdevant and Kreps.on another, (25) developped an application to 3π production.

Roughly they tried to get the realistic three-body scattering amplitude

$$T = T_1(z) + T_2(z) + T_3(z)$$

from the "off the energy shell"scattering amplitude for two particles $t_i(z)$, through the use of the coupled Faddeev integral equations

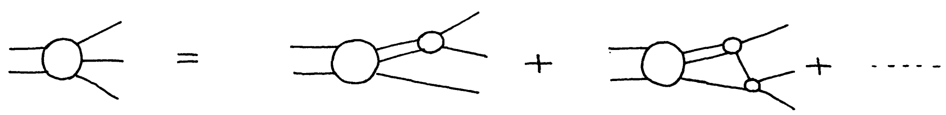
$$T_i(z) = t_i(z) + t_i(z)\,G_o(z)\,[T_j(z) + T_k(z)]$$

$$T_j(z) = t_j(z) + t_j(z)\,G_o(z)\,[T_i(z) + T_k(z)]$$

$$T_k(z) = t_k(z) + t_k(z)\,G_o(z)\,[T_i(z) + T_j(z)]$$

for more details see for instance (26)

This has the nice features of having the three-body unitarity built in and of taking into account all orders of rescattering in the final state.



The usual F.S.I. used only the first graph.

This model led finally to computational difficulties and also to trouble with the assumption concerning the propagator for three free particles $G_o(z)$.

The essential result was that the effect of these refinements becomes sensitive to experimental test only when the three-body unitarity saturates the amplitude, which is in fact the way a three π or a $K\bar{K}\pi$ resonance occurs, and this suggests caution in two cases. First when we use the F.S.I. to study the quantum numbers of some resonance extracted from the data $D \to K\bar{K}\pi$ or $\eta\pi\pi$, $E^\circ \to K\bar{K}\pi$, $A_2 \to 3\pi$

Second, if it occurs that in our formation experiments at some energy the $\bar{p}p \rightarrow 3$ body is almost saturated by some decay of a resonant state. The effect is especially sensitive at the crossing of two resonance bands inside the Dalitz plot limits.

This model has been nicely used by Hopkinson and Roberts (27) to extract the phase shift information from the reaction $\bar{p}p \rightarrow 3\pi$. Using the known P and D waves as input , they get the S, I = 0 phase shift and find a solution indicating an ε° at a mass $\simeq M_{\rho}$ and $\Gamma \simeq 480$ MeV, (Fig. 3), which may not be too bad due to the scarceness of the S wave contribution to the data. But they do not succeed in fitting the $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ data with ρ and f as input.

These efforts to solve this three-body problem could have given more results if it had not been for a more promising insight into the annihilation process which came in the form of dual models.

IV. - Duality and the Veneziano model (28)

Another aspect of annihilation appears when the momentum increases. Peripheral aspects begin to play a role marked by the forward - backward asymmetry of charge emission.

Several exchange models have been introduced but it seems that the C.L.A. model (29) has given some proof that it may reproduce most of the features in many annihilation channels (30,3I).

The amplitude then has the following form

$$|A| = \prod_{i=1}^{n-1} \left(\frac{g_i s_i + c a}{s_i + a}\right)\left(\frac{s_i + a}{a}\right)^{\alpha_i} \left(\frac{s_i + b_i}{b_i}\right)^{\alpha_i'(0)\, t_i}$$

It has the nice feature that for large intermediate subenergies s_i , it has a Regge behaviour

$$|A_i|_{s_i \rightarrow \infty} \simeq g_k \left(\frac{s_i}{a}\right)^{\alpha_i(0)} e^{\Omega_i t_i}$$

where $$\Omega_i = -\alpha_k'(0) \operatorname{Log} \frac{b_k}{a} + \alpha_k'(0) \operatorname{Log} \frac{s_i}{a}$$

and at small s_i , has a statistical behaviour

$$|A_i|_{s_i \rightarrow 0} \simeq c$$

In Fig. 4 and 5 are shown some nice results obtained by Chen (3I) using this model.

We can wonder to what extend the mass spectra can be influenced by this exchange model.

Take for instance the I,7 GeV/c $\rho\rho$ enhancement observed by Braun et Al. in $\bar{p}p \rightarrow 3\pi^+ 3\pi^-\pi^{\circ}$ at 5,7 GeV/c (32). Fig. 6.

Could the bump observed at the top of phase space be explained by a multi Regge exchange model which favours small transfers, i.e. low effective masse ? Even if a real $\rho\rho$ resonant effect exists, its share of the reaction must be very difficult to evaluate and the branching ratio is crucially model dependent.

I must quote along these lines a very interesting development of the C.L.A. model by de la Vaissière (33) in the $\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-$ reaction at 3,6 GeV/c - Where he includes in a simple way, following the ideas of Ranft (35) a certain number of resonances in the multiperipheral chain

Table II

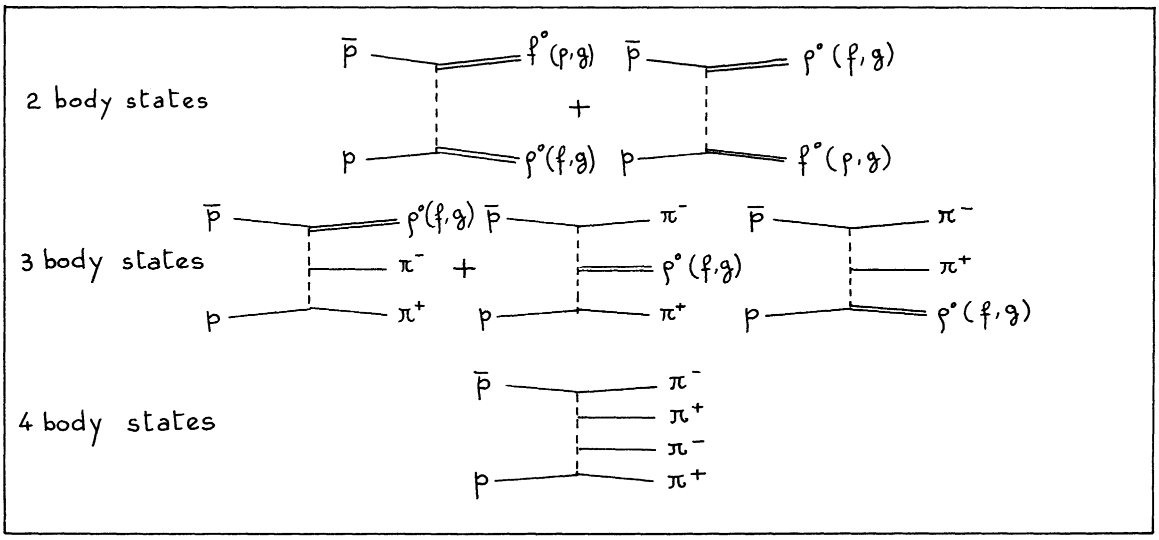


Table II shows the graphs entering in the analysis. Assuming some simple mass dependance for the coupling constant of different resonances, neglecting the exchange of Δ lines, this still rather crude model is able to reproduce quite nicely the mass and angular distribution and looks very promising Fig. 7 and 8.

The Regge exchangecould be formulated in some other ways and one of the most attractive is the Veneziano one.

Many applications of this model have now been made to annihilation processes (34, 35, 36, 37, 38, 39, 40,41), to be compared with the same F.S.I. analysis (15, 42, 43, 44, 45).

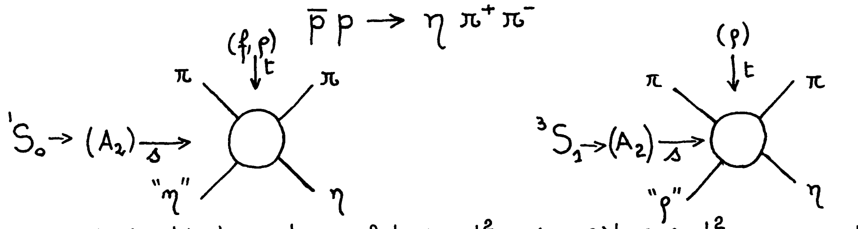
Though the Veneziano model may not be, strictly speaking, a model for analysis, and some believe it to be the real dynamics which underlay the strong interaction, some studies were conducted in parallel over the same sample of events and a review of the convergences, as well as divergences, may cast some lights on its utility as a tool for analysis.

