RAPPORTEURS TALKS

DYNAMICS OF THE TWO-BODY AND QUASI-TWO-BODY PROCESSES AT HIGH ENERGIES

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DYNAMICS OF THE TWO-BODY AND QUASI-TWO-BODY PROCESSES AT HIGH ENERGIES

A. Tavkhelidze

The present talk gives a short review of theoretical investigations on two-body and quasi-two-body processes at high energies. It is based mainly on papers submitted to this conference.

In spite of principle difficulties which are characteristic of a phenomenological approach, we have attempted to unify the reviewed material according to some principles borrowed from quantum field theory.

Development of existing technique in the theory of strong interactions is closely related to the method of dispersion relations introduced by Gell-Mann, Goldberger and Thirring.

In the papers by N. N. Bogolubov on the theory of dispersion relations the fundamental idea that the scattering amplitude is a unique analytic function has been introduced. This allows one to relate various physical processes. It is just this concept which helps us understand from a general point of view the existing phenomenological approaches as possible approximations to the theory of strong interactions.

This idea turned out to be most fruitful in the study of strong interactions at high energies. A number of fundamental asymptotic relations and bounds on the cross sections have been obtained. The concept that the scattering amplitude is a unique analytic function has served as an adequate tool for the introduction of Regge ideas to quantum field theory.

A relationship between the short range character of nuclear forces and the analyticity of the amplitude as a function of the momentum transfer variable has been found. This has led to the quasi-optical picture of the high energy scattering processes, which is rather closely related to the absorption sphere model in quantum mechanics.

Note that attempts to construct a theory of particle interactions, considering only high energies, are, in general, inconsistent.

Indeed, the analytic properties of the amplitude and the assumption of asymptotic Regge behaviour lead to the finite energy sum rules (FESR). They give integral relations between physical quantities at low and high energies. These relations are more restrictive for dual solutions of FESR.

At present there are only fragments of the theory which give more or less complete description of the observed phenomena.

Keeping in mind the above mentioned theoretical situation, we begin our talk reviewing the main experimental facts on two-body and quasi-two-body processes at high energies. Further we attempt to explain these facts by (1) Regge phenomenology, (II) quasi-optical approach, (III) unitarity condition at high energy. In conclusion we discuss some new ideas in the theory of strong interactions at high energies.

This talk has been written in collaboration with the Profs. O. Khrustalev, V. Matveev, R. Muradyan, V. Shelest.

Much work in reading the submitted papers and in preparing the final version of the text has been done by Drs. L. Jenkovsky, V. Garsevanishvili, S. Goloskokov, V. Savrin, L. Slepchenko, N. Tyurin.

Dr. K. Draxler has kindly agreed to read the English version of the manuscript.

Section I

Survey of the experimental situation

1. TOTAL CROSS SECTIONS

A general view on the behaviour of the total hadron scattering cross sections in the energy region 5—20 *GeV* can be obtained from the experimental data given in Fig. 1. In this region all cross sections decrease with energy. The cross sections for scattering of negative particles on protons are larger and decrease faster (Fig. 2). The ratio σ_{el}/σ_{tot} decreases with increasing energy and its magnitude varies between 20% and 25% (Fig. 3).

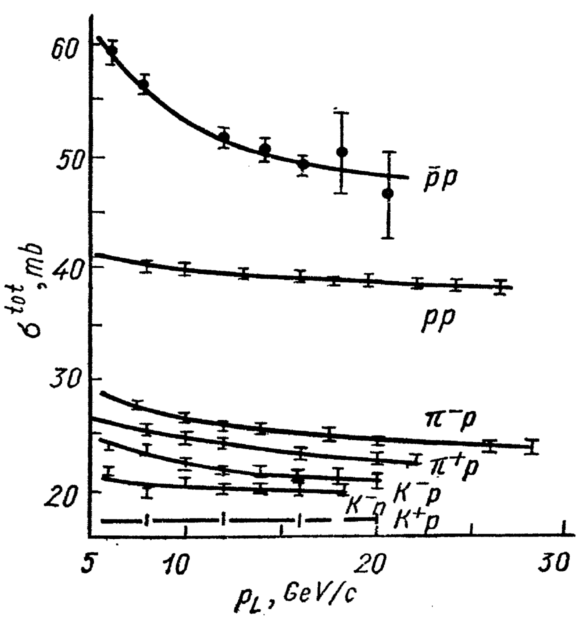
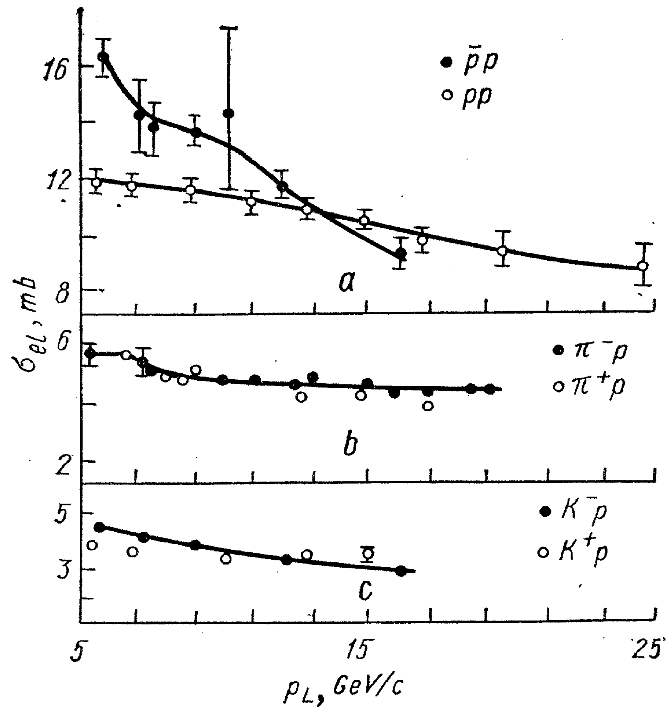


Fig. 1. *NN*, πN and *KN*-total cross sections [1,2].

The total cross sections of the reactions which proceed via nonzero, quantum number exchanges in the *t*-channel are by 2—3 orders of magnitude smaller than the corresponding elastic total cross sections and fall off rapidly with increasing energy. The cross sections for nucleon-nucleon scattering depend weakly on the isotopic spin. In Fig. 4 total cross sections for *pp*- and *pn*-scattering are shown. The magnitude of the *pn*-scattering cross sections ins determined by appropriate calculation using *pd*-scattering data as well as by direct measurement. Within experimental errors the following relation holds:

$$\sigma_{tot}(pn) \simeq \sigma_{tot}(pp), \qquad (1.1)$$

$$p_L \geqslant 5 GeV/c.$$



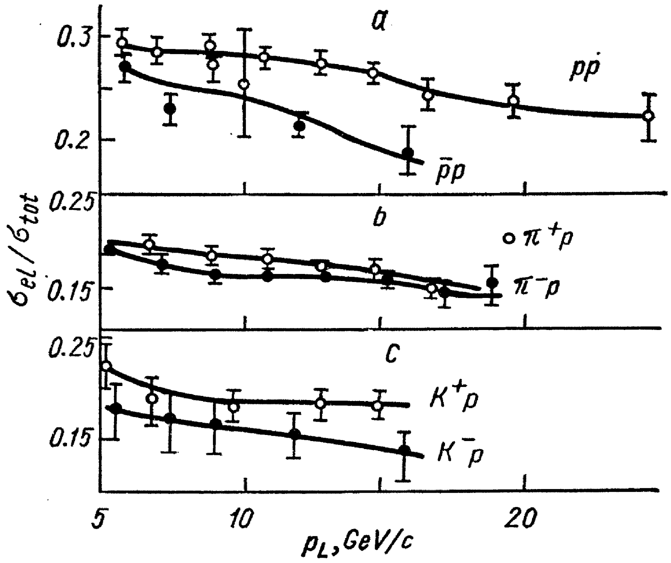


Fig. 3. The ratio σ_{el}/σ_{tot} [2, 4].

Fig. 2. Elastic scattering cross sections [2, 4].

For the antiproton reactions a weak dependence of the cross sections on the isospin can be seen. The character of the decrease of the pp- and $\bar{p}p$-scattering cross sections can be read off from Figs. 2*a* and 3*a*.

The behaviour of the total cross sections of kaon-nucleon scattering is shown in Figs. 1, 3*c*, 4*c*. The total cross sections of the K^+p and K^+n-scattering are approximately constant and equal to each other,

$$\sigma_{tot}(K^+p) \simeq \sigma_{tot}(K^+n). \tag{1.2}$$

Notice, that the $\pi^\pm p$ and $K^\pm p$-elastic scattering cross sections (Fig. 2) are approximately equal to each other,

$$\sigma_{el}(\pi^-p) \simeq \sigma_{el}(\pi^+p),$$
$$\sigma_{el}(K^-p) \simeq \sigma_{el}(K^+p), \tag{1.3}$$

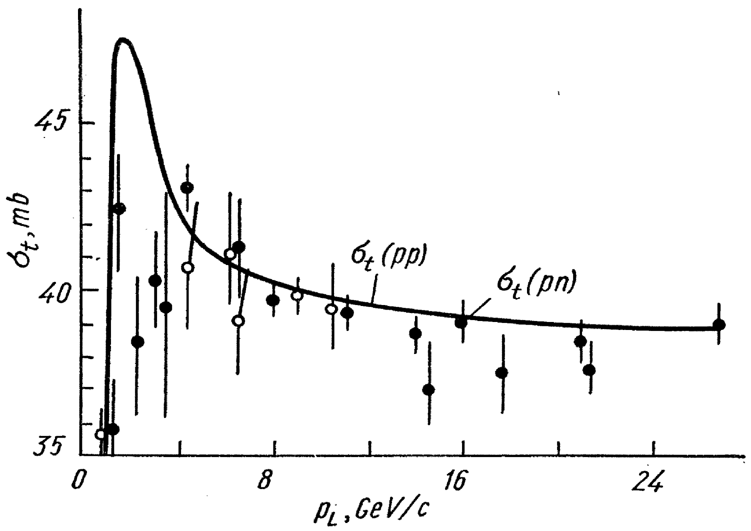


Fig. 4. pp and pn-total cross sections [3].

