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ON THE APPROACH TO SCALING OF INCLUSIVE DISTRIBUTIONS IN THE MULTIPERIPHERAL MODEL

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ABSTRACT

It is shown that the inclusive distributions of produced particles raise to their scaling limit in the MPM as a consequence of the assumed momentum transfer limitation. A quantitative parameter-free estimate of the effect reproduces satisfactorily the observed dependence on s, x and the mass. The rôle of non-leading Regge trajectories with intercept ~0.5 is found to be negligible. The relevance of inclusive phenomenology based on a six-point function approach and of its consequences (like the "Ferbel plots") is therefore questioned.

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Scaling of inclusive distributions is generally assumed to hold, mainly on the basis of theoretical attractiveness.

Phenomenologically, scaling is well satisfied for $pp \to \pi + X$ for values of $|x| \gtrsim 0.1$, but rather large corrections are required to reproduce the rise of the small x pion spectrum, as well as the more important energy dependence of the K and especially $\bar{p}$ yields [1].

The pattern suggested by these corrective terms is rather one of a "threshold effect" type (heavier particles, and especially particles the production of which requires a higher threshold showing more energy dependence) than one in which corrections to scaling are attributed, e.g., to the exchange of non-leading Regge trajectories with intercept ~0.5 in a Mueller diagram. Here, by "threshold effect", I mean rather a dynamical than a kinematical notion : there is plenty of energy at ISR to produce a $p\bar{p}$ pair, but a dynamical model that suppresses large momentum transfers is bound to suppress the production of massive particles. In the multiperipheral model (MPM) this suppression, that operates also at asymptotic s, turns out to be more severe at low energy and at small x [2], the scale of the approach to the asymptotic limit being given by $x < p_\perp^2 > s/m_c^4$ (m_c is the mass of the produced particle) *).

The one-particle inclusive spectrum can be computed in the MPM in terms of off-shell total cross-sections as shown in Fig. 1 (and Fig. 2) [4]. For fixed values of the momentum transfers, the dependence of these off-shell cross-sections on the variables M_ℓ^2 (or M_r^2) can be expanded in the MPM in terms of the same Regge singularities that determine the corresponding on-shell scattering **).

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*) The problem and the results of the present paper are very strictly connected to the problem of energy dependence of the large $p_\perp$ particles spectrum in the MPM [3]. In both cases one is studying the consequences of a non-vanishing minimum value of the momentum transfers in the loop integration of Fig. 1 that links the MP calculation to a Mueller expansion.

**) For simplicity, we will assume that for moderate values of t_ℓ, t_r the off-shell dependence can be factorized and linked with the propagators to give the function $f(t_\ell, t_r)$ that appears in the loop integration of Eq. (1).

However, as shown by Bassetto and Toller [5)], these singularities are not sufficient to exhaust the expansion of the inclusive distribution of Fig. 1 in terms of the variables $s_{ac} = (p_a+p_c)^2$ and $s_{bc} = (p_b+p_c)^2$ (Mueller expansion). For instance assuming that the M_ℓ^2 dependence of the left cross-section is completely represented by one Regge pole of intercept α, the Mueller expansion of the MP distribution of Fig. 1 contains a whole series of Regge poles of intercepts α, $\alpha-1$, $\alpha-2,\ldots$ (which I will improperly refer to as daughters) as well as a set of fixed poles. The physical origin of these extra singularities can be qualitatively understood by an examination of Fig. 1. The hypothesis that the a-c' cross-section is dominated by a Regge pole of intercept α is related to the t channel J plane structure of the $p_a p_a \to p_{c'} p_{c'}$ scattering. In order to obtain the J plane structure of the $p_a p_a \to p_c p_c$ scattering, a recoupling of angular momenta is needed. This recoupling is performed through the loop integration of Fig. 1, and generates the above-mentioned daughterlike structure, in which the residue of the n^{th} daughter is determined by the coefficient of the n^{th} order term in the power series expansion of the function $f(t_\ell,t_r)$ that appears in the loop integration *). [The origin of the fixed poles is related to the fact that, even if $p_a \cdot p_c$ and $p_b \cdot p_c$ are asymptotically large, finite values of M_ℓ^2 and M_r^2 still give a (non-leading) contribution to the loop integral.]

All this singularity structure is needed in the MPM to enforce the dynamical assumption of finite momentum transfers, as well as to satisfy transverse momentum conservation [5)].

