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THE REACTION $e^+e^- \rightarrow \mu^+\mu^-$ (OR $\tau^+\tau^-$)
WITH TWO NEUTRAL WEAK BOSONS
AT LEP ENERGIES

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At the present time, the standard Salam-Weinberg model of weak and electromagnetic interactions is in agreement with almost all the experimental data. The rates of neutral current-induced ν or $\bar{\nu}$ reactions on nucleons are now well accounted for with a Weinberg angle θ_W such that $\sin^2\theta_W = 0.24 \pm 0.02$ ^[1]. Recently, the discovery of parity violation in reactions of polarized electrons on nucleons at SLAC^[2] has provided a remarkable confirmation of the standard model with $\sin^2\theta_W = 0.20 \pm 0.03$. In the reactions $\nu_\mu(\bar{\nu}_\mu) + e^-$ there is still some controversy in the data^[1] but all experiments suffer from low statistics.

The standard model is also the "minimal" model, and its success at center-of-mass energies much smaller than the mass of the weak neutral boson is understandable, even if the structure of neutral current is actually more complicated. However, since we are concerned by the energy range which should be covered by the LEP machine, it is particularly interesting to review the predictions of models involving larger gauge groups, thus more than one neutral weak boson Z.

In Section 1, the model independent formulae giving the cross-section, the charge asymmetry and the longitudinal polarization of the final leptons are given for $e^+e^- \rightarrow \mu^+\mu^-$ (or $\tau^+\tau^-$) reaction. In Section 2, the general features of models based on the $(SU_2)_L \otimes (SU_2)_R \otimes U_1$ group are summarized. The predictions of two of them are presented in Section 3 for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ (or $\tau^+\tau^-$).

1 - GENERAL MODEL-INDEPENDENT FORMULAE .

The general formulae for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ are given by a straightforward extension of the calculations of R. Budny [3], involving three annihilation graphs, (via γ , Z_1 and Z_2). If only vector and axial couplings are present, the differential cross-section and the lepton longitudinal polarization are given by [4] :

$$(1) \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4S} \left[F_1(S) (1 + \cos^2 \theta) + 2 \cos \theta \cdot F_3(S) \right]$$

$$(2) P(\mu^+ \text{ or } \tau^+) = - \frac{F_4(S) (1 + \cos \theta)^2}{F_1(S) (1 + \cos^2 \theta) + 2 \cos \theta F_3(S)}$$

in which F_1 , F_3 and F_4 are functions of the center of mass energy squared S , and of the boson masses, widths and coupling constants.

In this report, we define the coupling constants g_{V1} , g_{A1} (for Z_1) and g_{V2} , g_{A2} (for Z_2) in such a way that the interaction lagrangian be written as :

$$e^2 z_i^\mu \cdot \bar{e} (g_{Vi} \gamma_\mu + g_{Ai} \gamma_\mu \gamma_5) e ; \quad (i = 1, 2).$$

(Note that in Ref. [3], e^2 is included in the coupling constants)

If $(M_k, \Gamma_k, k = 1, 2)$ are respectively the boson masses and widths, we define :

$$R_k = \frac{S}{S - (M_k - i \frac{\Gamma_k}{2})^2} \quad (k = 1, 2).$$

Then, F_1, F_3 are given by [5] :

$$(3) \quad F_1 = 1 + \sum_i (|R_i|^2 (g_{Vi}^2 + g_{Ai}^2)^2 + 2(\text{Re } R_i) g_{Vi}^2) \\ + 2(\text{Re } R_1^* R_2) (g_{V1} g_{V2} + g_{A1} g_{A2})^2$$

$$(4) \quad F_3 = \sum_i (4|R_i|^2 g_{Vi}^2 g_{Ai}^2 + 2(\text{Re } R_i) g_{Ai}^2 + 2(\text{Re } R_1^* R_2) (g_{V1} g_{A2} + g_{A1} g_{V2})^2)$$

and F_4 is similarly given by :

$$(5) \quad F_4 = \sum_i 2|R_i|^2 g_{Vi} g_{Ai} (g_{Vi}^2 + g_{Ai}^2) + 2(\text{Re } R_i) g_{Vi} g_{Ai} \\ + 2(\text{Re } R_1^* R_2) (g_{V1} g_{V2} + g_{A1} g_{A2}) (g_{V1} g_{A2} + g_{A1} g_{V2})$$

The terms in $|R_i|^2$ clearly result from the square of the Z_i amplitudes, the terms in $(\text{Re } R_i)$ from the γ - Z_i interference, and finally the term in $2(\text{Re } R_1^* R_2)$ from the $Z_1 - Z_2$ interference.

Formulae (1) to (5) allow to calculate event rates, charge asymmetry and polarization [4] as functions of S . In the following, the detector is assumed to be Solenoidal, with full azimuthal acceptance, and $30^\circ < \theta < 150^\circ$, θ being the angle with respect to the beam axis ; the luminosity of the machine is assumed to be : $\mathcal{L} = 10^{32} \left(\frac{E}{70 \text{ GeV}}\right)^2 \text{cm}^{-2} \text{S}^{-1}$.

The model dependent inputs are thus the coupling constants, masses and widths of the bosons Z_1 and Z_2 .

GENERAL FEATURES OF $(SU_2)_L \otimes (SU_2)_R \otimes U_1$ models

The extension of the usual $SU_2 \otimes U_1$ gauge group had been initially motivated by experimental results, namely : anomalous trimuon production in ν reactions and absence of parity violation in bismuth atoms. At the present time, all experimental data agree on trimuon rates compatible with conventional processes [6] and parity violation in $e N$ reactions is clearly proved [2].

However, the models based on the $(SU_2)_L \otimes (SU_2)_R \otimes U_1$ group have an interesting feature, namely the basic left-right symmetry, parity being spontaneously broken via the Higgs mechanism. A complete review of such models can be found in Ref. [5]. Their most important characteristics are summarized here :

- (a) The $(SU_2)_L$ and $(SU_2)_R$ groups have the same coupling constant :

$$g = \frac{e}{\sin \theta_W} . \text{ The } U_1 \text{ coupling constant is : } g' = \frac{e}{\sqrt{\cos 2\theta_W}} .$$

- (b) The generators of $(SU_2)_L$, $(SU_2)_R$ and U_1 being denoted by \vec{T}_L , \vec{T}_R and Y respectively, the electric charge is given by :

$$Q = T_{3L} + T_{3R} + \frac{Y}{2}$$

Left-handed leptons are classified in $(SU_2)_L$ doublets and $(SU_2)_R$ singlets, whereas right-handed leptons are classified in $(SU_2)_L$ singlets and $(SU_2)_R$ doublets. Both have $Y = -1$.

- (c) There are 4 charged boson states W_L^\pm and W_R^\pm and 3 neutral boson states : W_{3L} , W_{3R} and B , (B corresponding to U_1). The physical bosons are linear combinations of such states. The Higgs fields are chosen in such a way to give mass to charged bosons and to two neutral bosons, (the third one being the photon).