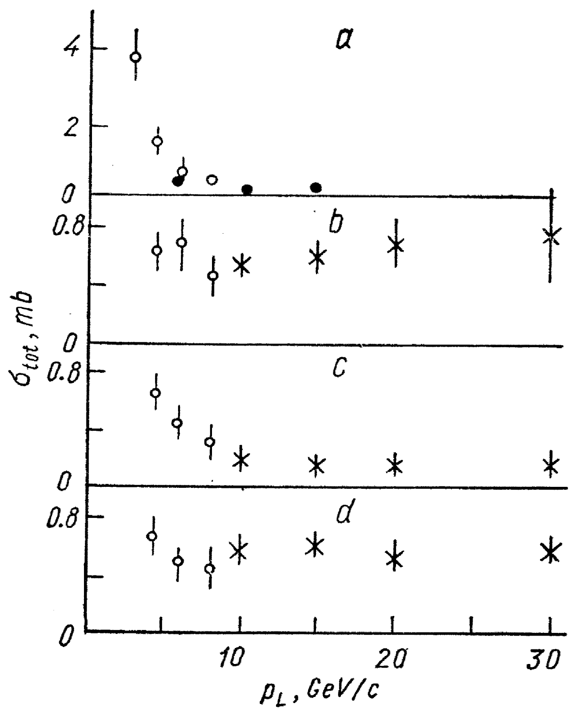
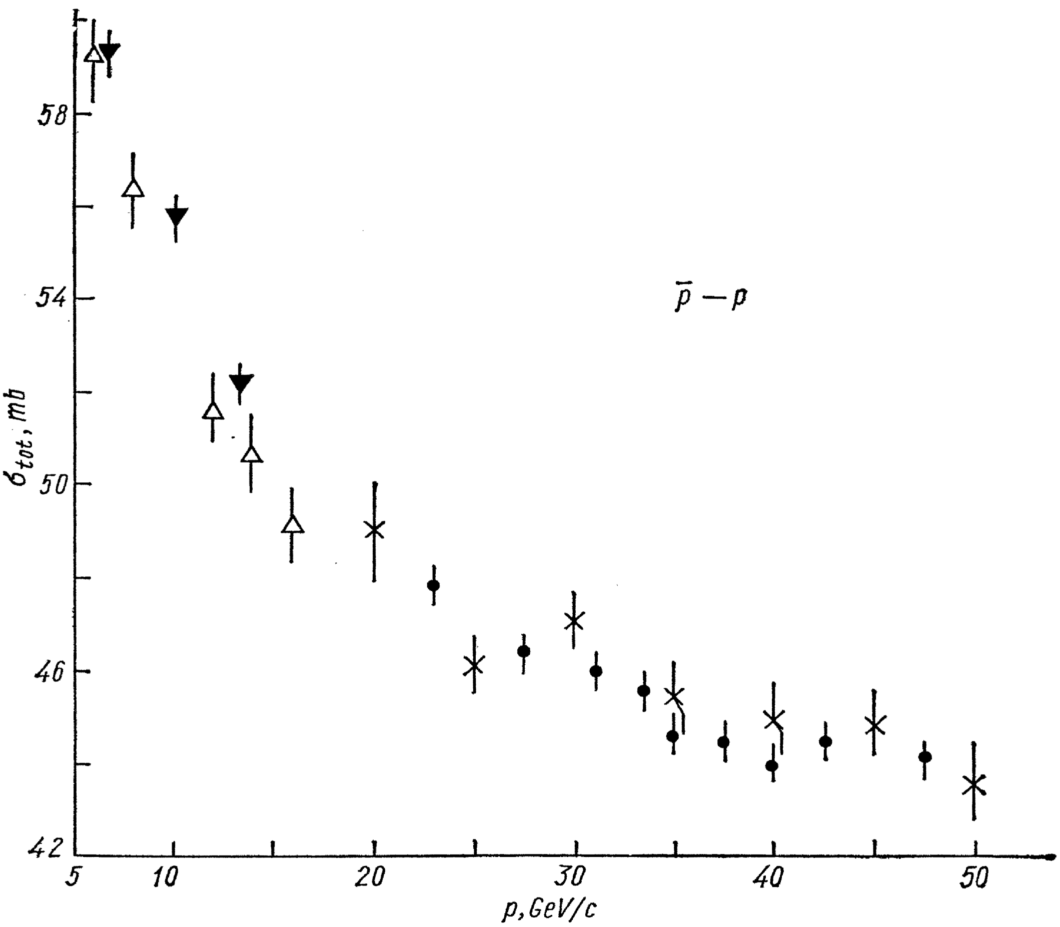
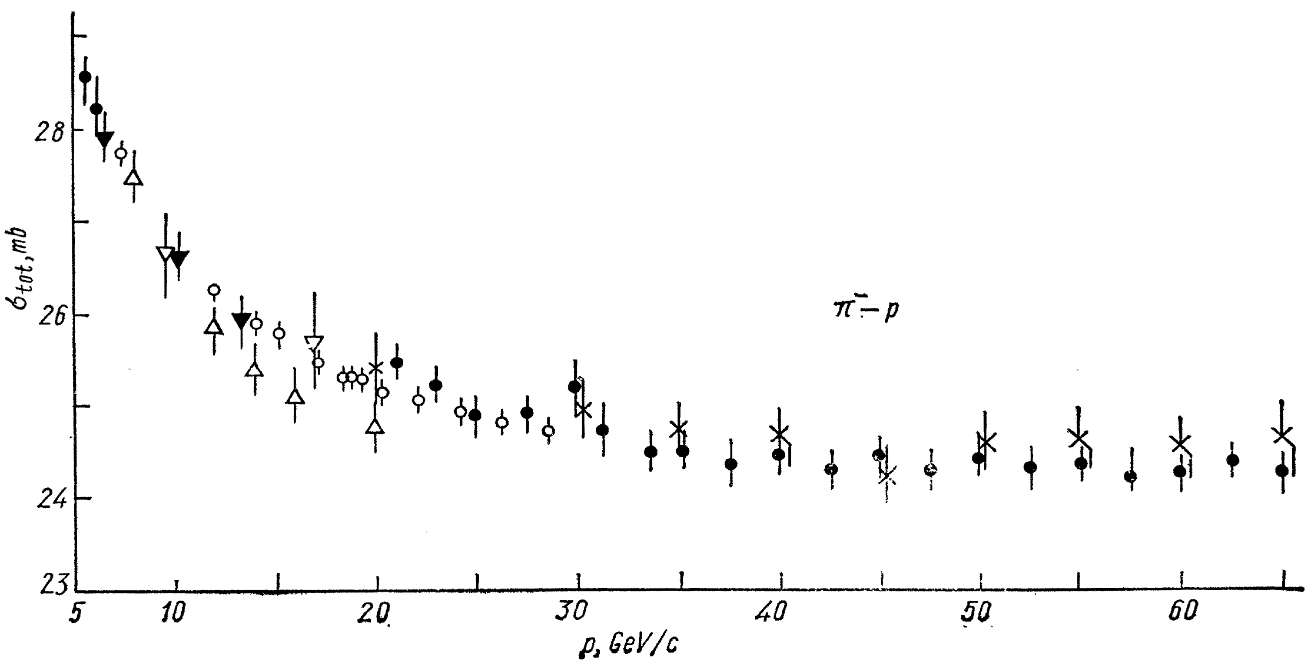


Fig. 5. Cross sections of quasi-two-body production of isospin $I = 1/2$ isobars in pp collisions [5]: a) N^* (2190), *b*) N^* (1400), *c*) N^* (1518), *d*) N^* (1688).



$\bar{p}-p$
σ_{tot}, mb
58
54
50
46
42
5
10
20
30
40
50
p, GeV/c



$\pi^{-}-p$
σ_{tot}, mb
28
26
24
23
5
10
20
30
40
50
60
p, GeV/c

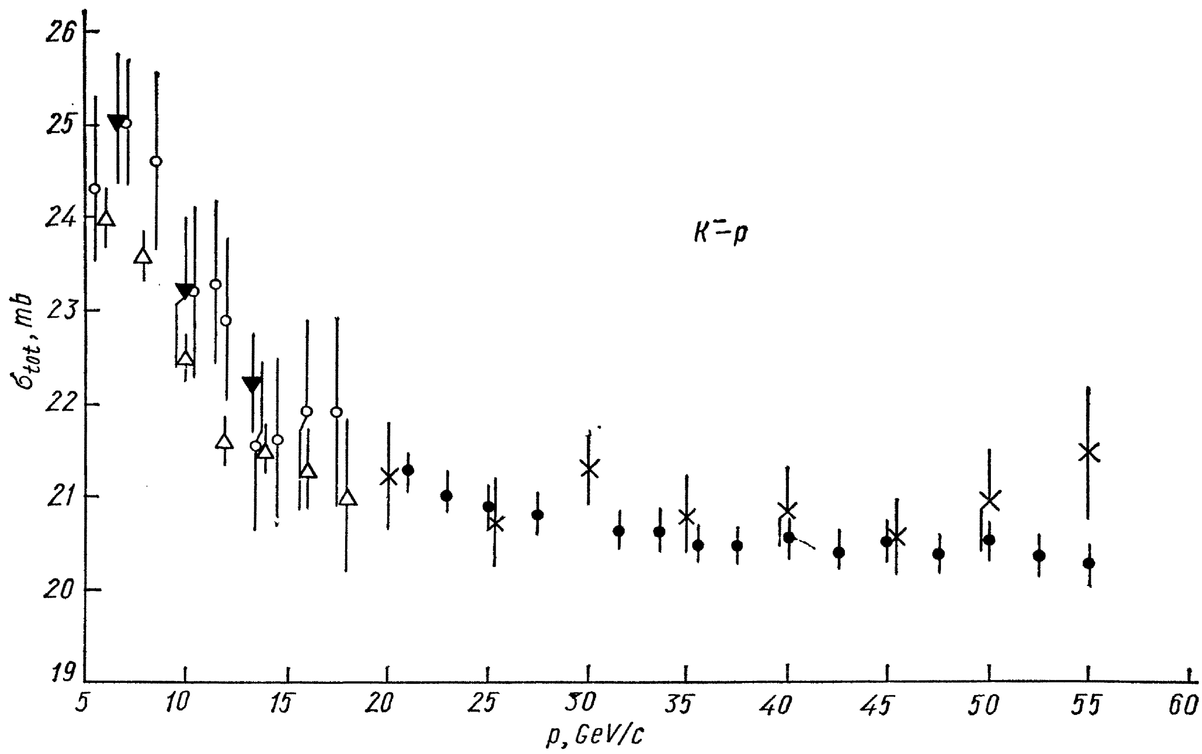


Fig. 6. $N\bar{N}$, πN and KN-total cross sections [6]. New data are shown by black points.

while for the total cross sections of these processes we have

$$\sigma_{\text{tot}}(A^-p) > \sigma_{\text{tot}}(A^+p),$$
$$A^\pm = (\pi^\pm, K^\pm). \tag{1.4}$$

Interesting data have been obtained on isobar production in quasi-two-body pp-scattering.

