ECFA/LEP 2
24 July, 1978

STUDY OF HEAVY-LEPTON POLARIZATION IN e^+e^- ANNIHILATION AT THE Z^0 POLE

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1. Introduction

The annihilation process into lepton pairs $e^+e^- \rightarrow \ell^+\ell^-$ in the proximity of the Z^0 peak yields information on the relative strength of the vector and axial couplings g_V and g_A through the cross-section energy dependence and the angular asymmetry of the final state leptons.

The measurement of lepton polarization, besides providing a more sensitive test of the vector-axial mixture in the interaction, is the only way to determine the relative sign of g_V and g_A, in the absence of polarized beams.

The production of a pair of heavy leptons and the study of their subsequent leptonic decays provides a practical and effective way to determine final state helicities at energies around the expected Z^0 mass. The reduction of the $\tau^+\tau^-$ rate by a factor $B^2{}_\ell$ with respect to $\mu^+\mu^-$ due to the branching ratio of the leptonic decays of the τ is not a serious limitation, given the large cross-sections expected at the peak Z^0 energy. This reduction factor is more than compensated by the exceptionally clean experimental signature of μ e events, as opposed to hadronic τ decays, and by the relative ease of detecting the decay leptons with a conventional apparatus. This is to be compared to the massive detector of a dedicated muon polarimeter[1)] which requires about 0.4 M_Z (GeV) metres of Fe to stop muons from the decay $Z^0 \rightarrow \mu^+\mu^-$. The purpose of this note is to study the experimental feasibility of a measurement of longitudinal polarization on heavy leptons produced near the Z^0 pole.

2. Polarized heavy lepton production

In the following the same conventions as in reference 1) are used, as well as the general approach. The formalism adopted is that of Budny[2)]. The Weinberg-Salam model[3)] is used to obtain numerical results.

The cross-section for unpolarized beams and unobserved final state helicities for the reaction

$$e^+e^- \to \tau^+\tau^- \qquad \tau \to \ell\nu\nu \qquad \ell = e,\ \mu \tag{1}$$

is in lowest order in the weak and electromagnetic interactions

$$\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\,\sigma_p \left\{ F_1(1 + \cos^2\theta) + 2F_3\cos\theta \right\} \; 2B_\ell^2 \tag{2}$$

where σ_p is the point-like cross-section and the set of functions F_i is given in the appendix.

The Lorentz boost in τ decay confines the final state leptons in a narrow collinearity distribution, as shown in Fig. 1a for M_Z = 77.3 GeV. This allows also the determination of the asymmetry arising from the second term in (2) (inset in Fig. 1a), which sets an additional constraint on the origin of the μe events. An additional experimental signature is represented by the large fraction of the total energy carried by the decay neutrinos, shown in Fig. 1b. For this case the average lepton momenta are about 13 GeV/c.

The longitudinal polarization P_L of the heavy leptons produced is given by

$$P_L(\cos\theta) = \frac{\mp F_4(1 + \cos\theta)^2}{F_1(1 + \cos^2\theta) + 2F_3\cos\theta} \qquad (\tau^\pm) \tag{3}$$

Averaging over the polar aperture of any forward-backward symmetric apparatus yields

$$\langle P_L(\tau^\pm)\rangle = \mp \frac{2\xi}{1+\xi^2} \tag{4}$$

where $\xi = g_A/g_V$ is the ratio of the axial and vector couplings. The function (4) is shown in Fig. 2a for τ^+; as an example the dependence of ξ on $\sin^2\theta_W$ in the W-S model is given in Fig. 2b.

Fig. 2a shows how the polarization determines the sign of g_A/g_V; although the polarization is maximal at $|\xi|$ = 1, it appears that even small values of g_V, as obtained by the current values of $\sin^2\theta_W$, yield sizeable polarization effects.

