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QUASIELASTIC ELECTRON AND NEUTRINO INTERACTIONS AND THE EXCLUSION EFFECTS

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ABSTRACT

The correlation functions which describe the exclusion effects in quasielastic electron nucleus and neutrino nucleus reactions are studied, showing similarity and difference between them. The calculation of the total structure factors in electron scattering is done for the cases of deuterium, ^{4}He and ^{12}C. The effects of the configuration mixing mechanism tend to deviate the values given by the simple shell model towards the supermultiplet relations, then showing the intermediate coupling nature. But they are not large, and practically only affect the spin flip terms, which contribute to the neutrino and muon capture reactions - due to the axial current - but insignificantly to the electron scattering.

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1. INTRODUCTION

The theory of quasielastic neutrino reactions in nuclei, when formulated in the framework of the closure approximation [1], separates the nuclear structure effects from the interaction for the elementary process in free nucleons. This is achieved by introducing three correlation functions depending on the structure of the target, which tend to vanish when the momentum transfer increases, then giving an incoherent cross-section. Their values suppress the differential cross-section, showing the Pauli exclusion effects, if the momentum transfer is small. In this context, Wigner supermultiplet symmetry [2] or simple spin orbit shell model [1] were assumed to obtain the behaviour of the correlation functions. A similar approach can be constructed for electron nucleus interactions, and sum rules have been obtained under specified kinematical conditions [3)–5)]. In particular, Bishop et al. [6] have tested several sum rules in the case of ^{16}O.

The present paper has two purposes. One is to obtain the differential cross-section of the electron nucleus reaction under the same approximations as those usually made in the corresponding neutrino case, in order to see how the nuclear structure effects can be separated, comparing similarity and differences between the two reactions, and studying how the exclusion effects are manifested. The other one is to investigate the modifications which affect the relevant correlation functions when realistic wave functions are used in the calculation. It must be remarked that from data of total muon capture rates, Bell and Llewellyn Smith [1] suggest that the very simplest shell model wave functions exaggerate greatly the effect of the spin orbit splitting.

A theoretical comparison between the two reactions was already undertaken by Løvseth [7] using a Fermi gas model, but then the exclusion effect does not appear in a "natural" way, corresponding to the introduction of antisymmetric wave functions. Similar considerations can be applied to Refs. 8) and 9).

In Section 2.1 we obtain the expressions giving the differential cross-section of the electron nucleus reaction in the one photon exchange approach using the impulse and closure approximations. In the general case this leads to consider six correlation functions, two of them corresponding to isoscalar-isoscalar terms, two to isovector-isovector terms and two to the interference. The behaviour of the cross-section in the limits of very small and very large momentum transfers is studied in Section 2.2. This reproduces, respectively, the Mott cross-section and the incoherent scattering. For nuclei of zero isospin, the isovector functions are related to the corresponding neutrino reaction ones, and the cross-section vanishes for $q^2 \to 0$ when the elastic reaction is subtracted, this being a clear manifestation of the expected Pauli exclusion effect. In Section 3 we are dealing with the applications. In Section 3.1 we study the deuteron case, where the functions depending on the nuclear structure are reduced to two of them, and the results of Ref. 10) have been used. It is pointed out that the equality to the sum of the proton and neutron cross-sections is practically valid for all values of q^2 when the elastic contribution is included. The case of ^{4}He is considered in Section 3.2, assuming a closed shell model. The effects of the configuration mixing mechanism are studied in Section 4 for the case of ^{12}C, and its implications on the neutrino and muon capture reactions are also considered. In Section 5 some discussion of the obtained results is given. Finally, the Appendix shows an interesting relation between some isoscalar and isovector correlation functions.

2. ELECTRON NUCLEUS CROSS-SECTION

2.1 Impulse and Closure Approximations

Let us consider the elementary reaction

$$e^-(p,\mu) + N(P,\lambda) \rightarrow e^-(p',\mu') + N(P',\lambda') \tag{1}$$

where N stands for proton and neutron and the four momentum and helicity are indicated in parenthesis. In the approximation of one photon exchange the invariant T matrix element is given by

$$\begin{aligned} T &= \frac{e^2}{q^2}\, \ell_\mu\, \bar{u}(P',\lambda')\left[F_1(q^2)\gamma^\mu + i\, F_2(q^2)\sigma^{\mu\nu}q_\nu\right] u(P,\lambda) \\ \ell_\mu &\equiv \bar{u}(p',\mu')\,\gamma_\mu\, u(p,\mu) \end{aligned} \tag{2}$$

where the four-momentum transfer is defined by $q = p-p' = P'-P$, and $\bar{u}(\vec{k},\mu)u(\vec{k},\nu) = 2M\,\delta_{\mu\nu}$. In the reference frame where the initial nucleon is at rest, we can write

$$\begin{aligned} T &= \frac{e^2}{q^2}\, \chi_{\lambda'}^{+}\, J\, \chi_\lambda \\ J &\equiv \alpha + \vec{\beta}\cdot\vec{\sigma} \end{aligned} \tag{3}$$

where the two spinors satisfy $\chi_{\lambda'}^{+}\chi_\lambda = \delta_{\lambda\lambda'}$. The parts of non-spin flip are defined, respectively, by

$$\alpha = 2M \left(1 - \frac{q^2}{4M^2}\right)^{-1/2} \ell^0 \; G_E(q^2)$$

$$\vec{\beta} = -i \left(1 - \frac{q^2}{4M^2}\right)^{-1/2} (\vec{q} \times \vec{\ell}) \; G_M(q^2) \tag{4}$$

where use has been made of $q^\mu \ell_\mu = 0$. $G_E(q^2)$ and $G_M(q^2)$ are the electric and magnetic Sachs form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{2M} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + 2M \, F_2(q^2) \tag{5}$$

We shall assume the dipole dependence for the electric form factor of the proton and the scaling law for the magnetic form factors

$$G_{Ep}(q^2) = \left(1 - \frac{q^2}{M_V^2}\right)^{-2} \qquad G_{Mp}(q^2) = (1 + \mu_p) \, G_{Ep}(q^2) \qquad G_{Mn}(q^2) = \mu_n \, G_{Ep}(q^2) \tag{6}$$

where $M_V^2 = 0.71 \text{ GeV}^2$ and μ_p (μ_n) is the anomalous magnetic moment of the proton (neutron). The electric neutron form factor $G_{E_n}(q^2)$ shows a systematic deviation from zero in the results of Ref. 11) for elastic electron deuteron scattering. For all values of q^2, $G_{E_n} < 0.05$.

Let us now consider the scattering with nuclei

$$e(p, \mu) + A(P_i, \lambda_i) \rightarrow e(p', \mu') + A'(P_f, \lambda_f) \tag{7}$$

where A' denotes the final nuclear state which can be bound, equal or different of the initial one A, or broken. In the impulse approximation the differential cross-section corresponding to a fixed angle Θ between the incident and outgoing electrons is given by

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \frac{1}{4ME} \int dE' \, \frac{e^4}{q^4} \, E'^2 d^3 P_f \, \frac{1}{2E' 2E_f} \, \delta(E_i + E - E' - E_f)$$

$$\overline{\sum_{\lambda_i \mu}} \sum_{\lambda_f \mu'} \langle \psi_i | \sum_{j=1}^{A} e^{-i \vec{q} \cdot \vec{x}_j} (J_j^{(1)+} + \tau_{3_j} J_j^{(2)+}) | \psi_f \rangle \langle \psi_f | \sum_{k=1}^{A} e^{i \vec{q} \cdot \vec{x}_k} (J_k^{(1)} + \tau_{3_k} J_k^{(2)}) | \psi_i \rangle \tag{8}$$

where ψ_i and ψ_f denote the initial and final nuclear wave functions, and the isoscalar $J^{(1)}$ and isovector $J^{(2)}$ operators have been introduced as $J^{(1)} = \frac{1}{2}(J_p + J_n)$ and $J^{(2)} = \frac{1}{2}(J_p - J_n)$. We are not interested in the detection of the final state and then a summation over all final nuclear states can be performed. But in general $\vec{q}$ depends on the channel and an assumption has to be made : it is considered that kinematics of processes (1) and (7) are equal, regarding the nucleus as a collection of free nucleons. Using then the closure approximation, we obtain