- (d) Higgs bosons χ_L ($T_L = \frac{1}{2}$, $T_R = 0$, $Y = -1$) and χ_R ($T_L = 0$, $T_R = \frac{1}{2}$, $Y = -1$) are introduced with vacuum expectation values λ_L and λ_R . They contribute to both neutral and charged boson masses. Neutral current violate parity only if $\lambda_R \neq \lambda_L$.
- (e) Higgs bosons δ_L ($T_L = 1$, $T_R = 0$, $Y = 0$) and δ_R ($T_L = 0$, $T_R = 1$, $Y = 0$) are introduced with vacuum expectation values b_L and b_R , contributing only to charged boson masses.
- (f) An additional Higgs boson ϕ ($T_L = T_R = \frac{1}{2}$, $Y = 0$) mixes W_L^\pm and W_R^\pm . It also affects the masses of the neutral bosons, and has no effect on parity violation in neutral currents.

In order to suppress right-handed charged currents at low energies, a very large mass has to be given to W_R^\pm . This can be achieved either by taking $b_R \gg b_L$, or by taking $\lambda_R \gg \lambda_L$ or both. We shall only consider here, as illustrations, the two following extreme cases :

1) Fritzsch - Minkowski - Mohapatra - Sidhu model (FMS) ^{-[7,8,9]}

In this model, λ_L and λ_R are equal and b_L set equal to 0, whereas b_R is very large. There is no parity violation in neutral currents since the lighter neutral boson (Z_A) is coupled to a purely axial current and the heavier one (Z_V) to a purely vector current. The predictions of this model for neutrino reactions coincide with those of the standard model with the same value of $\sin^2 \theta_W$. However, the recent SLAC experiment ^[2] now excludes this model.

2) De Rujula - Georgi - Glashow model (DGG) ^[10] :

In this model, λ_L is set equal to 0 and only λ_R is used to give mass to W_R^\pm . Parity is thus violated in neutral current processes. The two physical neutral bosons are now linear combinations of Z_A (purely axial) and Z_V (purely vector) states. The model parameters can be fitted to account for neutrino data ^[5] and two solutions (denoted by DGG - and DGG+ respectively) are found. However, the DGG+ solution would imply a reduction of parity vio-

lating effects in eN reactions relatively to the standard model and is excluded by the recent SLAC results [2].

3 - APPLICATION OF FMMS AND DGG MODELS TO $e^+e^- \rightarrow \mu^+\mu^-$ AT LEP ENERGIES.

	FMMS	DGG+	DGG-	S - W
$\sin^2\theta_W$	0.26 ± 0.07	$0.29 \pm \begin{matrix} 0.05 \\ 0.08 \end{matrix}$	$0.29 \pm \begin{matrix} 0.06 \\ - 0.10 \end{matrix}$	0.26 ± 0.03
$M_{Z_1}^2$ GeV/c ²	72 ± 10	$78 \pm \begin{matrix} 11 \\ 6 \end{matrix}$	$80 \pm \begin{matrix} 16 \\ - 6 \end{matrix}$	85 ± 3
$M_{Z_2}^2$ GeV/c ²	$106 \pm \begin{matrix} 8 \\ - 27 \end{matrix}$	$143 \pm \begin{matrix} 33 \\ - 13 \end{matrix}$	$222 \pm \begin{matrix} \infty \\ - 86 \end{matrix}$	∞

TABLE 1

Table 1 shows the range allowed for $\sin^2\theta_W$ and for the neutral boson masses from the fits to neutrino data performed in Ref.[5], respectively for FMMS, DGG± models and for the standard Salam-Weinberg model. In the first three models, the mass of the first boson is expected to be slightly smaller than in the standard model. The mass of the heavier boson, however, is not likely in the energy range of LEP 70 if FMMS and DGG+ models are discarded on the basis of SLAC results [2], but the present low energy data are still compatible with a mass of about 200 GeV resulting in observable effects at 70 GeV, especially in charge asymmetry.

In all models, the neutral boson widths have been calculated on the basis of 3 quark and 3 lepton doublets [9]. In the DGG± models, additional parameters have been fixed as explained in Appendix 1.

Figures 1 to 4 compare the FMMS model to the standard model for event rates and charge asymmetry. The effect of the heavier boson in charge asymmetry is the presence of a "dip" whose width strongly depends

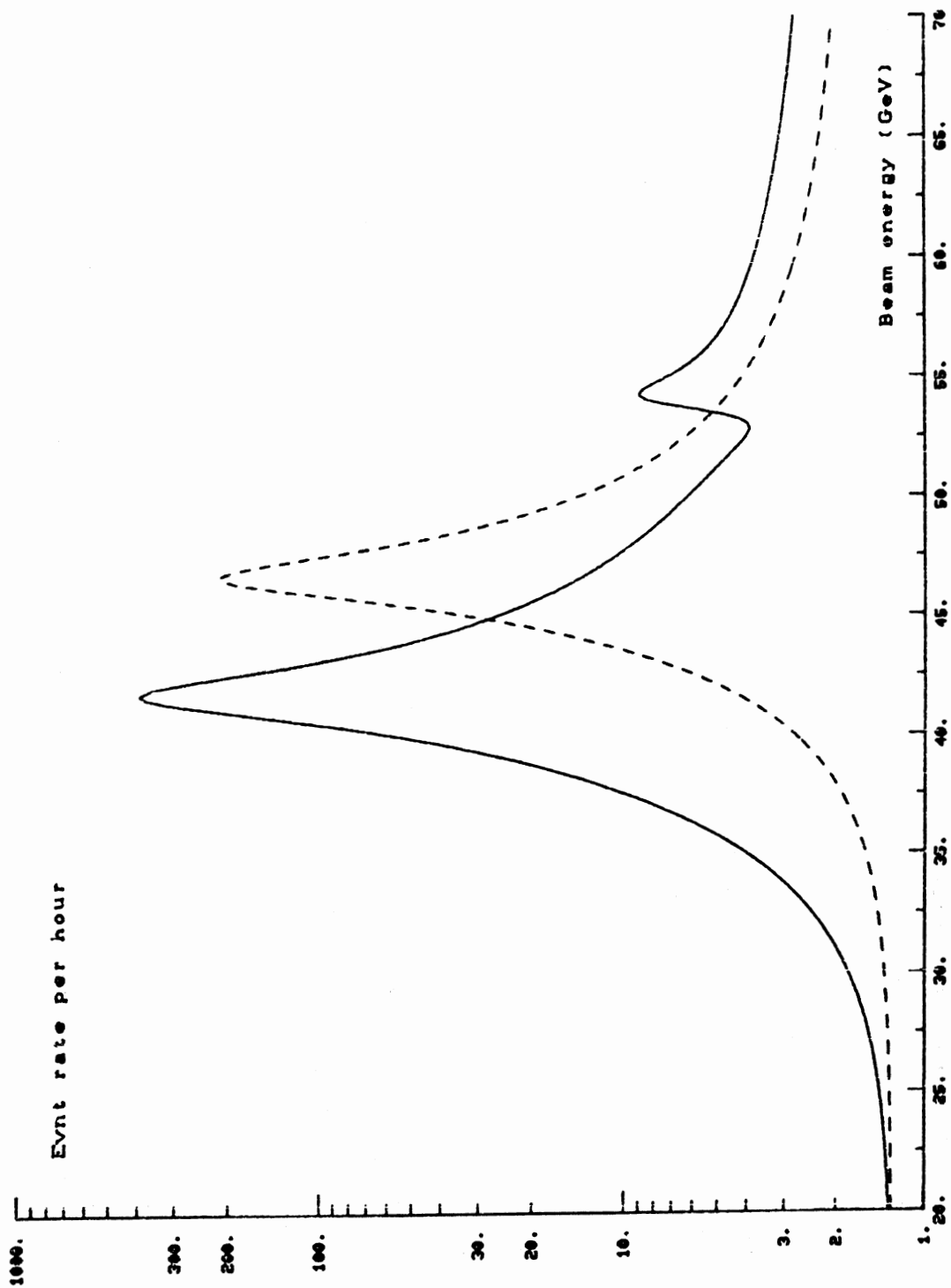


Figure 1 : Event rate per hour versus beam energy ($\sin^2\theta_w = 0.20$)

a) FMMS model (————)

b) Standard model (-----)