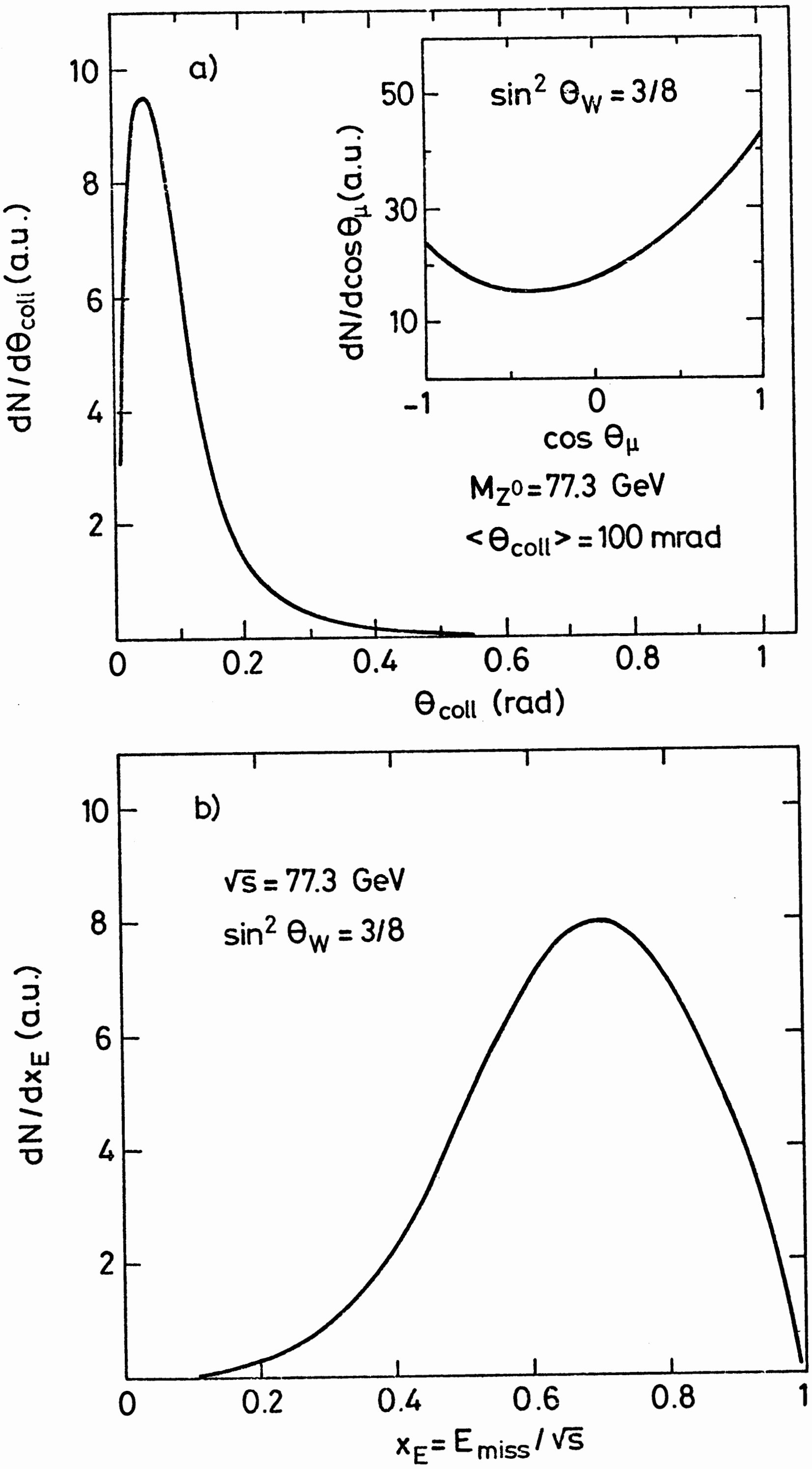

Figure 1.a) : Collinearity distribution for e events from pair production at $\sqrt{s}$ = 77.3 GeV. The inset shows the angular distribution of the muon in the lab system.

b) : Distribution of the normalized missing energy for the same reaction.