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{64E^3E'M^2\sin^4\theta/2} \frac{1}{1+\frac{2E}{M}\sin^2\theta/2} \langle \psi_i | \sum_{j,k} e^{-i\vec{q}\cdot(\vec{x}_j - \vec{x}_k)} (J_j^{(1)+} + \tau_{3_j} J_j^{(2)+})(J_k^{(1)} + \tau_{3_k} J_k^{(2)}) | \psi_i \rangle \tag{9}$$

where an average over the initial nuclear polarizations is understood. Taking the z axis along $\vec{q}$, from Eq. (3) several operators appear in the construction of the matrix elements in Eq. (9). The contribution of the incoherent terms $j = k$ is easily calculated. The $j \neq k$ terms can be expressed in terms of the following correlation functions, depending on the nuclear structure

$$D^{(1)}_{S,T,L}(q^2) = \langle \psi_i | \sum_{j\neq k} e^{-i\vec{q}\cdot(\vec{x}_j - \vec{x}_k)} (1, \sigma_{x_j}\sigma_{x_k}, \sigma_{z_j}\sigma_{z_k}) | \psi_i \rangle$$

$$D^{(2)}_{S,T,L}(q^2) = -\langle \psi_i | \sum_{j\neq k} e^{-i\vec{q}\cdot(\vec{x}_j - \vec{x}_k)} \tau_{3_j}\tau_{3_k} (1, \sigma_{x_j}\sigma_{x_k}, \sigma_{z_j}\sigma_{z_k}) | \psi_i \rangle \tag{10}$$

$$D^{(12)}_{S,T,L}(q^2) = -\langle \psi_i | \sum_{j\neq k} e^{-i\vec{q}\cdot(\vec{x}_j - \vec{x}_k)} \tau_{3_j} (1, \sigma_{x_j}\sigma_{x_k}, \sigma_{z_j}\sigma_{z_k}) | \psi_i \rangle$$

and then the result is

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{64E^3E'M^2\sin^4\theta/2} \frac{1}{1+\frac{2E}{M}\sin^2\theta/2} \Big\{ |\alpha^{(1)}|^2 (A + D_S^{(1)}) + |\vec{\beta}^{(1)}|^2 (A + D_T^{(1)}) \tag{11}$$

$$+ |\alpha^{(2)}|^2 (A - D_S^{(2)}) + |\vec{\beta}^{(2)}|^2 (A - D_T^{(2)}) - 2\left[\alpha^{(1)}\alpha^{(2)+}(N - Z + D_S^{(12)}) + \vec{\beta}^{(1)}\vec{\beta}^{(2)+}(N - Z + D_T^{(12)})\right]\Big\}$$

where A is the number of nucleons in the initial nuclear state, Z the number of protons and N of neutrons. From the value of the trace of the leptonic tensor, we obtain

$$\alpha^{(p)}\alpha^{(q)+} = 4M^2\left(1-\frac{q^2}{4M^2}\right)^{-1}\left[4E\left(E+\frac{q^2}{2M}\right)+q^2\right] G_E^{(p)} G_E^{(q)}$$

$$\vec{\beta}^{(p)}\vec{\beta}^{(q)+} = -q^2\left(1-\frac{q^2}{4M^2}\right)^{-1}\left[4E\left(E+\frac{q^2}{2M}\right)-q^2\left(1-\frac{q^2}{2M^2}\right)\right] G_M^{(p)} G_M^{(q)} \qquad (12)$$

with $p,q=1,2$ indicating the isoscalar and isovector components.

It is interesting to write the differential cross-section in a form which is a consequence of using the one photon exchange approximation

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_M}{d\Omega}\left\{A(q^2) + 2\,tg^2\frac{\theta}{2}\;B(q^2)\right\} \qquad (13)$$

where the Mott cross-section for the proton is given by

$$\frac{d\sigma_M}{d\Omega} = \left(\frac{e^2}{4\pi}\right)^2 \frac{\cos^2\theta/2}{4E^2\sin^4\theta/2}\left(1+\frac{2E}{M}\sin^2\frac{\theta}{2}\right)^{-1} \qquad (14)$$

Comparing with Eqs. (11) and (12) the structure functions $A(q^2)$, $B(q^2)$ are

$$A(q^2) = \left(1-\frac{q^2}{4M^2}\right)^{-1}\left[G_E^{(1)2}(A+D_S^{(1)}) + G_E^{(2)2}(A-D_S^{(2)}) - 2G_E^{(1)}G_E^{(2)}(N-Z+D_S^{(12)}) + B(q^2)\right]$$

$$B(q^2) = -\frac{q^2}{4M^2}\left[G_M^{(1)2}(A+D_T^{(1)}) + G_M^{(2)2}(A-D_T^{(2)}) - 2G_M^{(1)}G_M^{(2)}(N-Z+D_T^{(12)})\right] \qquad (15)$$

We see that the isovector terms present exclusion factors $(A-D_S^{(2)})$ and $(A-D_T^{(2)})$ similar to those which appear in weak interactions [1].

2.2 Behaviour of the Structure Functions

From Eq. (15) we can study the values of the structure functions for large values of $-q^2$. If $-q^2 \to \infty$, the nuclear matrix elements (10) go to zero, and then

$$A(q^2) \to \left(1-\frac{q^2}{4M^2}\right)^{-1}\left[Z\,G_{Ep}^2 + N\,G_{En}^2 + B(q^2)\right]$$

$$B(q^2) \to \frac{-q^2}{4M^2}\left[Z\,G_{Mp}^2 + N\,G_{Mn}^2\right] \qquad (16)$$

obtaining, as was expected, that the reaction is incoherent from each constituent in this limit. It can be remarked here that the effects of the nuclear structure in the magnetic terms - i.e., $B(q^2)$ - will not be very important for the behaviour of the cross-section at all values of q^2, because if $-q^2$ is small these terms do not contribute practically.

We are going to study now the value of the electric terms in the limit of small $-q^2$. It is easily seen that

$$D_S^{(1)}(0) = A(A-1) \qquad D_S^{(2)}(0) = A-(Z-N)^2 \qquad D_S^{(12)}(0) = (A-1)(N-Z) \qquad (17)$$

and therefore the isoscalar term $G_E^{(1)}$ presents a factor A^2, that is, the reaction for isoscalar photons is coherent and no reduction of the cross-section is present. However, for the isovector $G_E^{(2)2}$ and cross terms $G_E^{(1)}G_E^{(2)}$ the corresponding factors at $-q^2=0$ are $(N-Z)^2$ and $A(N-Z)$, respectively. From these results we obtain $A(q^2) \xrightarrow[-q^2\to 0]{} Z^2$, and then

$$\frac{d\sigma}{d\Omega} \xrightarrow[-q^2\to 0]{} Z^2\,\frac{d\sigma_M}{d\Omega}$$

reproducing the Mott cross-section for an object of Ze charge.

For nuclei of zero isospin, the functions $D_{S,T,L}^{(12)}$ vanish and furthermore the isovector functions $D_{S,T,L}^{(2)}$ are related to the corresponding ones to the neutrino reaction $D_{S,T,L}$ introduced in Ref. 1). This is so because only a scalar operator in the isospin space can contribute in this case and $\tau_{3j}\,\tau_{3k}$ and $2\,\tau_j^-\tau_k^+$ are equivalent. The structure functions are then

$$A(q^2) = \left(1-\frac{q^2}{4M^2}\right)^{-1}\left[G_E^{(1)2}(A+D_S^{(1)}) + G_E^{(2)2}(A-D_S^{(2)}) + B(q^2)\right]$$

$$B(q^2) = -\frac{q^2}{4M^2}\left[G_M^{(1)2}(A+D_T^{(1)}) + G_M^{(2)2}(A-D_T^{(2)})\right] \qquad (18)$$

On the basis of Wigner supermultiplet symmetry spin independent forces can be ignored and [12] $D_S^{(2)} = D_{T,L}^{(2)} = -D_{T,L}^{(1)}$. Taking into account spin orbit splitting this relation is valid for closed shell nuclei [13]. In the Appendix it is shown that $D_T^{(2)} = -D_T^{(1)}$ and $D_L^{(2)} = -D_L^{(1)}$ is satisfied for closed <u>sub</u>-shells; for example, the pure configuration $(1s_{\frac{1}{2}})^4(1p_{\frac{3}{2}})^8$ in ^{12}C verifies these relations, but not $D_S^{(2)} = D_{T,L}^{(2)}$.