Since $f(t_\ell,t_r)$ is a decreasing function of its arguments, the first daughter residue, proportional to f', is certainly negative. Hence in the assumption of Pomeron dominance in the variables M_ℓ^2 and M_r^2 the one-particle spectrum $f(x,s) = E(d^3\sigma/d^3p)$ **) is expected to have the following features :

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*) This expansion is equivalent to a power series expansion of the scaling function $G[(p_\perp^2/s_1),(p_\perp^2/s_2)]$ of Ref. 3).

**) We will consistently work from now on at fixed small $p_\perp$ (all the numerical analysis is performed at $p_\perp^2 = 1$) unless the value of $p_\perp$ is explicitly stated.

a) for $s \to \infty$, $x \simeq 0$ $\quad f(x) = f(0) - c|x| \quad c > 0$;

b) for $x \neq 0$, $s \to \infty$ $\quad f(x,s) = f(x) - c(x)/s \quad c(x) > 0$;

c) fox $x = 0$, $s \to \infty$ $\quad f(0,s) = f(0) - 2c\, s^{-\frac{1}{2}}$.

The constant c is proportional to $|f'/f|$, i.e., inversely proportional to $< p_\perp^2 >$, and we will see in a moment that it is proportional to m_c^2 and that, therefore, the effect of the daughters is particularly important for heavy particle production, as expected since these terms are nothing but a translation in the six-point function language of the dynamical thresholds typical of the MPM [6].

SOME KINEMATICS

The minimum values of t_ℓ and t_r in the loop integration of Fig. 1 are given, up to first order in m_a^2/s_{ac}, m_c^2/s_{ac}, m_b^2/s_{bc}, m_c^2/s_{bc}, but to all orders in M_ℓ^2/s_{ac} and M_r^2/s_{bc}, by

$$t_r^{min} = \frac{A_0' + A_0 C_0'}{1 - C_0 C_0'} + \frac{A_1' - B_1 A_0' + A_0 C_1' + A_1 C_0'}{1 - C_0 C_0'} + \frac{A_0' + A_0 C_0}{(1 - C_0 C_0')^2} \left(B_1 + B_1' + C_1 C_0' + C_1' C_0 \right)$$

with

$$A_0 = - \frac{m_c^2 M_\ell^2}{2 s_{ac}} \qquad A_1 = - 2 m_c^4 \frac{M_\ell^2 (M_\ell^2 + s_{ac})}{s_{ac}^3} + \frac{2 m_a^2 m_c^2}{s_{ac}}$$

$$B_1 = - \frac{m_c^2 M_\ell^2}{4 s_{ac}^2} \qquad C_0 = \frac{M_\ell^2}{M_\ell^2 + 2 s_{ac}}$$

$$C_1 = \frac{2 m_c^2}{s_{ac} + s_{ac}^2/M_\ell^2} - 2 \frac{m_a^2 s_{ac} M_\ell^6}{M_\ell^{10} + M_\ell^4 s_{ac}^3} + \frac{m_c^4 M_\ell^2}{4 s_{ac}}$$

with a similar expression for t_ℓ, and with the primed constants obtained from the unprimed with the substitution $a \rightleftarrows b$, $\ell \rightleftarrows r$.

The contribution of the exchange of two Regge poles of intercept α_ℓ and α_r to Fig. 1 is of the form

$$\int_{s_0^\ell} \frac{dM_\ell^2}{s_{ac}} (M_\ell^2 - s_0^\ell)^{\alpha_\ell} \int_{s_0^r} \frac{dM_r^2}{s_{bc}} (M_r^2 - s_0^r)^{\alpha_r} \int_{t_\ell^{min}, t_r^{min}} dt_\ell \, dt_r \, f(t_\ell, t_r) \quad (1)$$

The M^2 integrations are effectively cut-off at some value proportional to s_{ac} and s_{bc} from the t_ℓ, t_r limitation, and therefore the integral behaves asymptotically like $s_{ac}^{\alpha_\ell}$ $s_{bc}^{\alpha_r}$ as expected. However, non-leading terms appear automatically from the dependence of the boundary of the t_ℓ, t_r integration on s_{ac} and s_{bc}. A detailed quantitative evaluation must be performed numerically, but, in order to obtain a qualitative estimate, let us assume that the t_ℓ and t_r integrations can be factorized and that the M_ℓ^2 and M_r^2 integration can be done by the mean value theorem in which

$$\bar{M}_\ell^2 = c(\alpha_\ell) \, (s_{ac} - s_0^\ell) + s_0^\ell \quad ; \quad \bar{M}_r^2 = c(\alpha_r)(s_{bc} - s_0^r) + s_0^r \quad (2)$$