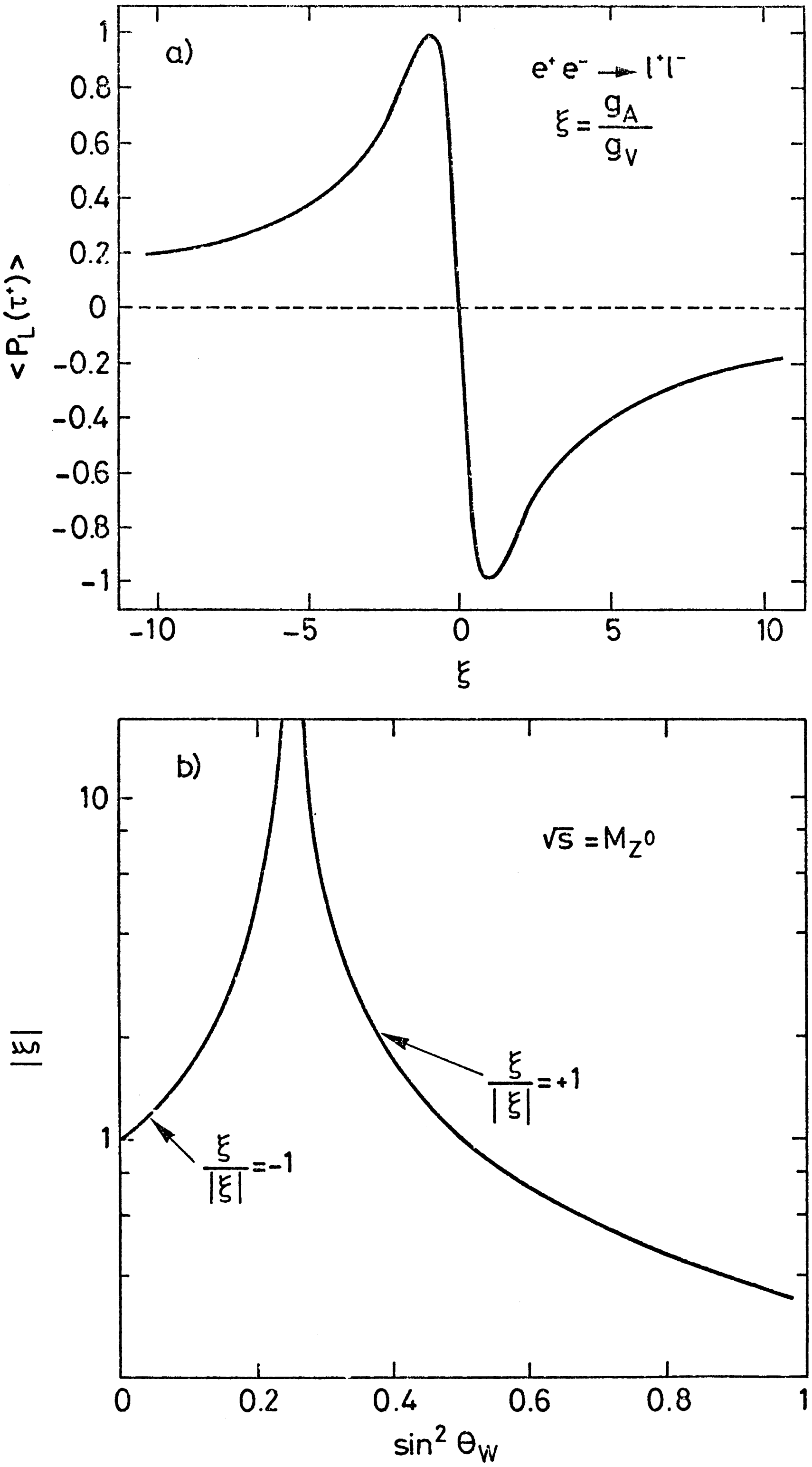

Figure 2.a) : Dependence of the average longitudinal polarization on the ratio of the axial to the vector couplings ξ. The polarization of a τ^+ is shown.

b) : Variation of the parameter ξ as a function of $\sin^2\theta_W$ in the W-S model. The left branch of the curve corresponds to negative ξ values.

3. Decay of polarized heavy leptons

In the measurement of the longitudinal polarization of muons at rest the relevant information on the spin direction is contained in the angular asymmetry of the decay electrons. The rapid decay in flight of heavy leptons transfers this information to the momentum distribution of the decay leptons. It is the final state momentum of the detected lepton which is then correlated with the τ spin. Assuming for the heavy lepton weak current the same V-A structure as for the muon and neglecting final state masses ($m_\ell/m_\tau \rightarrow 0$) the differential momentum spectrum of the decay lepton in the lab. system has the form[4)]

$$\frac{dn}{dX} = f(X,P_L) = a(X) \mp P_L\, b(X) \quad (\text{for } \tau^\pm) \tag{5}$$

where $X = P\ell/p_\tau$ is the normalized lepton momentum; the functions a and b are given in the appendix. Fig. 3a shows the momentum spectrum expected for τ^+ decay with $\sin^2\theta_W = 0.1$ or $\langle P_L \rangle = 0.88$.

The sensitivity of the momentum distribution to the polarization value is shown in Fig. 3b, where the quantity r(X), the fractional difference of $f(X, P_L)$ for two values of $\sin^2\theta_W$, is displayed:

$$r(X) = 1 - f(X)_{\sin^2\theta_W=0.5}\; f^{-1}(X)_{\sin^2\theta_W=0.1} \tag{6}$$

The function df/dP_L changes sign when crossing the critical value $X_c = 0.4215$, at which point the momentum distribution does not depend on the polarization. It follows that, rather than fitting the entire distribution, which requires an absolute energy calibration, a simple estimator of the polarization can be the quantity[5)]

