DEPENDENCE OF TRANSVERSE AND LONGITUDINAL RESOLUTION ON SOME PARAMETERS DEFINING THE SET-UP IN HOLOGRAPHY

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1. INTRODUCTION

With the aid of holographic techniques a good spatial resolution can be obtained over a large depth of field. This arises from the fact that these two quantities are decoupled in holography, which is not the case in standard photography. We have investigated the sensitivity of spatial holographic resolution to different parameters which characterize the optical set-up. Important parameters are:

ℓ : the distance between the object and the holographic plate,

θ : the angle between the reference beam and the object beam,

I_r/I_o : the ratio of the intensity of the reference beam to that of the object beam.

In this work a quantitative study has been made of the transverse resolution (R_{xy}) and of the longitudinal resolution (R_z) as a function of these parameters. Special attention was paid to the dependence of the spatial resolutions on the distance parameter ℓ in the different set-ups. It is known that the transverse resolution (R_{xy}) varies linearly with ℓ from the classical relation:

$$R_{xy} = 1.22 \frac{\lambda \ell}{D} , \tag{1}$$

where λ symbolizes the optical wavelength and D the effective diameter of the reference beam. Furthermore, one can show that the longitudinal resolution R_z is related to ℓ by

$$R_z = 8\lambda \frac{\ell^2}{D^2} \tag{2}$$

by calculating the intensity pattern resulting from the holographic image of two points situated along an axis perpendicular to the holographic plate. The application of this relation to bubble-chamber recordings remains, however, to be verified.

2. EXPERIMENTAL SET-UPS

All tests presented here were performed in the Applied Optics Institute (VUB-Brussels). The holograms were recorded on Agfa 8E56 plates, using an Ar-ion laser (supplied by Spectra-Physics) having a maximum power of $\sim$ 1 W. This laser was operated at a wavelength of λ = = 5145 Å throughout all tests.

In order to define the spatial resolutions in an unambiguous manner a USAF test target was used to serve as the object. This target consists of a glass plate on which chromium elements are evaporated. The size of these elements ranges from 500 to 1 µm. Figure 1a

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summarizes the most important characteristics of this target. In Fig. 1b a detail of the target is shown containing the smallest elements. This photograph was made from the screen of a TV monitor, connected with a vidicon-camera equipped with a microscope viewing the test target.

Basically two set-ups were tested in this work:

a) The "in-line" set-up, where the reference beam is parallel to the object beam. The test target was put perpendicularly to the incoming beam, and different values of ℓ were chosen. During these tests information was obtained on R_{xy} and R_z.

b) The "two-beam" set-up, where an angle θ_{ro} was imposed between the reference beam and the object beam. Tests were done at $\theta_{ro} \sim 30°$ and again different ℓ-values were taken. Furthermore, two different intensity ratios were tried between the two beams. With this set-up it was only possible to obtain information on R_{xy}.

3. IN-LINE TESTS

Figure 2 displays the set-up used to study in-line holography. At the recording stage the test target was placed at different distances ℓ from the holographic plate. At the replay stage the hologram was mounted on a device which could be moved along the three axes x, y, and z. This device was equipped with a micrometer on each axis. The real image of the hologram was viewed through a system consisting of a microscope equipped with a vidicon camera, connected with a TV monitor. Eight holograms were taken at different ℓ values.

3.1 Transverse resolution (R_{xy})

The best transverse resolution (r) of a hologram was defined as the size of the smallest elements which could still be distinguished in the real image (Fig. 3). The error (Δr) was defined by comparing the size of these elements (i) with that of the previous elements (i - 1) and that of the next elements (i + 1) on the target. Figure 4 shows the values of R_{xy} as a function of ℓ, obtained during this test. A straight line was fitted through these points, represented by the following equation*)

$$R_{xy} = a + b\ell \ . \qquad (3)$$

The value obtained for the slope factor b was then used to derive the effective beam diameter D appearing in Eq. (1):

$$D = 3.4 \pm 0.4 \text{ cm} \ .$$

3.2 Longitudinal resolution (R_z)

In order to compute the longitudinal resolution of a holographic image a statistical method was applied. Again the real image of the hologram was viewed through the chain consisting of a microscope, a vidicon camera, and a TV monitor**).