Then Eq. (1) can be recast in the form

$$s_{ac}^{\alpha_\ell} \left(1 - m_c^2 \left| \frac{f'(0)}{f(0)} \right| \frac{c_1(\alpha_\ell)(s_0^\ell - m_a^2) + c_2(\alpha_\ell) m_c^2 + c_3(\alpha_\ell) m_a^2 + \dots}{s_{ac}} \right)$$

$$s_{bc}^{\alpha_r} \left(1 - m_c^2 \left| \frac{f'(0)}{f(0)} \right| \frac{c_1(\alpha_r)(s_0^r - m_b^2) + c_2(\alpha_r) m_c^2 + c_3(\alpha_r) m_b^2 + \dots}{s_{bc}} \right) \quad (3)$$

which exhibits the expected features. [c_1, c_2, c_3 in (3) are positive constants that depend on the value of α_ℓ and α_r though the value of the coefficient $c(\alpha_\ell)$ and $c(\alpha_r)$ in (2)].

NUMERICAL EVALUATION

I have performed an accurate numerical evaluation of (1) with exact phase space and kinematics, assuming $f(t_\ell,t_r) = e^{a(t_\ell+t_r)}$. The value of a is fixed at $a \cong 5$ from the requirement that $< p_\perp^2 > = 0.1$ for pions, and is assumed to be the same also for other particles. Concentrating our attention on pp interactions, the thresholds s_o^ℓ and s_o^r have been chosen as follows :

a) for π production both $(m_p+m_\pi)^2$;

b) for K production $(m_p+m_\pi)^2$ and m_Λ^2 ;

c) for $\bar{p}$ production $(m_p+m_\pi)^2$ and $4m_p^2$.

The contribution of Fig. 1 exhausts the inclusive distribution only for small x and rather large s [such that $\bar{n}(s)$ is substantially larger than 4].

The distribution obtained evaluating (1) for $p+p \to \pi + X$ with $\alpha_\ell = \alpha_r = 1$ is shown in Fig. 3 (dashed lines) for two values of x, $p_\perp^2 = 0.1$. As expected these distributions raise monotonically, and have a maximum at $x = 0$. Remark also that the $x \neq 0$ curve asymptotizes faster, as expected.

NON-LEADING TERMS

We want to study now the importance of non-leading terms with $\alpha_M \simeq 0.5$ in (1). The first problem is to assess the relative strength of the P and M couplings. For a non-exotic channel (we have considered $\pi^- p$) a satisfactory representation of the average over the resonance region as well as of the threshold behaviour is obtained with a relative weight one to one, the scale factor being chosen equal to 1 GeV^2. I have (rather arbitrarily) somewhat reduced the M/P ratio to 3/4 to keep into account the possibility that some subchannels will be exotic *).

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*) A suppression mechanism external to the MPM must also operate to reduce the contributions of these non-leading terms that would otherwise appear through the inclusive sum rules also in the pp total cross-section [7].

The results of evaluating the integral (1) for the joint contribution P and M on the left and on the right are given in Fig. 3 for π, Fig. 4 for K and $\bar{p}$. A few remarks are in order.

A) - Even if the complete π distribution (P+M exchange) decreases to its asymptotic limit as expected due to the disappearance of the positive non-leading terms of intercept 0.5 in M_ℓ^2 and M_r^2, this effect is quite small and is completely overcome up to ISR energy by the rise due to the t_{min} effect.

B) - For heavier particle $(K,\bar{p})$ the rise due to the threshold is much more violent, and the effect of the M terms even less noticeable.

C) - Correspondingly, the asymptotic f(x) does show the inverted cusp minimum at $x = 0$ expected in the presence of positive non-leading terms [8], but even for π the effect is hardly noticeable : with our parameters, the maximum of f(x) happens at $x \simeq 0.005$ and is only 3% higher than f(0).

D) - The normalization of the different spectra can be adjusted by choosing a multiplicative coupling constant different for π, K and $\bar{p}$. The curves that we show are obtained using a universal constant, and therefore the difference in yield of the various particles is a consequence of their masses and is a function of $< p_\perp^2 >$ only. It is remarkable that, even if the $\bar{p}$ probably requires a larger coupling, the right order of magnitude of the $\pi : K : \bar{p}$ ratio is obtained automatically.