$$R = \frac{n_H}{n_L} \tag{7}$$

where

$$n_H = \int_{X_c}^{1} f(X)dX \qquad n_L = \int_{X_o}^{X_c} f(X)dX$$

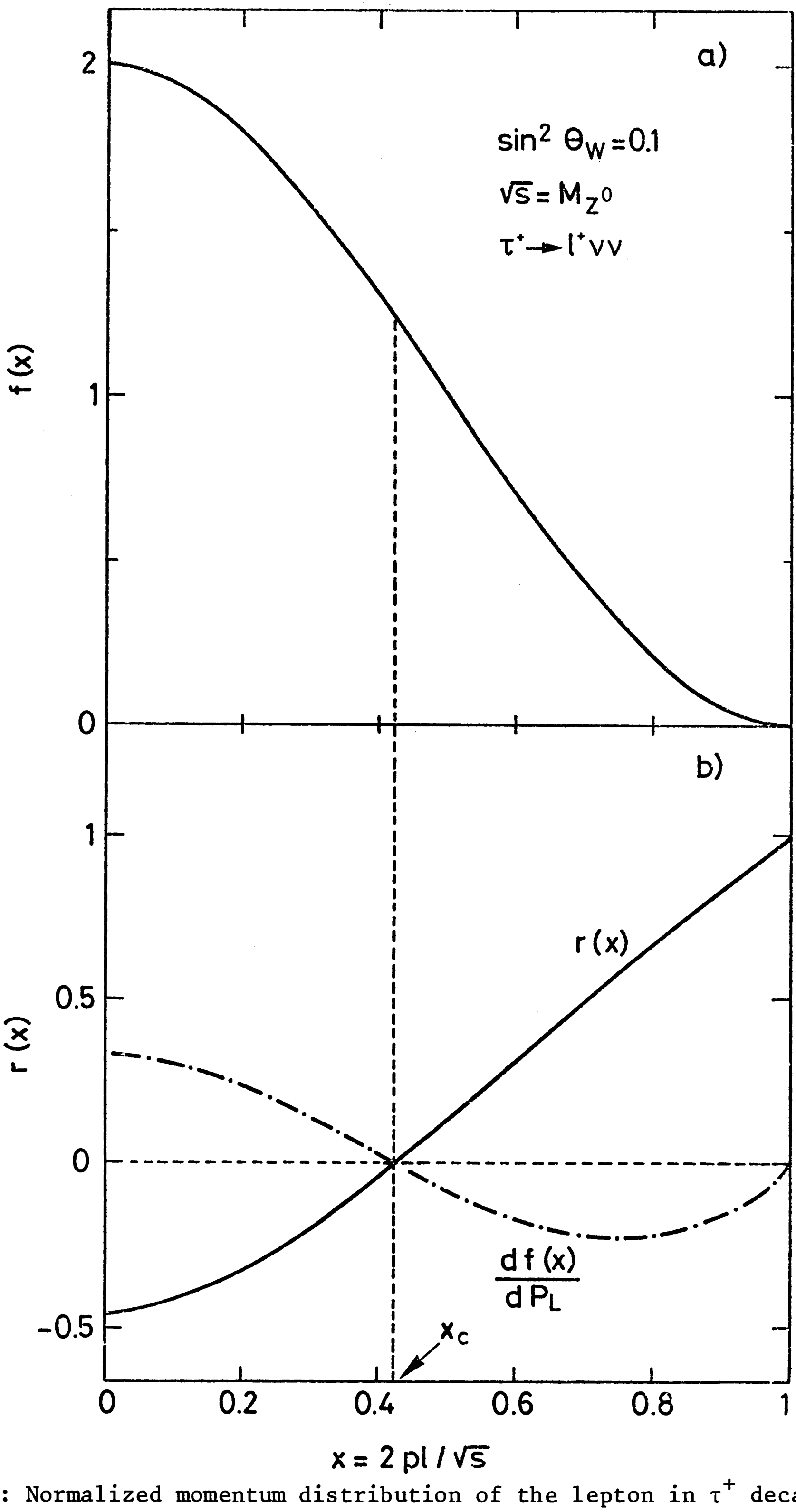

Figure 3.a) : Normalized momentum distribution of the lepton in τ^+ decay at $\sin^2\theta_W = 0.1$.

b) : The fractional difference r(X) of the distribution in a) for $\sin^2\theta_W = 0.1$ and $\sin^2\theta_W = 0.5$ (solid line). The dash-dotted line indicates the sensitivity of the spectrum in a) to the polarization. The critical value $X = X_c$ at which the derivative is zero is shown.

where the lower limit X_o on the momentum is set by considerations of detection efficiency and backgrounds. The variation of the average longitudinal polarization for a τ^+ and of the estimator R on $\sin^2\theta_W$ are displayed in Fig. 4. Two separate curves for R correspond to values of the lower limit $X_o = 0.05$ and $X_o = 0.1$. It is clear that the method is rather insensitive to the experimental momentum cut, which in turn is also dependent on the absolute energy calibration.

4. A polarization experiment

As a realistic example let us consider an experiment in which τ pairs from Z^o decay are detected through their e μ decays over a reasonable fraction of the solid angle. The leptonic branching ratio of the τ is taken to be $B_\ell \approx 0.2$, the width of the Z^o $\Gamma_Z = 1$ GeV, and the luminosity $\mathcal{L} = (M_Z/140)^2 \cdot 10^{32} cm^{-2} s^{-1}$.

The apparatus is supposed to be rather conventional, having a polar acceptance between 30^o and 150^o to the beams and full azimuthal coverage. It must be capable of measuring electron and/or muon momenta up to the beam value; momentum resolution is not a critical parameter provided it is sufficient to minimize (calculable) smearing effects on R at $X \simeq X_c$, around 17 GeV/c for $M_Z = 78$ GeV.