*) The constant a was introduced to take into account the intrinsic resolution of the measurements.

**) The microscope was equipped with an objective having magnification × 20; no ocular was used and the vidicon camera was operated without an objective.

The holographic plate was mounted on the device, which was movable along the three axes x, y, z, and which was equipped with a micrometer on each axis. Figure 5 illustrates the procedure of the measurements:

i) A given element of the target image was focused as accurately as possible (in this way the z-coordinate of the element was adjusted).

ii) Then the element was moved in the plane perpendicular to the microscope axis until it coincided with a cross marked on the TV screen (in this way the x- and y-coordinates were adjusted).

iii) Next the position of the element was recorded from the read-out on the three micrometers.

In each hologram thirty measurements were performed on different elements.

Next the measured coordinates of the elements were processed as follows. A plane was fitted through the 30 points, expressed by

$$z = \alpha x + \beta y - \gamma \ . \qquad (4)$$

This was done in practice by minimizing the sum of the squares of the distances of these points to a given plane defined by the parameters (α,β,γ):

$$F(\alpha,\beta,\gamma) = \sum_{i=1}^{n} D^2(x_i,y_i,z_i) = \min \qquad (5)$$

$$D(x_i,y_i,z_i) = \frac{\gamma - (\alpha x_i + \beta y_i - z_i)}{\sqrt{1 + \alpha^2 + \beta^2}} \ . \qquad (6)$$

After having fixed the parameters (α,β,γ) at their best value, Eq. (6) was used to compute the distances of the measured points to this "best" plane. These distances were found to be centred around zero with a given spread expressed by the corresponding $\text{r.m.s.}_{\text{hol}}$. This r.m.s. is caused by the deviation of the measured points from the best plane in the z-direction. Hence, it can only be a reflection of the longitudinal resolution (R_z) of the hologram and of the inherent measurement precision. In order to take this measurement precision into account the same procedure was repeated on the actual elements of the original test target. In this way the following intrinsic resolution was obtained:

$$\text{r.m.s.}_{\text{target}} = 17 \pm 3\ \mu\text{m} \ .$$

The longitudinal resolution for a given hologram was then defined as:

$$R_z = \sqrt{\text{r.m.s.}_{\text{hol}}^2 - \text{r.m.s.}_{\text{target}}^2} \ .$$

The error on R_z was computed using a Monte Carlo method.

The results of this test are shown in Fig. 6. From these data points one cannot conclude whether R_z really exhibits a parabolic variation as a function of ℓ as was suggested by Eq. (2), although the latter predicts the right order of magnitude of R_z within the range of ℓ-values under investigation. However, if one fits a parabola through these points, expressed by

$$R_z = \alpha\ell^2 \ , \tag{7}$$

one can derive again a value of the effective beam diameter D from Eq. (2):

$$D = 4.3 \pm 0.1 \text{ cm} \ .$$

This value is not in disagreement with that derived from the transverse resolutions.

4. TWO-BEAM HOLOGRAPHY

Figure 7 illustrates the set-up used to study spatial resolution in "two-beam" holography. At the recording stage the object beam was focused through the target onto a small spot of the holographic plate, to eliminate the zero-order component of the object beam. This was done by means of a converging lens (L_3). The reference beam was oriented at an angle of 30° with respect to the object beam. At the replay stage the object beam was blocked and the holographic plate was mounted in a similar way to that described in Section 3.

A rough measurement was made of the average intensities of the object beam and the reference beam. A test was done where both intensities were about equal and another one where the reference beam was about 15 to 20 times as intense as the object beam. Figure 8 summarizes the results for the transverse resolutions found for these two tests. One notices that these results vary between 15 and 35 μm for ℓ inferior to 20 cm. Moreover a pronounced astigmatism of the image was observed, which ranged typically between 50 and 500 μm. The presence of "ghost" images in some places of the hologram also increased the difficulty of analysing the holograms. A general conclusion about this specific set-up was that all results were definitely worse that those of the "in-line" set-up described in Section 3. Hence, if one really needs off-axis holography to perform high-resolution experiments, a different set-up should be used.