In the table III, IV, V have been sketched the main parameters implied in the analysis and the conclusion reached by the authors, as well as some features of the F.S.I. analysis to be compared, this for the three

reactions $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$
$\bar{p}p \rightarrow \eta^\circ\pi^+\pi^-$
$\bar{p}p \rightarrow \omega^\circ\pi^+\pi^-$

TABLE III

$\bar{p}p \rightarrow \eta\pi^+\pi^-$



Total Amplitude $L = f\left|\Sigma A_s\right|^2 + (1-f)\left|\Sigma A_T\right|^2 + \alpha$ Background

$$A_{^1S_0} = \beta_0 F_0 + \beta_1 F_1 + \beta_2 F_2$$

$$F_0 = V(111,\alpha_s,\alpha_t) + V(111,\alpha_u,\alpha_t) + V(111,\alpha_s,\alpha_u)$$

$$F_1 = V(112,\alpha_s,\alpha_t) + V(112,\alpha_u,\alpha_t)$$

$$F_2 = V(112,\alpha_s,\alpha_u)$$

A relation between the β exclude an unwanted $\eta\pi$ resonance at 870 $\frac{\text{MeV}}{c^2}$

$$A_{^3S_1} = \gamma_0 G_0 + \gamma_1 G_1 + \gamma_2 G_2$$

$$G_0 = V(112,\alpha_s,\alpha_t) - V(112,\alpha_s,\alpha_u) + V(112,\alpha_t,\alpha_u)$$

$$G_1 = V(223,\alpha_s,\alpha_t) + V(223,\alpha_u,\alpha_t) - V(223,\alpha_s,\alpha_u)$$

$$G_2 = V(213,\alpha_s,\alpha_t) + V(213,\alpha_u,\alpha_t)$$

Trajectories

$$\rho = \alpha(t) = 0.39 + 1.06\,t + i\,0.18\,(t-t_0)^{1/2}$$

$$f = \alpha(t) = 0.39 + 1.06\,t + i\,0.13\,(t-t_0)^{1/2}$$

$$A_2 = \alpha\,(s \text{ or } u) = 0.20 + 1.06\,x + i\,0.18\,(x-x_0)^{1/2}$$

Fit without satellites (only F_0 and G_0) $\chi^2 = 70/35$ cells

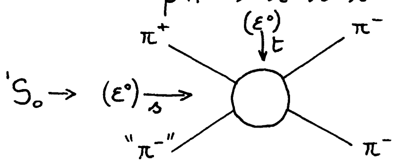
Fit with satellites $\chi^2 = 40/35$ cells

Results of Fit

	Veneziano	F.S.I.
Fitted parameters	$^1S_0/^3S_1 = 0.60$	1.20
	$\beta_0 = \gamma_0 = 1$	
	$\beta_1 = 1.9 \pm 0.3$	
	$\beta_2 = 1.9 \pm 0.3$	
	$\gamma_1 = 1.1 \pm 0.3$	
	$\gamma_2 = 1.8 \pm 0.4$	
	$\frac{^1S_0 \rightarrow A_2\pi}{^3S_1 \rightarrow A_2\pi} \simeq 1$	5

TABLE IV

$\bar{p}n \rightarrow \pi^+ \pi^- \pi^-$



Total Amplitude $\quad L = \alpha \left| A(s,t) \right|^2$

$$A_{^1S_0} = \left[0.885\,(s,t) - 0.034 \right] \frac{\Gamma(1-\alpha(s)) \cdot \Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

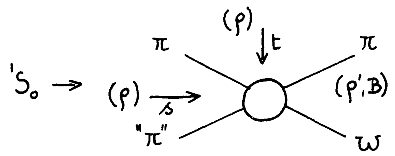
Trajectory $\quad \alpha_\varepsilon(s) = 0.483 + 0.885\,s + i\;0.28\,(s - 4\mu^2)^{1/2}$

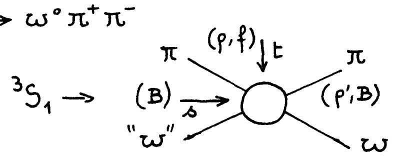
Hypothesis $\quad \rho$ and f° are decoupled

Fitted parameters $\quad \alpha$ (Normalisation) and width of ε°

TABLE V

$\bar{p}\,p \rightarrow \omega^\circ \pi^+ \pi^-$





$$A_{^1S_0} = \beta_0\, B_0 + \beta_1\, B_1 + \beta_2\, B_2 + \beta_3\, B_3$$

$$B_0 = V(112, \alpha_s, \alpha_t) + V(112, \alpha_t, \alpha_u) + V(112, \alpha_u, \alpha_s)$$

$$B_1 = V(223, \alpha_s, \alpha_t) + V(223, \alpha_t, \alpha_u) + V(223, \alpha_u, \alpha_s)$$

$$B_2 = V(224, \alpha_s, \alpha_t) + V(224, \alpha_u, \alpha_t)$$

$$B_3 = V(224, \alpha_s, \alpha_u)$$

Trajectories $\quad \alpha_B(x) = -0.56 + 1.06\,x + 0.28\,i\,(x - x_0)^{1/2}$

$\qquad\qquad \alpha_{\rho'}(x) = 0.39 + 1.06\,x + 0.38\,i\,(x - x_0)^{1/2}$

$A_{^3S_1}$ has 5 independent spin Amplitudes

$$A_{^3S_1} = e_{4\mu} e_{3\nu} \left[-g^{\mu\nu} A_1 - P_1^\mu P_2^\nu A_2 - P_2^\mu P_1^\nu A_3 + P_1^\mu P_1^\nu A_4 + P_2^\mu P_2^\nu A_5 \right]$$

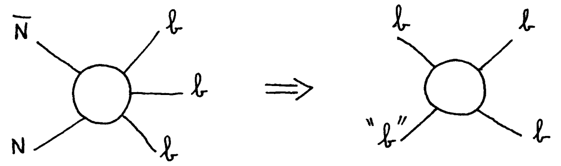
Good fit only with satellites - especially in $^1S_0$ - 10 parameter fit

Results of Fit

Veneziano	F.S.I
$^1S_0 / ^3S_1 \simeq 0.09$	0.25
B comes from $^3S_1$	B comes from $^3S_1$
$B + \rho_A$	$B + \rho_V$
"ρ" Bump due to ε° Dip at 950 MeV/c^2 and bump at 1.05 GeV/c^2 not reproduced	Good ρ signal Adequate δ_0° reproduce the dip and bump structure

Some assumption have been added to the original Veneziano formulation

- The fact that $\bar{p}p$ or $\bar{p}n$ at rest annihilate in well defined initial states simplifies the application of the Veneziano formula from a 5 point to a 4 point function and avoids most of the troubles coming from the N and $\bar{N}$ spins.



at the expense of dealing with an "Off mass shell" boson. As Lovelace remarks (34) the Regge trajectory exchanged cannot depend on the external mass, but the relative amount of the amplitude may vary and its significance is not obvious.

- The resonances are normaly represented by zero-width poles - This unrealistic and non- unitary technique is turned round by introducing an imaginary part to the trajectory- For instance

$$\alpha_\rho(s) = 0.39 + 1.06\, s + i\ 0.18\,(s - s_0)^{1/2}$$

This is of course just a trick and may appear as just an arbitrary parameter , but on the other side it does not allow an individual adjustment of the width along the same trajectory.

- We see also that all attempts to fit the data, in order to get goodness comparable to the one obtained by F.S.I, have to decouple some trajectory, the ρ in $\bar{p}n \to 3\pi$; or introduce satellites terms, in $\bar{p}p \to \eta\pi\pi$ (table IV) where the part of the satellites is larger than the leading trajectory.

This tendency is emphasized by Altarelli and Rubinsteim (35) or Barger (36): in the $\bar{p}n \to \pi^+\pi^-\pi^-$ fit they introduce a general form of formula

$$H(s,t) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} C_{nm} \frac{\Gamma(n-\alpha(s))\,\Gamma(m-\alpha(t))}{\Gamma(n+m-\alpha(s)-\alpha(t))}$$

An important question is how far we should go in applying Veneziano. Should we look more for general features, or should we try to reproduce finer details at the expense of adding many parameters and questionable assumptions. In fig. 9 you can see for instance the difference between a fit with only leading trajectories and with satellites.

I have the feeling that the first solution gives a more general insight into physics, especially if we relate crossed reactions. On the other end it may help also to reproduce some features that were not easy to obtain by the isobaric model, for instance the shape of ρ° in $\bar{p}p \to \pi^+\pi^-\pi^{\circ}$ (Fig. I0).

