ECFA/LEP 29
10 August, 1978

USE OF τ HADRONIC DECAY MODES TO STUDY POLARIZATION NEAR THE Z° POLE AT LEP

J.E. Augustin
Laboratoire de l'Accélérateur Linéaire, Orsay, France.

I. INTRODUCTION

The aim of this note is to assess the possibilities offered by the two-body or quasi-two body semi leptonic decay modes of the τ in order to measure the polarization of the τ produced in e^+e^- annihilation in the vicinity of the Z° pole.

These τ decay modes have been studied by Tsai[1] and also by Thacker and Sakurai[2]. Recent estimates[7] are presented in table 1.

Some results on $\tau^+\tau^-$ production are recalled in section II.

In section III, the π-ν decay mode is shown to have very visible effects both on the pion energy spectrum and on the pion angular distribution. Finally in section IV we give some remarks on the ρ-ν decay mode.

II. $e^+e^- \to \tau^+\tau^-$ (3,4,5)

With the following assumptions :

1. One neglects effects of the τ mass, which are at most of order $\frac{2m_\tau}{\sqrt{S}}$ where $\sqrt{S} = 2E$ is the center of mass energy in the e^+e^- collision.

2. One sums over the spin orientations of one of the final τ particles, let's say τ^+ spins. Then the cross section depends only upon the scattering

TABLE 1. (from ref. 7)

τ Branching ratios in %

Decay mode	B
$\nu_\tau + \nu_e + e^-$	16.41
$\nu_\tau + \nu_\mu + \mu^-$	15.97
$\nu_\tau + \pi^-$	9.8
$\nu_\tau + K^-$	0.62
$\nu_\tau + \rho^-$	23.01
$\nu_\tau + K^{*-}$	1.57
$\nu_\tau + A_1^-$	9.34
$\nu_\tau + Q^-$	0.41
$\nu_\tau + u\bar{d} > 1.1$ GeV	21.27
$\nu_\tau + \bar{u}S > 1.1$ GeV	1.54

Life time : $2{,}53.10^{-13}$ sec.

angle θ and the τ^- helicity h_-

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8S} \left(F_1 \, (1 + \cos^2\theta) + 2 \cos\theta \, F_3 + h_- \, (1 + \cos\theta)^2 \, F_4\right)$$

with : $F_1 = 1 + 2 v^2 \, \mathrm{Re}\,\chi + (a^2 + v^2)^2 \, |\chi|^2$: $\sigma_{TOT} = F_1 \, \sigma_{point}$.

$$F_3 = 2a^2 \, \mathrm{Re}\,\chi + 4a^2v^2 \, |\chi|^2$$

$$F_4 = 2 \, av \, (\mathrm{Re}\,\chi + |\chi|^2 \, (a^2 + v^2))$$

with : $\mathrm{Re}\,\chi = gm^2 \dfrac{S(S-m^2)}{(S-m^2)^2 + \Gamma^2 m^2}$ $\quad m = m_{z°} \quad \Gamma = \Gamma_{z°}$

$$|\chi|^2 = g^2 m^4 \frac{S^2}{(S-m^2)^2 + \Gamma^2 m^2}$$

In the Weinberg-Salam model :

$$gm^2 = \frac{1}{16 \sin^2\theta_w \cos^2\theta_w} \; ; \quad m = m_{z°} = \frac{37.3}{\sin\theta_w \cos\theta_w} \; (\mathrm{GeV})$$

$$a = 1$$

$$v = 4 \sin^2\theta_w - 1$$

$$\Gamma = \Gamma_{z°} = m.\alpha.gm^2 \, 16 \, (1 - 2 \sin^2\theta_w + \tfrac{8}{3} \sin^4\theta_w)$$

(if only six quarks and three lepton doublets are present).

For example : $\Gamma = 3$ GeV at $\sin^2\theta_w = .2$

The mean polarizations are : $\langle P_- \rangle = \dfrac{F_4}{F_1}$, $\langle P_+ \rangle = -\dfrac{F_4}{F_1}$.

At the Z_o peak : $\dfrac{F_4}{F_1} \simeq \dfrac{2av}{a^2 + v^2}$

III. DECAY $\tau^- \to \pi^-\nu$

The angular distribution for this decay with a helicity h_- for the τ is given by Tsai[1] :

$$dw^{\pm} = B_{\pi} \, (1 \mp h_{\pm} \cos\theta^*) \, d\cos\theta^*$$

One simple way to use this result is to look at the total pion spectrum in the final state.

$$\text{Then :} \quad E_{\pi} = \frac{E}{2}(1+\varepsilon) + \frac{E}{2}(1-\varepsilon)\beta_{\tau} \cos\theta^* \approx \frac{E}{2}(1+\cos\theta^*) \quad \left(\varepsilon = \frac{m_{\pi}^2}{m_{\tau}^2}\right)$$

$$\text{So that :} \quad \frac{dN}{dE_{\pi}^-} \propto 1 + \langle P_- \rangle \left(\frac{E_{\pi} - E/2}{E/2}\right) = 1 + \frac{F4}{F1}\left(\frac{E_{\pi} - E/2}{E/2}\right)$$

$$\text{and :} \quad \frac{dN}{dE_{\pi}^+} \propto 1 - \langle P_+ \rangle \left(\frac{E_{\pi} - E/2}{E/2}\right) = 1 + \frac{F4}{F1}\left(\frac{E_{\pi} - E/2}{E/2}\right)$$

The slope of the pion spectrum, irrespective of the charge, will give a measure of the sign and value of the v/a ratio of the Z_o (fig. 1).