5. CONCLUSIONS

Quantitative measurements were made of spatial resolutions in "in-line" holography and in "off-axis" holography. With "in-line" holography transverse resolutions were measured between 2 and 10 μm for distances of ℓ between the object and the holographic plate ranging from ∿ 4 to 40 cm.

In the same set-up longitudinal resolutions were measured between 20 and 300 μm. Above ℓ = 35 cm it was very difficult to perform good measurements of this quantity. This was due to the fact that at such distances one is practically unable to determine a best focus of the image. From the data presented here one can conclude that high-resolution experiments should be done for optical distances reduced as much as possible. Furthermore, the large values of R_z (compared to R_{xy}) should be kept in mind if one wants to perform very accurate measurements of coordinates. Indeed, at very short distances (ℓ ∿ 4 cm), the minimal r.m.s. of such measurements appeared to be ∿ 20 μm.

When using a set-up for off-axis holography worse results were obtained and a significant astigmatism of several hundred μm was observed throughout the images. It was impossible to measure transverse resolutions below 15 µm for distances of ℓ inferior to 20 cm. Owing to the poor quality of the real image in this set-up (astigmatism, ghost images, etc.), it turned out to be impossible to measure the corresponding longitudinal resolution. One concludes here that the specific off-axis set-up used in this test is most probably not applicable in high-resolution experiments. If off-axis holography were really required in an experimental set-up, more investigations should be carried out in this field in order to obtain a deeper insight into all problems met so far.

a)

b)