It is worth mentioning that the results of Veneziano analysis are not always consistent with those obtained by F.S.I. I do not mention the $\bar{p}\,n \to \pi^+\pi^-\pi^-$ analysis, where Jengo and Remiddi (37) have shown that introducing the ρ' and ε° hypothesis in the isobaric model gives a prediction not too far from Veneziano Fig. II. But consider for instance the A_2 production in $\bar{p}p \to \eta\,\pi^+\pi^-$, the F.S.I. model gives

$$\frac{{}^1S_0 \to A_2\,\pi}{{}^3S_1 \to A_2\,\pi} \simeq 5$$

Whereas Veneziano gives 1 (39)

And in $\bar{p}p \to \bar{K}K\,\pi$, F.S.I. gives

$$\frac{{}^1S_0 \to A_2\,\pi}{{}^3S_1 \to A_2\,\pi} \simeq 7$$

Whereas Veneziano gives 0 (4I).

Another example $\bar{p}p \to \omega\,\pi^+\pi^-$ in F.S.I. gives $\omega\rho \to 25\,\%$ and evidence for ρ_V at I250 MeV/c^2, Veneziano gives essentiely no $\omega\rho$ production , replaced by $\omega\,\varepsilon^{\circ}$ and a ρ_A at I250 MeV/c^2. Where is the truth ?

V - CONCLUSION

Now at the time to reach some conclusion, I feel rather embarassed. First I apologize for all the nice and important work I did neglect, but this talk does not intend to be an exhaustive survey, rather it should raise question and discussion.

It is trivial to say that in spite of some progress we are far from having a clear view of what to use as a model for analysis. The same could be said for the whole field of strong interaction.

My feeling is that dual models may be the more promising, as they permit connexion with many crossed channels πp, $K\,p$ ..., Veneziano formalism is a way to tackle the problem, first results are not too discouraging and work could be continued along these lines. Some attempts are now being done to go beyond the four-point function, and it looks worthwhile to extend the duality ideas to annihilations in flight and more complicated final states, with $n\pi$ and $K\bar{K}\,n\pi$. Data are already in our hands.

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29) A Reggeized Multipheral Model for inelastic processes at High Energy
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30) The annihilation reaction $\bar{p}p \to 2\pi^+ + 2\pi^-$ at high Energy and a multi-Regge Model.
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33) Analysis of the annihilation reaction $\bar{p}p \to 2\pi^+ \, 2\pi^-$ at 3,6 GeV/c in terms of a multiperipheral model including resonances.
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4I) A study of the $\bar{K}K \pi$ final states for $N\bar{N}$ annihilations at rest in The Veneziano language.
C. Ben fatto - Cassandro - Lusignoli - Nicoli - N.C. 1 A (I97I) 255

42) Production of three charged Pions in $\bar{p}+n$ Annihilations at rest.
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43) $\bar{p}p$ annihilation at rest into $\eta \pi \pi$
P. Espigat et Al. N.P. B 36 (I97I) 93

44) Production of three pions $\bar{p}p$ annihilation at rest.
A. Foster et Al. N.P. B 6 (I968) I07

45) The annihilation at rest $\bar{N}N \longrightarrow K\bar{K}\pi$
A. Bettini et Al. N.C. 68 A (I969) II99

FIGURE CAPTION

Fig. I : Extracted from (I5)

Histogram of the effective mass $\pi^+\pi^-$ in $\bar{p}p \rightarrow \omega^\circ\pi^+\pi^-$ at rest. Bumps are hidden by ρ° and f° contribution. The dip at $\simeq$ 950 MeV/c2 is clearly visible.

Fig. 2 : Extracted from (I5)

Shape of the δ_0° phase shift introduced to best fit the data.

Fig. 3 : Extracted from (32)

δ_0° phase shift from the three-body analysis of $\bar{p}p \rightarrow \pi^+\pi^-\pi^\circ$ at rest.

Fig. 4 : Extracted from (23)

Histograms of transverse momentum distributions, for various multiplicities at 5,7 GeV/c $\bar{p}p$ annihilations. Solid lines are the fit with C.L.A. model.

Fig. 5 : Extracted from (23)

Histograms of single pion angular distribution in the c.m. π^- and reflected π^+ distributions are added. Dashed lines are from the model.

a) $2\pi^+ \; 2\pi^-$ at 3,28 GeV/c
b) $2\pi^+ \; 2\pi^-\pi^\circ$ at 3,28 GeV/c
c) $2\pi^+ \; 2\pi^-$ at 5,7 GeV/c
d) $2\pi^+ \; 2\pi^-\pi^\circ$ at 5,7 GeV/c

Fig. 6 : Extracted from (24)

Effective mass $2\pi^+2\pi^-$ distribution selected for $2\pi^+2\pi^-$ systems having two $\pi^+\pi^-$ mass combinations in the various $M_{\pi^+\pi^-}$ indicated. Curves on a, b and c are phase space predictions. Curve on d is (phase space + B.W.) fit.

Fig. 7 : Extracted from (25)

Effective mass distribution of $\pi^+\pi^-$ from $\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-$ at 3,6 GeV/c. Dashed line is the prediction of the simple C.L.A. model without resonance. Full line best fit with ρ and f.

Fig. 8 : Extracted from (25)

Outer left : Angular distribution of $\pi^\pm$ in the production c.m. in $\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-$ at 3,6 GeV/c. Dashed line is the prediction of the model.
Three righ : Angular distribution of the $\pi^+\pi^-$ combinations in the center of mass system with several selection criteria. Full lines are prediction of the model.

Fig. 9 : Extracted from (39)

Effective mass distribution for $\eta\pi^\pm$ and $\pi^+\pi^-$ in the reaction $\bar{p}p \rightarrow \eta\pi^+\pi^-$ at rest.
Solid lines are Veneziano best fit. Dotted lines are the results of Veneziano fit without satellites terms.

Fig. IO : Extracted from (37)

Comparison between various models and the experimental $\pi^+ \pi^-$ mass^2 distribution in $\bar{p}p$

Model I = Basic Veneziano

Model 2 = Virasoro formulation

Model 3 = Simple isobaric formula

Fig. II : Extracted from (37)

Comparison between the Veneziano model and the B.W. model with some initial assumptions, for the mass squared distributions in $\bar{p}n \rightarrow \pi^+ \pi^- \pi^-$ at rest.