More information can be gained by looking at the bi-dimensional distribution of pions in energy and angle. One finds (Appendix 1) that the forward-backward assymmetry of the π^- will be a varying function of the pion energy.

$$A(E_{\pi}) = \frac{3}{4} \cdot \frac{F3 + C(E_{\pi})\, F4}{F1 + C(E_{\pi})\, F4} \cos u$$

where $C(E_{\pi})$ varies from -1 to +1 when E_{π} goes from minimum to maximum, and cos u is equal to + 1 at minimum and maximum of E_{π}, ans is always near 1 at energies around the Z° peak.

The figure 2 shows the angular distribution of pions for $E_{\pi} < \frac{E}{2}$ and $E_{\pi} > \frac{E}{2}$. These asymmetries give an independent measurement of the τ helicity.

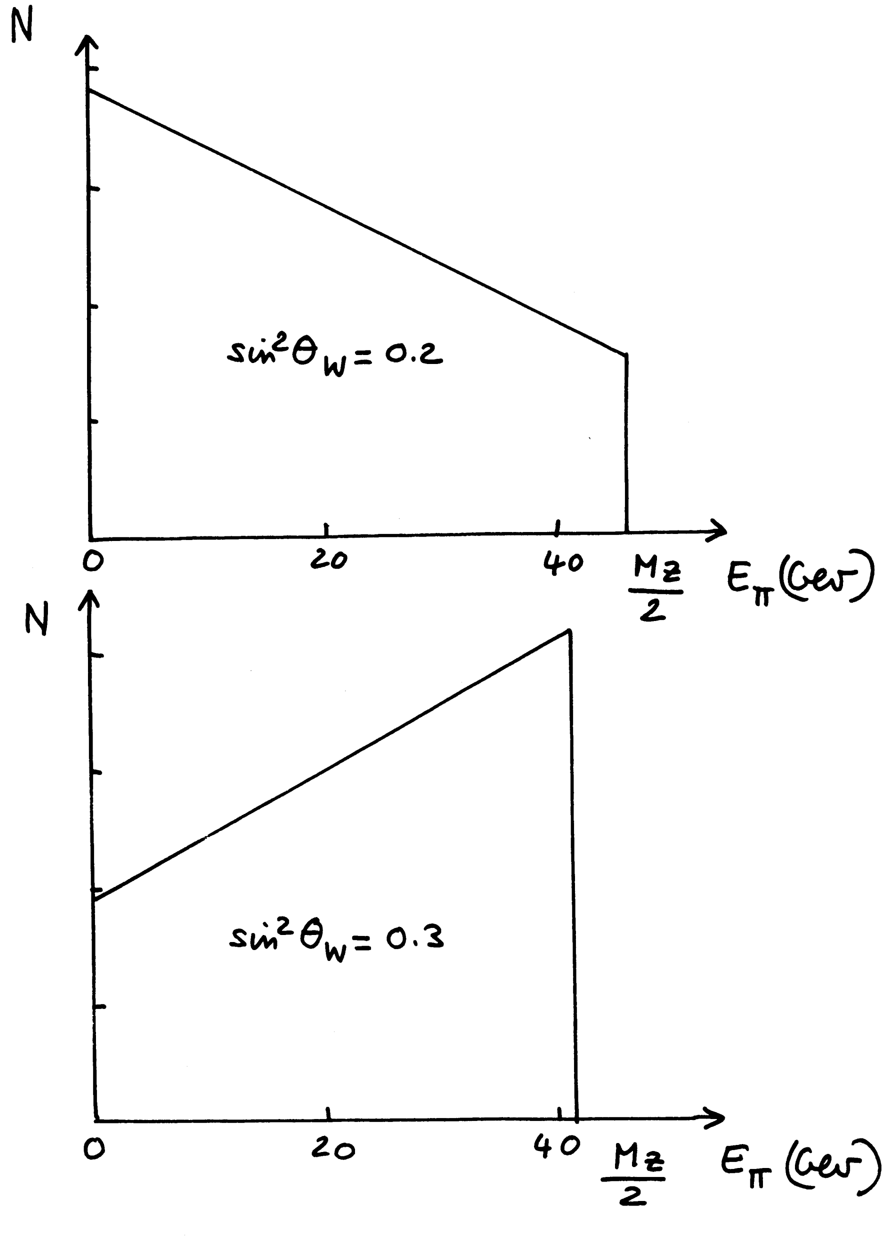

Figure 1 : Pion energy spectrum

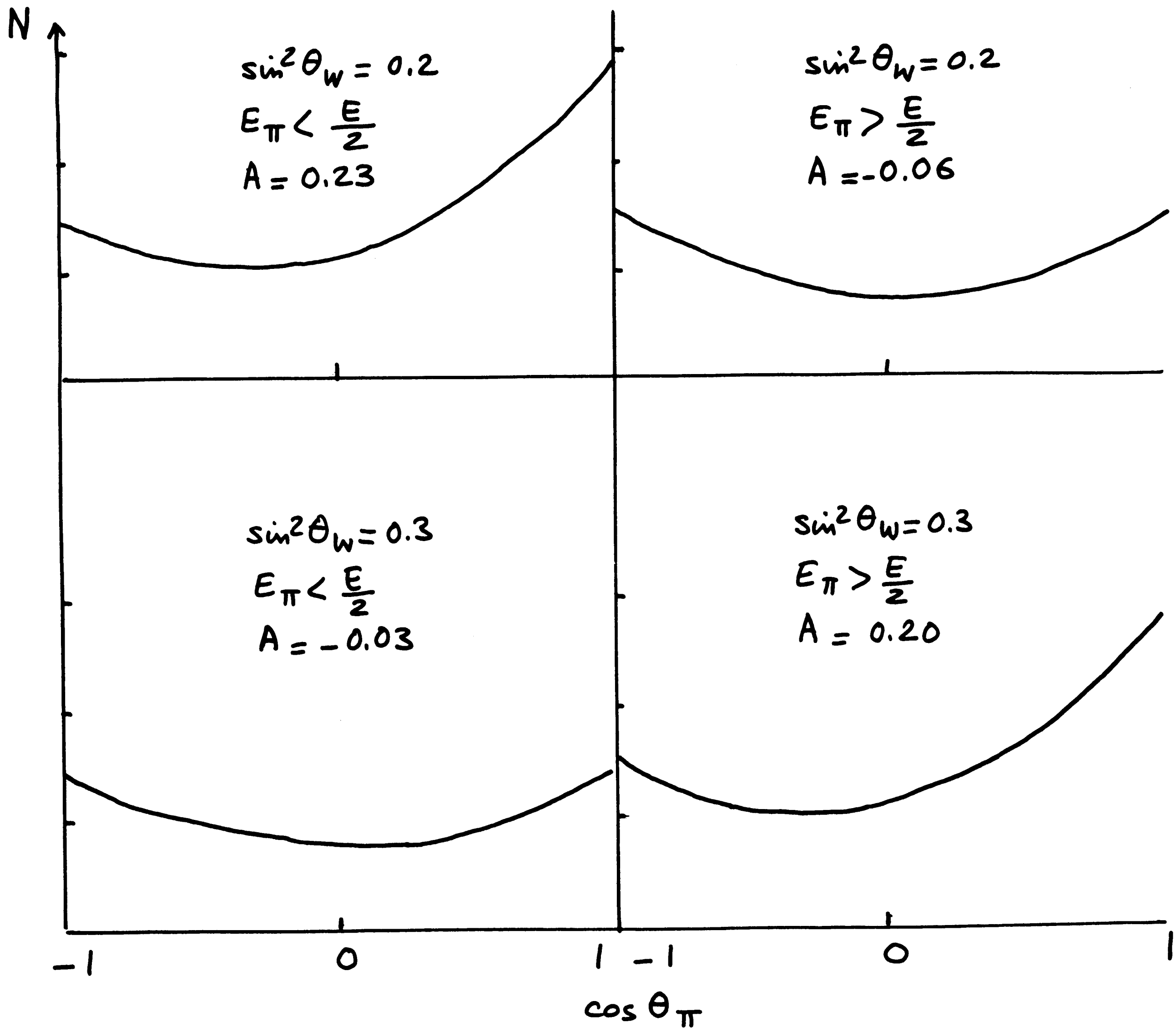

Figure 2 : Pion angular distribution

Another way to use these distributions could be to cut the two-dimensional plot E_π, $\cos\theta_\pi$ into four quadrants. As an example, for $\sin^2\theta_W = .2$, one gets the ratios :

E_π \ $\cos\theta_\pi$	−1 to 0	0 to +1
$\frac{E}{2}$ to E	22 %	19 %
0 to $\frac{E}{2}$	23 %	36 %

In terms of counting rates necessary for a meaningful measurement, the reaction $e^+e^- \rightarrow \tau^+\tau^-$ will be signed by a leptonic (e or μ) decay mode of one of the τ's, with a branching ratio of 32 %. The fraction of events with a π-ν decay of the other τ is 2 × 10 % × 32 % = 6.4 % of all produced τ-pairs.

In the Weinberg-Salam model, with the realistic Z° width of 3 GeV, the number of π-ν decays produced at the top of the Z° is ∿ 17/hour at a luminosity of $5.10^{32}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$.

In a hundred hours of running time, an absolute precision of 4 % is reached on the τ polarization, or a relative precision of 10 % for $\sin^2\theta_W = .2$.