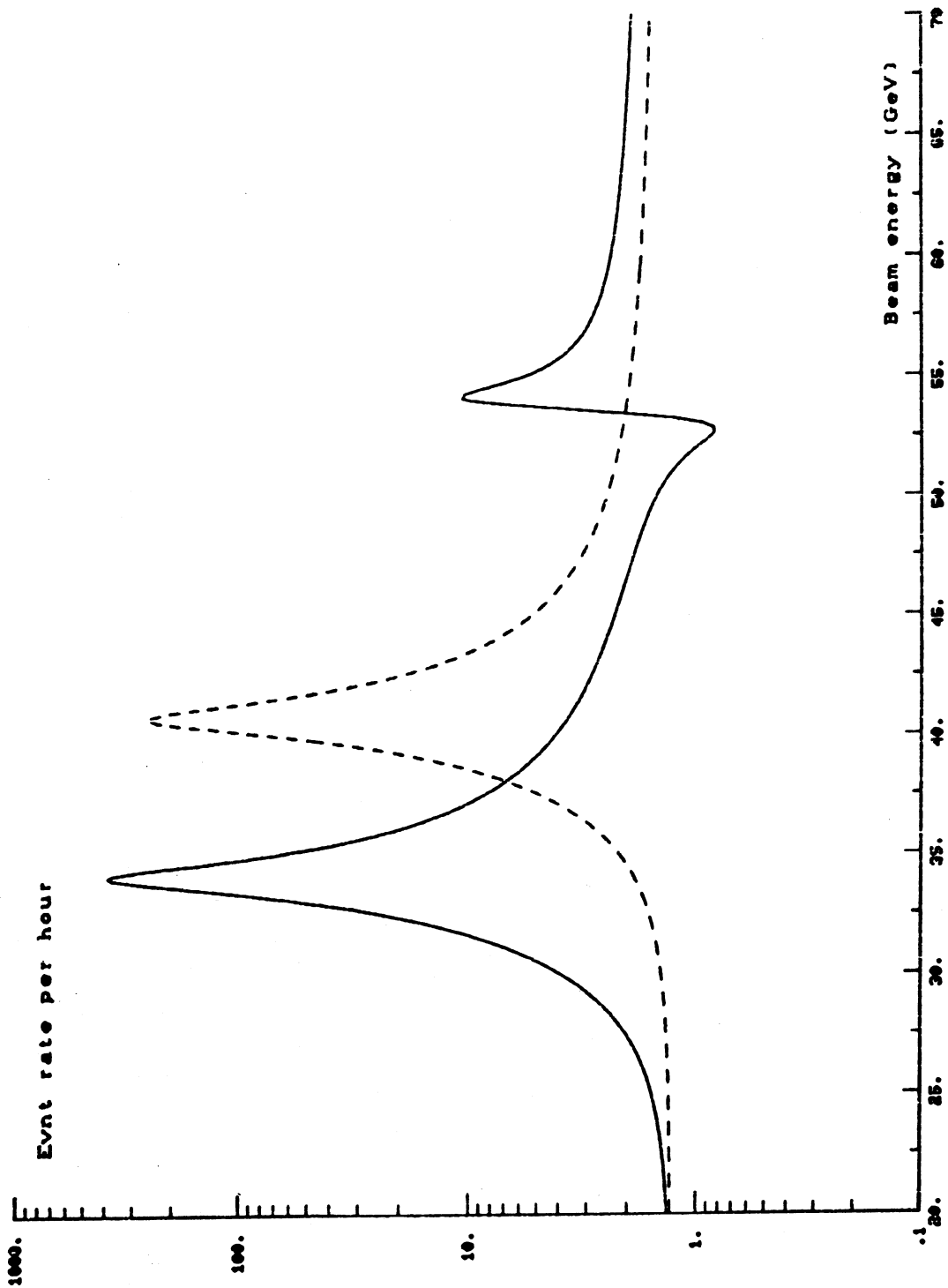


Figure 2 : Event rate per hour versus beam energy ($\sin^2\theta_w = 0.30$)

a) FMMS model (————)

b) Standard Model (-----)