As shown in Fig. 5 the cross sections for the production of the isospin $I = {}^1/_2$ isobars N^* (1400), N^* (2190), N^* (1518), N^* (1688) are approximately constant and amount to about 10% of the magnitude of the pp scattering cross section at the same energies. Recent experiments performed at Serpukhov by the IHEP — CERN Collaboration [6] and IHEP group [6a] (Fig. 6) have given a number of important results on the behaviour of the total cross sections for scattering of π^-, K^- and antiprotons on protons and deuterons. It has been found that in the region $p_L = 25 \div 65$ GeV/c the total cross sections for $\pi^\pm p$ and K^-p-scattering are almost constant. Assuming isospin invariance of the interaction, the cross sections for π^+p scattering have been determined from the data on π^-d-scattering with the help of the usual Glauber correction formula. The differences of the cross sections $\sigma_{\pi^-p} - \sigma_{\pi^+p}$ and $\sigma_{K^-p} - \sigma_{K^+p}$ (extrapolated) are essentially non-vanishing.

2. ANGULAR DISTRIBUTIONS

It is common practice to treat separately the following parts in the angular distribution: (I) very forward scattering (Coulomb interference), (II) forward scattering (diffraction peak), (III) large momentum transfers and large angles, (IV) backward scattering (see, e. g. Fig. 17). The measurements of the various differential cross sections give information on forward and backward

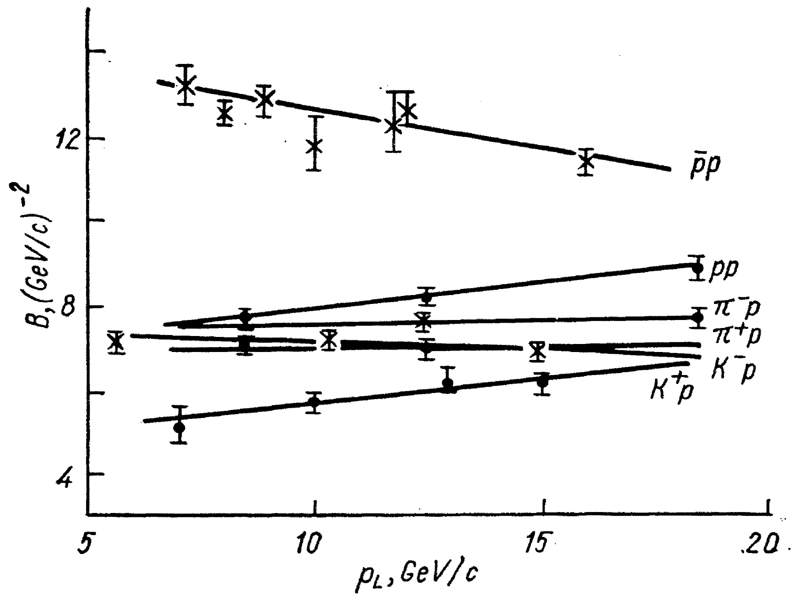


Fig. 7. Energy dependence of diffraction peak slopes [2, 4, 7].

diffraction peak slopes, the dip-bump structure in the angular distributions, the ratio $\alpha =$ $=$ Re T (0)/Im T (0), the energy dependence of the elastic scattering characteristics, etc.

a) Forward scattering $|t| <$ < 0.5 $(GeV/c)^2$. The differential cross sections of all elastic processes are sharply peaked in the forward direction, being parametrized rather well by the following formula

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{Bt}. \qquad (1.5)$$

The forward differential cross sections lie between 160 $mb/(GeV/c)^2$ for $\bar{p}p$-scattering and 20 $mb/(GeV/c)^2$ for K^+p-scattering at $p_L = 10$ GeV/c and decrease slowly with energy. For these quantities the following relation is fulfilled:

$$\left(\frac{d\sigma}{dt}\right)_0 (A^-p) \geqslant \left(\frac{d\sigma}{dt}\right)_0 (A^+p),$$

$$A^- = (\pi^-, K^-, \bar{p}),\ A^+ = (\pi^+, K^+, p). \qquad (1.6)$$

The differential cross sections of the exchange processes $\pi^-p \to \pi^0 n$ and $K^-p \to$ $\to \bar{K}^0 n$ are two orders of magnitude smaller than the corresponding elastic scattering differential cross sections.

The dependence of the slope parameter B on the energy for various processes is shown in Fig. 7. From Fig. 7 it is seen, that the following inequality holds:

$$B(A^-p) > B(A^+p). \qquad (1.7)$$

It follows from the relations (1.6) and (1.7) that the cross section difference $\frac{d\sigma}{dt}(A^-p) - \frac{d\sigma}{dt}(A^+p)$ may change the sign with increasing momentum transfer

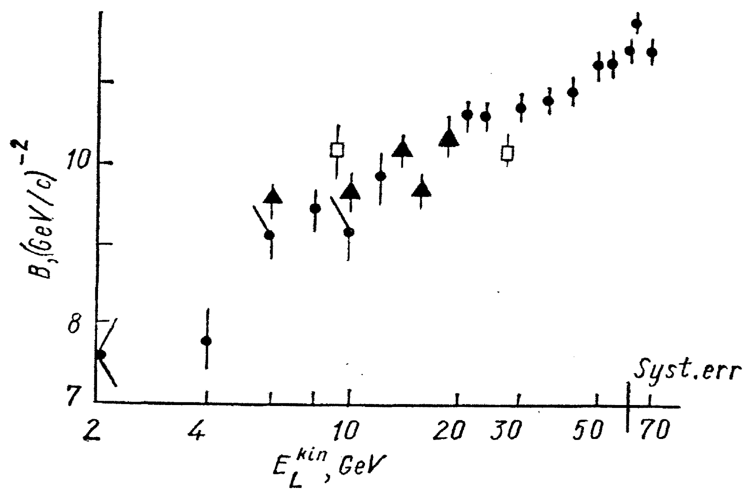


Fig. 8. Diffraction peak slope for pp-scattering [8].

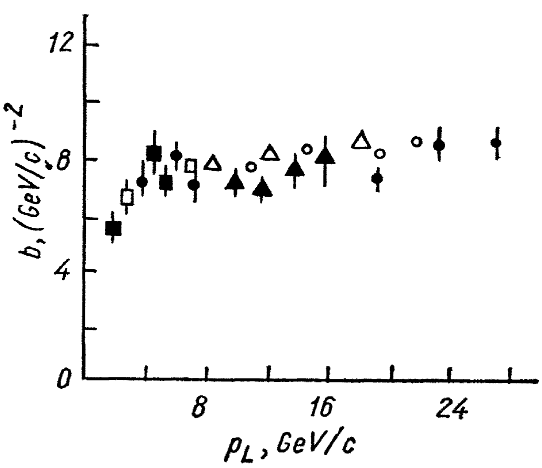


Fig. 9. Comparison of pp and pn-scattering diffraction peak slopes [9].

(cross-over phenomenon). The cross-over effect is observed for all elastic processes at $|t| \simeq 0{,}15 \div 0{,}20$ $(GeV/c)^2$ (Fig. 12).

The measurements show that with increasing energy the quantity $B(\bar{p}p)$ decreases while $B(pp)$ increases. Note that the recent experiments at the Serpukhov accelerator (up to 70 *GeV*) have shown a further shrinking of the diffraction peak in the *pp*-elastic scattering (see Fig. 8).

The experimental data show that the following relation between the *pp* and *pn*-elastic differential cross section (Fig. 9)

$$\frac{d\sigma}{dt}(pn) \simeq \frac{d\sigma}{dt}(pp) \qquad (1.8)$$

holds. We have already remarked that the corresponding total cross sections are approximately equal.

The angular distributions for the isobar production processes $pp \to pN^*_{I=1/2}$ are also strongly peaked in the forward direction. The quantity B, however, differs appreciably for the various isobars. The B — value for the production of N^* (1400) is twice as large as $B(pp)$, whereas the corresponding B — value for the production of N^* (1520) and N^* (1690) are only half of $B(pp)$. Note that the quantity B for elastic $\pi^{\pm}p$ and K^-p-scattering is almost energy independent, while for elastic K^+p-scattering it increases with energy (Fig. 7).

Information on the energy dependence of the ratio $\alpha = \mathrm{Re}\, T(s, 0)/\mathrm{Im}\, T(s, 0)$ can be obtained by measuring the differential cross section in the region of the Coulomb interference ($|t| = 0 \div 0.05\,(GeV/c)^2$). The main features of the high energy behaviour of α are as follows (see Fig. 10):

(I) the quantities $\alpha(\pi p)$, $\alpha(pp)$ are negative and lie in the interval $(0{,}4 \div 0{,}1)$ for incident momenta $p_L = (2 \div 25)$ GeV/c.

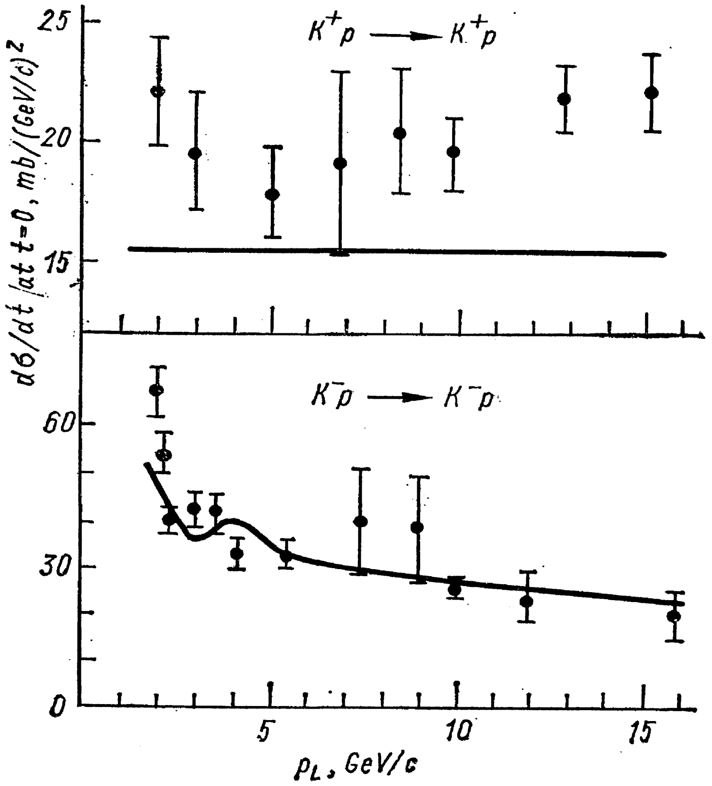


Fig. 10. Comparison of experimental data on $(d\sigma/dt)_0$ with optical theorem values for $K^{\pm}p$-scattering [2].