The coherent contribution to $A(q^2)$ in the limit $-q^2 \to 0$, which physically corresponds to the behaviour of the elastic channel $A_{el}(q^2)$, is completely due to the isoscalar term for $N = Z$ nuclei, as it is seen from Eq. (17). This is not present in the neutrino reaction and has nothing to do with exclusion principle effects. It will be interesting to compare the "quasielastic" structure function $A_{qel}(q^2) = A(q^2) - A_{el}(q^2)$ with the incoherent quantity $ZA_p(q^2) + NA_n(q^2)$ in order to see the exclusion effects. This will be the programme undertaken in the applications. In the limit $-q^2 \to \infty$. $A_{qel}(q^2)$ presents the correct behaviour because the elastic contribution $A_{el}(q^2) \xrightarrow[-q^2\to\infty]{} 0$ due to the nuclear form factor.

3. <u>APPLICATIONS</u>

3.1 <u>The Deuteron Case</u>

For the deuteron $(J = 1,\ T = 0)$ it is immediately verified that $D_S^{(1)} = D_S^{(2)} = 2D_S$, $D_T^{(1)} = D_T^{(2)} = 2D_T$ and $D_L^{(1)} = D_L^{(2)} = 2D_L$ due to the isospin structure. Furthermore, if the d wave is neglected $D_S = 3D_T = 3D_L$.

From Eq. (18) the structure function $A(q^2)$ can be written as

$$A(q^2) = A_p(q^2) + A_n(q^2) + \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left[2\, G_{E_p}(q^2)\, G_{E_n}(q^2)\, D_S(q^2) - \frac{q^2}{4M^2}\, 2\, G_{M_p}(q^2)\, G_{M_n}(q^2)\, D_T(q^2) \right] \qquad (19)$$

and similarly for $B(q^2)$. As $G_{E_n} \approx 0$ and considering the behaviour of $D_T(q^2)$ we see that in the case of deuterium the nuclear structure is not important for the sum of elastic and quasielastic cross-sections. In a sufficient approximation

$$\left.\frac{d\sigma}{d\Omega}\right|_d \approx \left.\frac{d\sigma}{d\Omega}\right|_p + \left.\frac{d\sigma}{d\Omega}\right|_n \qquad (20)$$

In the literature this sum rule has been applied to obtain the structure of the neutron from the quasielastic cross-section, using the so-called "area method"; for large momentum transfers this is the same because the elastic cross-section does not contribute practically. In this sense we can see the study of the magnetic structure of the neutron in Ref. 14). However, if it is applied to lower and lower momentum transfers, negative neutron cross-sections can be obtained as it happens in Ref. 15). It must be remarked that in the case of deuterium we obtain that this sum rule is practically valid at all values of q^2, but applied to the total events, including elastic. In this context a great deviation only can be explained by significant values of the electric neutron form factor.

Figure 1a corresponds to the value $A(q^2)$ calculated as A_p+A_n, or taking into account the nuclear structure functions D_S, D_T with $G_{E_n}=0$ and $G_{E_n} = -\mu_n(-q^2/4M^2)(1-q^2/M^2)^{-1}G_{E_p}$ [11),16)]. We see that practically there are no differences among the four different cases. The two functions D_S and D_T have been taken from Ref. 10), where they were applied to the neutrino deuteron reaction, and correspond to the result of using generalized Hulthèn wave functions [17)] for the s and d waves.

The values of $A_{qel}(q^2)$, Fig. 1c, have been obtained from $A(q^2)$ and the experimental results [11)] of $A_{el}(q^2)$, Fig. 1b. In this zone of small values of q^2 three values of $A_{qel}(q^2)$ obtained from experimental results of the quasielastic differential cross-section [15)] are also given for comparison. They correspond to $-q^2 = 0.0584, 0.0974, 0.179\ \mathrm{GeV}^2$, and have been obtained taking, at the same point $-q^2$, values of the cross-section corresponding to different angles θ and fitting $A(q^2)$ and $B(q^2)$ in order to reproduce the behaviour $A(q^2)+2\mathrm{tg}^2\frac{\theta}{2}B(q^2)$. A reasonable agreement is found.

Due to the sum rule for the total deuteron cross-section, Eq. (20), in this particular case we can say, loosely speaking, that the values of the elastic cross-section are a measure of the exclusion factors due to nuclear structure for the quasielastic scattering. For $-q^2=0.1\ \mathrm{GeV}^2$ the reduction is only about 10%.

3.2 Application to ^{4}He

^{4}He is a double closed shell nucleus corresponding to the configuration $(1s_{\frac{1}{2}})^4$. In Eq. (18) the correlation functions are given by

$$D_S^{(1)}(q^2) = \langle \psi(^4He) | \sum_{i \neq j} e^{-i \vec{q}\cdot(\vec{x}_i - \vec{x}_j)} | \psi(^4He) \rangle$$

$$D_S^{(2)} = D_T^{(2)} = -D_T^{(1)} = -\langle \psi(^4He) | \sum_{i \neq j} \frac{1}{3} \vec{\tau}_i \cdot \vec{\tau}_j \, e^{-i \vec{q}\cdot(\vec{x}_i - \vec{x}_j)} | \psi(^4He) \rangle \tag{21}$$

The operators in (21) are scalars under $O(3)\times SU(2)_T$. As in this particular case only $\ell = 0$ waves are present for each nucleon within the nucleus, the substitution

$$e^{-i\vec{q}\cdot(\vec{x}_i - \vec{x}_j)} \rightarrow j_0(|\vec{q}||\vec{x}_i|)\, j_0(|\vec{q}||\vec{x}_j|)$$

can be made. In this way we obtain immediately

$$D_S^{(1)}(q^2) = 3\, D_S^{(2)}(q^2) = 12 \left| \int_0^\infty dr\, r^2\, R_{1s}^2(r)\, j_0(|\vec{q}|r) \right|^2 \tag{22}$$

where $R_{1s}(r)$ is the radial wave function of the nucleon in the 1s shell. Using the harmonic oscillator solutions with scale parameter $b = 1.39$ fm, which reproduces the root mean square charge radius, the result is

$$D_S^{(1)}(q^2) = 3\, D_S^{(2)}(q^2) = 12\, e^{-\frac{1}{2} b^2 |\vec{q}|^2} \tag{23}$$

With this structure, the function $A(q^2)$ can be written in a simple way as

$$A(q^2) = 4 \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left\{ G_E^{(1)2} \left(1 + 3 e^{-\frac{1}{2} b^2 |\vec{q}|^2}\right) + \left[G_E^{(2)2} - \frac{q^2}{4M^2} \left(G_M^{(1)2} + G_M^{(2)2}\right)\right] \left(1 - e^{-\frac{1}{2} b^2 |\vec{q}|^2}\right) \right\} \tag{24}$$

Figure 2a corresponds to the values of $A(q^2)$ calculated with $G_{E_n} = 0$ and $G_{E_n} \neq 0$ using the same dependence as in the deuteron case; Fig. 2b is the experimental elastic form factor taken from Ref. 18), and Fig. 2c is obtained as $A_{qel}(q^2) = A(q^2) - A_{el}(q^2)$. The broken line corresponds to the incoherent behaviour $2A_p(q^2) + 2A_n(q^2)$. We see that the quasielastic structure function $A_{qel}(q^2)$ presents the expected exclusion when it is compared to the one corresponding to the sum of protons and neutrons. For $-q^2 = 0.1$ GeV^2 the reduction is of the order of 15%.

4. CORRELATION FUNCTIONS IN ^{12}C

In this section our purpose is to show how the correlation functions can be modified when the configuration mixing mechanism is introduced in the description of the wave function. To this end, the nucleus ^{12}C has been chosen due to its known large deformation and because it is a good example of seeing how the relations given in Section 2.2 are broken when both Wigner supermultiplet symmetry and simple closed subshell description are destroyed.

^{12}C is described by the care $(1s_{\frac{1}{2}})^4$ adding eight nucleons in the 1p shell coupled to $J=0$, $T=0$. Cohen and Kurath [19], using effective interactions in this shell, have obtained the wave function

$$|1p^8(0,0)\rangle = C_1\,|1p_{3/2}^8\rangle + C_2\,|1p_{3/2}^6(10)\,1p_{1/2}^2(10)\rangle + C_3\,|1p_{3/2}^6(01)\,1p_{1/2}^2(01)\rangle + C_4\,|1p_{3/2}^5(\tfrac{1}{2}\,\tfrac{1}{2})\,1p_{1/2}^3(\tfrac{1}{2}\,\tfrac{1}{2})\rangle + C_5\,|1p_{3/2}^4(00)\,1p_{1/2}^4(00)\rangle \tag{25}$$

where the amplitude values are

$$C_1 = 0.6124 \qquad C_2 = 0.6246 \qquad C_3 = 0.2610$$

$$C_4 = 0.2548 \qquad C_5 = 0.3190$$

which show the large deviation of the pure configuration $(1p_{\frac{3}{2}})^8$. This wave function has been applied, with success, to static processes [19], partial muon capture to ^{12}B and beta decay $^{12}B \rightarrow {}^{12}C$ [20].