With these assumptions, and considering the energy spectrum of only one detected lepton, the fractional uncertainty on R ($X_o = 0.05$) obtained in 100 hours of running time is shown in Fig. 4b. In the worst case R is measured to a 3% precision in such a short time.

Considering now the running time needed to measure the polarization to a given fractional error σ_P/P, including in the calculation the sensitivity of $\langle P_L \rangle$ on R, we obtain

$$t = 1.39 \cdot 10^{-4} \, g(P_L) \, R(P_L) \left[1 + R(P_L)\right]^2 \left(\frac{1}{P}\frac{dP_L}{dR}\right)^2 \left(\frac{\sigma_{P_L}}{P_L}\right)^{-2} \left(\mathcal{L}\int_{\Delta\Omega} \frac{d\sigma}{d\Omega}\, d\Omega\right)^{-1} \tag{8}$$

where t is the running time in hours, $R(P_L)$ and $g(P_L)$ are given by equations (A10) and (A11) and we consider the contributions of both leptons.

Numerical results from equation (8) are shown in Fig. 5 for the range of $\sin^2\theta_W$ which can be explored with a 70 GeV machine. As can be seen a running time of the order of 10^2 hours yields a 10% measurement of the τ longitudinal polarization almost everywhere. Such a precision allows a valuable cross-check

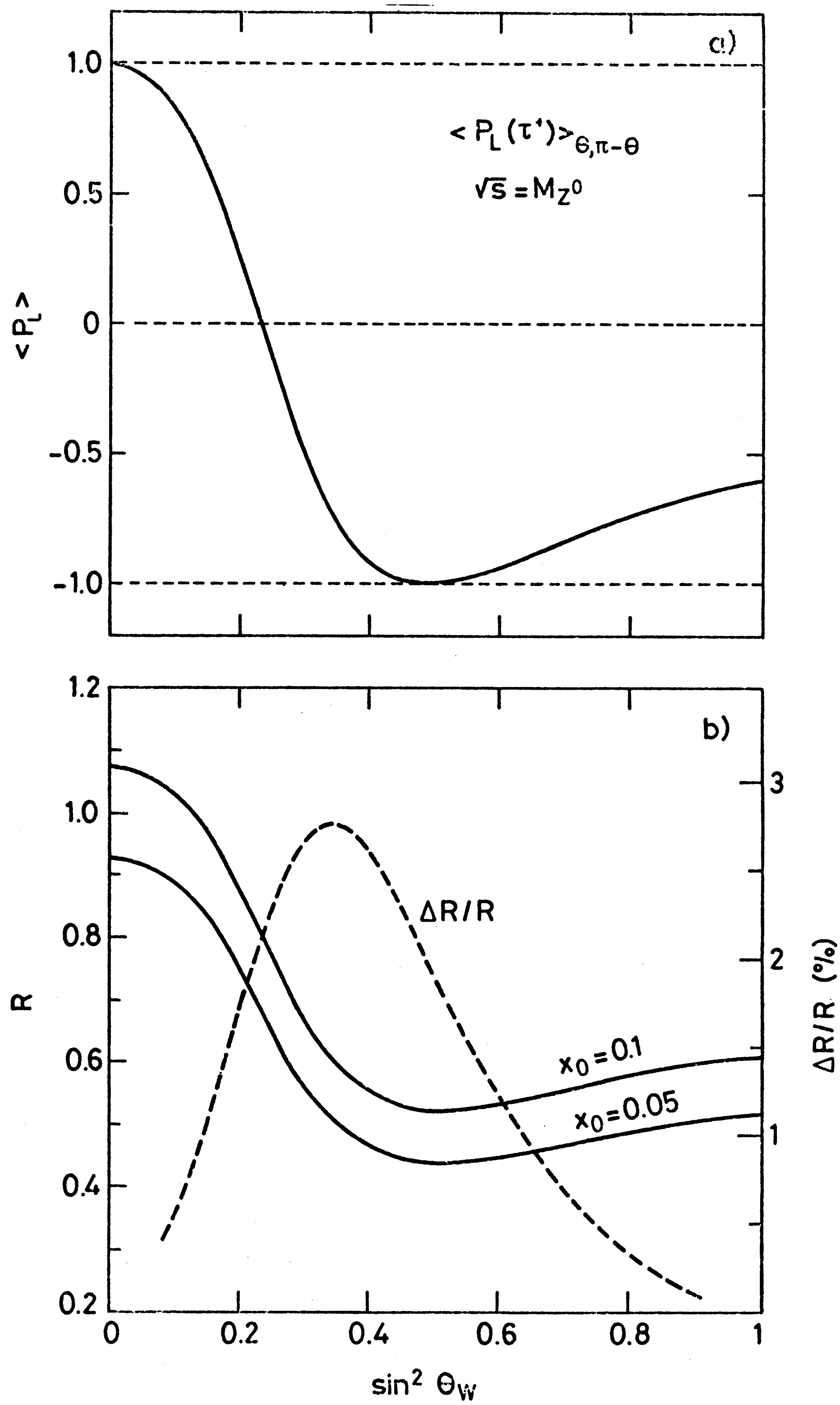

Figure 4.a) : The τ longitudinal polarization averaged over a symmetric polar interval as a function of $\sin^2\theta_W$.

b) : Dependence of the polarization estimator R on $\sin^2\theta_W$ for two values of the momentum cut X_o (solid lines). The dotted line shows the fractional error on R which can be obtained in 100 hours of running time (see text).