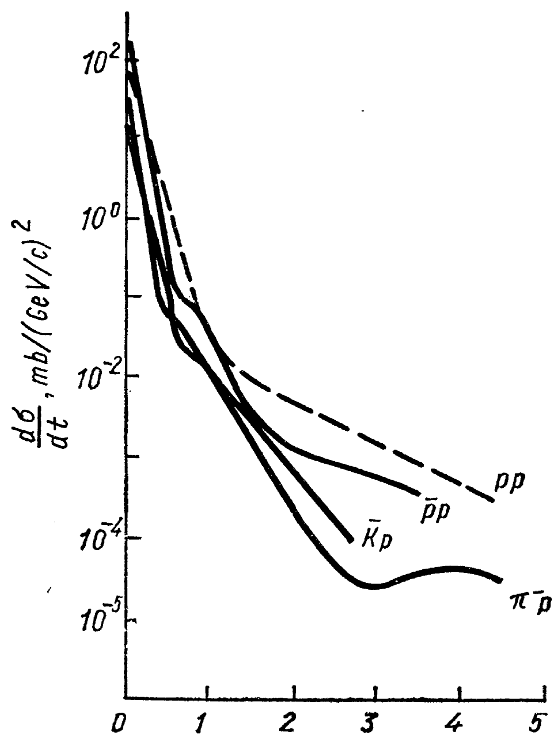


Fig. 11. Differential cross sections in the diffraction peak region.

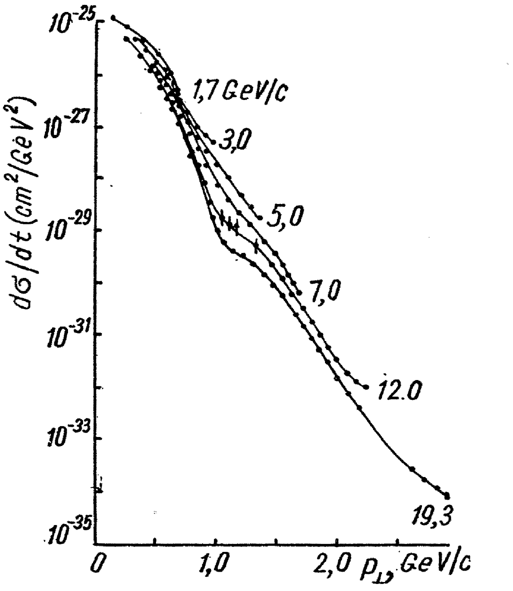


Fig. 12. Differential cross section of pp-scattering at large transverse momenta [10].

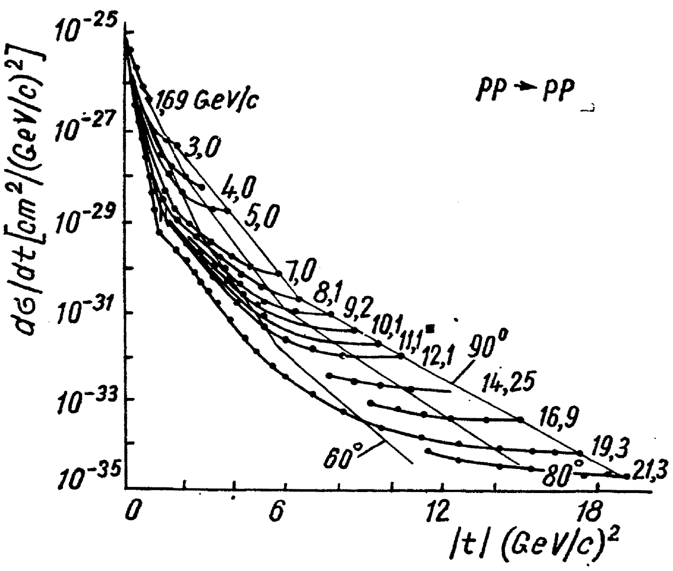


Fig. 13. Differential cross section of pp-scattering at large momentum transfer [10].

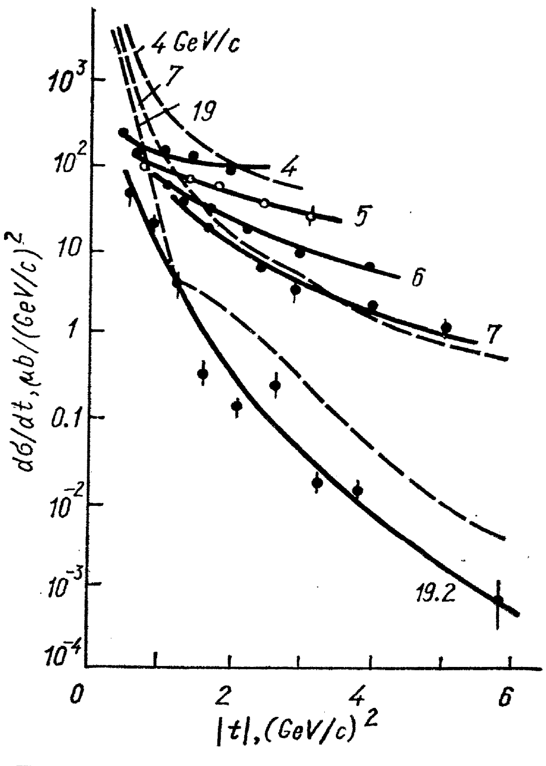


Fig. 14. Differential cross section of $pp \to pN^*$ (1512) process at large momentum transfer [11].

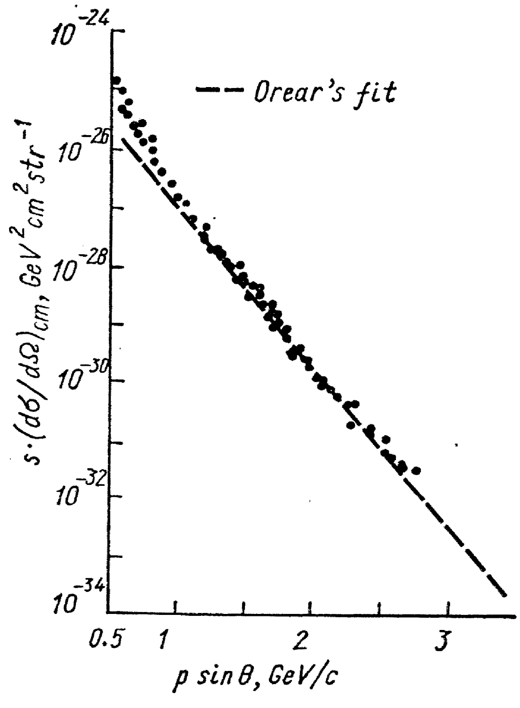


Fig. 15. Differential cross section of pp-scattering at large angles [10].

(II) the inequality $|\alpha (A^- p)| > |\alpha (A^+ p)|$ is fulfilled.

(III) $|\alpha (\pi p)|$ and $|\alpha (pp)|$ are monotonically decreasing functions of energy.

(IV) in the K^+p-scattering the α-value differs appreciably from zero $\alpha (K^+p)| \simeq 0{,}45$, while $\alpha (K^-p) \approx 0$ in the whole energy region $p_L = (3 \div 20)$ GeV/c.

(V) for π^-p-charge exchange the measurements give $\alpha \simeq 1$.

b) Structure in the diffraction region. Almost all diffractive elastic scattering processes are characterized by a dip-bump structure in the angular distributions at high energies. The angular distribution in the diffraction region exhibites the following features:

(I) in pp (pn) elastic scattering a break is observed in the differential cross section at $|t| = 0{,}8 \div 1{,}1$ $(GeV/c)^2$ (see Fig. 11).

(II) the dip at $|t| = 0{,}45 \div 0{,}60$ $(GeV/c)^2$ is followed by a bump at $|t| = 0{,}8$ $(GeV/c)^2$ in the $\bar{p}p$-scattering.

(III) in the angular distribution of $\pi^\pm p$-scattering a dip is observed at $|t| \simeq 0{,}6 \div 0{,}7$ $(GeV/c)^2$ and a bump at $|t| = 1{,}2 \div 1{,}4$ $(GeV/c)^2$.

(IV) the K^-p-differential cross section has a dip at $|t| \simeq 0{,}8 \div 1{,}0$ $(GeV/c)^2$.

(V) the angular distribution of the K^+p-scattering is a monotonically decreasing function of the momentum transfer.

(VI) in the charge exchange process $\pi^-p \to \pi^0 n$ at $|t| \simeq 0{,}04$ and $1{,}00$ $(GeV/c)^2$ maxima and at $|t| \simeq 0{,}6$ $(GeV/c)^2$ a minimum are observed. Note, that the angular distributions of the scattering of negatively charged particle on protons show a sharper dip-bump structure.

c) Large momentum transfers and large angles. In the region of large momentum transfers the cross sections decrease more slowly with increasing t (Figs. 12, 13).

There is a dip in the angular distribution of π^-p-scattering at $|t| \simeq 3{,}0$ $(GeV/c)^2$ which seems to be energy-independent.

At higher energies the differential cross sections for elastic pp-scattering as well as for isobar production processes seem to decrease at about the same speed (Fig. 14).

The behaviour of the differential cross section at large angles $60° < \theta < 120°$ has first been described by Orear (Fig. 15) in terms of the following empirical formula:

$$s \frac{d\sigma}{d\Omega} \simeq A e^{-p_\perp/b}. \qquad (1.9)$$

Recently, some deviations from the Orear formula have been revealed. Nevertheless, it describes rather well the decrease of the differential cross section.

d) Backward scattering. Most differential cross sections possess

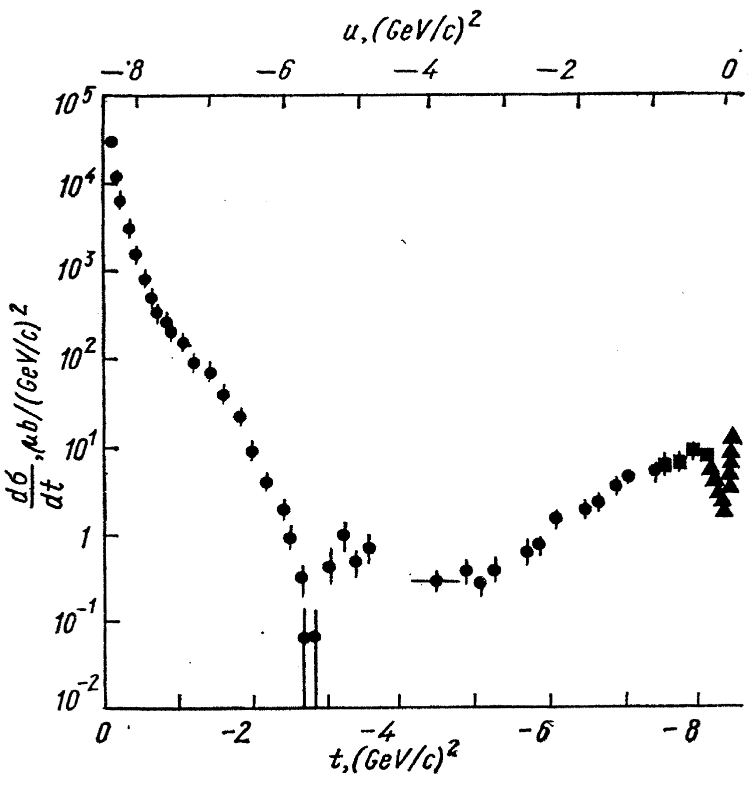


Fig. 16. Differential cross section of $\pi^\pm p$ scattering in the angular interval $\theta = 0° \div 180°$ [12].