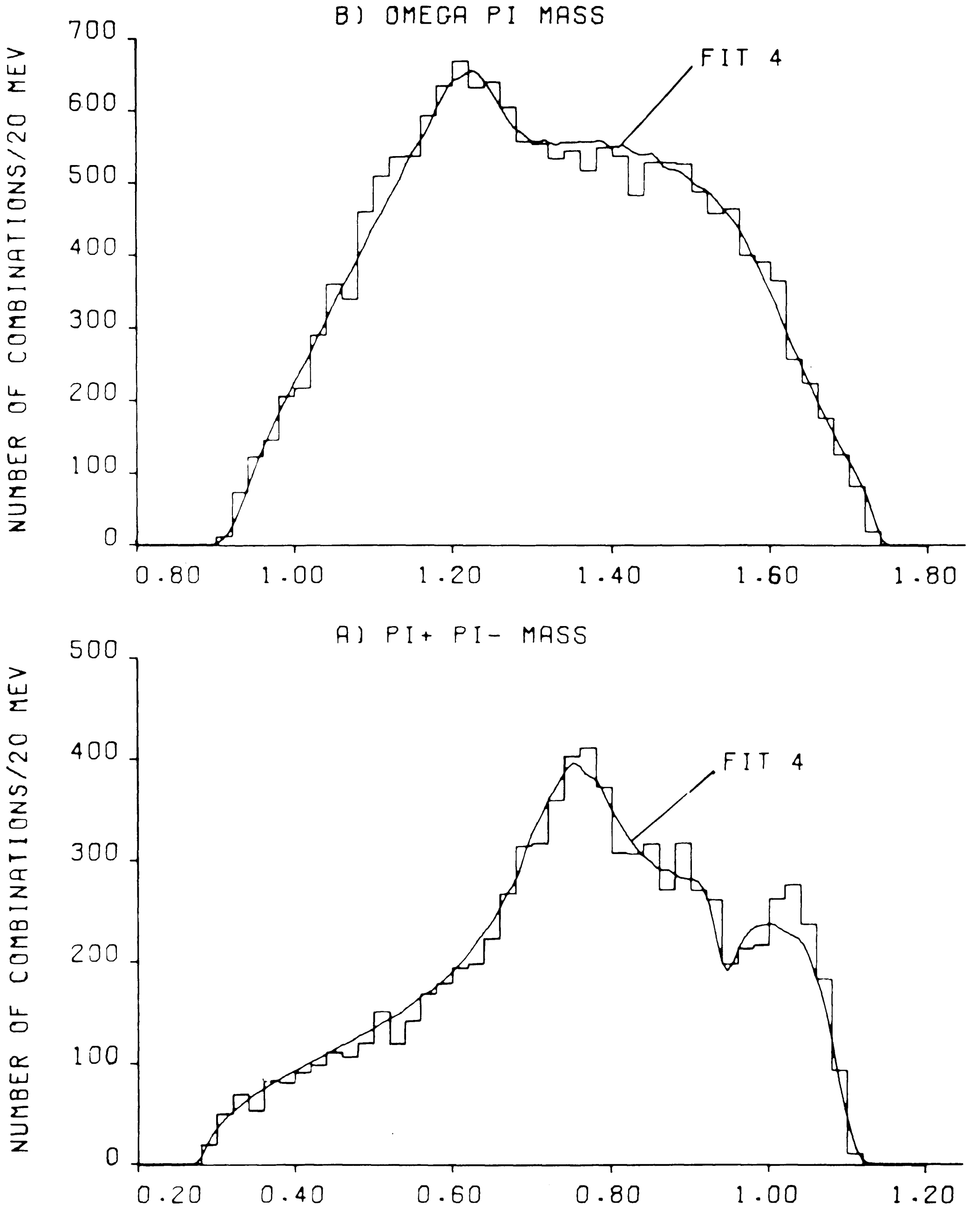

B) OMEGA PI MASS
FIT 4
NUMBER OF COMBINATIONS/20 MEV
700
600
500
400
300
200
100
0
0.80
1.00
1.20
1.40
1.60
1.80
A) PI+ PI- MASS
FIT 4
NUMBER OF COMBINATIONS/20 MEV
500
400
300
200
100
0
0.20
0.40
0.60
0.80
1.00
1.20

Fig. 1

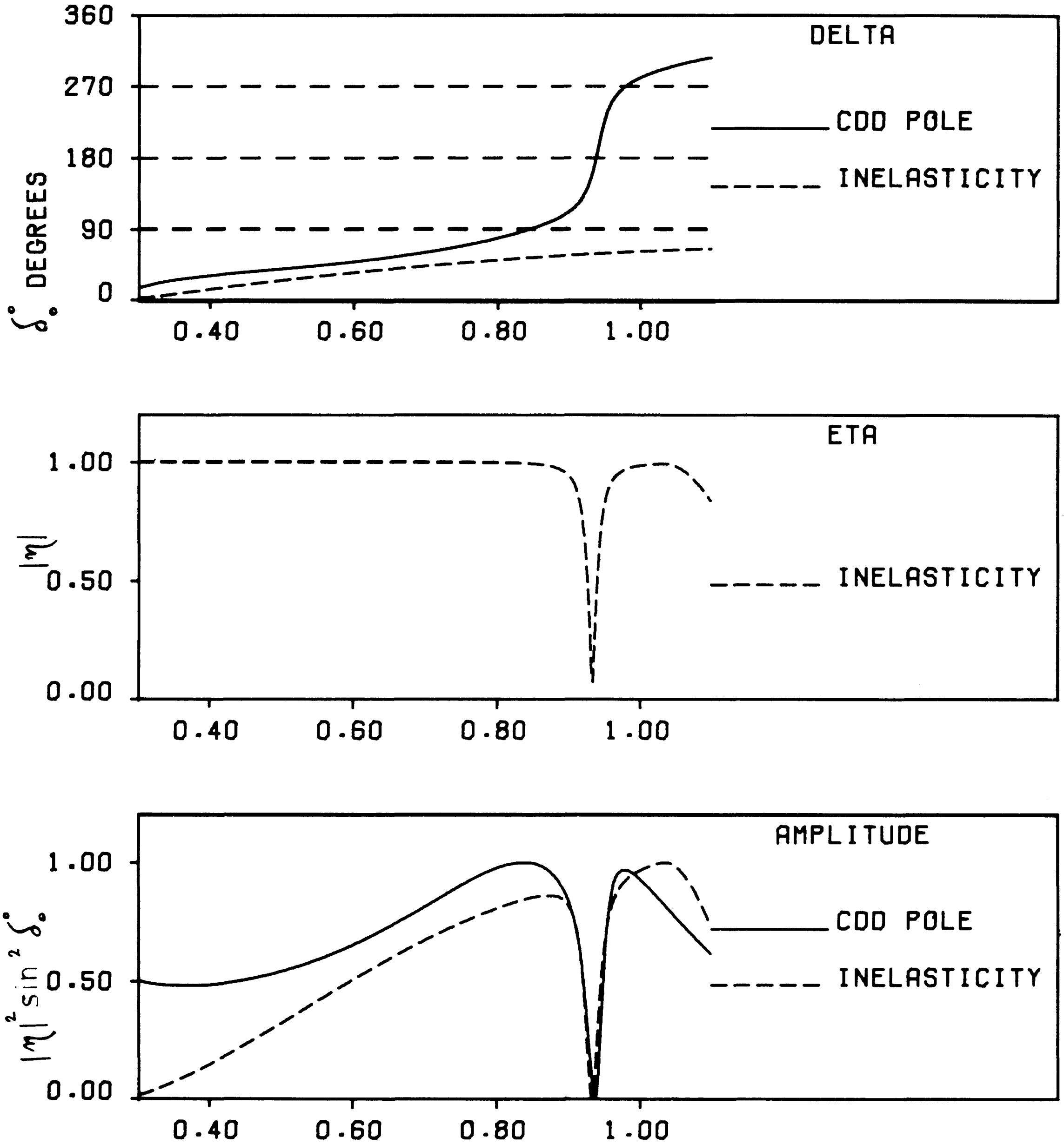

360
270
180
90
0
δ_0^0 DEGREES
DELTA
CDD POLE
INELASTICITY
0.40
0.60
0.80
1.00
ETA
1.00
0.50
0.00
$|\eta|$
INELASTICITY
0.40
0.60
0.80
1.00
AMPLITUDE
1.00
0.50
0.00
$|\eta|^2 \sin^2 \delta_0^0$
CDD POLE
INELASTICITY
0.40
0.60
0.80
1.00