E) - Since the non-vanishing x value scales faster, it can happen as a transitory phenomenon that $x = 0$ is indeed a minimum for heavy particles (see Fig. 4). This can be understood if one remarks that especially for $\bar{p}$ production the large difference in the two thresholds makes it more convenient to have one of the two subenergies much larger than the other at finite over-all energy. [A similar effect has been assumed in Ref. 3) for the large $p_\perp$ case.]

ENERGY CONSERVATION SUM RULES

It is well known that

$$\sum_c \int \frac{d^3\sigma^c_{ab}}{d^3p} \frac{E_c}{\sqrt{s}} d^3p = \sigma_{ab}(s) \tag{4}$$

Since in a first approximation the total pp cross-section is constant going from conventional accelerator to ISR energy, whereas the contributions to (4) of the π, K and $\bar{p}$ spectra increase, one can wonder how (4) can be satisfied *). Phenomenologically, Sivers [9] has recently observed that the increase of the π, K and $\bar{p}$ contributions to (4) is balanced by a small decrease in the p inelasticity. This observation fits very well in the present scheme in which the observed energy variations of the contributions to (4) are attributed to t_{min} effects. In fact, the proton is usually the leading (or next to leading) particle in the MPM, and the main contribution to its spectrum comes from the diagram of Fig. 2. In this diagram,

$$t_{min} = - m_p^2 \frac{(M^2 - m_p^2)^2}{s^2}$$

when $s \to \infty$ the relevant values of M^2 increase proportionally to s, say, $M^2 \simeq Rs$, and the corresponding

$$t_{min} = - R^2 m_p^2 + 2 \frac{m_p^4 R}{s} + O(s^{-2})$$

decreases to its asymptotic value, thus causing a depression of the leading particle effect **).

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*) This question has been repeatedly asked dealing with the non-leading terms corresponding to the exchange of meson trajectories [7], and its solution is not straightforward in the MPM, since it amounts to build in somehow exchange degeneracy in the optical theorem.

**) This obviously does not happen if the exchange in Fig. 2 is the Pomeron : in this case the leading particle effect actually becomes more and more important, corresponding to the development of a peak around $x \simeq 1$ of the proton inclusive spectrum . Indeed, this effect has been recently proposed [10] to explain the observed raise of σ_{pp} through the p contribution to (2).

CONCLUSION

A) - A very natural interpretation of the energy dependence and shape of one particle spectra is seen to be provided by the MPM.

B) - Whereas a Mueller translation of the MP dynamical threshold requirement is possible, it is not straightforward. Hence a phenomenology that starts from a guess at the six point level is likely to be misleading, as it is, in my opinion, the wide-spread habit to fit $f(0,s)$ with a straight line in an $s^{-1/4}$ plot [or $f(x,s)$ with a straight line in a $s^{-\frac{1}{2}}$ plot]. A comparison with Fig. 3 shows that this procedure can grossly over estimate the asymptotic limit of the π distribution, and is hard to reconcile with the sum rule (2).

C) - The phenomenological study of the rôle (and even sign) of the non-leading Regge trajectories in a Mueller diagram is very hard. Also for such "safe" quantities, like the difference between π^+ and π^- yield, the different thresholds associated to them can somehow blur the picture.

D) - The basic assumption that defines the MPM [11] is that the observed $p_\perp$ limitation stems from a projection on the transverse direction of a momentum transfer limitation. Hence from the known value of $< p_\perp^2 >$ one can predict the size of a variety of phenomena that are related to the limitation of the time and longitudinal components of the momentum transfers, such as the ratio of multiplicities of different mass particles, the shape of $f(x)$, the energy dependence of $f(x,s)$ and the asymmetry in $f(x)$ around $x = 0$ when the masses of the incoming particles are different [12]. The fact that all these effects are reasonably well predicted in terms of the known value of $< p_\perp^2 >$ might lead one to suspect that there is some truth in the assumption of invariant momentum transfer limitation.

It is a pleasure to thank D. Amati and A. Bassetto for helpful conversations.

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FIGURE CAPTIONS

Figures 1 and 2 — Multiperipheral contributions to one-particle inclusive distributions.

Figure 3 — Calculated π spectra as a function of s at various x, fixed $p_\perp^2 = 1$ in arbitrary units. The dashed line represents the P contribution alone, the solid line the complete $(P+M)$ one.

Figure 4 — Same for K and $\bar{p}$ yields. Only complete $(P+M)$ distributions are shown. The units are 10 times smaller than in Fig. 3 for K and 50 times smaller for $\bar{p}$.