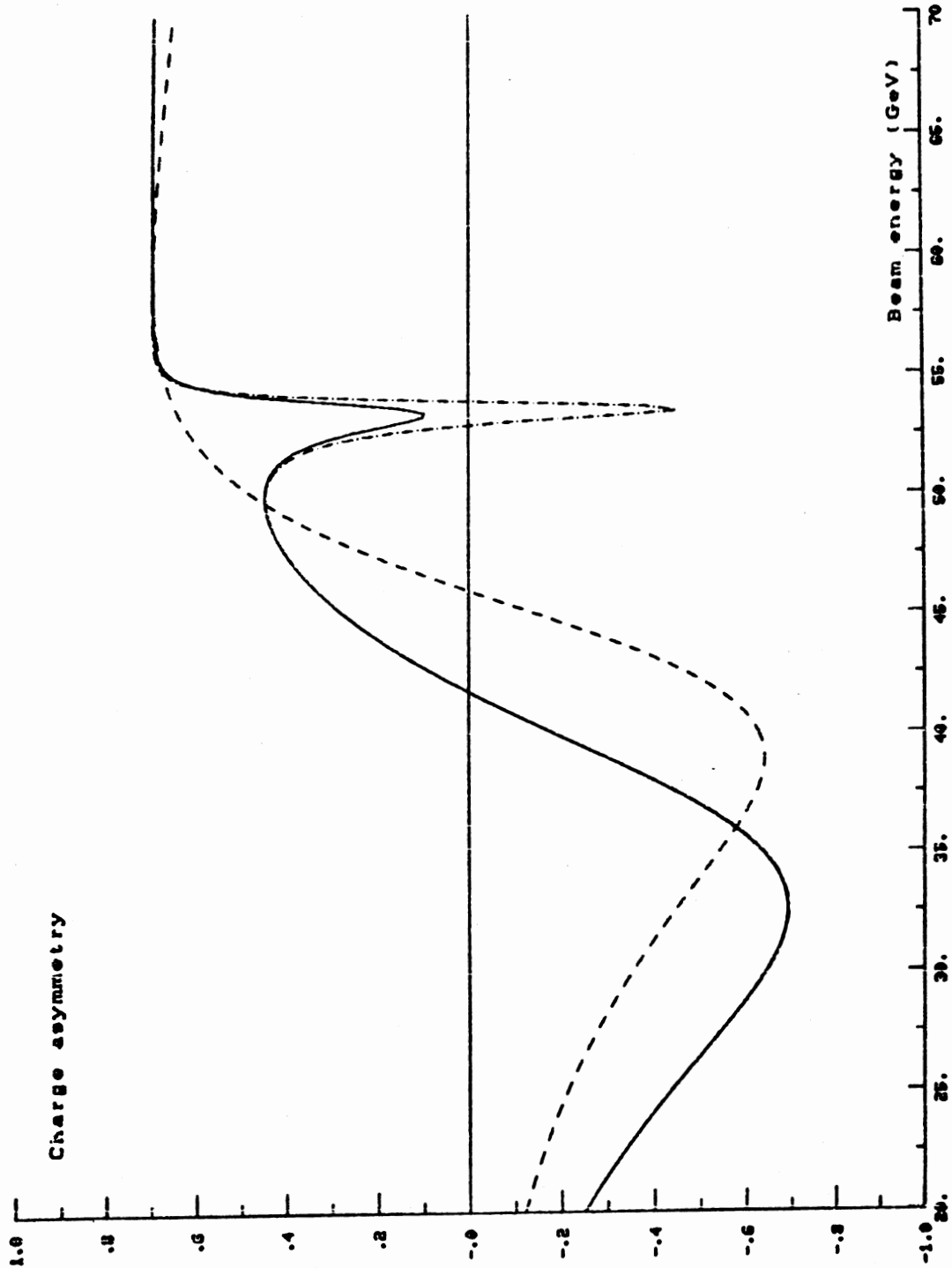


Figure 3 : Charge asymmetry versus beam energy ($\sin^2\theta_w = 0.20$)

- a) FMMS model with boson widths calculated
($M(Z_2) = 107.8 \text{ GeV}/c^2$, $\Gamma(Z_2) = 2.7 \text{ GeV}/c^2$) (————)
- b) FMMS model with $\Gamma(Z_2)$ fixed at $1 \text{ GeV}/c^2$ (-.-.-.-.-)
- c) Standard model (-----)

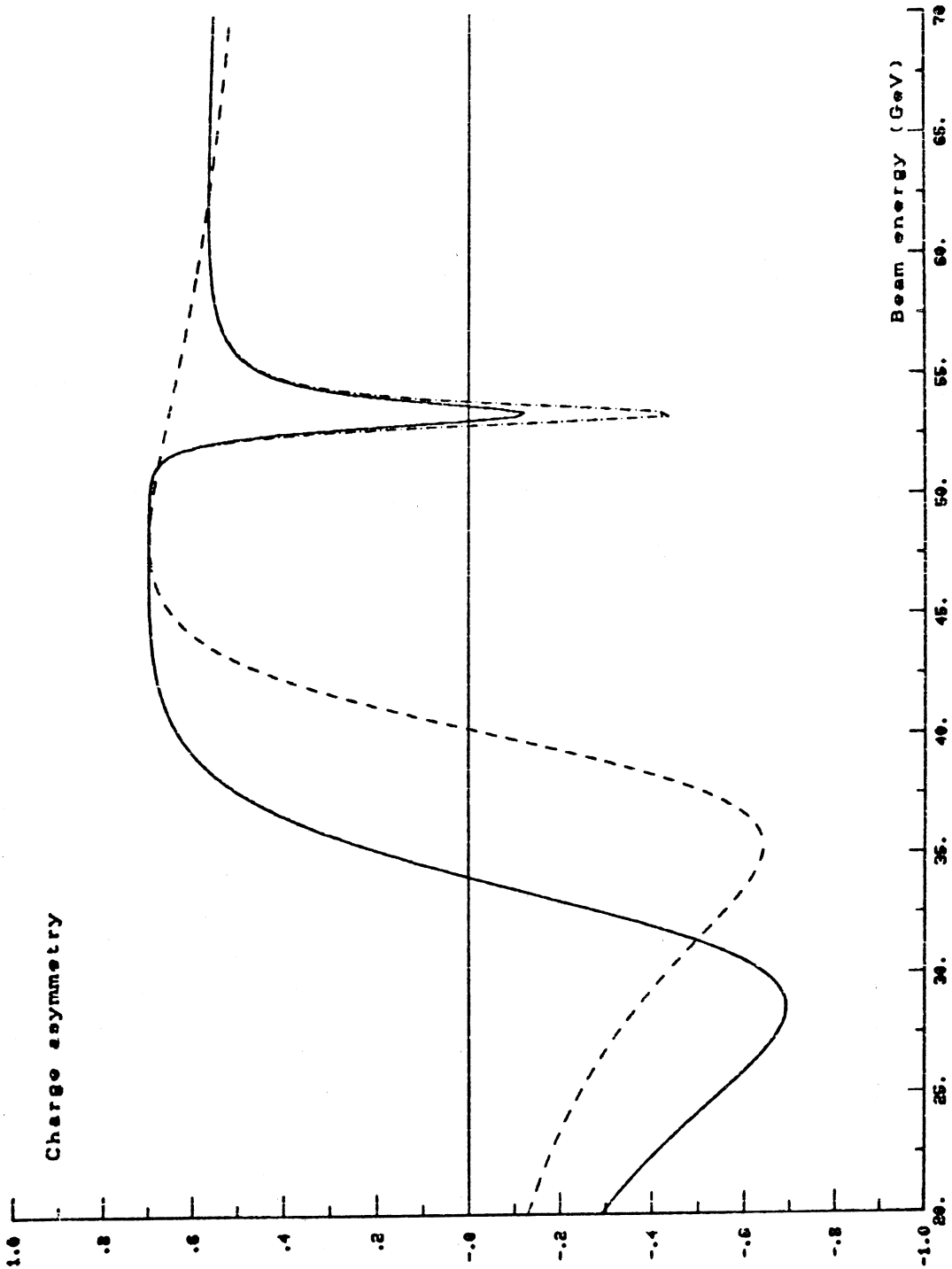


Figure 4 : Charge asymmetry versus beam energy ($\sin^2\theta_w = 0.30$)

- a) FMMS model with boson width calculated
($M(Z_2) = 107.8 \text{ GeV}/c^2$, $\Gamma(Z_2) = 1.8 \text{ GeV}/c^2$) (————)
- b) FMMS model with $\Gamma(Z_2)$ fixed at $1 \text{ GeV}/c^2$ (-.-.-.-.-)
- c) Standard model (-----)

on the boson width. Of course, the FMMS model predicts no final lepton polarization, since parity is conserved ; ($g_{V1} = 0$ and $g_{A2} = 0$) ; (see formula (5)).

Figures 5 to 7 compare DGG± models to the standard model for event rates, charge asymmetry and lepton polarization. In DGG± models the dip effect in charge asymmetry is not concentrated at the heavy boson mass as in the FMMS model ; significant effects can be seen below the resonance energy; they are not sensitive to the width of the heavier boson. As far as polarization is concerned, the predictions depend on the deviation from $\frac{1}{4}$ of $\sin^2 \theta_W$; (for $\sin^2 \theta_W = \frac{1}{4}$, all the preceding models predict no effect). Here also, significant effects of the heavier boson can be seen below the resonance energy.

The preceding features of the DGG- solution allow for significant deviations from the standard model at a beam energy of 70 GeV, even if the heavier boson is out of the range of LEP 70.