Fig. 2

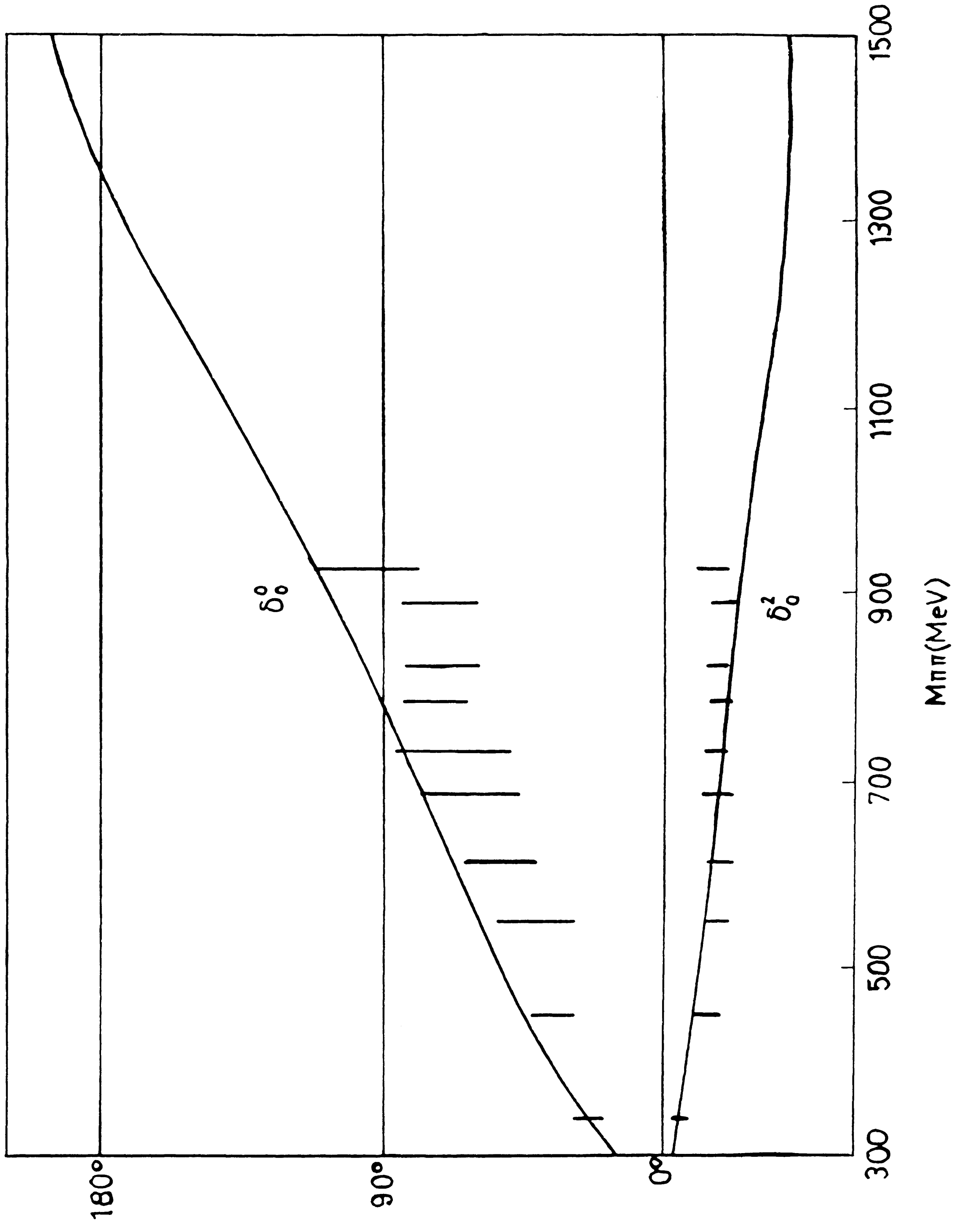

180°
90°
0°
300
500
700
900
1100
1300
1500
Mππ(MeV)
δ_0^0
δ_0^2

Fig. 3

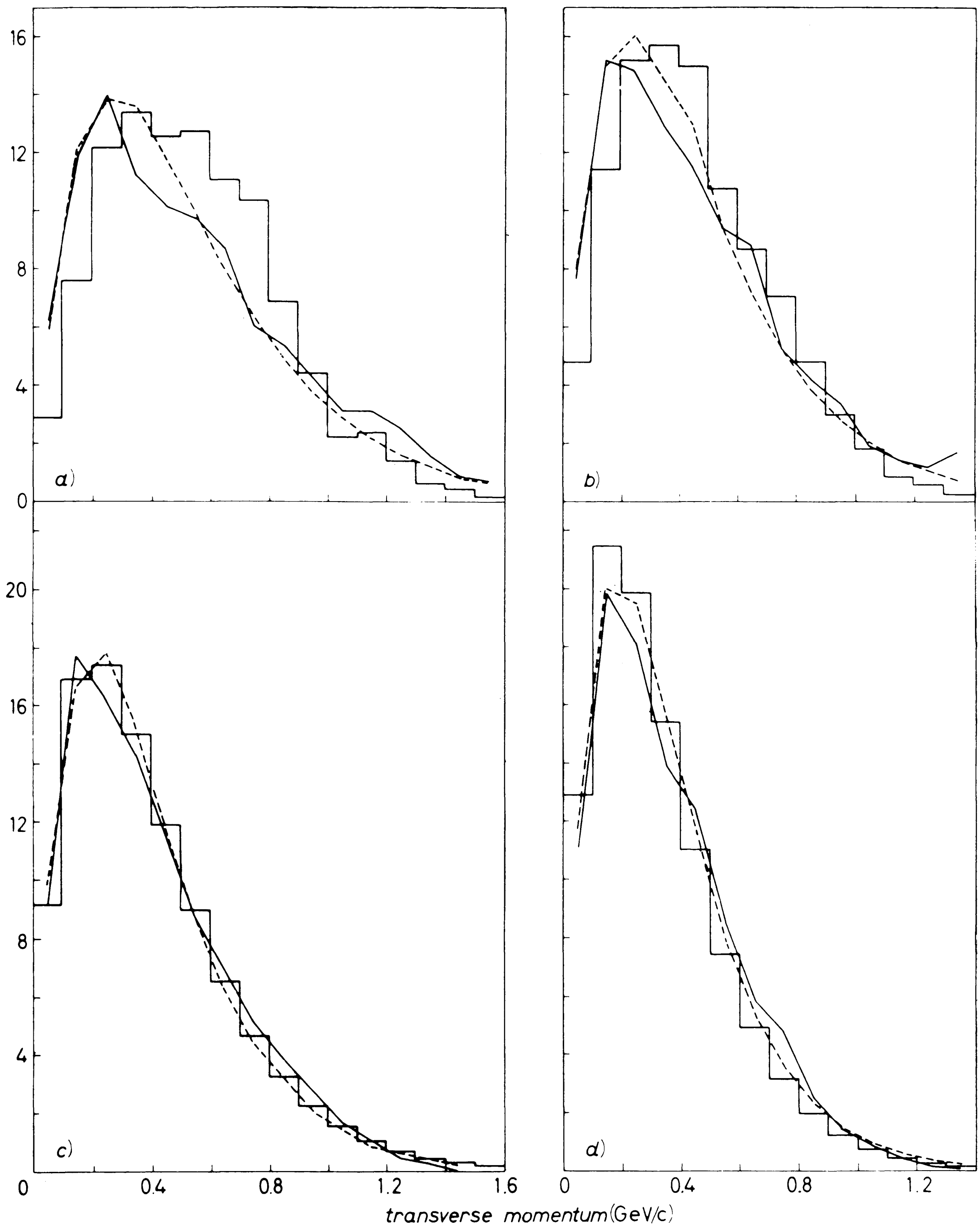

16
12
8
4
0
20
16
12
8
4
0
a)
b)
c)
d)
0.4
0.8
1.2
1.6
0
0.4
0.8
1.2
transverse momentum (GeV/c)