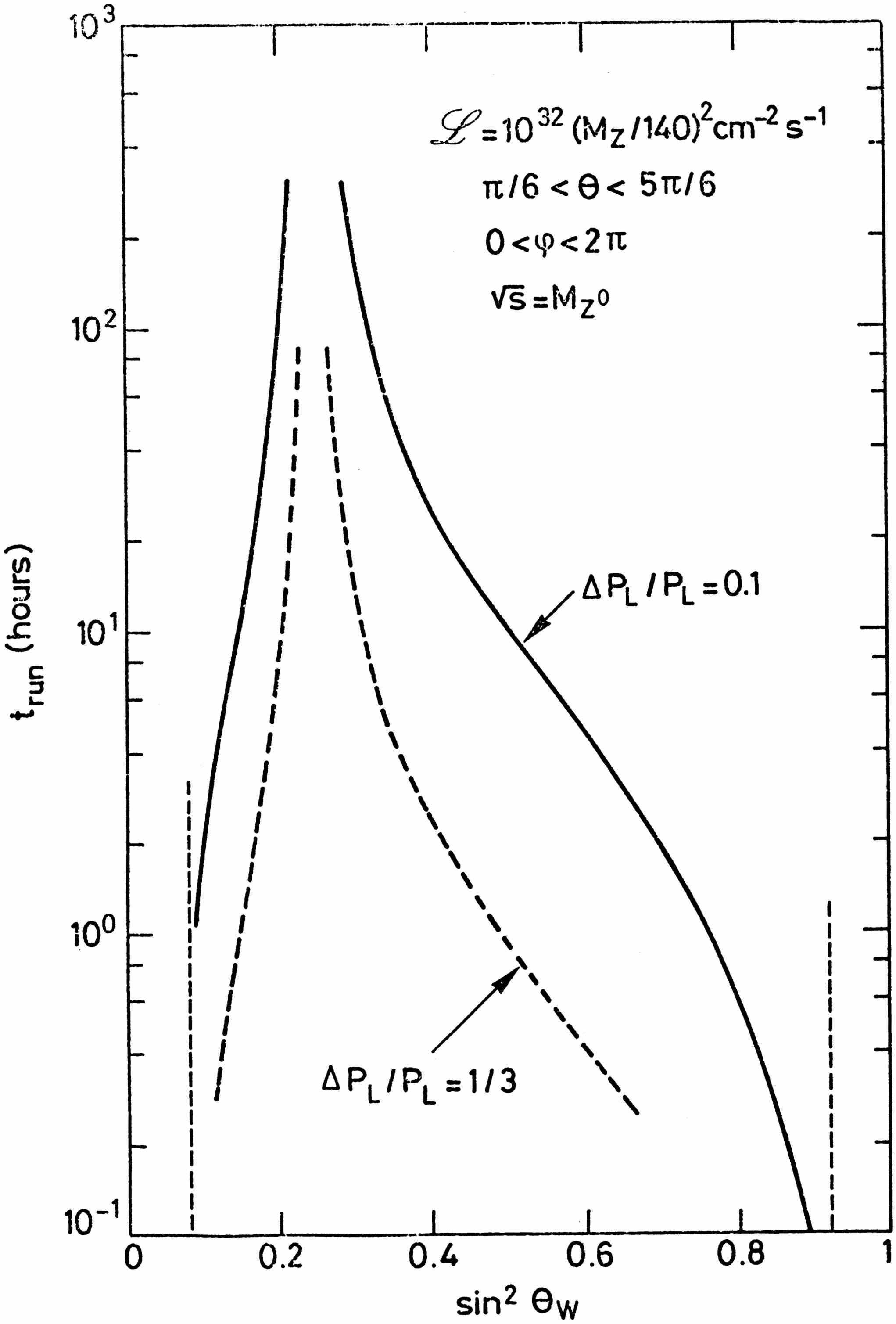

Figure 5 : Running time as a function of $\sin^2\theta_W$ for given fractional errors on the polarization. The vertical dashed lines delimit the interval of $\sin^2\theta_W$ which can be explored with a 70 GeV machine.

of the g_A/g_V ratio obtained by an asymmetry measurement. The curve labelled $\Delta P_L = P_L/3$ shows the minimal running time needed for a determination of the sign of g_A/g_V, which can be achieved in most cases in a few hours of running. The number of e μ events at the Z^0 peak corresponding to the case $\Delta P_L/P_L = 0.1$ is given in Fig. 6. Also shown is the statistics expected in 100 hours of running time.