More realistic experimental conditions are under study with the help of a Monte-Carlo simulator. Some results are shown on figs 5 and 6.

One sees that these $\tau \rightarrow \pi\nu$ measurements are one of the best candidates for measurement of the sign of the $\frac{v}{a}$ ratio of the Z_o.

More over, if it exists other heavy leptons with a mass greater than 4 GeV, and lower than $\frac{m_{Z^\circ}}{2}$, the rapid disappearance[7] of their π-ν mode would leave this τ decay mode a unique tool to study these polarization phenomena.

IV. THE $\tau \rightarrow \rho\nu$ DECAY MODE

The interest in this mode is its 23 % branching fraction. The angular distribution for this decay is complicated by the vector structure of the ρ. Two

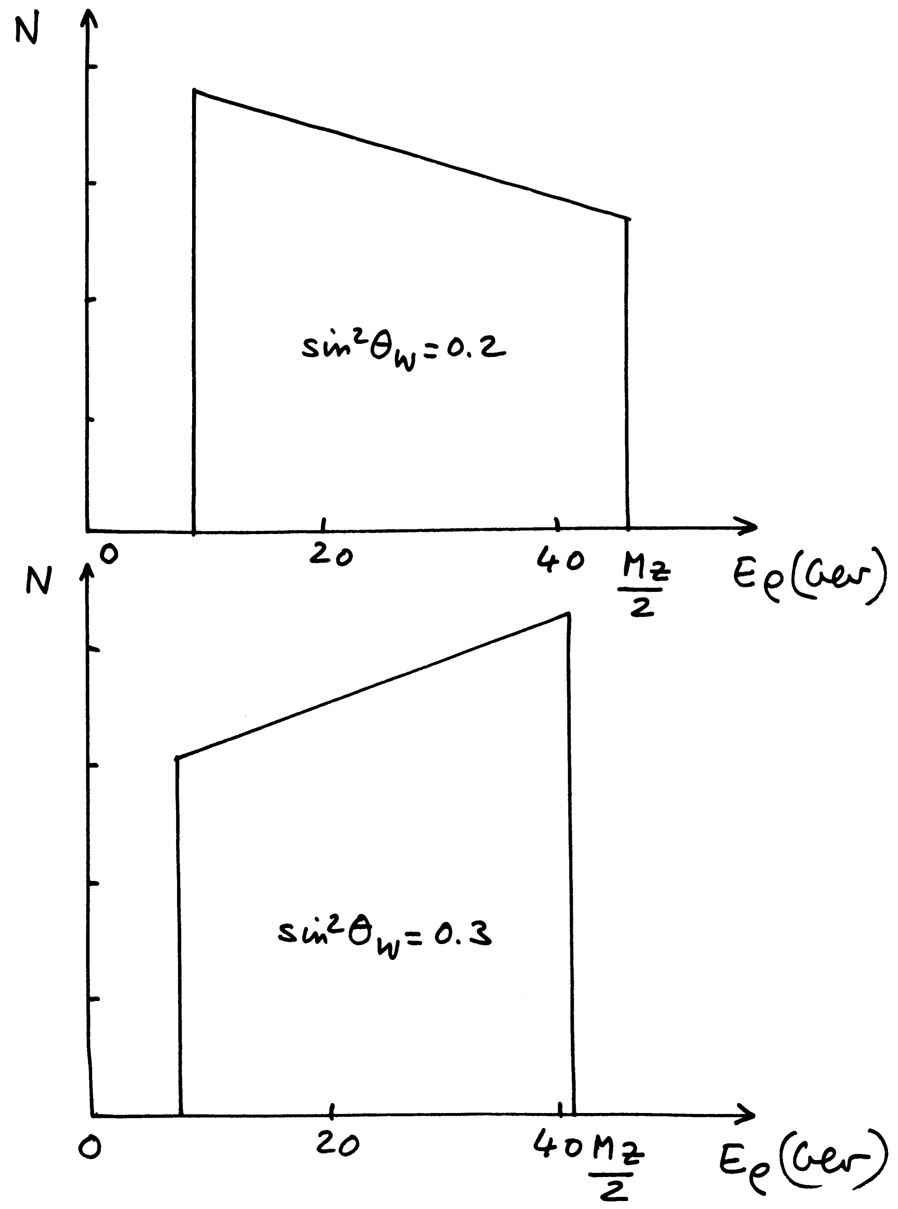

Figure 3 : Rho energy spectrum

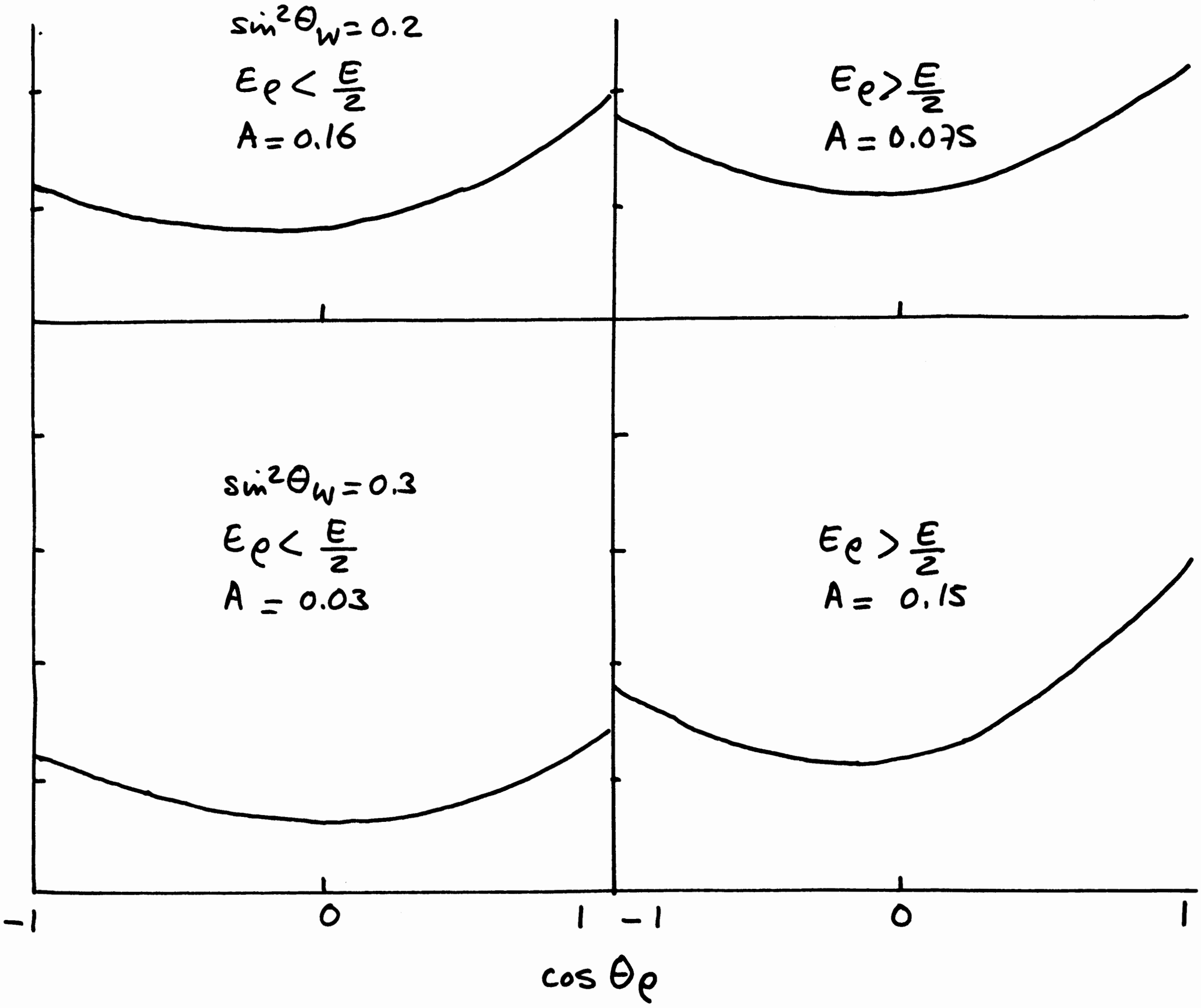

Figure 4 : Rho angular distribution

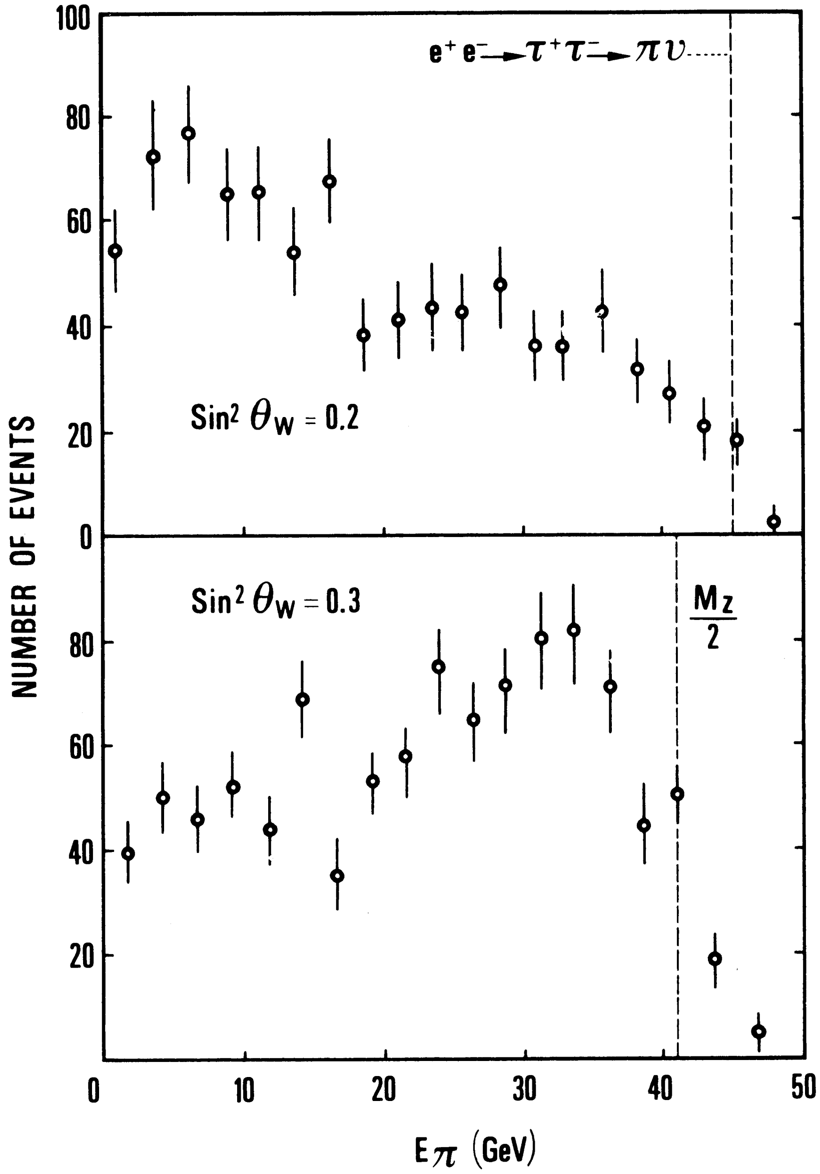

Figure 5 : Energy spectrum of pions from τ^+ or $\tau^- \rightarrow \pi\nu$ in the Weinberg-Salam theory for 2 values of $\sin^2\theta_W$. The pion energy is measured in a calorimeter with a resolution $\Delta E \sim .5\sqrt{E(\mathrm{GeV})}$