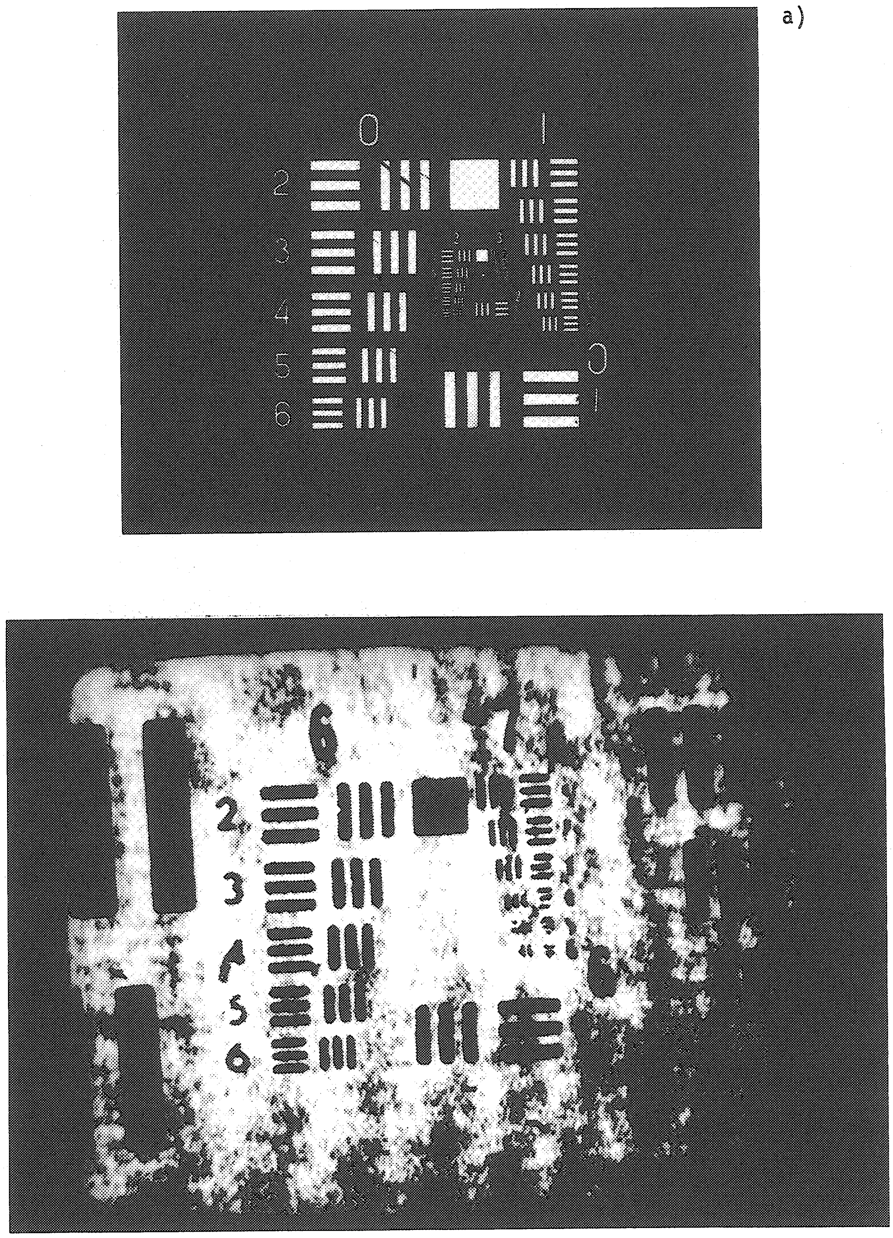

Fig. 1

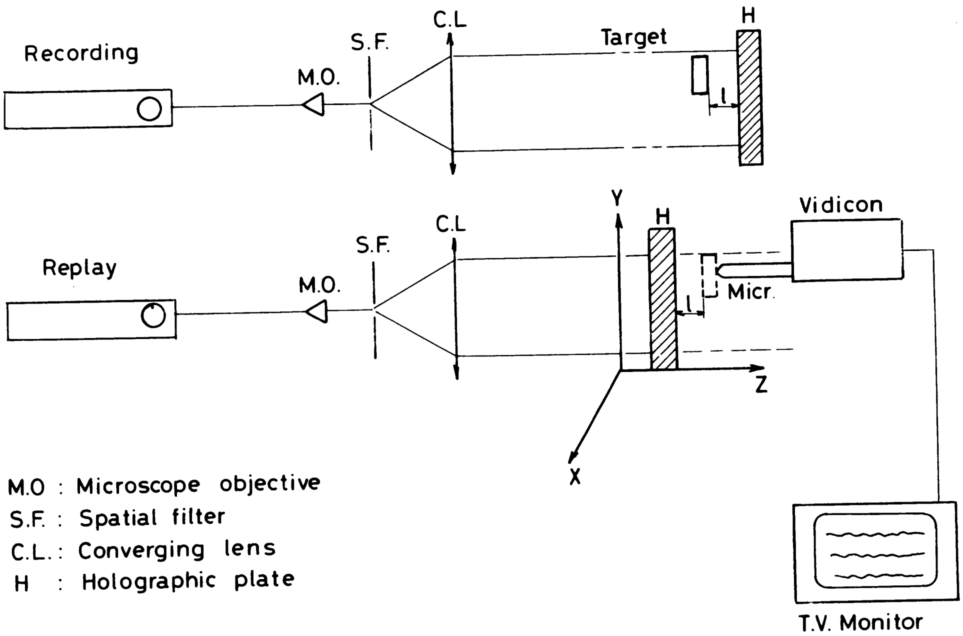
Recording
M.O.
S.F.
C.L
Target
H
l
Replay
M.O.
S.F.
C.L
Y
H
Vidicon
Micr.
l
Z
X
M.O : Microscope objective
S.F. : Spatial filter
C.L.: Converging lens
H : Holographic plate
T.V. Monitor