Our procedure of calculation of the correlation functions takes advantage of considering that only the scalar part, under $O(3)\times SU(2)_T$, of the operators contributes. Then, with the state coupled to $(0,0)$, the reduction to matrix elements between states of two nucleons is very easy and independent of the particular operator.

Under $O(3)$ the scalar operators associated to D_S, D_T and D_L are obtained under the substitution

$$e^{-i\vec{q}.(\vec{x}_1-\vec{x}_2)} \longrightarrow 4\pi \sum_{\ell} j_\ell(|\vec{q}|r_1)\, j_\ell(|\vec{q}|r_2)\; S_\ell(1,2)$$

$$e^{-i\vec{q}.(\vec{x}_1-\vec{x}_2)}\sigma_{x_1}\sigma_{x_2} \rightarrow 4\pi \sum_{\ell\ell' L} i^{\ell-\ell'} \frac{\sqrt{(2\ell+1)(2\ell'+1)}}{2L+1}\, j_\ell(|\vec{q}|r_1)\, j_{\ell'}(|\vec{q}|r_2)\; \frac{1}{2}\, C(1\ell L|{-1}0)\, C(1\ell' L|10)\,[1+(-1)^{\ell-\ell'}]\; T_L(1,2) \tag{26}$$

$$e^{-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}\sigma_{z_1}\sigma_{z_2} \rightarrow 4\pi \sum_{\ell\ell' L} i^{\ell-\ell'} \frac{\sqrt{(2\ell+1)(2\ell'+1)}}{2L+1} j_\ell(|\vec{q}|r_1)\, j_{\ell'}(|\vec{q}|r_2)$$
$$C(1\ell L100)\, C(1\ell' L100)\; T_L(1,2)$$

where $S_\ell(1,2)$ is the scalar constructed from the spherical harmonics of rank ℓ

$$S_\ell(1,2) = \sum_m (-1)^m\, Y_\ell^m(\hat{r}_1)\, Y_\ell^{-m}(\hat{r}_2)$$

and $T_L(1,2)$ is the scalar

$$T_L(1,2) = \sum_k (-1)^k\, \mathcal{Y}^k_{L(\ell)}(\hat{r}_1)\, \mathcal{Y}^{-k}_{L(\ell')}(\hat{r}_2)$$

where $\mathcal{Y}^k_{L(\ell)}(\hat{r})$ is a tensor of rank L given by

$$\mathcal{Y}^k_{L(\ell)}(\hat{r}) = \sum_m C(1\ell L|m,k-m)\, Y_\ell^{k-m}(\hat{r})\,\sigma_m$$

Then use of Eq. (15.5) of Ref. 21) gives the matrix elements of S_ℓ and T_L between states of two nucleons coupled to a given value of the angular momentum J. The isospin dependence of the isovector functions is factorizable in a simple way. The value of $1/3\, \vec{\tau}_i\cdot\vec{\tau}_j$ between states of two nucleons coupled to a given value of the isospin $T=0, 1$ is given by $(-1)^{1+T}/2T+1$.

If the two-nucleon matrix element corresponds to an expected value of the operator between states of non-equivalent particles j_1 and j_2, the antisymmetry has been taken into account using the prescription [21] of adding the exchange term with a sign $(-1)^{j_1+j_2-J-T}$.

Collecting all these indications and performing a straightforward reduction [21] to two nucleons matrix elements, we have obtained the correlation functions given in Table I, as function of the radial integrals $I_\ell(x,y)$ defined by

$$I_\ell(x,y) = \int_0^\infty dr\; r^2\, R_x^*(r)\, R_y(r)\, j_\ell(|\vec{q}|r) \qquad (27)$$

where $x,y = s,p$ indicate the shell in which the radial function $R(r)$ has to be taken. This is interesting because until now the potential well has not been specified. For each function, the first row indicates the result for the simple shell model, the second one the corresponding value when the wave function (25) is used.

A regularity in Table I is given by the terms of crossing between the two shells, represented in the last three columns. The direct contributions $I_o(s,s)I_o(p,p)$ and $I_o(s,s)I_2(p,p)$ disappear in D_T and D_L because then the value $L=0$ in Eq. (26) is not possible, which is necessary for the single particle operator $\mathcal{Y}^k_{L(\ell)}(\hat{r})$ between states coupled to (0,0). $D_S^{(2)}$ is eliminated by a similar reason in the isospin space. Furthermore, in $D_S^{(1)}$ the same value of ℓ for the two particles is needed, as is seen in the first expression of Eq. (26). Then $I_o(s,s)I_o(p,p)$ does not contribute. This last reason is also applied to $I_o(p,p)I_2(p,p)$ for $D_S^{(1)}$ and $D_S^{(2)}$. The exchange contribution $I_1^2(s,p)$ can be calculated separately as follows. If we define K_1 (K_2) as the coefficient of fractional parentage of separating a $p_{\frac{3}{2}}$ $(p_{\frac{1}{2}})$ nucleon in the wave function $(1p)^8$ of $^{12}C(0,0)$, we obtain for this contribution to the correlation functions

$$K_1^2\sum_{J,T}(2J+1)(2T+1)f(s_{1/2}\;p_{3/2},JT) + K_2^2\sum_{J',T'}2(2J'+1)(2T'+1)f(s_{1/2}\;p_{1/2},J'T')$$

where the function f indicates the expected value of the operators between two nucleon states. The cross term K_1K_2 vanishes due to orthogonality. As we are studying the exchange term, the isoscalar operators can be written as

$$D^{(1)}\to K_1^2\sum_{J,T}(-1)^{J+T}(2J+1)(2T+1)f_{exch}(s_{1/2}\;p_{3/2},J) - 2K_2^2\sum_{J'T'}(-1)^{J'+T'}(2J'+1)(2T'+1)f_{exch}(s_{1/2}\;p_{1/2},J')$$

whereas the isovector operators are given by

$$D^{(2)}\to K_1^2\sum_{J,T}(-1)^{J}(2J+1)f_{exch}(s_{1/2}\;p_{3/2},J) - 2K_2^2\sum_{J'T'}(-1)^{J'}(2J'+1)f_{exch}(s_{1/2}\;p_{1/2},J')$$

As $\sum_T(-1)^T(2T+1)=-2$ and $\sum_T 1=2$, $D^{(1)}$ and $D^{(2)}$ are equal but opposite, independent of the particular operator under O(3) and of the configuration mixing. The J dependence of the function f is given in the form of a Racah coefficient [21] and we have

$$\sum_J(2J+1)\begin{Bmatrix} J & j_2 & \frac{1}{2} \\ L & j_2 & \frac{1}{2}\end{Bmatrix} = 1$$

where $j_2=\frac{3}{2},\frac{1}{2}$ and $L=1$ for D_S and $L=0,1,2$ for the other operators. Using this relation, it is immediately seen that the j_2 dependence in f_{exch} is always $(-1)^{j_2+\frac{1}{2}}(2j_2+1)$ for the three operators. As $K_1^2+K_2^2=1$, the result is the same in the three cases, and independent of the particular configuration mixing used to describe the wave function of ^{12}C. This is the result which can be observed in Table I.

Another result, independent of the details of the wave function, is the dependence $\ell = 0$ of the electric correlations. In Eq. (17) their values in the forward direction have been given independently of the model. As electron scattering is dominated by these terms at small values of $-q^2$, we shall see that the configuration mixing does not affect greatly the total structure function $A(q^2)$. However, in the neutrino reaction the spin flip terms are important in this zone due to the axial current, and the results can be sensitive to the particular model.

If we use the harmonic oscillator potential well, the radial integrals (27) can be solved in an analytic way, and then the correlation functions are each of the form

$$e^{-\frac{1}{2} b^2 |\vec{q}|^2} \left\{ C(0) + C(1) \left(\frac{1}{2} b^2 |\vec{q}|^2 \right) + C(2) \left(\frac{1}{2} b^2 |\vec{q}|^2 \right)^2 \right\} \quad (28)$$

where the parameter $b = 1.66$ fm reproduces the empirical root mean square charge radius. In Table II we give the values of the C's for the six functions; in each case the first row is the simple shell model and the second one the result of using the wave function (25). The vanishing value of $C(1)$ in $D_S^{(2)}$ is due to the use of the harmonic oscillator wave functions, as was noticed in Ref. 1).