Fig. 4

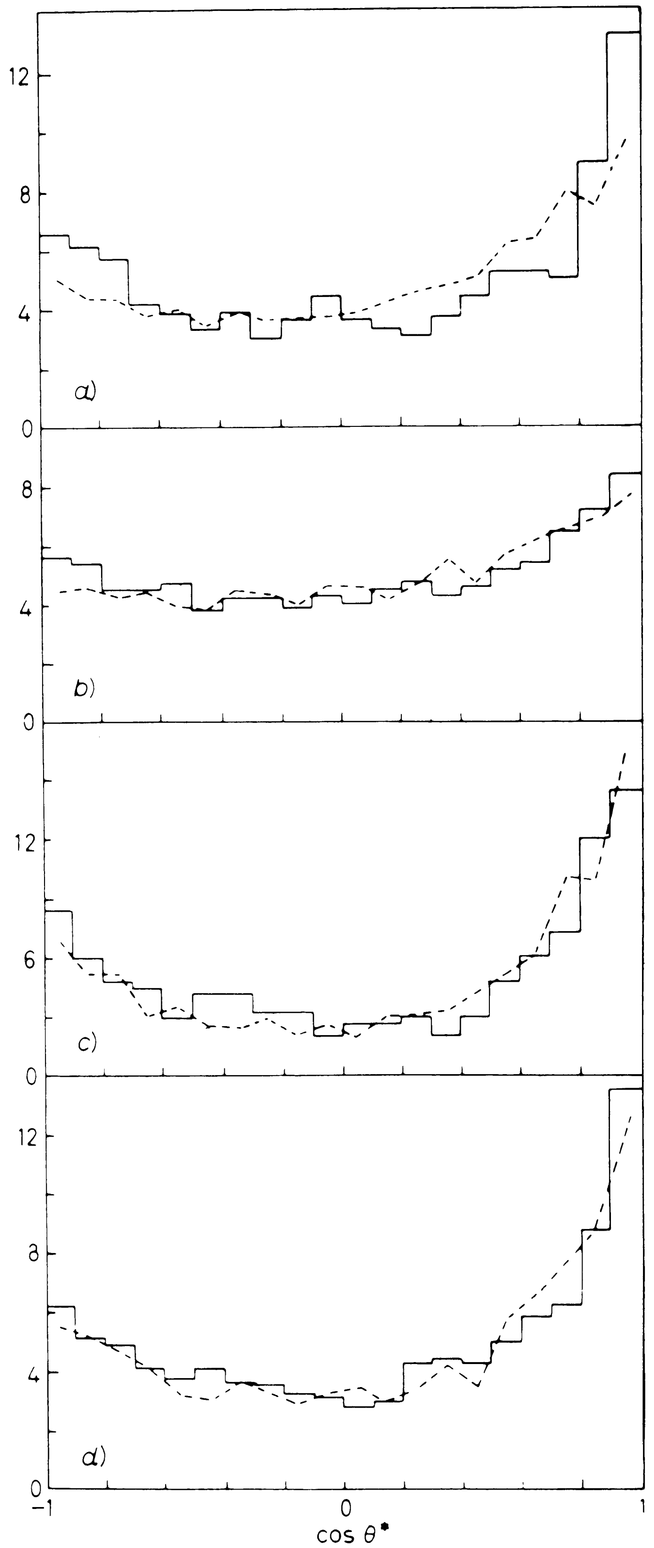


Fig. 5

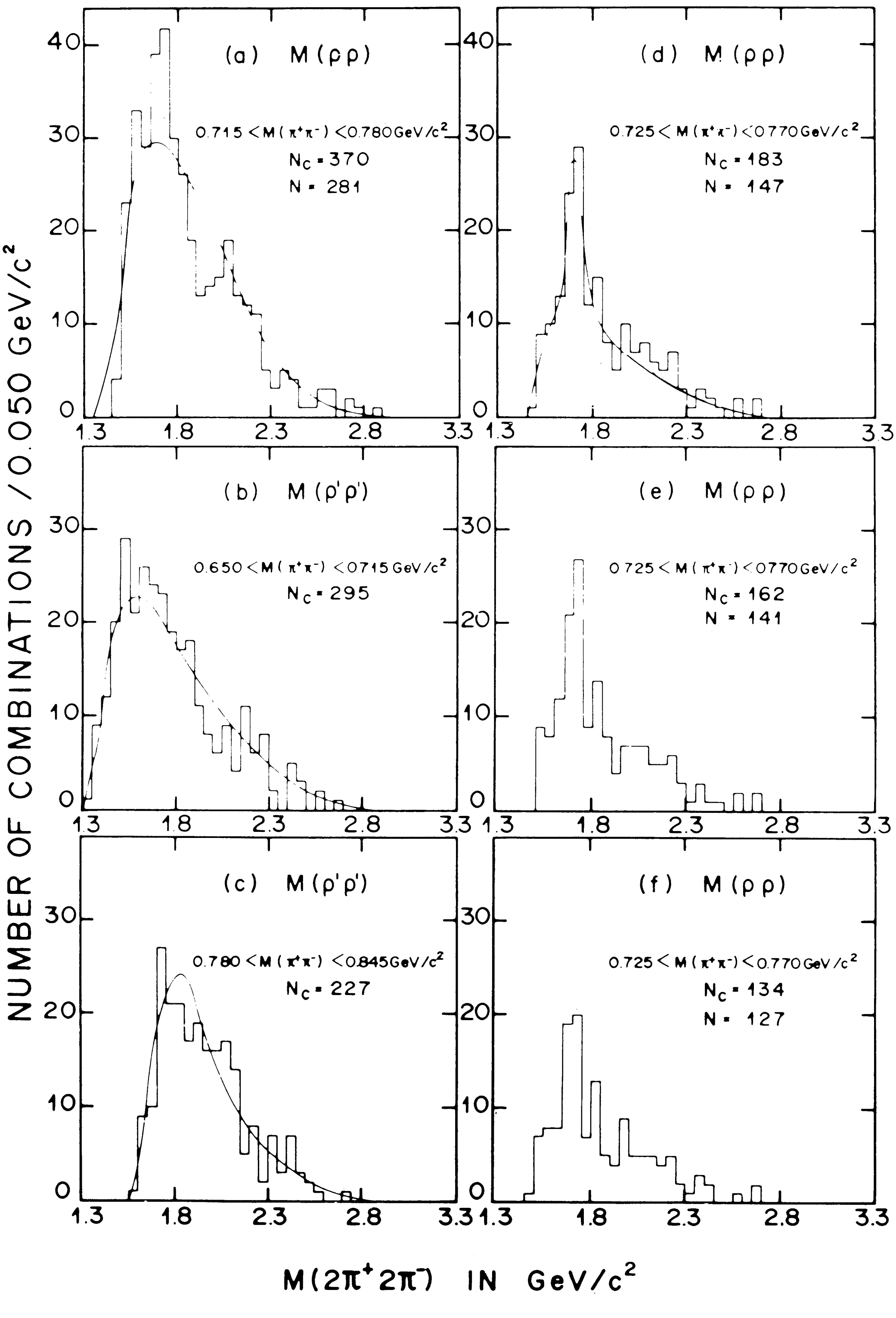
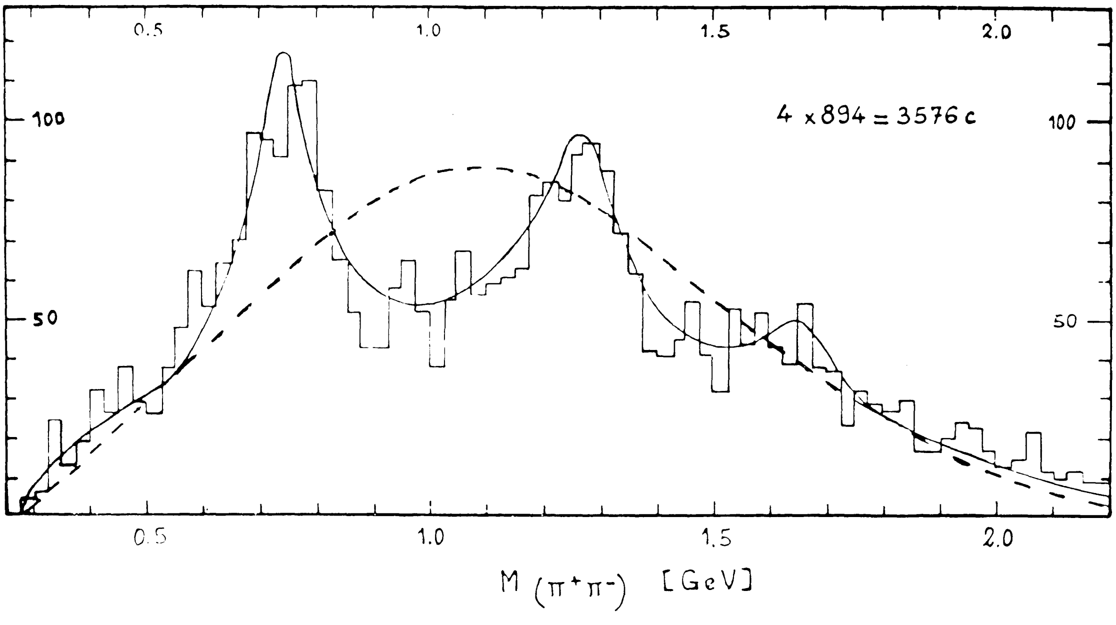

(a) M(ρρ)
0.715 < M(π⁺π⁻) < 0.780 GeV/c²
N_C = 370
N = 281
(d) M(ρρ)
0.725 < M(π⁺π⁻) < 0.770 GeV/c²
N_C = 183
N = 147
(b) M(ρ'ρ')
0.650 < M(π⁺π⁻) < 0.715 GeV/c²
N_C = 295
(e) M(ρρ)
0.725 < M(π⁺π⁻) < 0.770 GeV/c²
N_C = 162
N = 141
(c) M(ρ'ρ')
0.780 < M(π⁺π⁻) < 0.845 GeV/c²
N_C = 227
(f) M(ρρ)
0.725 < M(π⁺π⁻) < 0.770 GeV/c²
N_C = 134
N = 127
NUMBER OF COMBINATIONS / 0.050 GeV/c²
M(2π⁺2π⁻) IN GeV/c²

Fig. 6


0.5
1.0
1.5
2.0
100
4 x 894 = 3576 c
100
50
50
0.5
1.0
1.5
2.0
M (π⁺π⁻) [GeV]

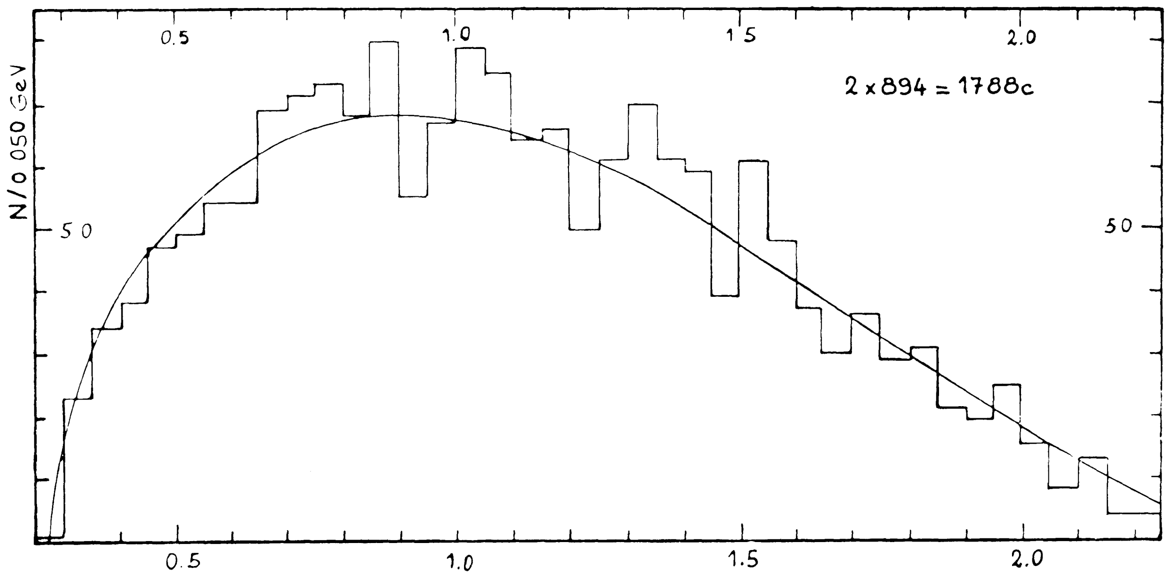

N/0.050 GeV
0.5
1.0
1.5
2.0
2 x 894 = 1788c
50
50
0.5
1.0
1.5
2.0
M (π± π±) [GeV]