Fig. 2

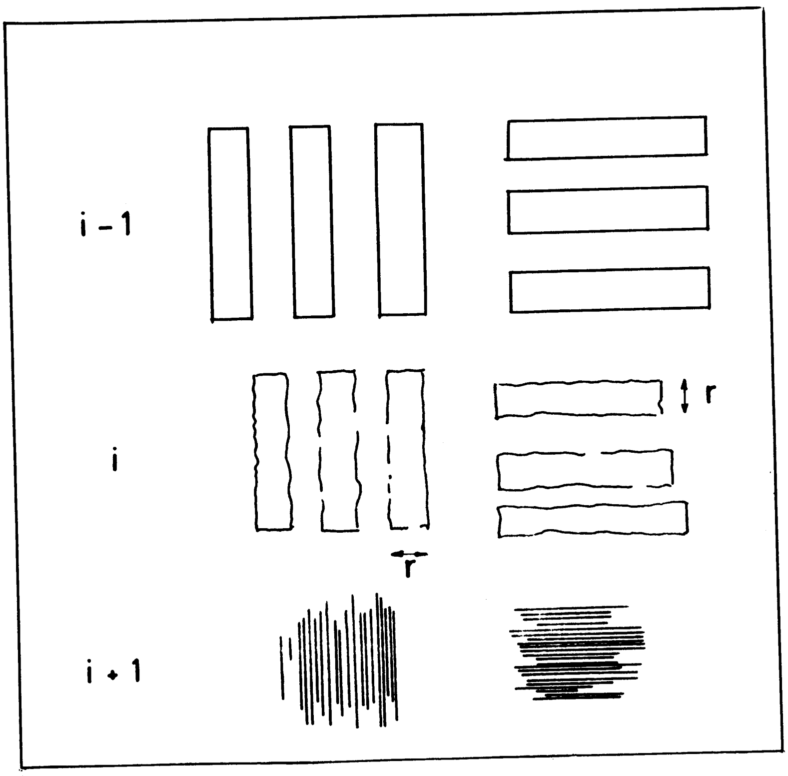
i - 1
i
r
r
i + 1

Fig. 3

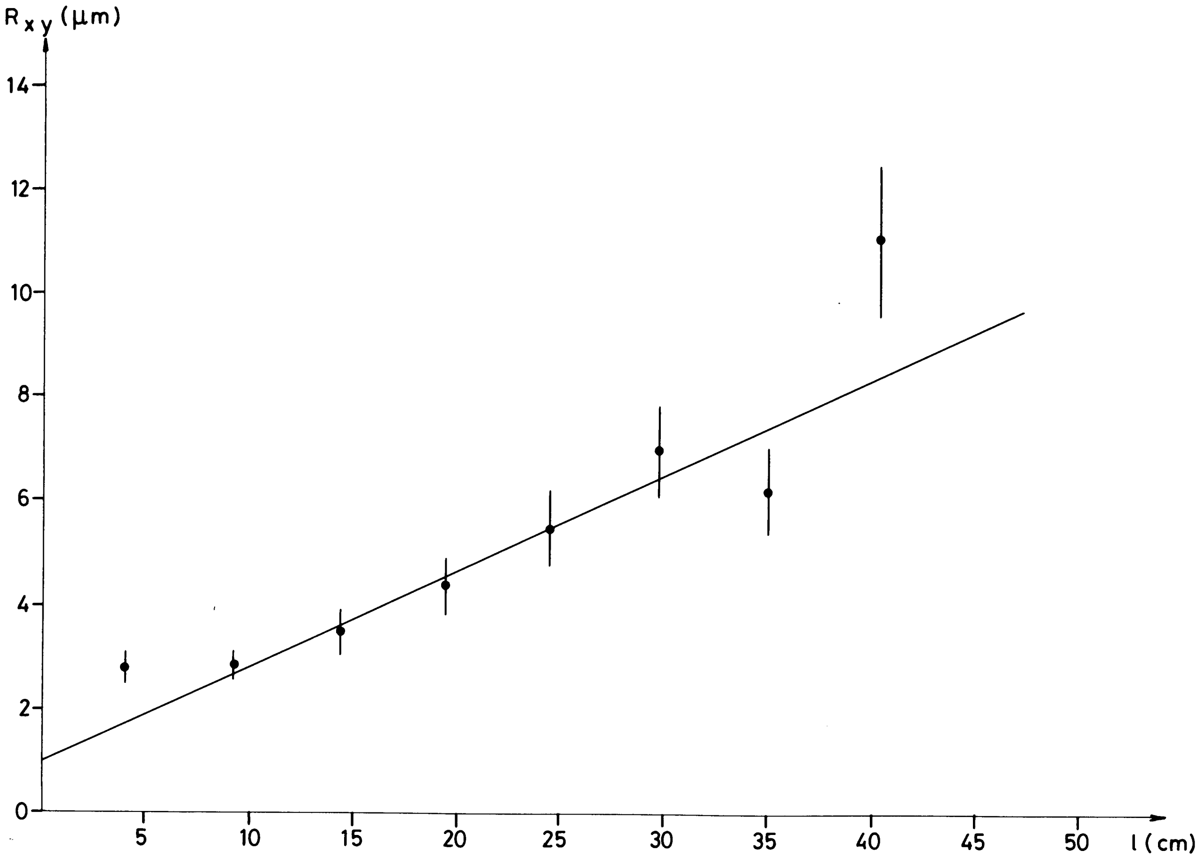
R_{xy} (μm)
14
12
10
8
6
4
2
0
5
10
15
20
25
30
35
40
45
50
l (cm)