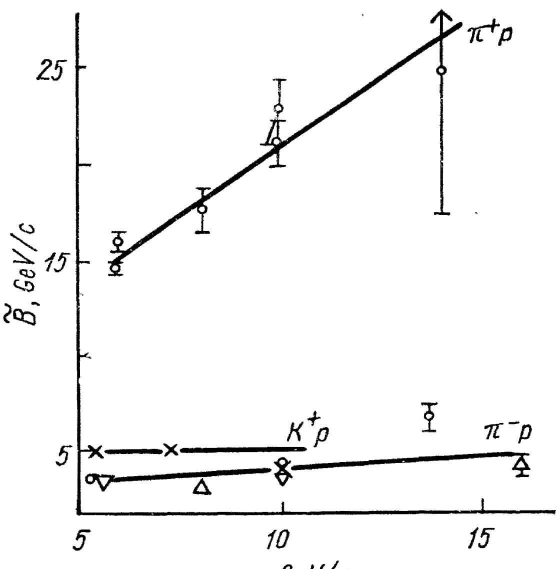


Fig. 17. Energy dependence of backward peak slopes [13].

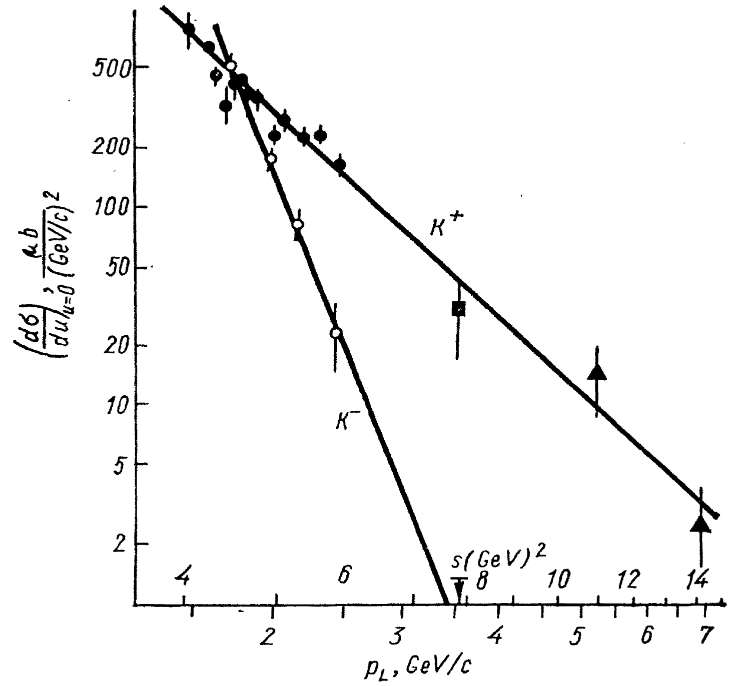


Fig. 18. Energy dependence of $(d\sigma/du)_0$ in $K^{\pm}$ p-backward scattering ($\theta \sim 180°$) [12].

a sharp backward peaking. However, the backward values are 2 — 3 orders of magnitude lower than the corresponding forward values (Fig. 16).

A typical property of the backward peaks is its rapid decrease with increasing energy. The backward scattering peak is rather well described by the formula:

$$\frac{d\sigma}{du} = \left(\frac{d\sigma}{du}\right)_{u=u_{\max}} e^{\tilde{B}u}, \quad \theta \sim 180°. \qquad (1.10)$$

The values of the backward differential cross sections cover the range from 1 $mb/(GeV/c)^2$ for $pn \to np$ scattering to 0,001 $mb/(GeV/c)^2$ for K^-p-scattering. The quantity $(d\sigma/du)_{u_m}$ for the charge exchange processes is of the same order of magnitude. It is interesting to compare the following experimentally established inequality for backward scattering,

$$\left(\frac{d\sigma}{du}\right)_{u_m}(A^-p) < \left(\frac{d\sigma}{du}\right)_{u_m}(A^+p), \qquad (1.11)$$

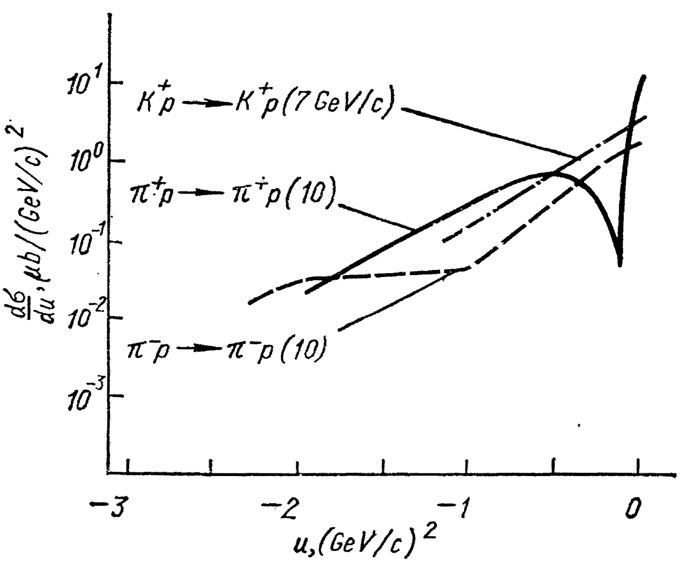


Fig. 19. Backward scattering differential cross sections ($\theta \sim 180°$) [2].

with the corresponding inequality Eq (16) for forward scattering. The energy dependence of the slope parameter $\tilde{B}$ differs appreciably for the various reactions. The following inequality:

$$\tilde{B}(A^-p) < \tilde{B}(A^+p) \qquad (1.12)$$

holds.

It is worthwhile to note, that the forward and backward peak slope parameters for K^+p-scattering are equal to each other,

$$\tilde{B}(K^+p) \simeq B(K^+p). \qquad (1.13)$$

While the forward scattering peaks for $\pi^{\pm}p$-scattering are appro-

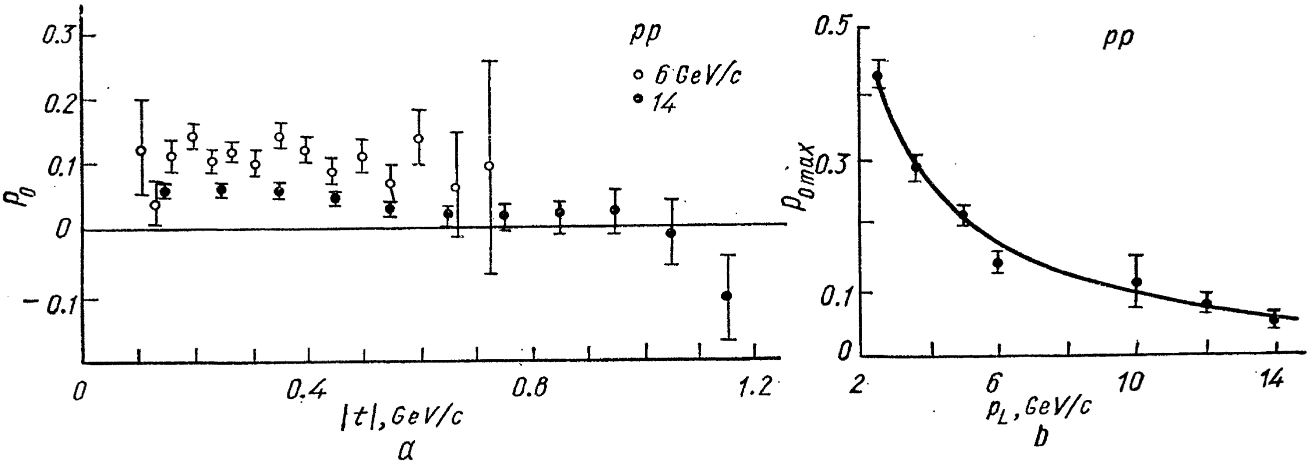


Fig. 20. *a*) Polarization parameter for *pp*-scattering at $P_L = 6$, and 14 *GeV/c*. *b*) Energy dependence of the maximum of polarization for *pp*-scattering [2].

ximately equal to each other and do not depend on energy, the situation for the backward peaks is different. The slope parameter $\tilde{B}$ (π^-p) does not depend on energy and is about twice as small as B (π^-p). $\tilde{B}$ (π^+p), however, increases with energy and is about twice as large as B (π^+p) (Fig. 17).

Because of the rapid decrease of the differential cross sections (Fig. 18) for K^-p and $\bar{p}p$ scattering near $\theta \sim 180°$, these processes are not yet observed above 4—5 *GeV/c*. Note that in the differential cross sections for π^+p-scattering as well as for $\pi^-p \to \pi^0 n$ charge exchange scattering, a dip at $|u| \simeq 0.2$ $(GeV/c)^2$ is observed. The differential cross sections for K^+p- and π^-p-scattering do not show such a dip-bump structure (Fig. 19).

3. POLARIZATION MEASUREMENTS

In Figs. 21—22 the results of polarization measurements are shown. In general, the polarization parameters decrease with increasing energy, the most rapid decrease being observed in the *pp*-scattering (Fig. 20).