Fig. 7

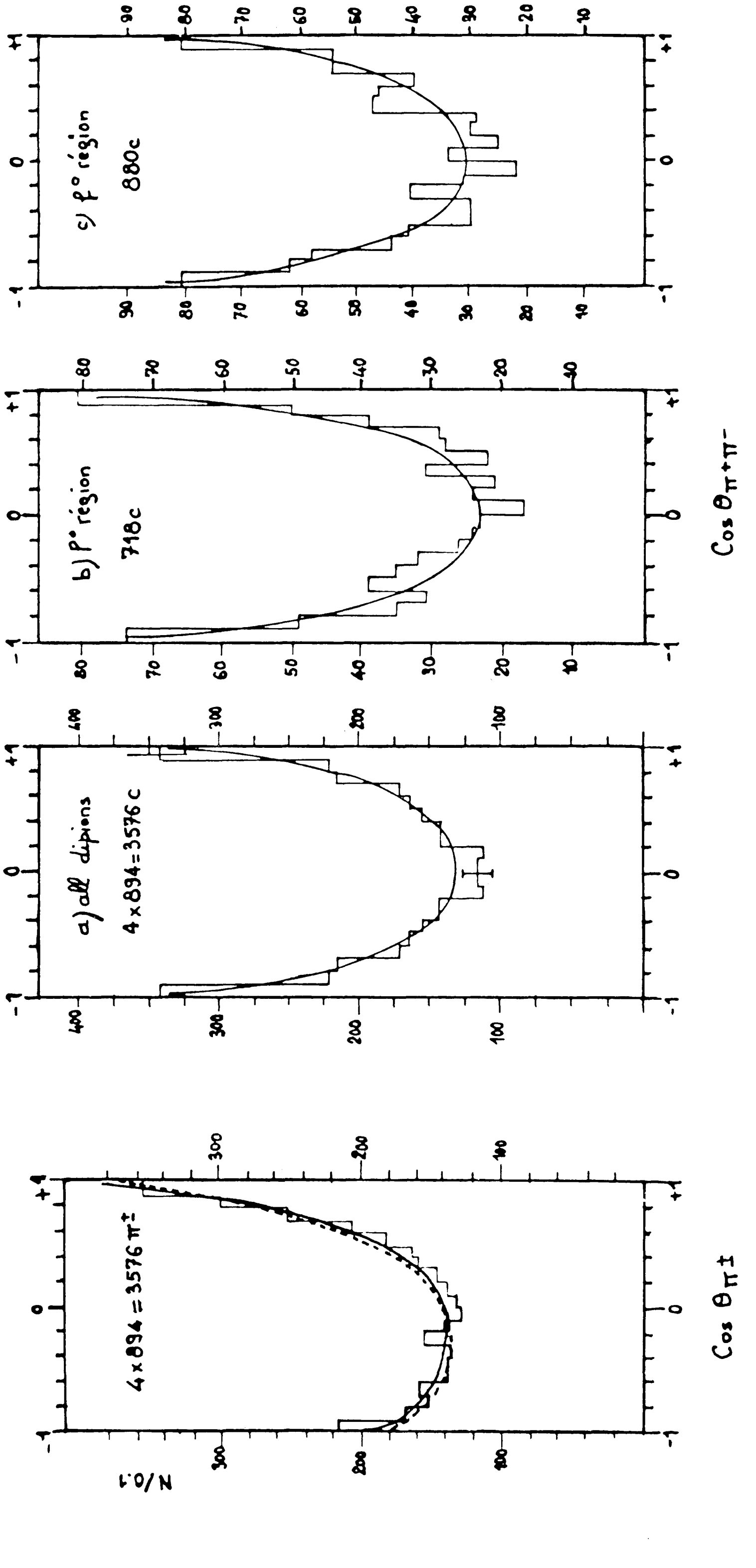

c) ρ° région
880c
b) ρ° région
718c
a) all dipions
4 x 894 = 3576 c
4 x 894 = 3576 π±
N/0.1
Cos θπ+π-
Cos θπ±

Fig. 8

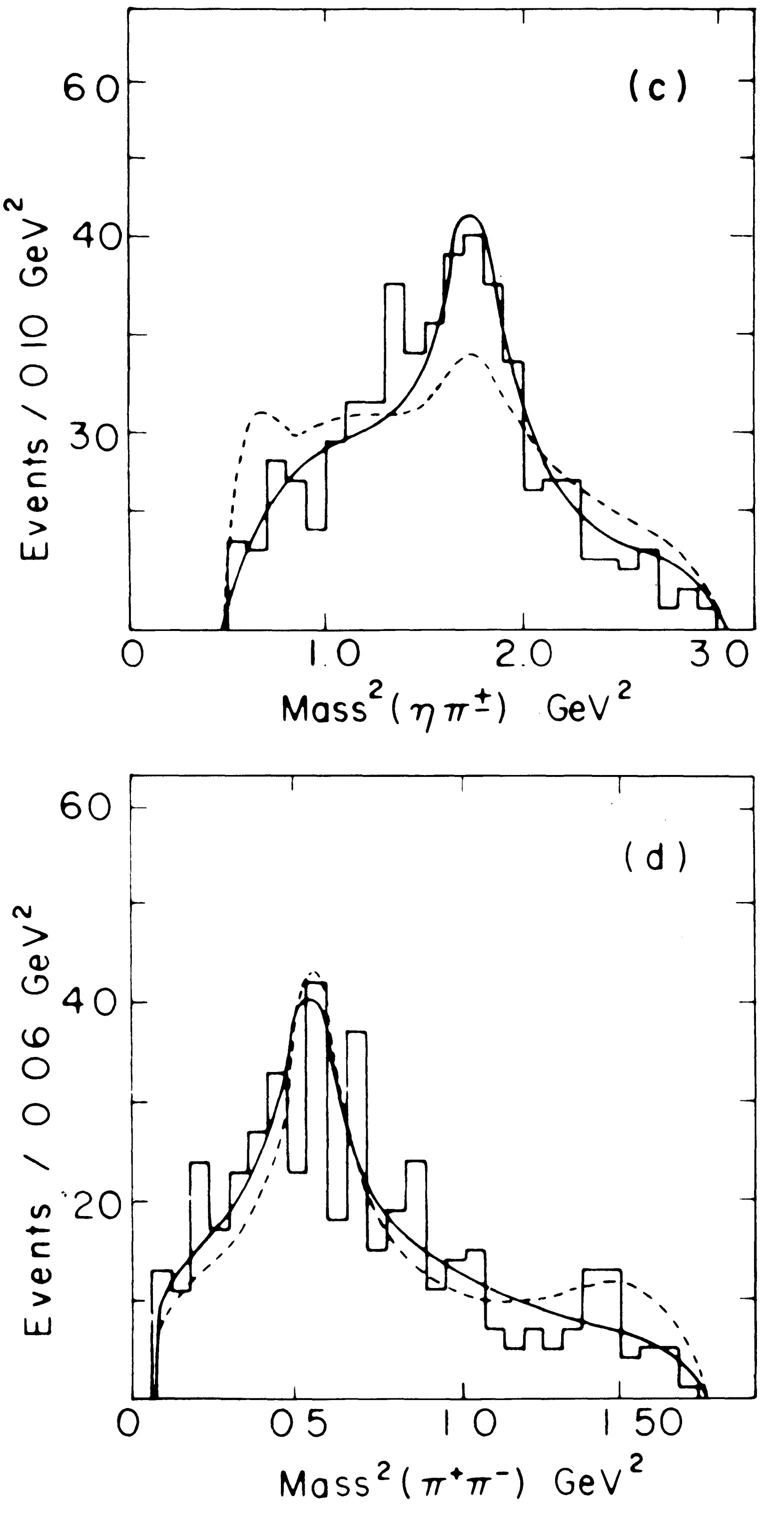

(c)
60
40
30
Events / 0.10 GeV²
0
1.0
2.0
3.0
Mass²(ηπ±) GeV²
(d)
60
40
20
Events / 0.06 GeV²
0
0.5
1.0
1.50
Mass²(π⁺π⁻) GeV²