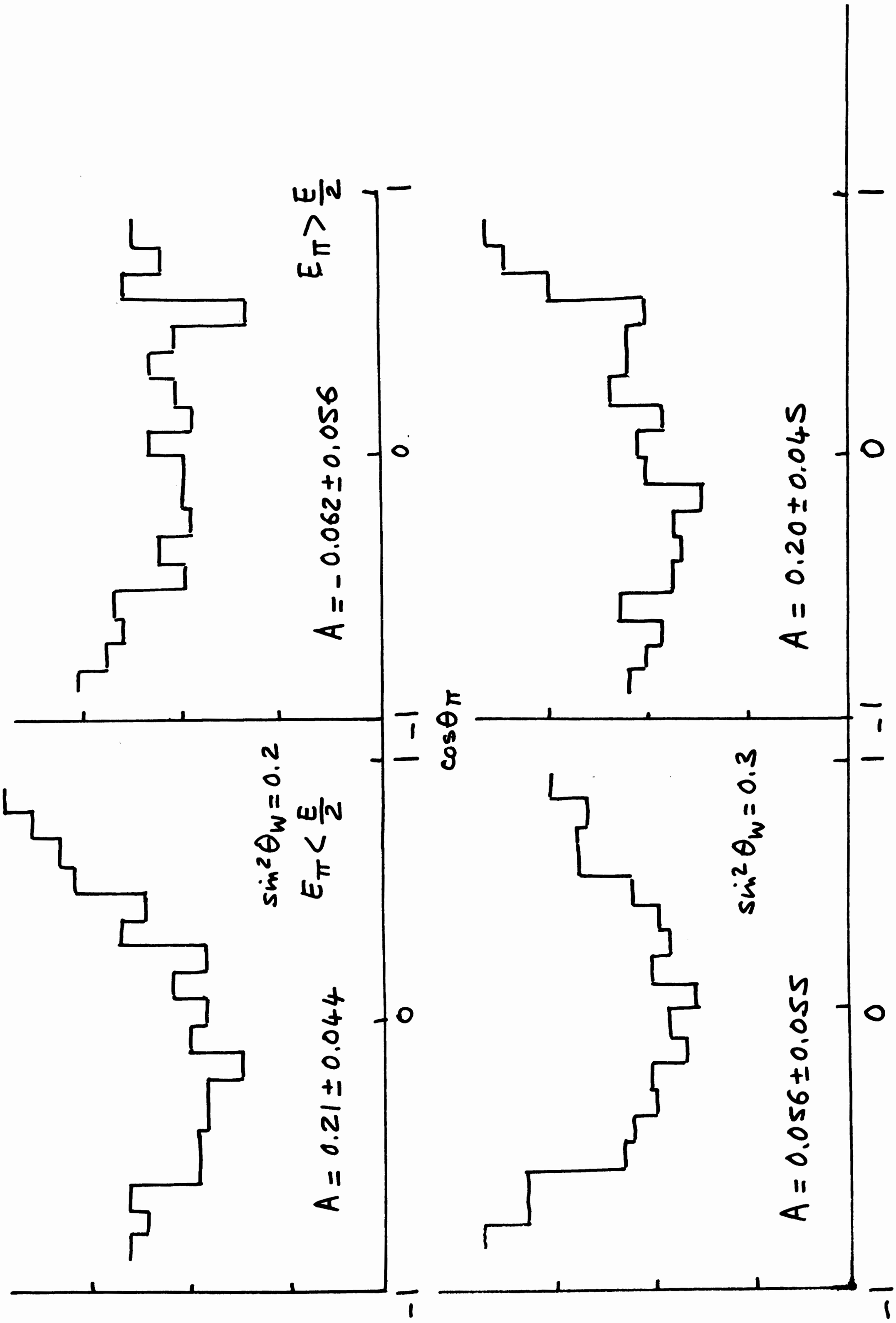

Figure 6 : Monte Carlo generation of the pion distribution for $|\cos\theta| < 0.9$

different amplitudes, with ρ's of helicity 0 and -1 contribute. The ρ angular distribution, following Tsai[1] result, is given in appendix 2. We have :

$$dW^{\pm} = B_{\rho} \, (1 \mp a \, h_{\pm} \cos\theta^{*}) \, d \cos\theta^{*}$$

$$\text{with} \quad a = \frac{m_{\tau}^2 - 2m_{\rho}^2}{m_{\tau}^2 + 2m_{\rho}^2} \simeq 0{,}46.$$

In terms of $\rho^{\pm}$ energy and angle distributions, (Figs 3 and 4) the effects are very similar to the $\pi^{\pm}$ ones, except that the polarization effects are decreased by the coefficient a. This loss of information will be partially recovered by the higher branching fraction of the ρ mode, but the number of produced τ's necessary to reach a given precision on the polarization using the ρ mode will be of order

$$\frac{B\pi}{B\rho} \, a^{-2} \simeq 2$$

times greater than using the π mode : it will be only a useful complement of information. For the A_1 mode, a ≃ .14 and this mode is useless. The π - ν mode thus remains the most important mode to measure τ polarization.

A useful conversation with A. COURAU is aknow ledged.

REFERENCES

1 - TSAI Y.S., Phys. Rev. D4, 2821 (1971)

2 - THACKER H.B. and SAKURAI J.J., Phys. Lett. 36B, 103 (1971)

3 - BUDNY R. and McDONALD A., Phys. Rev. D15, 81 (1977) and D16,3150 (1977)