As far as the cross-section is concerned, the main effect is the shift of the lighter boson peak relatively to the prediction of the standard model with $\sin^2 \theta_W = 0.24 \pm 0.02$ namely $M(Z_1) = 87 \pm 3 \text{ GeV}/c^2$. However, $\sin^2 \theta_W$ being essentially known from $\nu + \text{nucleon}$ reactions, its present value may be affected by QCD effects so that the discovery of the Z_1 peak at e.g. $M(Z_1) = 79 \text{ GeV}/c^2$ would not be a decisive argument against the Salam-Weinberg theory. However, once the Z_1 mass is known, it is possible to compare the standard model and a two-boson-model, both accounting for the same measured mass. As an example, we assume $M(Z_1) = 79 \text{ GeV}/c^2$ and compare the DGG- models ^(*) and the standard model for effects in the charge asymmetry A. The variation of A with the beam energy is shown in figure 8, both for $M(Z_2) = 214 \text{ GeV}/c^2$ and for an infinite value of $M(Z_2)$ (standard model). On the basis of the statistical error only ($\Delta A = \sqrt{\frac{1-A^2}{N}}$, N being the number of observed events at 70 GeV), the number of hours of a 70 GeV run necessa-

(*) The DGG models have 3 parameters, namely $\sin^2 \theta_W$ and the angles α and β defined in Ref.[10]. Here, β has been fixed to 0 in order to agree with SLAC data on polarized electron scattering. Only α and θ_W are varied in order to keep $M(Z_1)$ equal to $79 \text{ GeV}/c^2$.

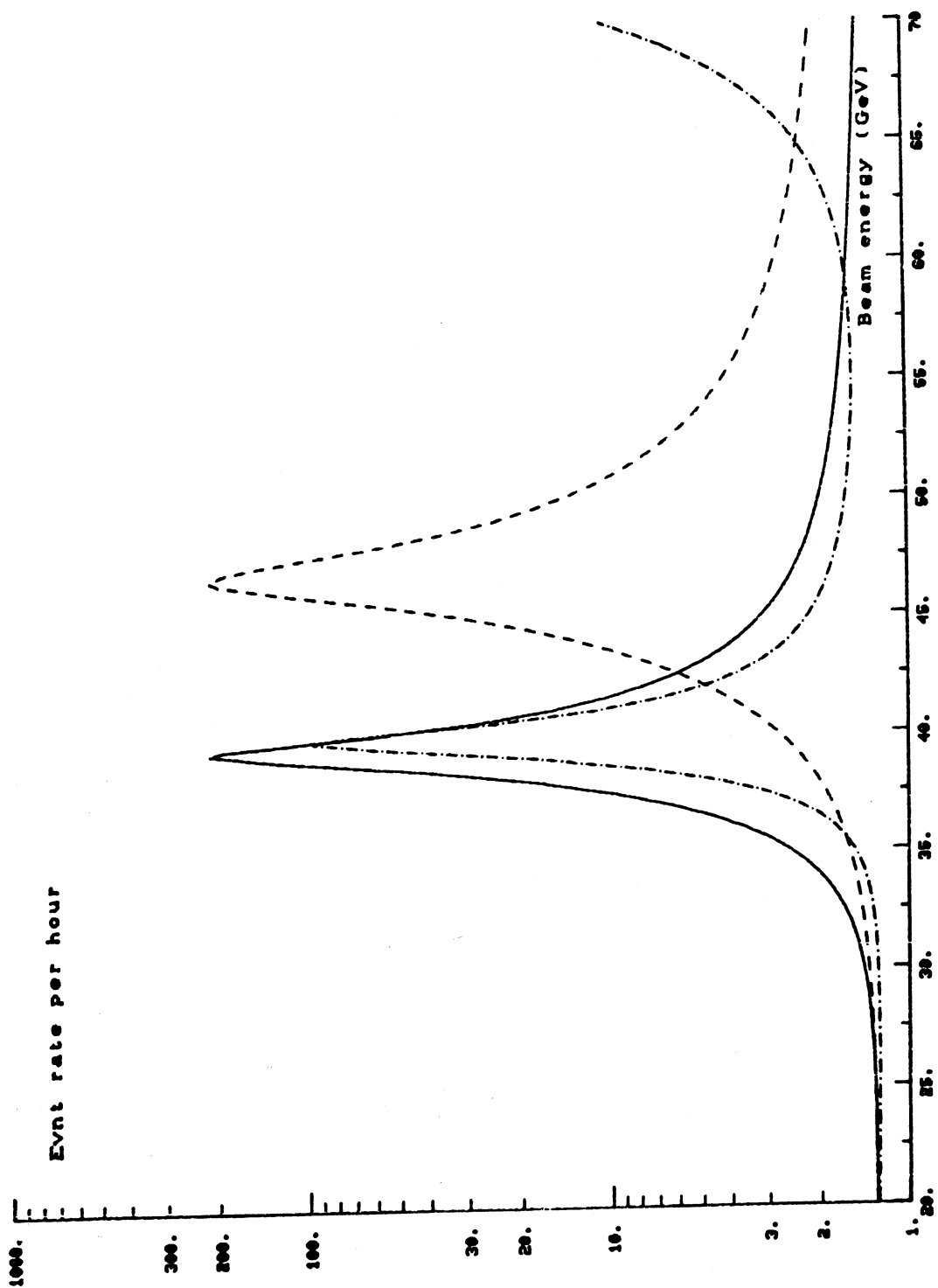


Figure 5 : Event rate per hour versus beam energy

- a) DGG - model (————)
- b) DGG + model (-.-.-.-)
- c) Standard model ($\sin^2\theta_w = 0.20$) (-.-.-.-)

For the parameters of DGG \pm models, see appendix 1.