Fig. 4

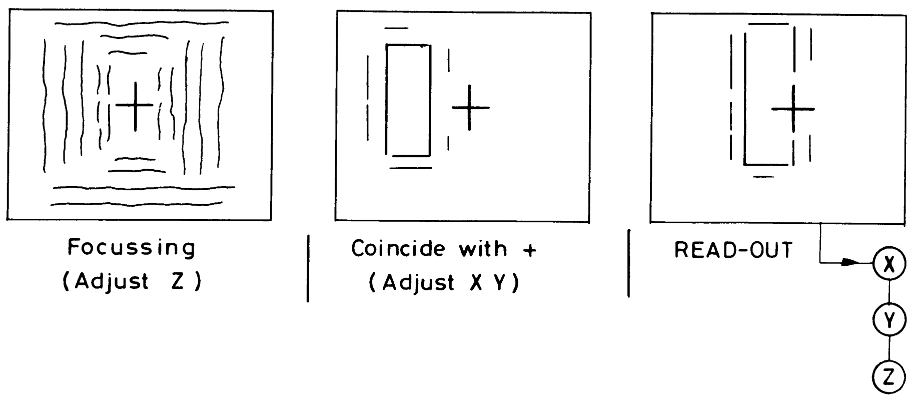
Focussing
(Adjust Z)
Coincide with +
(Adjust X Y)
READ-OUT
X
Y
Z