4 - BUDNY R., Phys. Lett. 55B, 227 (1975)

FIELD J.H. and RICHTER B. in CERN-76-18 (1976)

5 - ELLIS J. and GAILLARD M.K. in CERN-76-18 (1976)

6 - KONIUK R., LEROUX R., ISGUR N., Phys. Rev. D17, 2915 (1978)

7 - TSAI Y.S., SLAC-PUB-2105 and Phys. Rev. Lett. (1978)

8 - TSAI Y.S., Erratum to Ref 1. in Phys. Rev. D13, 771 (1976)

APPENDIX 1

$$e^+e \rightarrow \tau^+\tau^- \quad (\tau^- \rightarrow \pi^-\nu)$$

The angular distribution is :

$$d\sigma = \frac{2\pi\ \alpha^2 B_\pi}{4\ S}\left[F_1\ (1+\cos^2\theta) + 2\ F_3\ \cos\theta + F_4\ (1+\cos\theta)^2\cos\theta^*\right] d\ \cos\theta\ d\ \cos\theta^*\ d\phi^*$$

By a change of variables to the pion energy E_π, the pion angle $\cos\theta_\pi$, and the azimuthal angle of the τ around the pion, and integrating over this last angle one gets[6]

$$\frac{d\sigma}{d\cos\theta_\pi\ dE_\pi} \propto G_1 + 2\ G_2\ \cos\ \theta_\pi + G_3\ \sin^2\ \theta_\pi$$

with : $G_1 = (F_1 + C(E_\pi)\ F_4)\ (1 + \cos^2 u)$

$G_2 = (F_3 + C(E_\pi)\ F_4)\ \cos\ u$

$G_3 = -\frac{1}{2}\ (F_1 + C(E_\pi)\ F_4)\ (3\ \cos^2 u - 1)$

where : $C(E_\pi) = \dfrac{2\ E_\pi - E\ (1+\varepsilon)}{P_\tau\ (1-\varepsilon)} \approx \dfrac{E_\pi - E/2}{E/2}$

$\cos\ u = \dfrac{2\ E\ E_\pi - m_\tau^2 - m_\pi^2}{2\ P_\tau\ P_\pi} \approx 1$

Integration over $\cos\theta_\pi$ yields the energy spectrum of § III.