The polarization parameters in the π^-p- and π^+p-scattering have opposite signs. P_0 ($\pi^-p \to \pi^0 n$) seems to be positive. Recent measurements covering a momentum transfer interval $0 \leqslant |t| \leqslant 2.0$ $(GeV/c)^2$ show that the polarizations in the $\pi^\pm p$-scattering are approximately mirror-symmetric and do not change sign. There is a maximum near $|t| \simeq 1.0$ $(GeV/c)^2$. In the range $|t| \simeq 1 \div 2$ $(GeV/c)^2$ the polarization slowly decreases. The

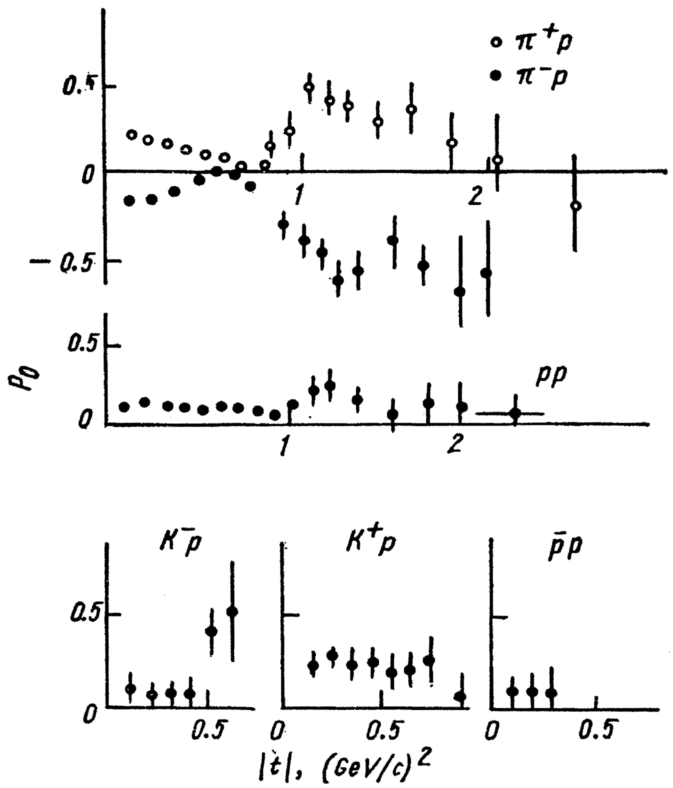


Fig. 21. Polarization parameters for *pp*, $\bar{p}p$, $\pi^\pm p$ and $K^\pm p$-scattering [14].

measurements of the polarization parameter in the pp-scattering seem to favour the positive sign. It is also interesting to note that the polarization in the K^-p and K^+p-scatterings have the same sign.

There are only a few polarization measurements in the backward scattering region. In the π^+p-scattering at $p_L = 3.3$ GeV/c an appreciable structure is observed. The polarization in the np-backward scattering is negative and a monotonic function of $|t|$ [15] (Fig. 21).

Section II

Regge-Phenomenology

1. MULTI-REGGE-POLE APPROXIMATION (MRPA)

In describing the experimental data on the high energy particle scattering up to $p_L \leqslant 20$ GeV/c the most widespread approach is based on the multi-Regge-pole approximation for the scattering amplitude (MRPA). In the framework of MRPA one assumes that the asymptotic behaviour of the scattering amplitude of the process $a + b \to c + d$ at $|z_t| \gg 1$ is determined by a finite number of t-channel Regge-poles [1],

$$T(s, t) = -\sum_i \beta_i(t) \left[\frac{1 + \tau_i e^{-i\pi\alpha_i(t)}}{\sin \pi\alpha_i(t)}\right] \nu^{\alpha_i(t)}, \tag{2.1}$$

$$\nu = \frac{s - u}{2s_0},$$

where $\alpha_i(t)$, $\beta_i(t)$ and $\tau_i = \pm 1$ are the trajectory, the residue function and the signature of the i-th boson Regge-pole, respectively. In the case of meson-baryon backward scattering the asymptotic behaviour of the amplitude at $|z_u| \gg 1$ is determined by the u-channel fermion Regge trajectory contributions

$$T(s, u) = -\sum_i \beta_i^{\pm} \left[\frac{1 + \tau_i e^{-i\pi(\alpha_i^{\pm} - 1/2)}}{\sin \pi(\alpha_i^{\pm} - 1/2)}\right] (\nu')^{\alpha_i^{\pm} - 1/2}, \tag{2.2}$$

$$\nu' = (s - t)/2s_0.$$

The poles in the summation of Eq. (2.2) enter in complex conjugate pairs, according to the usual approach. Note, that there is a relationship between corresponding trajectories and residue functions given by the McDowell symmetry

$$\begin{aligned} \alpha^+(\sqrt{u}) &= \alpha^-(-\sqrt{u}), \\ \beta^+(\sqrt{u}) &= -\beta^-(-\sqrt{u}). \end{aligned} \tag{2.3}$$

Indices $\pm$ in Eq. (2.3) correspond to the values of $P = \pm 1$. The exchange degeneracy assumption leads in the case of fermion Regge-poles to a difficulty which is connected with the parity doubling of the baryon resonances. In the paper by Halzen, Kumar and Martin [2] for πN-scattering an attempt is made to avoid parity doublets by introducing kinematical cuts in the j-plane.

The Regge-pole residue functions factorize

$$\beta_{ab \to cd} = \gamma_{a\bar{b}}(t) \cdot \gamma_{\bar{c}d}(t). \tag{2.4}$$

The vertex functions $\gamma(t)$ are usually assumed to be smooth (exponentially decreasing) functions of t.

a) Classification of Regge-poles. The quantum numbers of the dominant boson Regge-poles (with zero strangeness) are given in the table 1. When considering exchange processes, e. g. $np \to pn$, or resonance production processes of the type $\pi^+ p \to \rho^0 \Delta^{++}$, one introduces also poles with quantum numbers of the π, A_1 and B-mesons, respectively.

In the meson-baryon backward scattering processes the dominant trajectories are

$$\left.\begin{array}{l} T = {}^1/_2 : N_\alpha,\ N_\gamma \\ T = {}^3/_2 : \Delta_\delta \end{array}\right. \quad \text{(non-strange exchanges)}$$

and

$$T = 1,\ S = -1 : \sum_{\alpha,\gamma} \quad \text{(strangeness exchange).}$$

In data fitting one has to introduce a number of additional poles: P'', ρ', ω' etc. (see the paper of Barger and Phillips [4]). Note that in the framework of MRPA, cut and background contributions are assumed to be small.

Table 1

Quantum Numbers / Poles	T	P	C	τ
P, P'	0	+	+	+
ρ	1	—	—	—
ω, φ	0	—	—	—
A_2	1	+	+	+

The main idea of the Regge-pole model is the connection between the behaviour of the trajectory function $\alpha(t)$ in the scattering region $t < 0$ and the resonance spectrum in the crossed channel $t > 0$. This idea does not contradict experimental data. It turns out, that the linear approximation for $\alpha(t)$ against t is good enough.

Such a behaviour extrapolated to the region of large $t > 0$ leads to the infinitely rising trajectories.

A number of papers is devoted to the explanation of the mechanism of the infinitely rising trajectories. Among them we note the paper by Tiktopoulos [5], where the author, starting from the quasipotential equation, obtains an infinitely rising trajectory as a consequence of the choice of the quasipotential, which increases at high energies. In the paper of Shirkov submitted to the conference [6], a self-sustaining regime for Regge trajectories is discussed. In this approach the asymptotic growth of the real part Re $\alpha(s)$ is completely determined by the growth of the dispersion integral of Im $\alpha(s)$. Such a regime is possible when the total width along the trajectory increases proportionally with the mass

$$\Gamma_{\text{tot}}(s) \to \text{const}\sqrt{s}, \quad s \gg 1, \tag{2.5}$$

and the infinite growth of the trajectory is due to the transitions between the levels lying on the trajectory.

Notice, that the experimental data on the baryon trajectory widths do not contradict this relation, while it fails for the meson trajectories in the region of the energies investigated.

The Pomeranchuk pole has a special place in this scheme. In spite of its exceptional role the problem of its dynamical nature is completely unclear. There are large uncertainties in the determination of its parameters. In the paper by Barger and Cline [7] the reaction $\gamma p \to \Phi p$ is discussed. Assuming vector meson dominance and using the quark model for the particles involved, it is established that the only trajectory which contributes to this reaction is the Pomeranchuk trajectory. Analyzing the experimental data on the vector meson photoproduction, the authors conclude that $\alpha'_P = {}^1/_2$.

The paper of Fudjisaki [8] is devoted to the investigation of the self-reproducability of the Pomeranchuk singularity. The trajectory function obtained in this paper is of the form $\alpha(t) = 1 + \gamma\sqrt{t}$; earlier such a behaviour was discussed also by Schwartz [9].

b) Unequal mass case and spin consideration. The Regge trajectory $\alpha(t)$ and the residue function $\beta(t)$ in the case of the scattering of two spinless particles of equal masses are real analytic functions, having only the dynamical cuts with branch points coinciding with the thresholds. They are regular at $t = 0$. In the general case of the scattering of unequal mass spinning particles the trajectory $\alpha(t)$ remains unchanged, but additional singularities appear in the residue functions at $t = 0$ and at thresholds and pseudothresholds. These complications and their significance for the phenomenological analysis have been intensively studied in recent years. (See the review talk by Bertocchi at the Heidelberg conference 1967). Note that the most complete analysis of the singularity structure of the helicity amplitudes is given in the paper by Cohen — Tannoudji et al. [10] (see also [11]). From the phenomenological point of view the most interesting problem is the knowledge of the behaviour of the residue functions at $t = 0$, since it lies closer to the s-channel physical region.

The parity-conserving t-channel helicity amplitude for the reaction $a + c \to b + d$ can be written as follows:

$$f^{\sigma}_{\lambda\mu} \sim \beta^{\sigma}_{\lambda\mu}(t)\left(\frac{s}{p_t \cdot p_t'}\right)^{\alpha(t)-\lambda_m}, \tag{2.6}$$

$$\lambda = \lambda_a - \lambda_{\bar{c}}, \quad \mu = \lambda_{\bar{b}} - \lambda_d, \quad \lambda_m = \max\{|\lambda|, |\mu|\},$$

where λ_i denote the particle helicities, p_t and p_t' the initial and final momenta in the t-channel c. m. frame, $\sigma = P \cdot \tau$ the natural parity of the trajectory, τ the signature and P the usual parity. Taking into account the factorization theorem

$$\beta^{\sigma}_{\lambda\mu} = \beta^{\sigma}_{\lambda\lambda} \cdot \beta^{\sigma}_{\mu\mu}, \tag{2.7}$$

one can show [12], that for the three possible t-channel mass configurations EE ($m_a = m_c$, $m_b = m_d$), UE ($m_a \neq m_c$, $m_b = m_d$) and UU ($m_a \neq m_c$, $m_b \neq m_d$), the residues behave like

$$\beta^{\sigma}_{\lambda\mu}(t) \sim t^{N(\lambda,\mu,\sigma)}, \tag{2.8}$$

where the power N is equal to

$$N_{EE} = \frac{1}{4}[1 - \sigma(-1)^{\lambda+M+n} + (1 - \sigma(-1)^{\mu+M+n})],$$

$$N_{UE} = \frac{1}{2}\left[\frac{1}{2}(1 - \sigma(-1)^{\mu+M+n}) + |M - |\lambda|| - \alpha^0(t)\right], \tag{2.9}$$

$$N_{UU} = \frac{1}{2}[|M - |\lambda|| + |M - |\mu||] - \alpha^{(0)}(t),$$

$\alpha^{(0)}(t)$-parent trajectory, respectively. Here M is the well-known Toller quantum number, with respect to which the trajectories are classified, $n = 0, 1, \ldots$ for the «parent», first «daughter», etc.