In Figure 3 we give the results of $1-A^{-1}D_T^{(2)}$ and $1-A^{-1}D_L^{(2)}$ when (25) is used, compared with the shell model version - broken lines - and with $1-A^{-1}D_S^{(2)}$, in order to show the breaking of the supermultiplet relation. It is interesting to observe that the intermediate coupling nature of (25) is manifested through the deviation of the T, L values from the simple shell model towards the S value.

In Figure 4 the values $1+A^{-1}D_T^{(1)}$ and $1-A^{-1}D_T^{(2)}$, using the wave function (25), are plotted, compared with the result for closed subshell - the same for both ones - and with $1-A^{-1}D_S^{(2)}$. The same comparison for the functions $1+A^{-1}D_L^{(1)}$ and $1-A^{-1}D_L^{(2)}$ is given in Fig. 5.

When the results obtained for the correlation functions are applied to Eq. (18), the structure function $A(q^2)$ of ^{12}C given in Fig. 6a is obtained. It can be pointed out that the exclusion effect is so important in this case that total values $A(q^2)$ are suppressed in a zone of $-q^2$ with respect to the incoherent contribution $6A_p(q^2) + 6A_n(q^2)$ - broken line. The values of $A_{el}(q^2)$, Fig. 6b, have been taken from the experimental results [22] of the elastic charge form factor. Then Fig. 6c gives the quasielastic contribution $A_{qel}(q^2)$.

We can apply the obtained correlation functions to the calculation of the reduced muon capture rate [1] in ^{12}C, although the result is very sensitive to the choice of the average neutrino momentum $\nu = |\vec{q}|$. In our case, the use of the wave function (25) implies a variation with an effective power $\simeq 2.8 - 2.9$. If we take $\nu = 80$ MeV we obtain $\Lambda_r(^{12}C) = 0.185$, to be compared to 0.209 for the simple shell model and to 0.113 for the shell model $D_S^{(1)}$. The experimental result is 0.125±0.005.

We have studied in detail the behaviour of the six relevant quantities $1+A^{-1}D^{(1)}_{S,T,L}$, $1-A^{-1}D^{(2)}_{S,T,L}$ separately for each of the states used as a basis in Eq. (25), which will be denoted as 1,2,3,4,5. We summarize the results.

$1+A^{-1}D_S^{(1)}$ decreases quickly from the value 12 for $-q^2=0$ towards the asymptotic value 1; for $-q^2=0.06$ GeV^2, 1.2 is already obtained. The differences among the five states are not significant.

$1-A^{-1}D_S^{(2)}$ only manifests some difference among the five states in a zone around 0.07 GeV^2. For this value, 0.845 is obtained for the state 5 and 0.823 for 4 which give the extreme contributions.

For the other quantities the most significant differences appear at $-q^2 \to 0$. For $1+A^{-1}D^{(1)}_{T,L}(0)$ we obtain 0.297 for the states 1, 2 and 3, 0.148 for 5 and 0.074 for 4. Concerning $1-A^{-1}D^{(2)}_{T,L}(0)$, the values 0.37 for 4, 0.297 for 1, 0.278 for 2, 0.198 for 5 and 0.173 for 3 are obtained.

5. CONCLUSION

From this study it has been shown that the structure functions which describe the quasielastic electron nucleus scattering are given in terms of several correlation functions, from which the exclusion effects are manifest for small values of the four-momentum transfer squared $-q^2$, when compared to the corresponding incoherent quantities of protons and neutrons. This behaviour is also the expected one for the neutrino nucleus reaction comparing with the free neutron cross-section, as it has been emphasized in the literature. The assumptions and approximations of the theory are similar in both cases, in particular the use of impulse and closure. It must be remarked that closure also includes the contribution of the elastic electron scattering, which gives the coherent behaviour Z^2 if $-q^2 \to 0$, and it has to be subtracted. For $T=0$ nuclei the elastic reaction is only due to isoscalar photons and they have not an equivalent in weak interactions.

Our expressions for the structure functions have been applied to the cases of deuteron, ^{4}He and ^{12}C. In particular, in the deuteron case, the obtained reduction has been observed experimentally, showing from comparison that the theoretical approach used here can be valid in a sufficient approximation.

The effects of the configuration mixing mechanism on the correlation functions have been studied in the case of ^{12}C, and a comparison with the results of the simple shell model is given in Tables I and II. They are not important for the scalar functions $D_S^{(1)}$ and $D_S^{(2)}$; on the other functions the over-all effects are showing a deviation towards the well-known relation given by the supermultiplet symmetry. The differences between the predictions of the different models are stronger for very small values of $-q^2$, and they appear in the neutrino and muon capture reactions where the spin flip terms are important due to the axial current. The relation given here $D_{T,L}^{(1)} = -D_{T,L}^{(2)}$ for a closed subshell description is not badly broken when one introduces the configuration mixing; both quantities are deviated in the same sense from the simple description.

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APPENDIX

Here we show the relations $D^{(1)}_{T,L} = -D^{(2)}_{T,L}$ due to a closed subshell. The quantities in which we are interested are the expected values of the relevant operators between configurations $j^{2(2j+1)}$. With the operator acting on two different particles, this is reduced to [21]

$$\sum_{\substack{\text{possible}\\ J,\,T}} (2J+1)(2T+1)\, f(j^2, JT) \tag{A.1}$$

where f denotes the expected value of the operator between two equivalent nucleon states coupled to J, T. Only values with $J+T=\text{odd}$ are possible due to antisymmetry. The sum in (A.1) can be extended to all values of J, T introducing the factor $\frac{1}{2}[1-(-1)^{J+T}]$. Then the isoscalar operators give

$$D^{(1)} \rightarrow \frac{1}{2}\sum_{J,T}[1-(-1)^{J+T}]\,(2J+1)(2T+1)\, f(j^2, J)$$

while the isovector operators are given by

$$D^{(2)} \rightarrow -\frac{1}{2}\sum_{J,T}[1-(-1)^{J+T}]\,(-1)^{1+T}(2J+1)\, f(j^2, J)$$

which is reduced to

$$D^{(1)} \rightarrow \sum_J (-1)^J (2J+1)\, f(j^2, J) + 2\sum_J (2J+1)\, f(j^2, J)$$

$$-D^{(2)} \rightarrow \sum_J (-1)^J (2J+1)\, f(j^2, J)$$

Then the equality is valid for the functions depending on the angular momentum in which $\sum_J (2J+1) f(j^2,J) = 0$. For $D_{S,T,L}$ this quantity is always proportional to

$$\sum_J (2J+1)(-1)^J \begin{Bmatrix} J & j & j \\ L & j & j \end{Bmatrix} = (2j+1)(-1)^{2j}\delta_{L,0} \tag{A.2}$$

For the D_S functions the value $L=0$ - it corresponds to ℓ in the first line of Eq. (26) - it is possible, and the relation does not hold. However, it is easily seen that the tensor $\mathcal{Y}^0_0(\hat{r})$ in Eq. (26) does not contribute when parity restrictions are taken into account. Then the relation is valid for D_T and D_L.

	$I_o^2(s,s)$	$I_o^2(p,p)$	$I_o(p,p)I_2(p,p)$	$I_2^2(p,p)$	$I_o(s,s)I_o(p,p)$	$I_o(s,s)I_2(p,p)$	$I_1^2(s,p)$
$D_S^{(1)}$	12. 12.	56. 56.	0. 0.	-8. -5.73	64. 64.	0. 0.	-16. -16.
$D_T^{(1)}$	-4. -4.	-4.44 -5.11	1.78 -0.214	-9.78 -10.11	0. 0.	0. 0.	-16. -16.
$D_L^{(1)}$	-4. -4.	-4.44 -5.11	-3.56 0.428	-15.11 -10.62	0. 0.	0. 0.	-16. -16.
$D_S^{(2)}$	4. 4.	8. 8.	0. 0.	8. 10.12	0. 0.	0. 0.	16. 16.
$D_T^{(2)}$	4. 4.	4.44 5.28	-1.78 -1.20	9.78 10.66	0. 0.	0. 0.	16. 16.
$D_L^{(2)}$	4. 4.	4.44 5.28	3.56 2.41	15.11 13.12	0. 0.	0. 0.	16. 16.

TABLE I : Isoscalar and isovector correlation functions in ^{12}C as function of the radial integrals defined in Eq. (27). The first row is the value for the simple shell model, the second one for Eq. (25).