Fig. 9

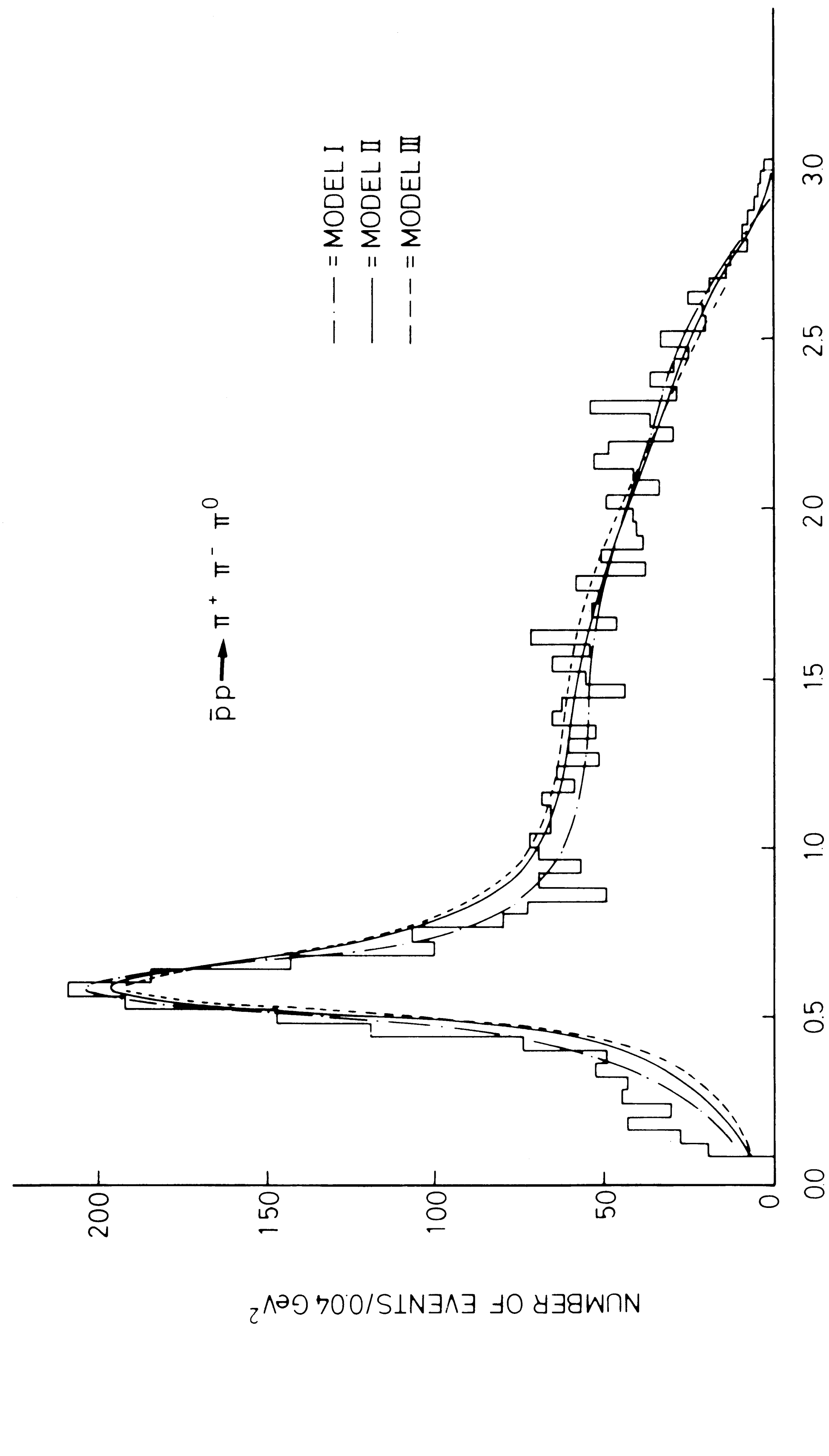
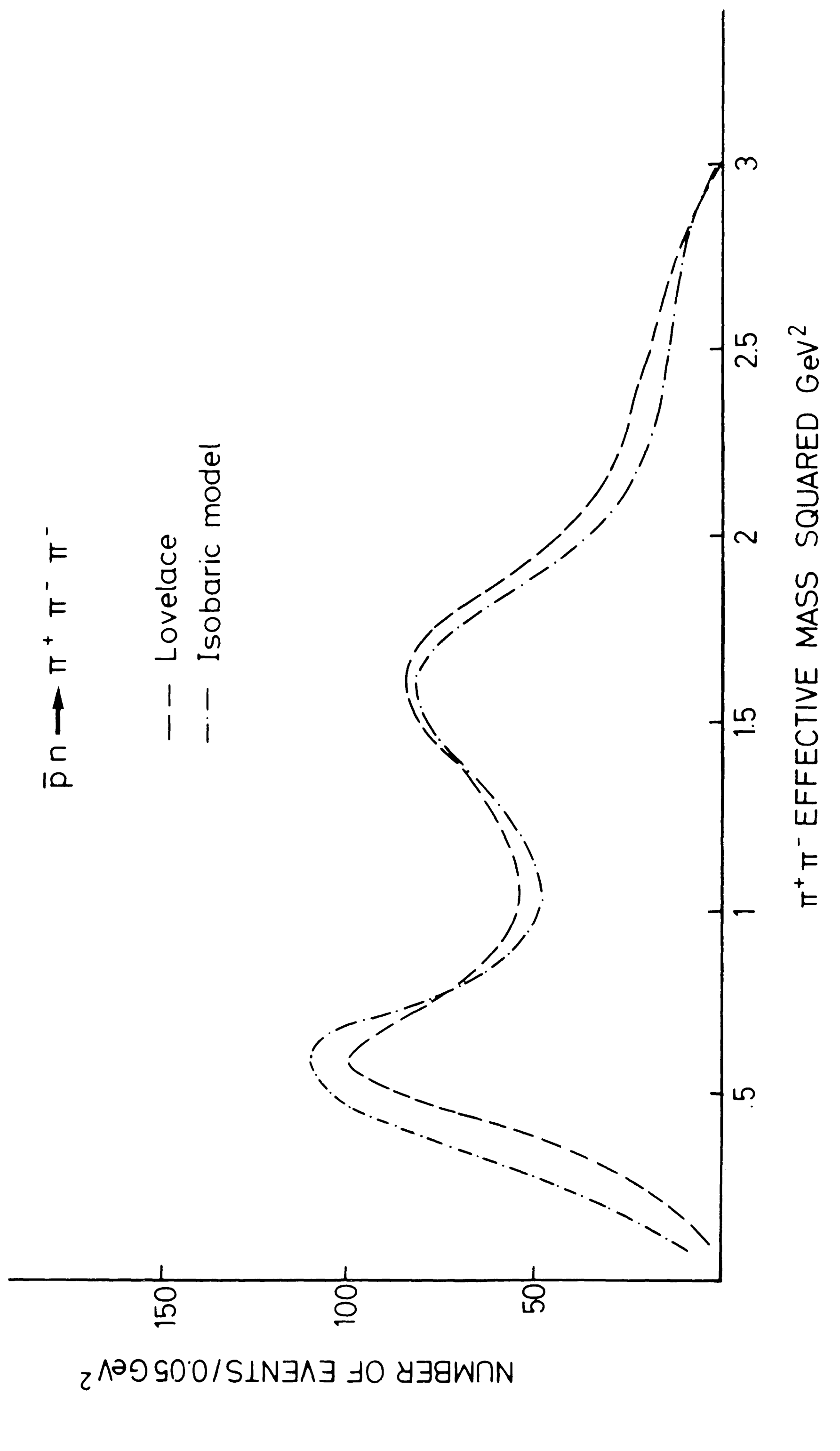

p̄p → π⁺ π⁻ π⁰
= MODEL I
= MODEL II
= MODEL III
π⁺ π⁻ EFFECTIVE MASS SQUARED GeV²
NUMBER OF EVENTS/0.04 GeV²
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
50
100
150
200

Fig. 10



$\bar{p}n \rightarrow \pi^+\pi^-\pi^-$
Lovelace
Isobaric model
NUMBER OF EVENTS/0.05 GeV^2
150
100
50
.5
1
1.5
2
2.5
3
$\pi^+\pi^-$ EFFECTIVE MASS SQUARED GeV^2

Fig. 11