Thus, the formulae Eqs. (2.8) — (2.9) allow one to determine immediately the behaviour of each helicity amplitude at $t = 0$. This is an important point in the phenomenological analysis.

In the paper by Capella [13] (submitted to the conference), which is devoted to the problem of the pion trajectory, arguments are given that the pion belongs to the nonconspiring ($M = 0$) parent ($n = 0$) trajectory. Such a classification seems to be more natural, though there are supporters of the classification with $M = 1$.

A number of papers is devoted to the study of consequences of the hypothesis of the s-channel helicity conservation in the diffractive production of hadrons [14]. Such a hypothesis leads to interesting experimental consequences. In the case of πN-scattering the s-channel helicity amplitudes are related to the invariant amplitudes A and B in the following manner:

$$\begin{aligned} f^{(s)}_{\frac{1}{2}0;\,\frac{1}{2}0} &\xrightarrow[s\to\infty]{} A + \frac{s}{2m} B, \\ f^{(s)}_{-\frac{1}{2}0;\,\frac{1}{2}0} &\xrightarrow[s\to\infty]{} \frac{\sqrt{-t}}{2m}(A + mB). \end{aligned} \tag{2.10}$$

The amplitudes $A\ (s,\ t)$ and $B\ (s,\ t)$ possess the usual Regge asymptotics $A \to s^{\alpha(t)}$, $B \to s^{\alpha(t)-1}$. It is easily seen, that the spin-nonflip amplitude $f^{(s)}_{\frac{1}{2}0;\,\frac{1}{2}0}$ will be larger than the spin-flip amplitude $f^{(s)}_{-\frac{1}{2}0;\,\frac{1}{2}0}$ only in the case, when the leading Regge-pole (e. g. Pomeranchuk-pole) does not contribute to the invariant amplitude A. An experimental verification of this hypothesis is difficult to obtain since a direct measurement of the Wolfenstein parameters R and A is needed. Preliminary data [15] on $\pi^- p$-scattering at 6 GeV/c, however, do not contradict the condition $A\ (s,\ t) = 0$. Similarly, in order to check the s-channel helicity conservation hypothesis in NN-scattering, polarization experiments are necessary. In particular, using the continuum momentum sum rules, Barger and Phillips [4] have shown that the t-channel spin-nonflip amplitude A' ($\approx A + \nu \cdot B$) and spin-flip amplitude B are connected by the relation $A' \simeq \nu \cdot B$, i. e. $A = 0$ for the P and P'-trajectories. This means that the crossing-symmetric amplitude $A^{(+)} \equiv {}^1/_2\,(A_{\pi^- p} + A_{\pi^+ p})$ satisfies unsubtracted dispersion relations, leading thus to a number of sum rules. It is interesting to note, that some relations between the isobar decay constants in this approach agree with the $SU\,(2) \otimes SU\,(2) \cdot T_9$-symmetric theory of strong coupling and differ from the corresponding $SU\,(6)$ predictions.

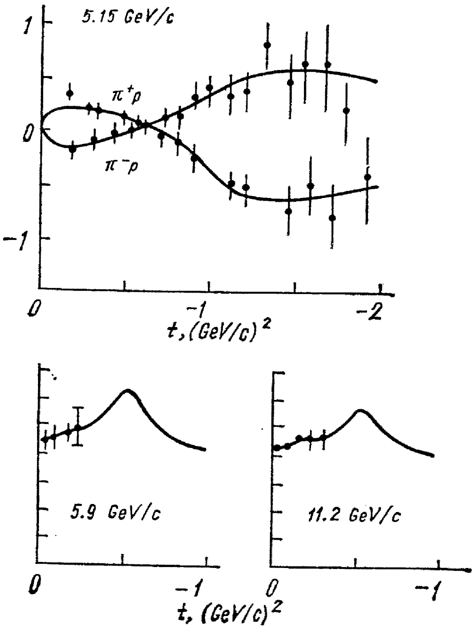
We note, in conclusion, that the notion of the «non-essential» complications arising in the treatment of spin effects become gradually a notion of the past and a number of interesting discoveries in the theory is connected with deeper understanding of the role of the spin.

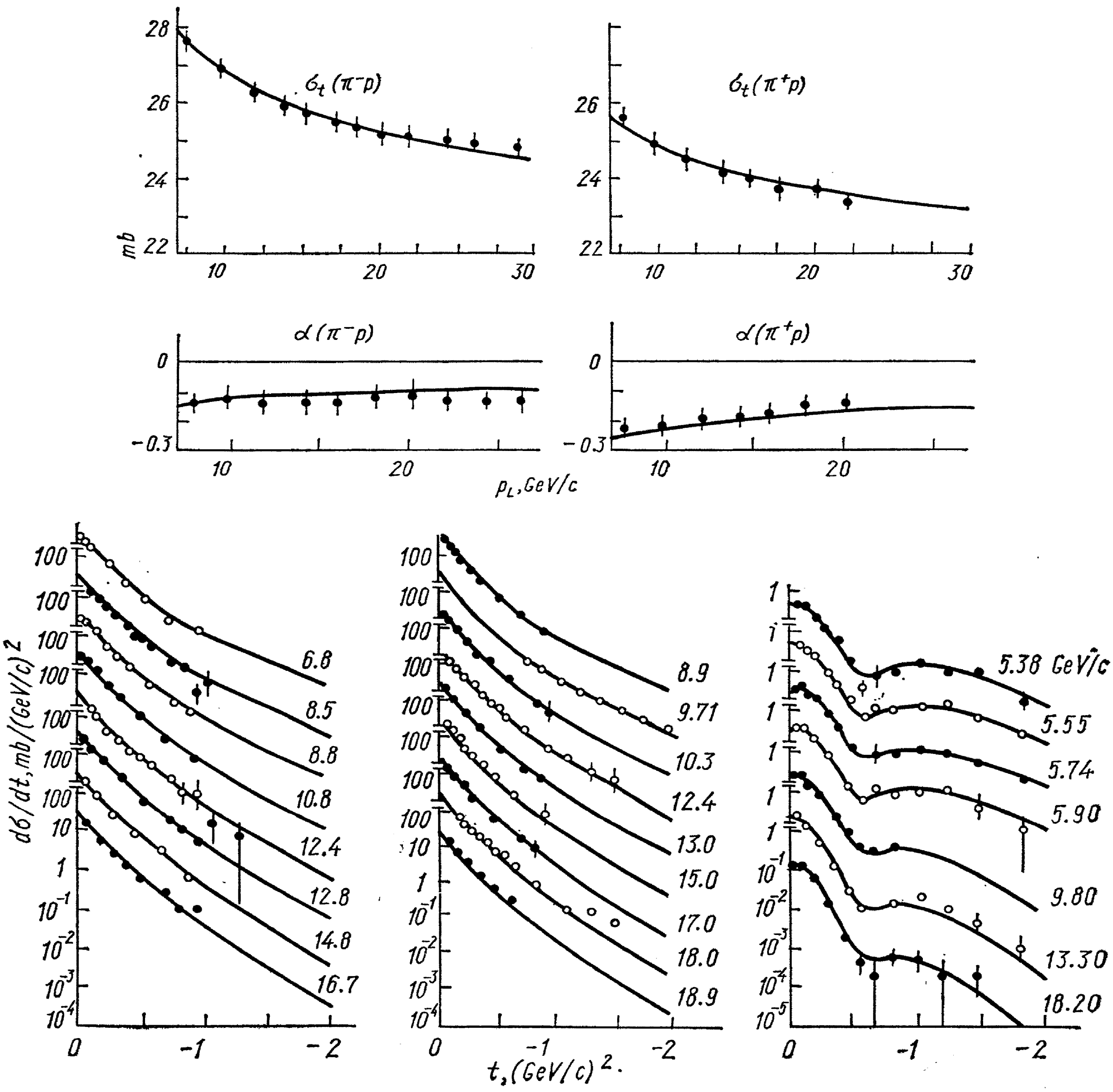
c) Finite energy sum rules. In order to determine the parameters of MRPA finite energy sum rules (FESR) [16] are widely used. We do not deal here with the problems connected with the further development of FESR which led to the construction of dual models, since they are discussed in the rapporteur's talk by Prof. Veneziano.

Finite energy sum rules serve to connect the Regge parameters with the scattering amplitude at low and intermediate energies. For the description of the low energy amplitude, phase-shift analysis data are often used. The contribution of the intermediate energy region (2—4 GeV) is sometimes calculated assuming the validity of the Regge approximation above 2 GeV. This assumption depends on the low momenta sum rules. Its application to the higher momenta sum rules can lead to a contradiction.

Aznauryan and Soloviev [17] have developed a method of taking into account the intermediate energy region (2 — 4 GeV) without using this assumption. Solving the corresponding extremum problem, the authors were able to estimate the maximum of uncertainty which can arise from the region of intermediate energies. The method has been applied to the determination of the spin rotation parameters in the πN-scattering above 4 GeV. Preliminary experimental data at 6 GeV agree with the results obtained.

Fig. 22. Comparison of MRPA — predictions with experimental data on πN-scattering [4].





In the paper [18] by Steiner submitted to the conference, the region of the convergence of the sum rules with respect to t is studied. An estimate is given, $t \geqslant -0{,}5\ GeV^2$ for FESR, and $t \geqslant 0{,}2\ GeV^2$ for CMSR.

The problem of taking into account cut contributions to the FESR is discussed in the papers by Rivers; Jenkovsky et al. [19] and others. Schrempp [20] makes use of the K-matrix-formalism to account for the cut contributions.

d) Predictions and shortcomings of MRPA. In the framework of the five-pole (P, P', P'', ρ, ρ')-model Barger and Phillips [4] have given a complete analysis of the πN-scattering. The fitting of the data for the πN elastic and charge-exchange scattering is performed using the continuum momentum sum rules (CMSR) in the region of energies up to 30 GeV and momentum transfers in the

$$0 < t < -2\,(GeV/c)^2 \text{ — interval (Fig. 22).}$$

The authors predict the behaviour of the spin rotation parameters R and A (Fig. 23). Note that, from the analysis of FESR [16] some restrictions on these parameters are obtained (Aznauryan, Soloviev [17]).