The angular distribution yields a forward-backward assymmetry[6]

$$A(E_\pi) = \frac{\displaystyle\int_0^1 \frac{d\sigma\ d\ \cos\theta_\pi}{d\ E_\pi\ d\ \cos\theta_\pi} - \int_{-1}^0 \frac{d\sigma\ d\ \cos\theta_\pi}{d\ E_\pi\ d\ \cos\theta_\pi}}{\displaystyle\int_{-1}^0 \frac{d\sigma}{dE_\pi\ d\ \cos\theta_\pi}\ d\ \cos\ \theta_\pi} = \frac{6G_2}{6G_1 + 4G_3} = \frac{3(F_3 + C(E_\pi)\ F_4)\ \cos\ u}{4\ (F_1 + C(E_\pi)F_4)}$$

$A(E_\pi)$ depends upon the pion energy :

at $E_\pi = E_\pi\ \min$: $A_{min} = \frac{3}{4}\ (\frac{F3 - F4}{F1 - F4})$

at $E_\pi = E_\pi\ \max$: $A_{max} = \frac{3}{4}\ (\frac{F3 + F4}{F1 + F4})$

At the top of the Z_o : $A_{min} \simeq \frac{-2av}{a^2 + v^2}$, $A_{max} \simeq \frac{2av}{a^2 + v^2}$

$$\cos u = \frac{\vec{P}_\pi . \vec{P}_\tau}{|P_\pi| |P_\tau|}$$ is always positive if E > 11.3 GeV, and

at 2 E $\sim M_{Z^o}$, one gets cos u $\geqslant$ .83.

Such dependance of the assymmetry in the angular distribution as a function of the particle energy should also be useful for purely leptonic decay modes of the τ.

APPENDIX 2 : $\tau^- \to \rho^- \nu_\tau$, $\rho^- \to \pi^- \pi^\circ$

The squared matrix element is given by Tsai[1] in term of dot products :

$$|M|^2 \propto 2(p_1.Q)\,(p_2.Q) - (p_1.p_2)Q^2 - m_\tau \left[2\,(W.Q)\,(p_2.Q) - (W.p_2)Q^2 \right]$$

where P_1, P_2, q_1, q_2 are the four momenta of the τ^-, ν_τ, π^-, π° respectively, $Q = q_1 - q_2$ and W is the four vector which reduces to the three dimensionnal polarization vector $\vec{W}$ in the rest frame of τ^-.

If α^* and ϕ^* are the ρ decay angles in its rest frame and θ_ρ the ρ production angle in the τ frame we have :

$$P_1.Q = P_2.Q = \frac{m_\tau^2}{2}\,(1 - \eta)\,\beta^* \cos\alpha^*$$

$$P_1.P_2 = \frac{m_\tau^2}{2}\,(1 - \eta)$$

$$m_\tau(W.P_2) = \frac{m_\tau^2}{2}\,(1 - \eta)\,W\cos\theta_\rho$$

$$m_\tau(W.Q) = -\frac{m_\tau^2}{2}\,(1 + \eta)\,\beta^*\,W\cos\theta_\rho\,\cos\alpha^* - m_\tau m_\rho \beta^*\,W\sin\theta_\rho\,\sin\alpha^*\,\cos\phi^*$$

and $Q^2 = -\,m_\rho^2\,\beta^{*2}$

where $\beta^* = \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{1/2}$, $\eta = \frac{m_\rho^2}{m_\tau^2}$ and $\vec{W}$ is along the z axis.

so that

$$|M|^2 \propto \cos^2\alpha^*(1 + W\cos\theta_\rho) + \frac{m_\rho^2}{m_\tau^2}\sin^2\alpha^*\,(1 - W\cos\theta_\rho) - 2\,\frac{m_\rho}{m_\tau}\,W\sin\theta_\rho\,\sin\alpha^*\,\cos\alpha^*\,\cos\phi^* \qquad \text{(A2.1)}$$

It is worth noticing that this can be expressed, for W = 1, as

$$|M|^2 \propto \left|\, d^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}}(\theta_\rho)Y_1^\circ(\alpha^*) + \frac{\sqrt{2}m_\rho}{m_\tau}\, d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_\rho)Y_1^{-1}(\alpha^*, \phi^*)\right|^2$$

where the two helicity states of the ρ are evident.

The formula A2.1 has been used for M.C. generation of events. When integrated on the ρ decay angles, it gives the ρ production angular distribution used in section 4 :

$$\frac{d\,W}{d\,\cos\theta_\rho} \alpha \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right) + \left(1 - \frac{2m_\rho^2}{m_\tau^2}\right) W \cos\theta_\rho$$

It may be useful to note here that the π^- energy angle spectrum in the τ^- rest frame given in ref 1 has a term missing[8] which changes its shape : the formula 2 - 20 of this paper should have its last line changed to :

$$\pm \frac{(\vec{W}.\vec{q}_1)\omega_1}{q_1^2} \left[16M_L^2 \left(\omega_1 - \frac{M_L^2 + M_\rho^2}{4M_L}\right)^2 + M_\rho^2 \left(1 - \frac{4M_\pi^2}{M_\rho^2}\right) \left(3M_L^2 - M_\rho^2 - \frac{M_L^3}{\omega_1}\right) \right]$$

so that the coefficient in the square braket is not definite positive, and in fact changes sign in the ω_1 range. Note also that the ρ^- and A_1^- decay modes are incorrect in réf 6.