Fig. 5

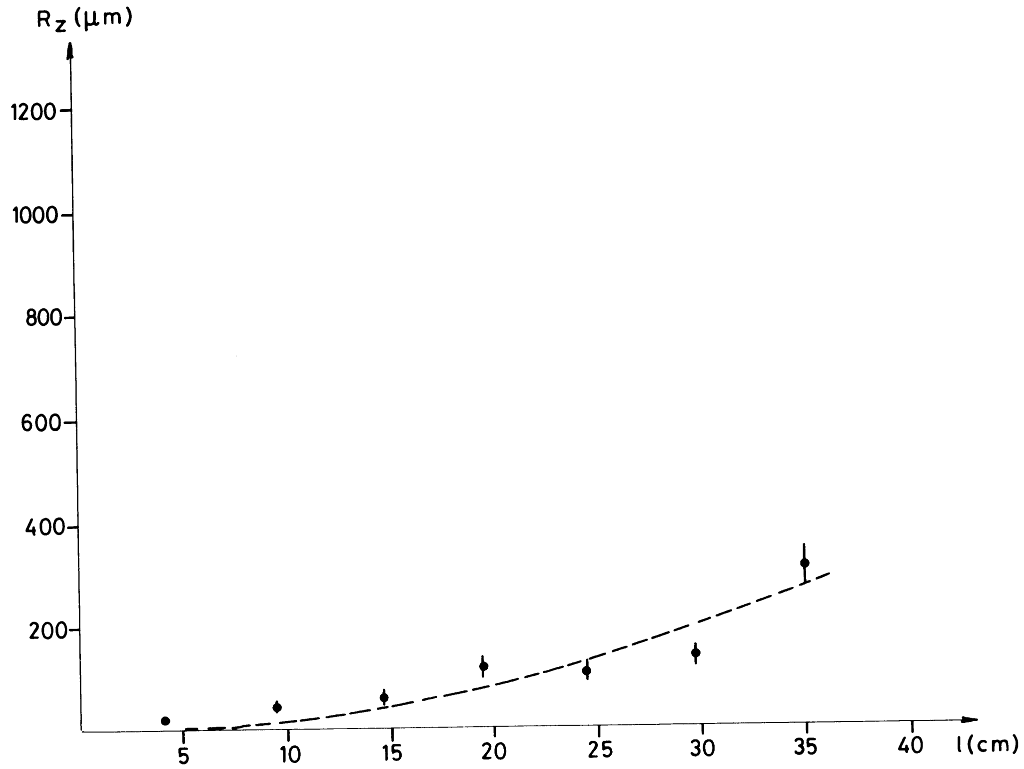
Rz (µm)
1200
1000
800
600
400
200
5
10
15
20
25
30
35
40
l(cm)

Fig. 6

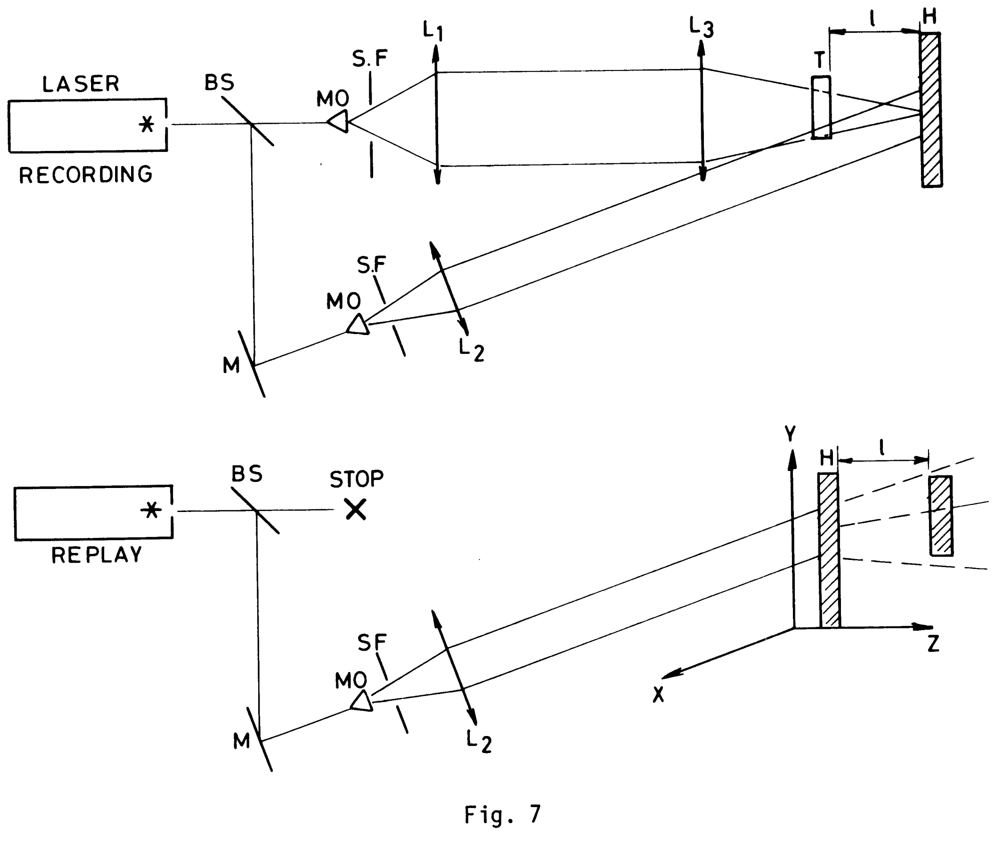
LASER
RECORDING
BS
MO
S.F
L1
L3
T
l
H
S.F
MO
M
L2
REPLAY
BS
STOP
Y
H
l
SF
MO
M
L2
X
Z