	C(0)	C(1)	C(2)
$D_S^{(1)}$	132. 132.	-64. -64.	5.33 5.59
$D_T^{(1)}$	-8.44 -9.11	-1.78 -2.01	-1.78 -1.67
$D_L^{(1)}$	-8.44 -9.11	-3.56 -1.78	-1.78 -1.80
$D_S^{(2)}$	12. 12.	0. 0.	1.78 2.01
$D_T^{(2)}$	8.44 9.28	1.78 1.41	1.78 1.90
$D_L^{(2)}$	8.44 9.28	3.56 2.62	1.78 1.78

TABLE II : Values of the C's defined in Eq. (28) for the different correlation functions in ^{12}C. The first row is for the simple shell model, the second one for the wave function of Eq. (25).

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FIGURE CAPTIONS

Figure 1 a) Total structure function $A(q^2)$ in deuterium and incoherent value $A_p(q^2)+A_n(q^2)$ for different values of the electric form factor of the neutron.

b) Elastic structure function $A_{el}(q^2)$ in deuterium.

c) Quasielastic structure function $A_{qel}(q^2)$. The points have been obtained from the experimental results of Ref. 15).

Figure 2 a) Total structure function $A(q^2)$ in ^{4}He for different values of the electric form factor of the neutron.

b) Elastic contribution $A_{el}(q^2)$ in ^{4}He.

c) Quasielastic structure function $A_{qel}(q^2)$ in ^{4}He. The broken line is the corresponding incoherent quantity $2A_p(q^2) + 2A_n(q^2)$.

Figure 3 Isovector exclusion functions $1 - 1/12\ D^{(2)}_{T,L}$ in ^{12}C using configuration mixing, compared with the simple shell model result – broken lines – and $1 - 1/12\ D^{(2)}_S$.

Figure 4 Exclusion factors $1 + 1/12\ D^{(1)}_T$ and $1 - 1/12\ D^{(2)}_T$ in ^{12}C using configuration mixing, and its value (the same for both quantities) in the simple shell model – upper broken line – compared with $1 - 1/12\ D^{(2)}_S$.

Figure 5 Same as Fig. 4, but corresponding to the functions $1 + 1/12\ D^{(1)}_L$ and $1 - 1/12\ D^{(2)}_L$.

Figure 6 Same as Fig. 2, but applied to the case of ^{12}C. The corresponding incoherent quantity is $6A_p(q^2) + 6A_n(q^2)$.