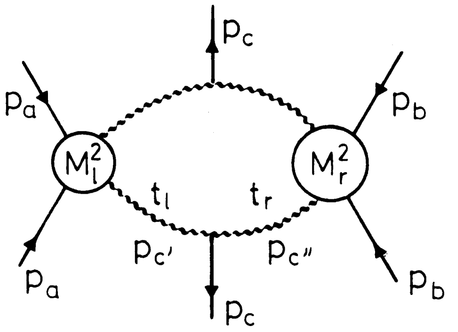

Fig. 1

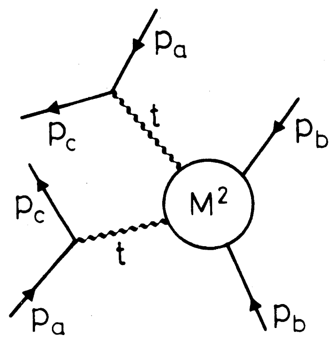

Fig. 2

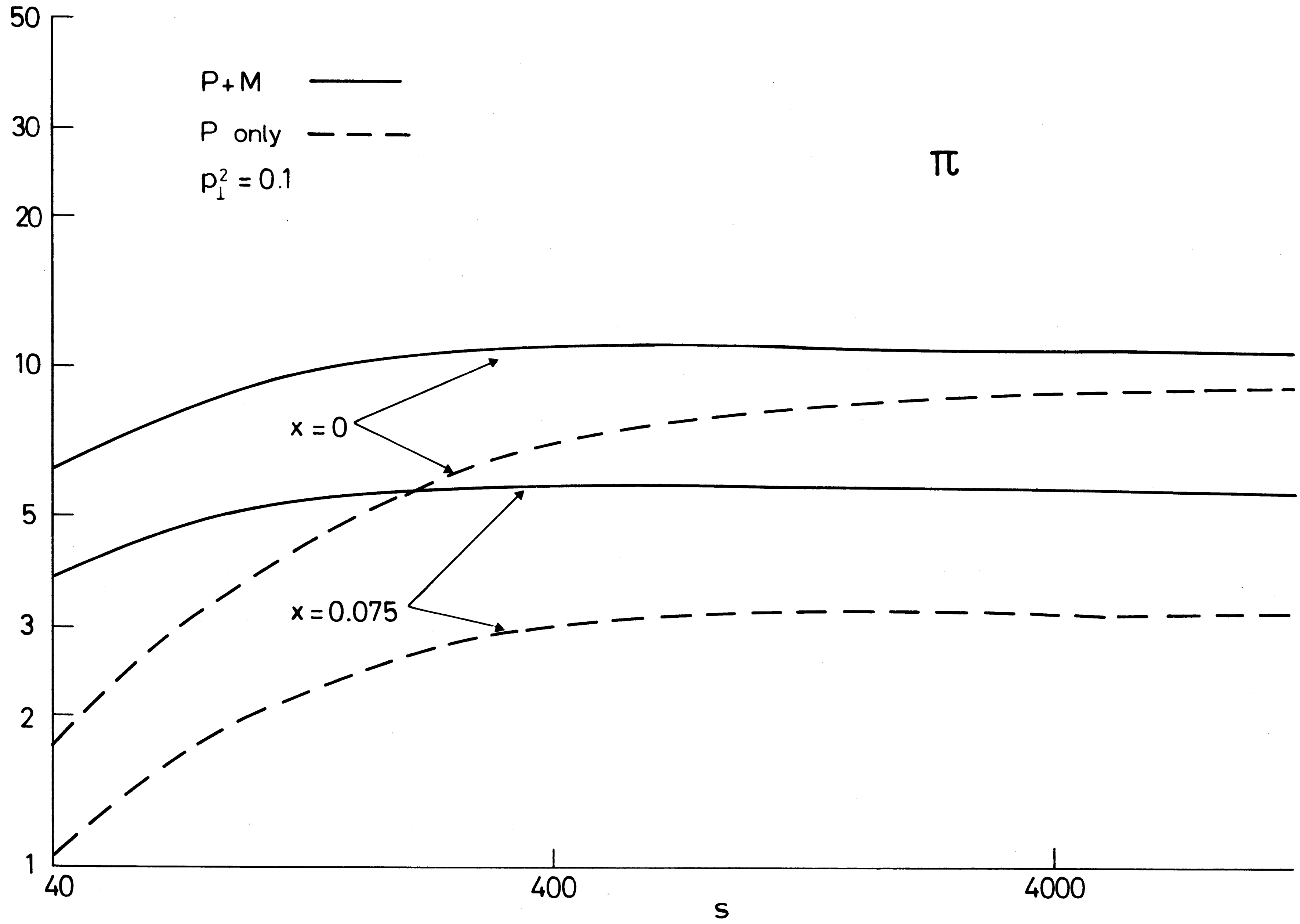

Fig. 3

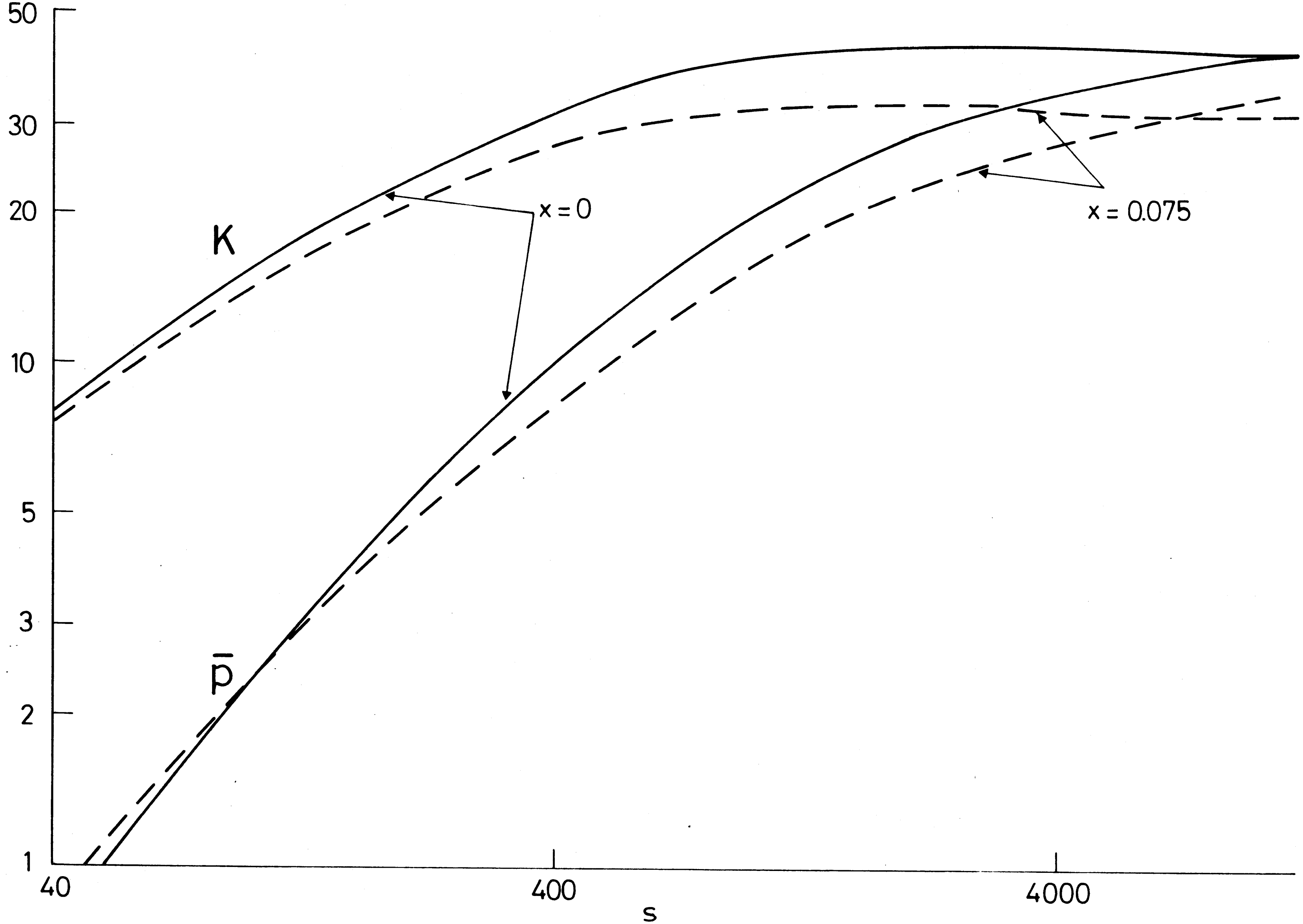

Fig. 4