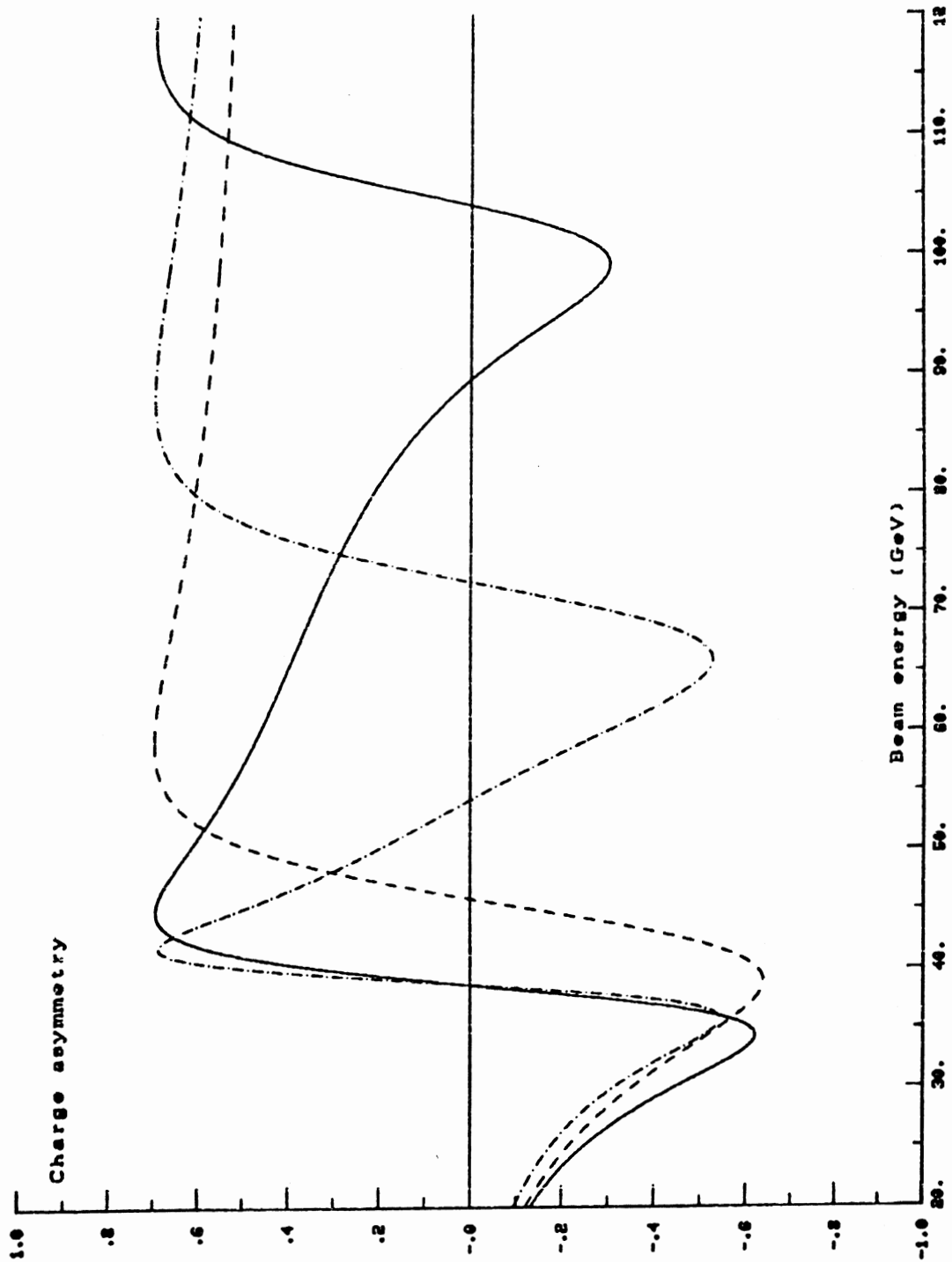


Figure 6 : Charge asymmetry versus beam energy

a) DGG - model (————)

b) DGG + model (-.-.-.-)

c) Standard model ($\sin^2\theta_w = 0.20$) (-.-.-.-)

For the parameters of DGG \pm models, see appendix 1.

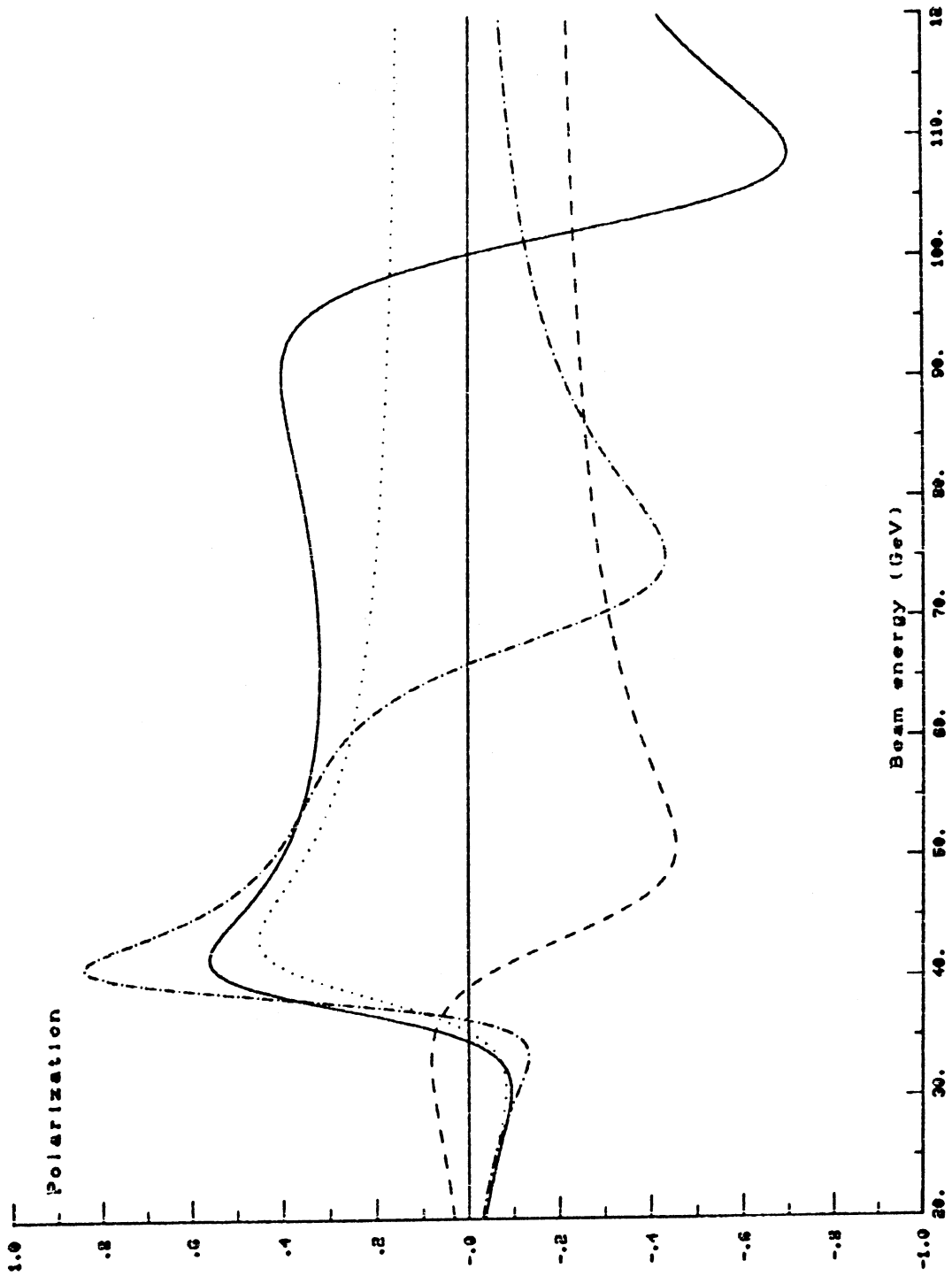


Figure 7 : Longitudinal polarization of the final lepton versus beam energy

- a) DGG - model (————)
- b) DGG + model (-.-.-.-)
- c) Standard model ($\sin^2\theta = 0.20$) (-----)
- d) Standard model ($\sin^2\theta = 0.30$) (.....)

For the parameters of DGG - models, see appendix 1.