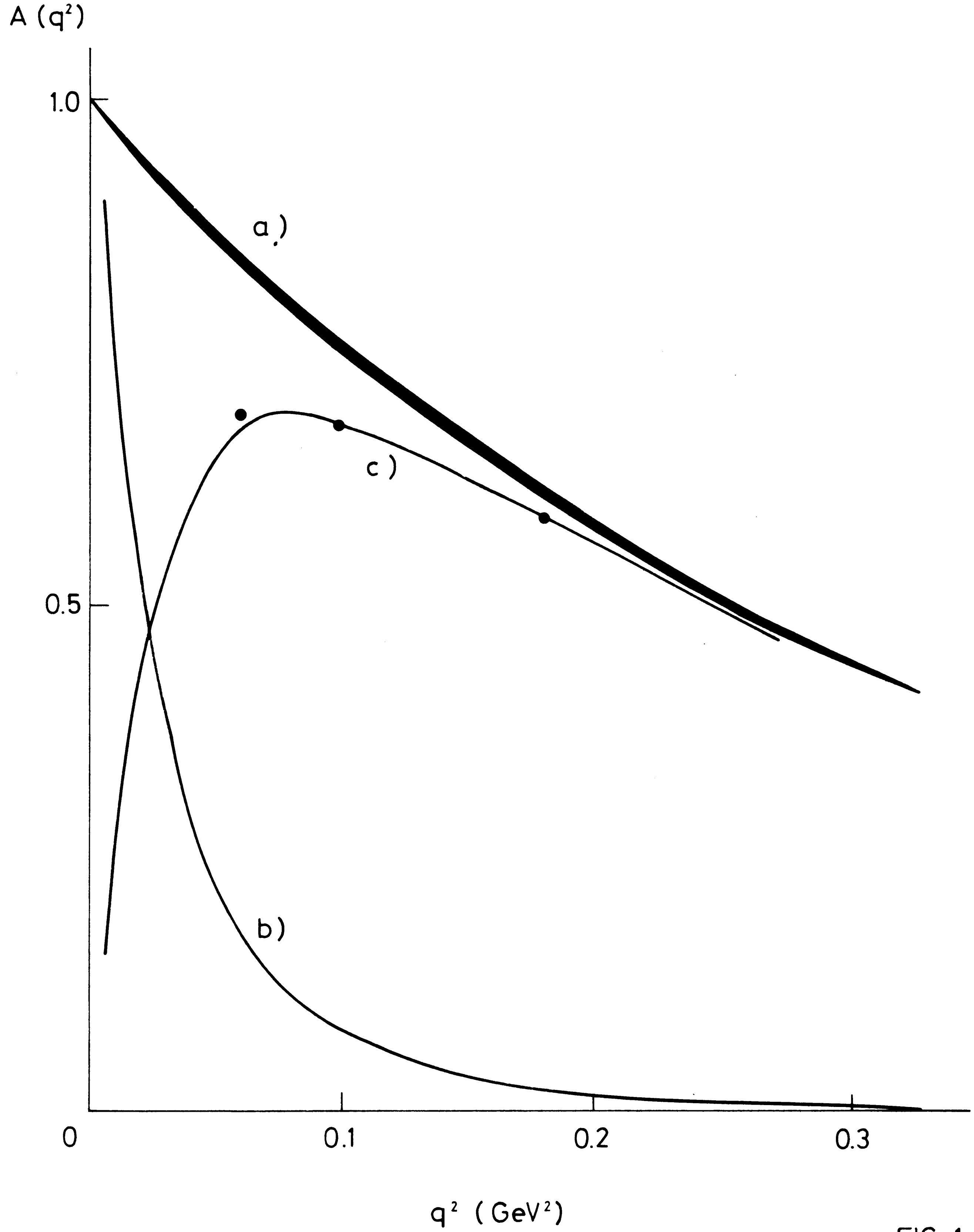

FIG. 1

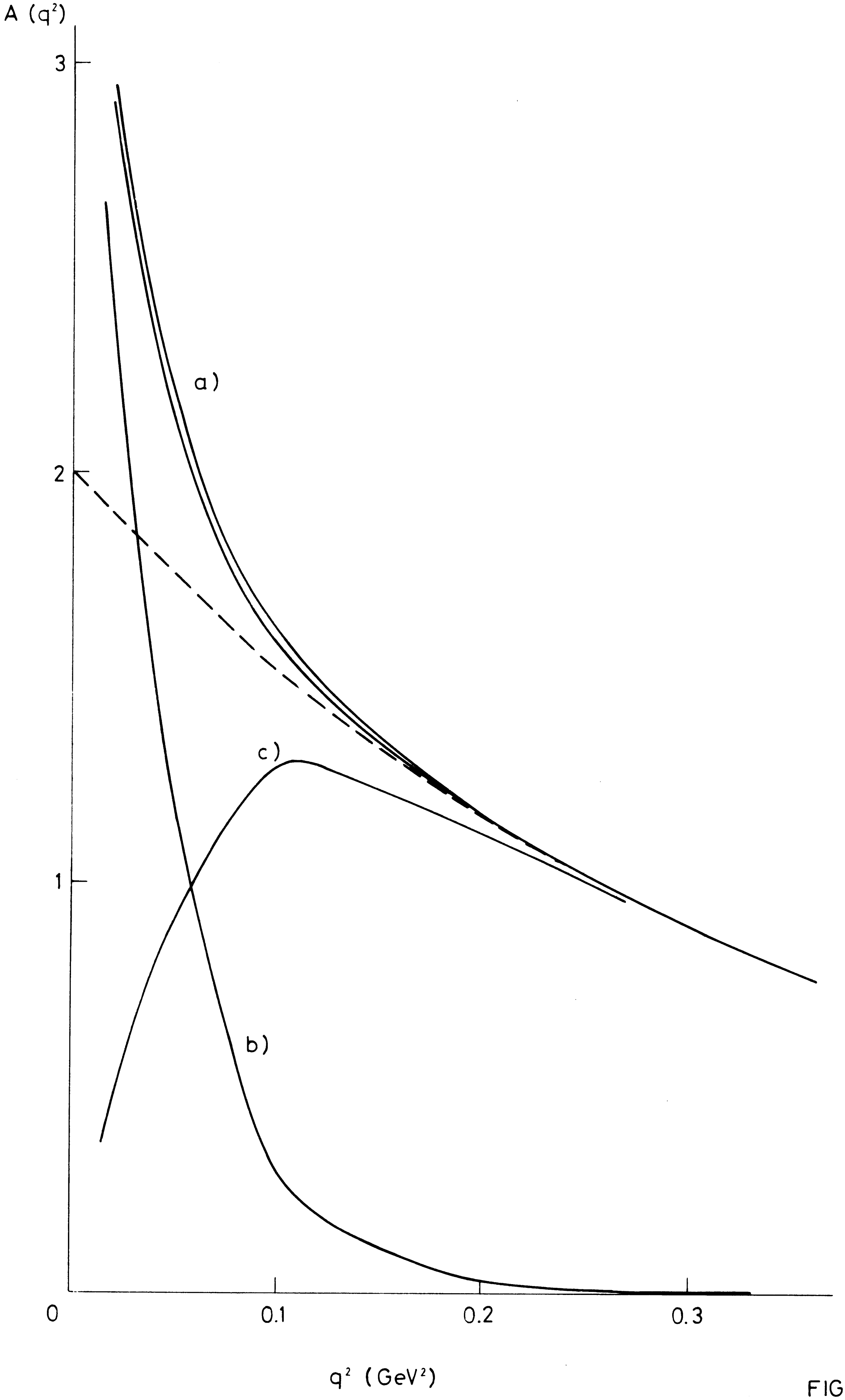

FIG. 2

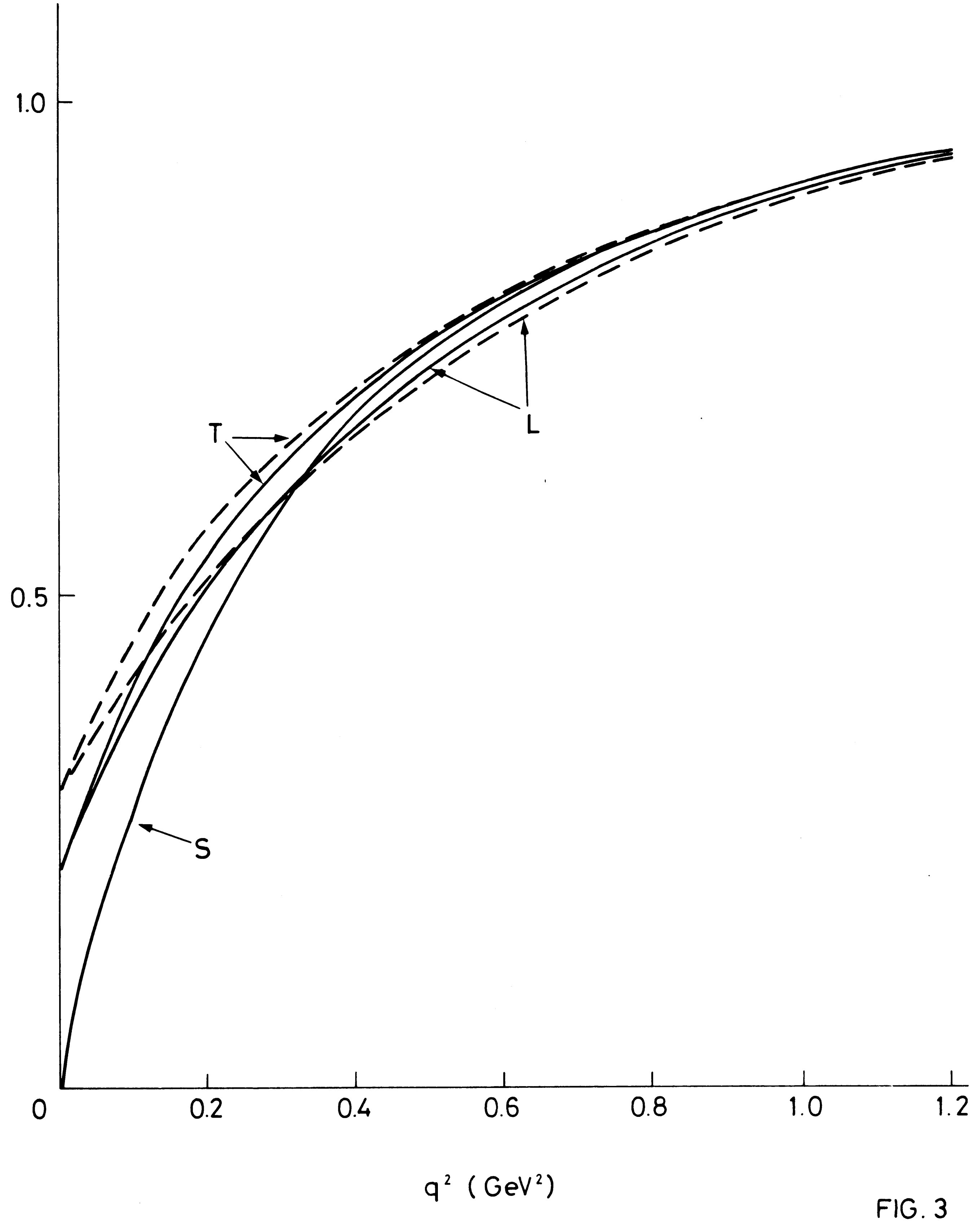

FIG. 3