The above values show that, besides the considerations of this note, the study of the weak properties of heavy leptons at LEP-70 has considerable intrinsic validity if such a copious source as the neutral weak boson is within reach of the machine.

5. Conclusions and remarks

It has been shown in the previous sections that heavy-lepton pair production at the neutral vector boson mass can be easily detected and measured through the leptonic decay mode. This reaction is particularly suited for a determination of the τ helicity; no specialized apparatus is needed for this measurement, which is compatible with the physics programme of a general detector. In addition the expected rate of e μ events is very high, yielding a minimum of $6 \cdot 10^3$ events in 100 hours of running time, which makes a detailed investigation of the τ properties a physically interesting programme in itself. The information on the τ spin alignment can be easily extracted from the lepton decay spectrum; good sensitivity can be achieved in the region $0.2 < \sin^2\theta_W < 0.4$ where the physical value is most probably found. More generally the value of the longitudinal polarization allows the determination in a model-independent way of the relative sign of the axial and vector weak couplings, as well as their absolute ratio[6].

Although in this case the need to observe final state polarizations can be circumvented by the use of polarized beams[7], it is generally true[8] that some of the information which can be obtained by observing final state helicities is not accessible by simply controlling the longitudinal beam polarization and vice versa. This point is relevant to the study of possible contributions from non-conserved currents.

Acknowledgements

I wish to thank G. Barbiellini, L. Camilleri and J.H. Field for very useful discussions.