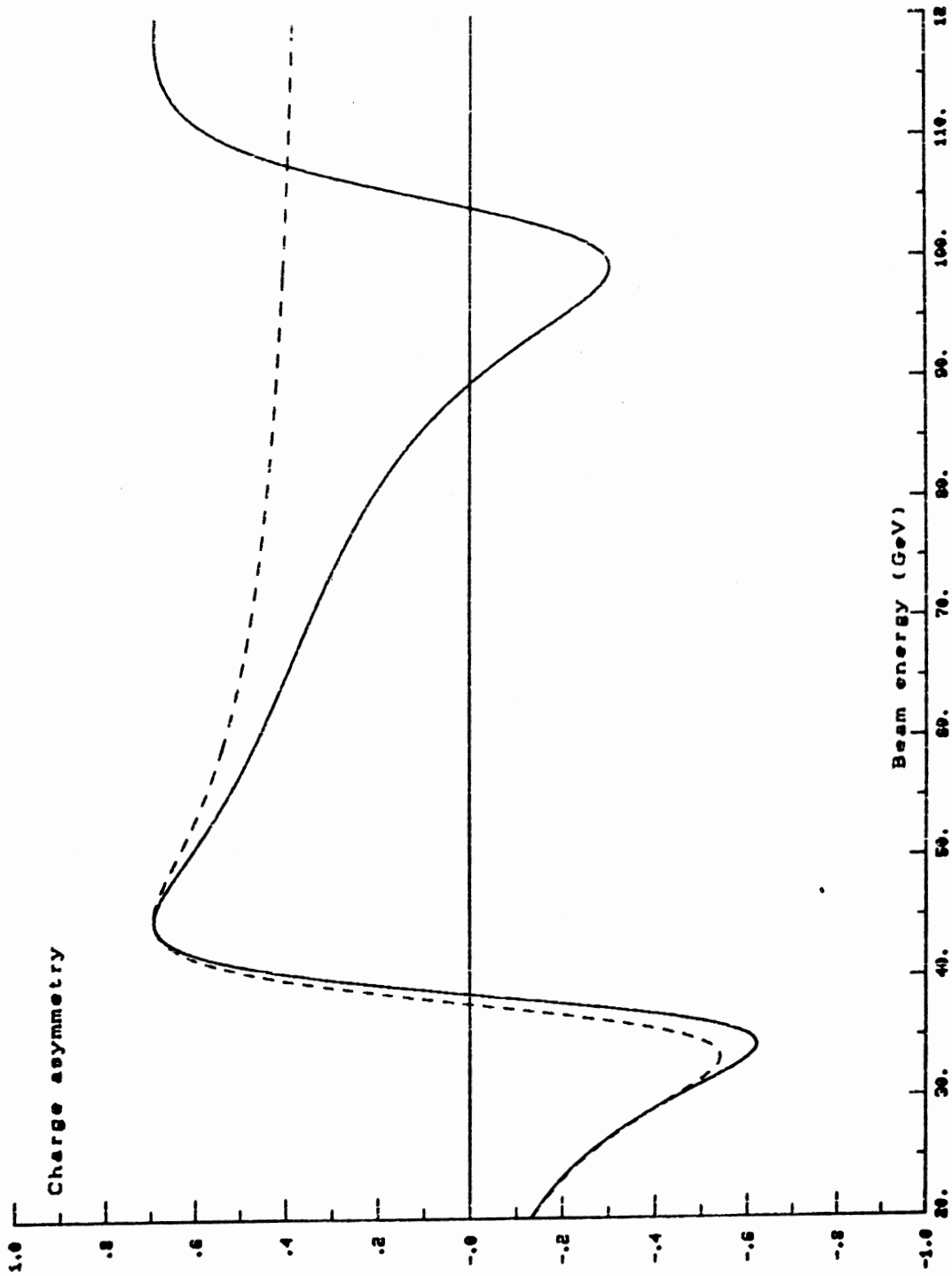


Figure 8 : Charge asymmetry versus beam energy, for $M(Z_1) = 79 \text{ GeV}/c^2$

a) DGG - model with $M(Z_2) = 214 \text{ GeV}/c^2$ (————)

b) Standard model (-----)

ry to obtain a 2 s.d. discrepancy with the standard model is plotted in figure 9 as a function of $M(Z_2)$. Assuming that the LEP machine can be operated at 100 GeV with a luminosity $4 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$, the number of hours of a 100 GeV run necessary to observe the same deviation is also indicated in figure 9. Similarly, the 90% confidence intervals for A obtained at 70 GeV are shown in figure 10 as functions of $M(Z_2)$ both for a 200 h run and for a 1000 h run.

It can be concluded that indirect effects of a second boson can be detected by LEP 70, only if $M(Z_2) \lesssim 260 \text{ GeV}/c^2$.

CONCLUSION.

The present data on $\nu + \text{nucleon}$ and $e^- + \text{nucleon}$ scattering (found to be in good agreement with the Salam-Weinberg model) impose severe constraints to $(SU_2)_L \otimes (SU_2)_R \otimes U_1$ models, so that no deviation from the Salam-Weinberg model is predicted in $e^+e^- \rightarrow \mu^+\mu^-$ reactions for beam energies lower than about 50 GeV. However, if there exists a second neutral boson Z_2 , its effects in charge asymmetry can be detected by LEP 70 provided its mass is lower than $\sim 260 \text{ GeV}/c^2$.