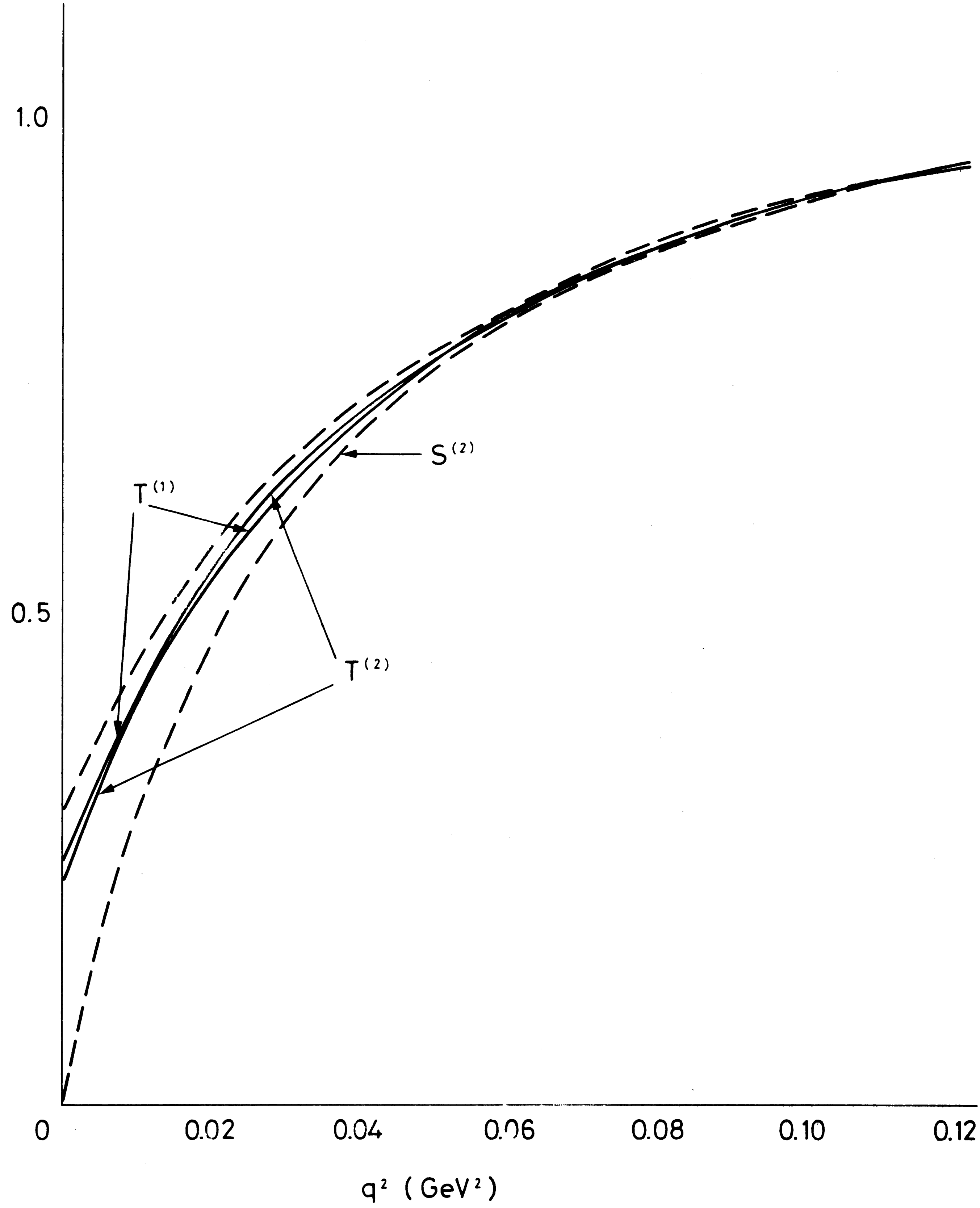

FIG. 4

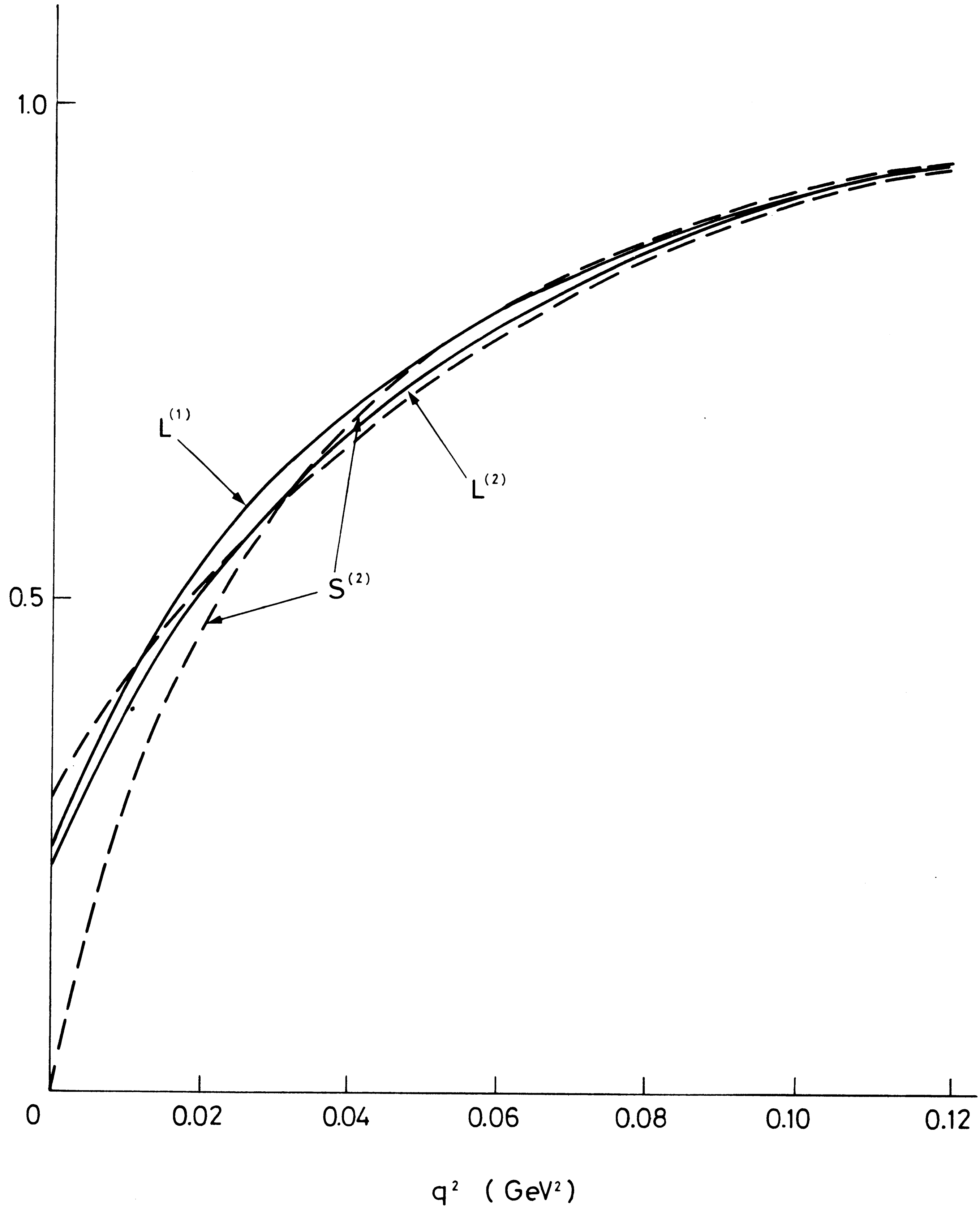

FIG. 5

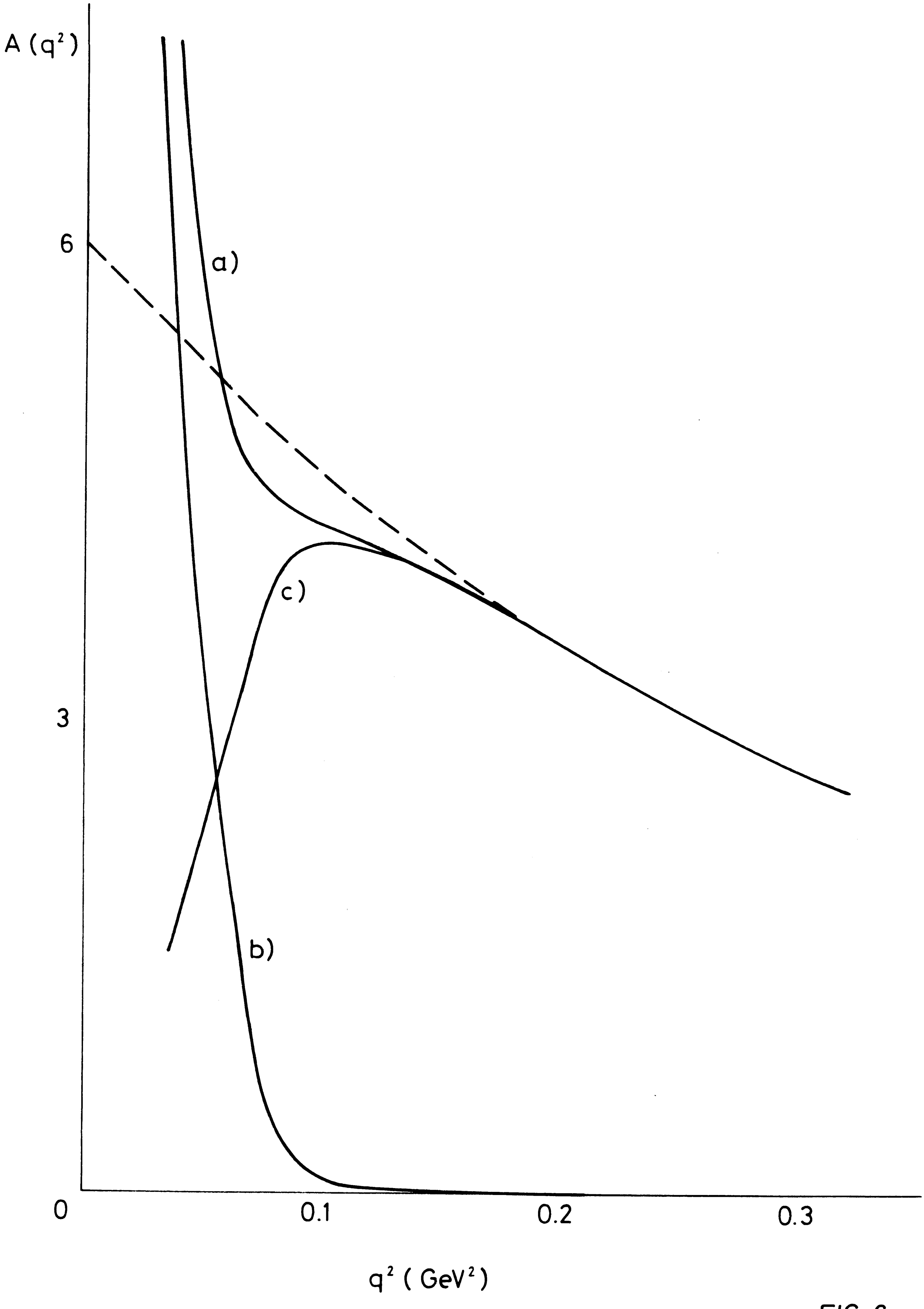

FIG. 6