Fig. 7

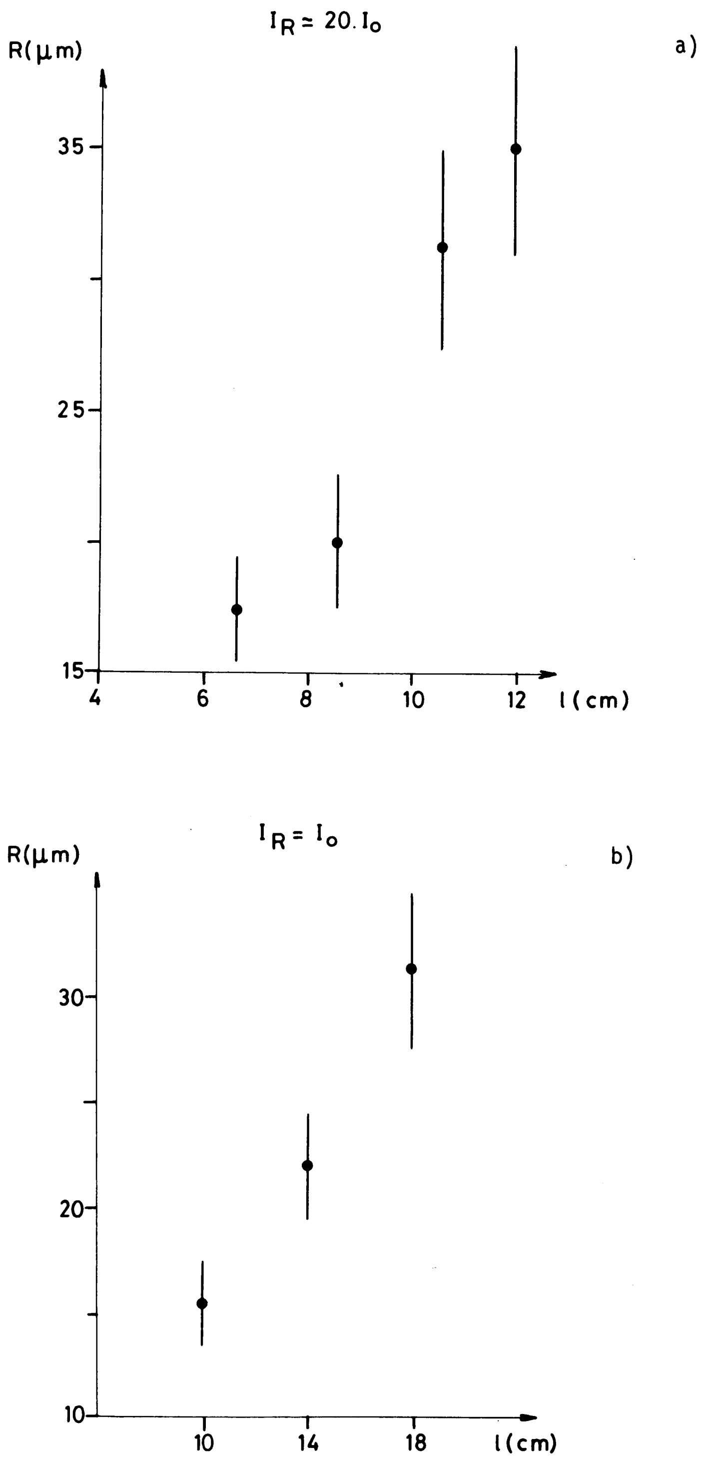

Fig. 8