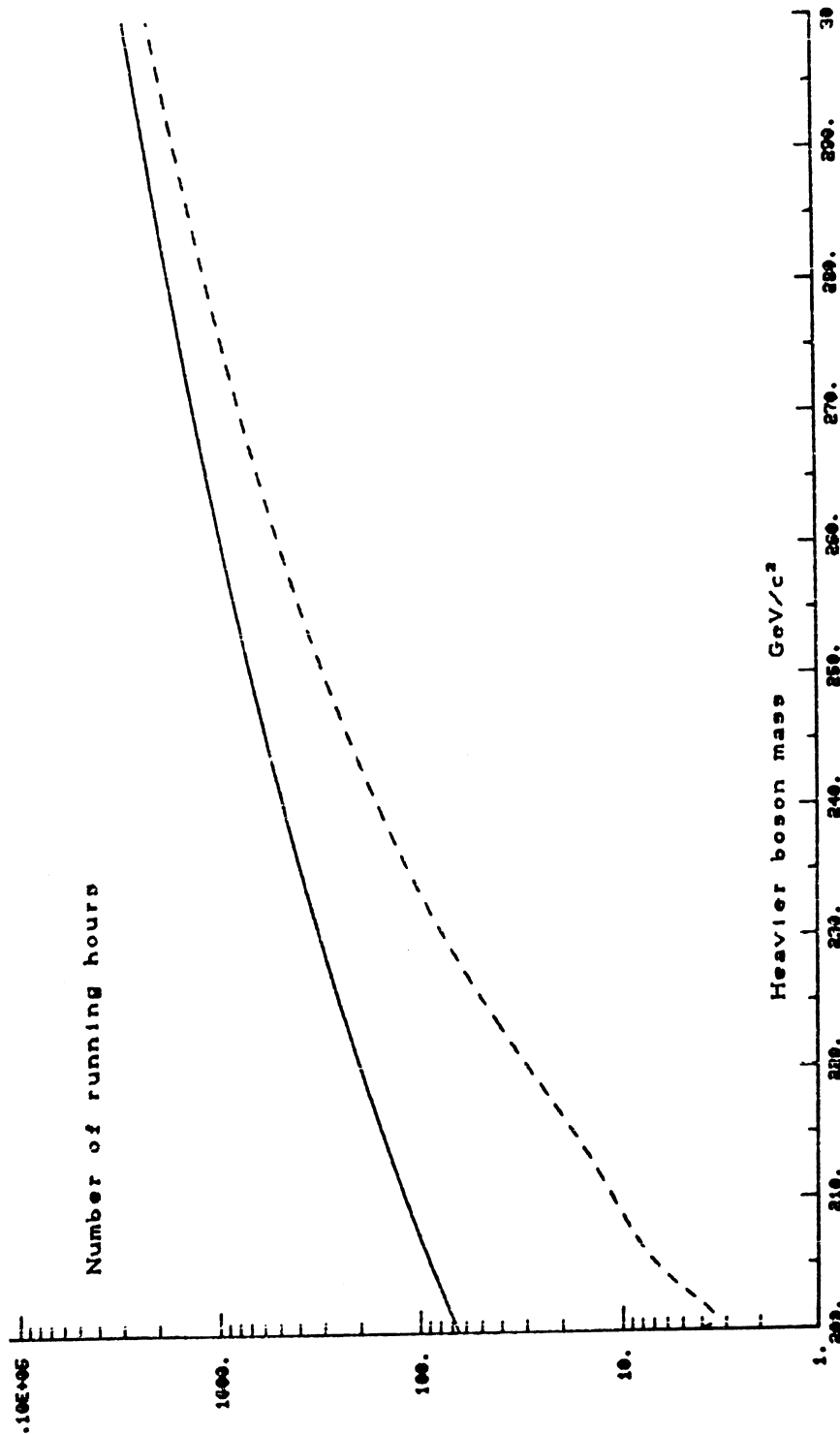


Figure 9 : Number of hours of run necessary to obtain a 2 s.d. discrepancy with the standard model, versus the mass of the heavier boson (DGG - model with $M(Z_1) = 79 \text{ GeV}/c^2$)
a) Run at 70 GeV with a luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ (—————)
b) Run at 100 GeV with a luminosity of $0.4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ (-----)

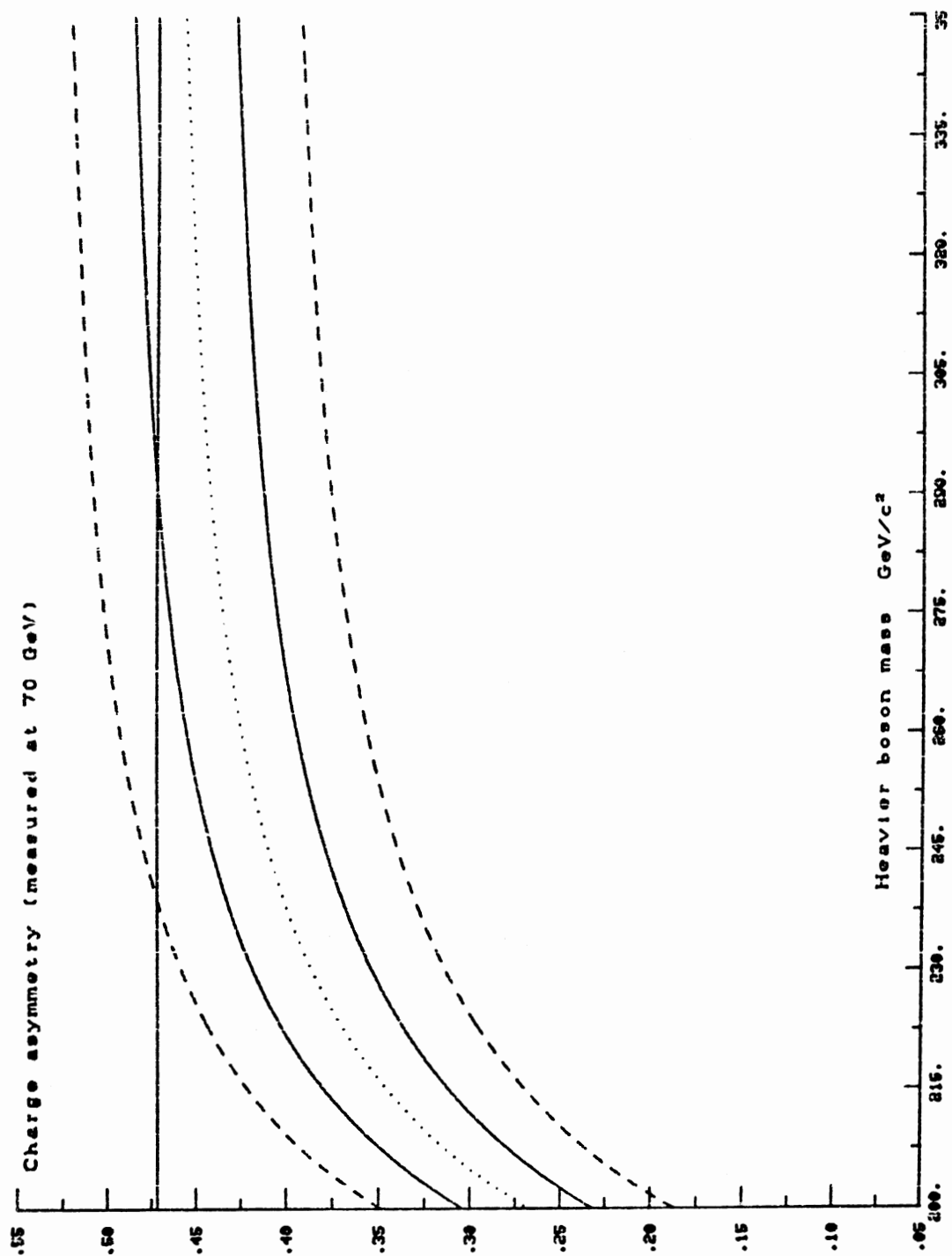


Figure 10 : 90% confidence interval in charge asymmetry, versus the mass of the heavier boson.

(DGG - model with $M(Z_1) = 79 \text{ GeV}/c^2$)

a) 1000 h run at 70 GeV (————)

b) 200 h run at 70 GeV (-----)

The dotted line corresponds to the prediction of the DGG-model.

APPENDIX :

- a) FMMS model : Apart from $\sin^2\theta_W$, this model uses a parameter ϵ defined in Ref.[8]. This parameter (which has no effect on parity conservation in the model) has been taken equal to 0.
- b) DGG model : Apart from $\sin^2\theta_W$, this model uses two angles α and β defined in Ref. [10]. The curves shown in figures 5 to 7 have been calculated using $\sin^2\theta_W=0.30$ and
 $\sin^2\alpha = 0.16$, $\sin^2\beta = 0$ for solution DGG- ;
 $\sin^2\alpha = 0.40$, $\sin^2\beta = 0.23$ for solution DGG+

Those values are close to the ones fitted in Ref. [5], and thus agree with neutrino data.

The curves shown in figures 9 and 10 have been calculated with $\beta = 0$, and by varying α and θ_W simultaneously in order to keep $M(Z_1) = 79 \text{ GeV}/c^2$. The following table shows their corresponding variations together with those of $M(Z_2)$, of event rates per hour and of charge asymmetry at a beam energy of 70 GeV.

$\sin^2\alpha$	$\sin^2\theta_W$	$M(Z_2)$ GeV/c^2	Rate/h	Charge Asymmetry
0.16	0.300	214	1.44	0.34
0.12	0.315	251	1.53	0.42
0.08	0.326	313	1.60	0.45
0.04	0.336	451	1.66	0.46
0.00	0.343	∞	1.72	0.47

(standard model)

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