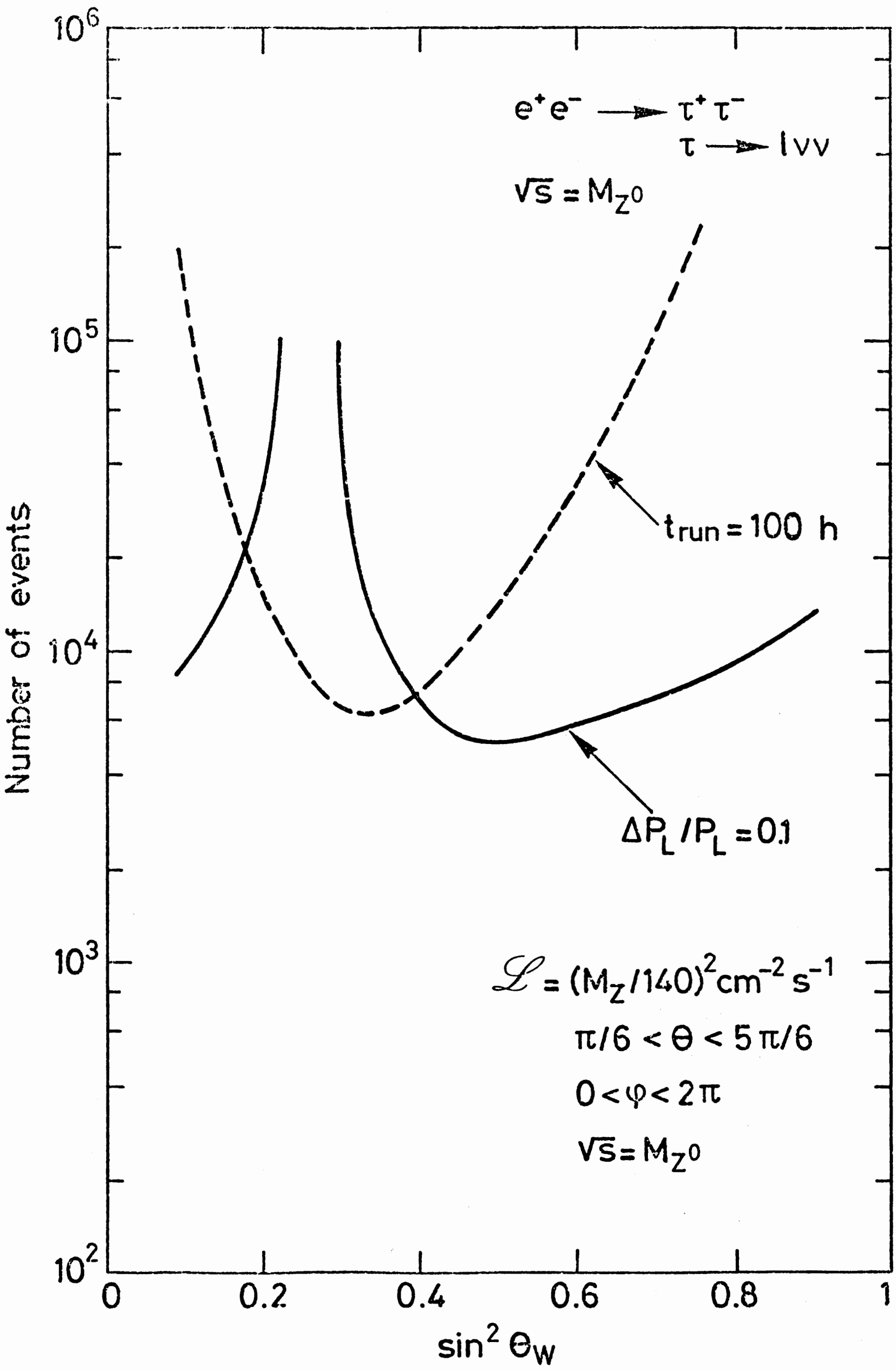

Figure 6 : Number of events at the Z^0 pole as a function of $\sin^2\theta_W$. The solid line corresponds to the case $\Delta P_L/P_L = 0.1$ of Figure 5. The dotted line shows the expected number of events in 100 hours of running time.

APPENDIX

For unpolarized beams the differential cross-section for τ pair production with $\tau^{\pm}$ helicity $h_{\pm}$ is:

$$\frac{128\pi}{3\sigma_P}\frac{d\sigma}{d\Omega} = 2F_1(1+\cos^2\theta) + 4F_3\cos\theta - h_+h_-(1-\cos\theta)^2(F_1 - F_3) + (h_- - h_+)(1+\cos\theta)^2 F_4 \qquad (A1)$$

where

$$F_1 = 1 + 2g_V{}^2 R_e(R) + (g_V{}^2 + g_A{}^2)^2 |R|^2$$

$$F_3 = 2g_A{}^2 \quad Re(R) + 4g_V{}^2 g_A{}^2 |R|^2 \qquad (A2)$$

$$F_4 = 2g_V g_A \quad Re(R) + 2g_V g_A (g_V{}^2 + g_A{}^2) |R|^2$$

with

$$R = \frac{s}{e^2(s - M_Z^2 + iM_Z\Gamma_Z)}$$

From (A1) and (A2), averaging over the helicity of one τ, we obtain the longitudinal polarization of the other:

$$P_L(\tau^{\pm}) = \frac{\mp F_4(1+\cos\theta)^2}{F_1(1+\cos^2\theta) + 2F_3\cos\theta} \qquad (A3)$$

Averaging over any interval $(\theta, \pi-\theta)$ corresponding to the angular acceptance of a forward-backward symmetric detector we obtain

$$\langle P_L(\tau^{\pm})\rangle = \mp\frac{F_4}{F_1} \qquad (A4)$$

At the Z^0 pole we obtain from (A2):

$$\langle P_L \rangle \;=\; \mp \frac{2g_V g_A (g_V{}^2 + g_A{}^2) s/\Gamma_Z^2}{1 + (g_V{}^2 + g_A{}^2)^2 \; s/\Gamma_Z^2} \qquad (s = M_Z^2) \qquad \text{(A5)}$$

and for a reasonably narrow width

$$P_L \;=\; \mp \frac{2\xi}{1 + \xi^2} \qquad \xi \;=\; g_A/g_V \qquad (s \gg \Gamma^2) \qquad \text{(A6)}$$

• The normalized lepton momentum distribution in the laboratory from τ decays in flight has the form

$$f(X) \;=\; a(X) \mp P_L \; b(X) \qquad (\tau^{\pm}) \qquad \text{(A7)}$$

where

$$a(X) \;=\; \frac{5}{3} - 3X^2 + \frac{4}{3} X^3$$

$$b(X) \;=\; \frac{1}{3} - 3X^2 + \frac{8}{3} X^3 \qquad X = P_\ell / P_\tau \qquad \text{(A8)}$$

$$b(X) \;=\; 0 \quad \text{at} \quad X \;=\; X_c$$

Defining:

$$R \;=\; \frac{n_H}{n_L} \qquad n_H \;=\; \int_{X_c}^{1} f(X)dX \qquad n_L \;=\; \int_{X_o}^{X_c} f(X)dX \qquad \text{(A9)}$$

The dependence of R on the longitudinal polarization is given by:

$$R(P_L) \;=\; \frac{A \mp BP_L}{C \mp DP_L} \qquad (\text{for } \tau^{\pm}) \qquad \text{(A10)}$$

with A = 0.36186, B = -0.8666, C = 0.55493, D = 0.07011 for X_o = 0.05.

If N_D is the number of events used in the determination of R and N the number in the whole distribution:

$$N_D = n_L + n_H \qquad X_o < X < 1$$
$$N = g(P_L)N_D \qquad 0 < X < 1 \tag{A11}$$

where

$$g(P_L) = (0.91679 \pm 0.01655\, P_L)^{-1} \qquad (\text{for } \tau^{\pm})$$

The statistical error on R is given by:

$$\sigma_R = \sqrt{\frac{R(1+R)}{n_L}} = \sqrt{\frac{R(1+R)^2}{N_D}} \tag{A12}$$

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