In the case of πN-scattering predictions for the cross sections, polarizations and $\alpha = \mathrm{Re}\,T\,(s,0)\,/\mathrm{Im}\,T\,(s,\,0)$ are given up to $p = 300\ GeV/c$. With the help of the P, P', ρ, ω, A_2-pole model the data on the cross sections and polarizations in the KN-scattering are fitted (Plaut, Carreras, Donachie) [21, 22]. A comparison of two possible mechanisms for the residue functions (Chew-mechanism and non-compensation mechanism) has been made for the case of K^-p-scattering [22]. The polarization in the K^-p-elastic scattering is positive and vanishes at $|t| \approx \approx 1.0\ (GeV/c)^2$, while the polarization in the K^-p-charge exchange has a clear-cut structure and vanishes at $|t| \simeq 0{,}5\ (GeV/c)^2$ (Fig. 24, 25). Up. to $p_L = 50\ GeV/c$ a negative polarization is predicted in the K^+p-scattering, but this contradicts the new experimental data [1, 14] (see Section 1, Fig. 22). In describing the πN and KN-scattering data, various models introducing the secondary poles (ρ, A_2'', dipoles, etc.) have been used [23—25]. The results of the data fitting are given in Fig. 26. In Fig. 27*g* the charge ecxhange differential cross sections are presented, which are almost identical in all above-mentioned papers, while the polarizations are rather sensitive to the models used.

The experimental data are fitted satisfactorily, but the prediction in the region of larger t differ widely. In the paper [26] charge-exchanges in the NN-system have been considered. π, B, B' (conspirator)-trajectories are used. The comparison of the differential cross sections of the $p\bar{p} \to n\bar{n}$ and $pn \to np$-processes with experimental data (Fig. 27) at $p_L = 3 \div 8\ GeV/c$ has been performed. Note, that in the papers on πN and KN-charge exchange polarizations, a simultaneous analysis of the elastic and exchange processes has not yet been done. The quasi-two-body processes $0^-\frac{1}{2}^+ \to 0^-\frac{3}{2}^+$ have also been considered (Gizbert — Studnicki, Golemo) [27, 28] (ρ, A_2-exchanges). The differential cross sections of the reactions $\pi N \to \pi\Delta$, $\eta\Delta$; $KN \to K\Delta$ have been fitted in the momentum region $p_L = 3 \div 16\ GeV/c$ [27] (Fig. 28). Vector meson and Δ^{++}-isobar production processes

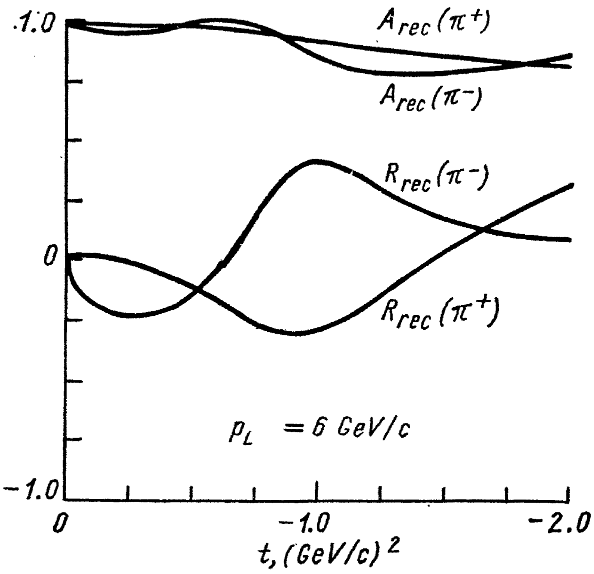


Fig. 23. Spin rotation parameters R and A [4].

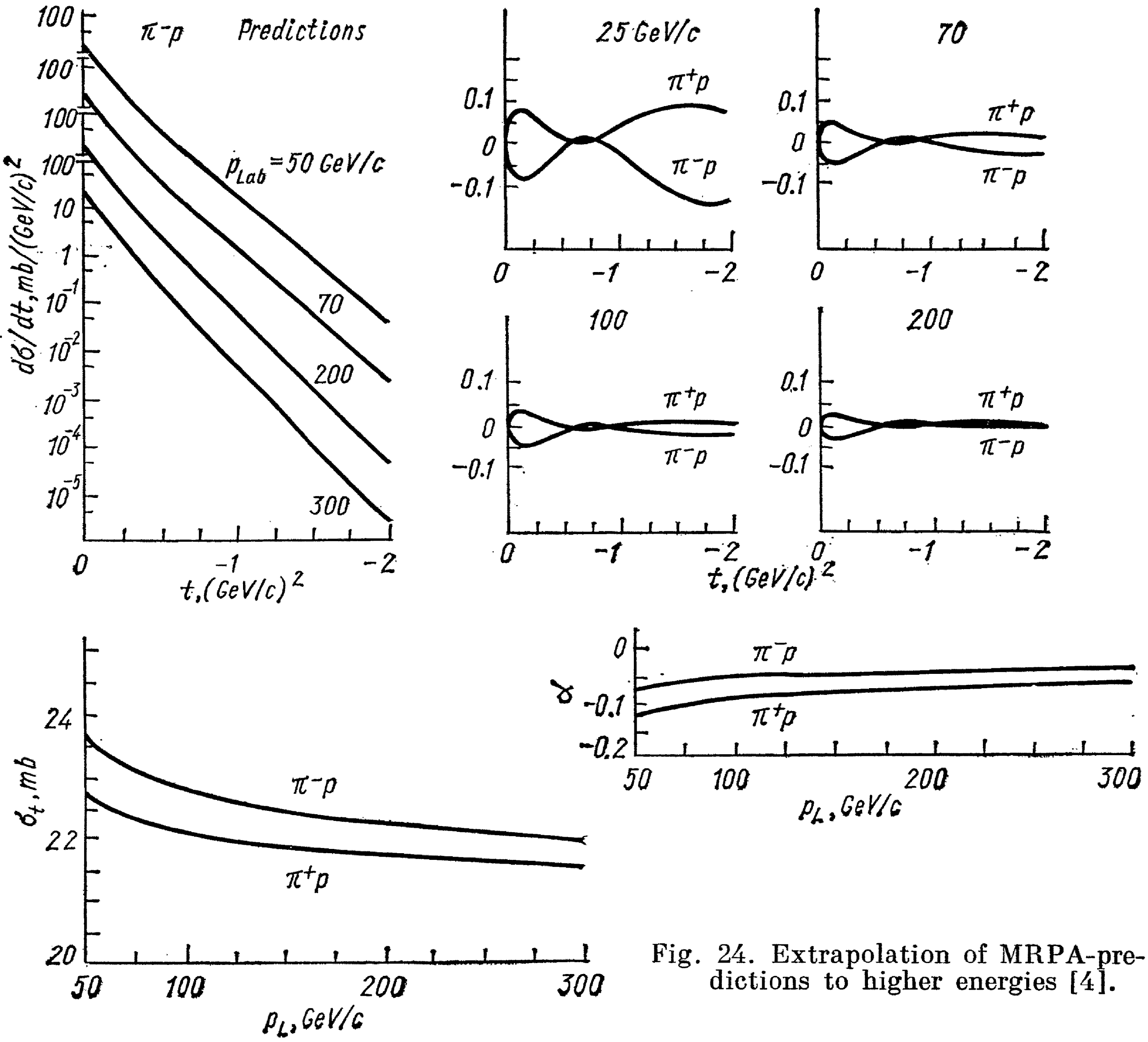


Fig. 24. Extrapolation of MRPA-predictions to higher energies [4].

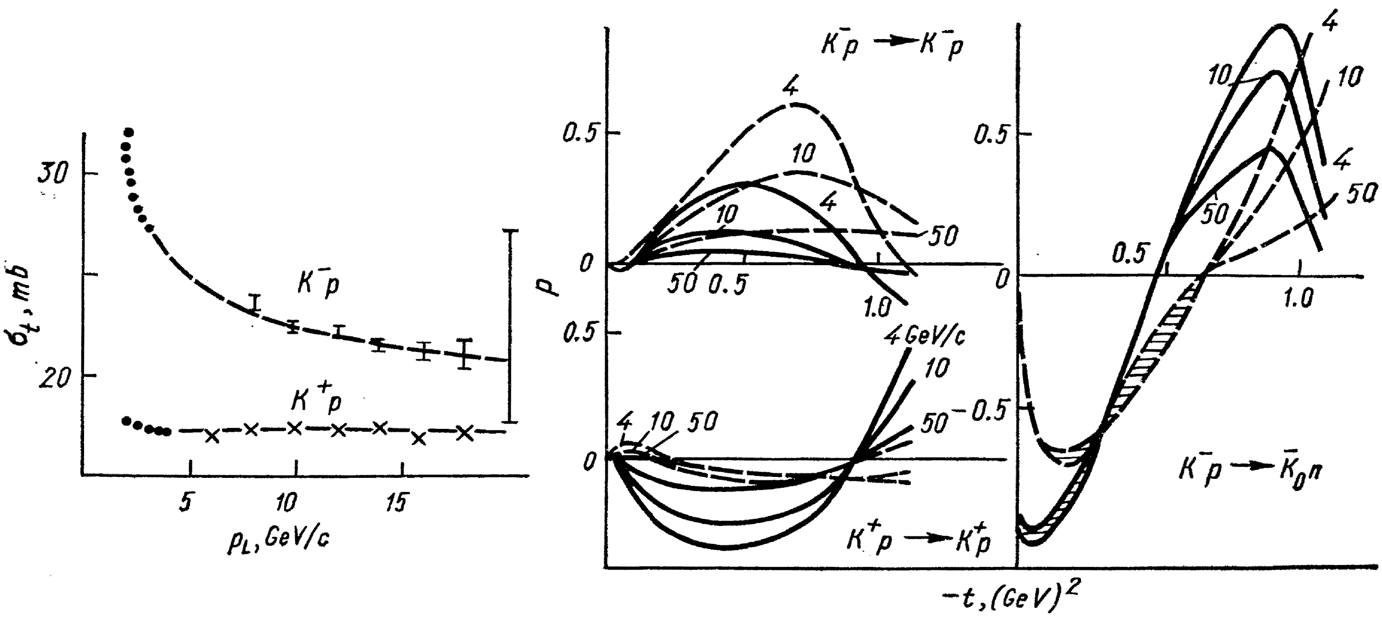


Fig. 25. Total cross sections and polarizations for scattering in MRPA.

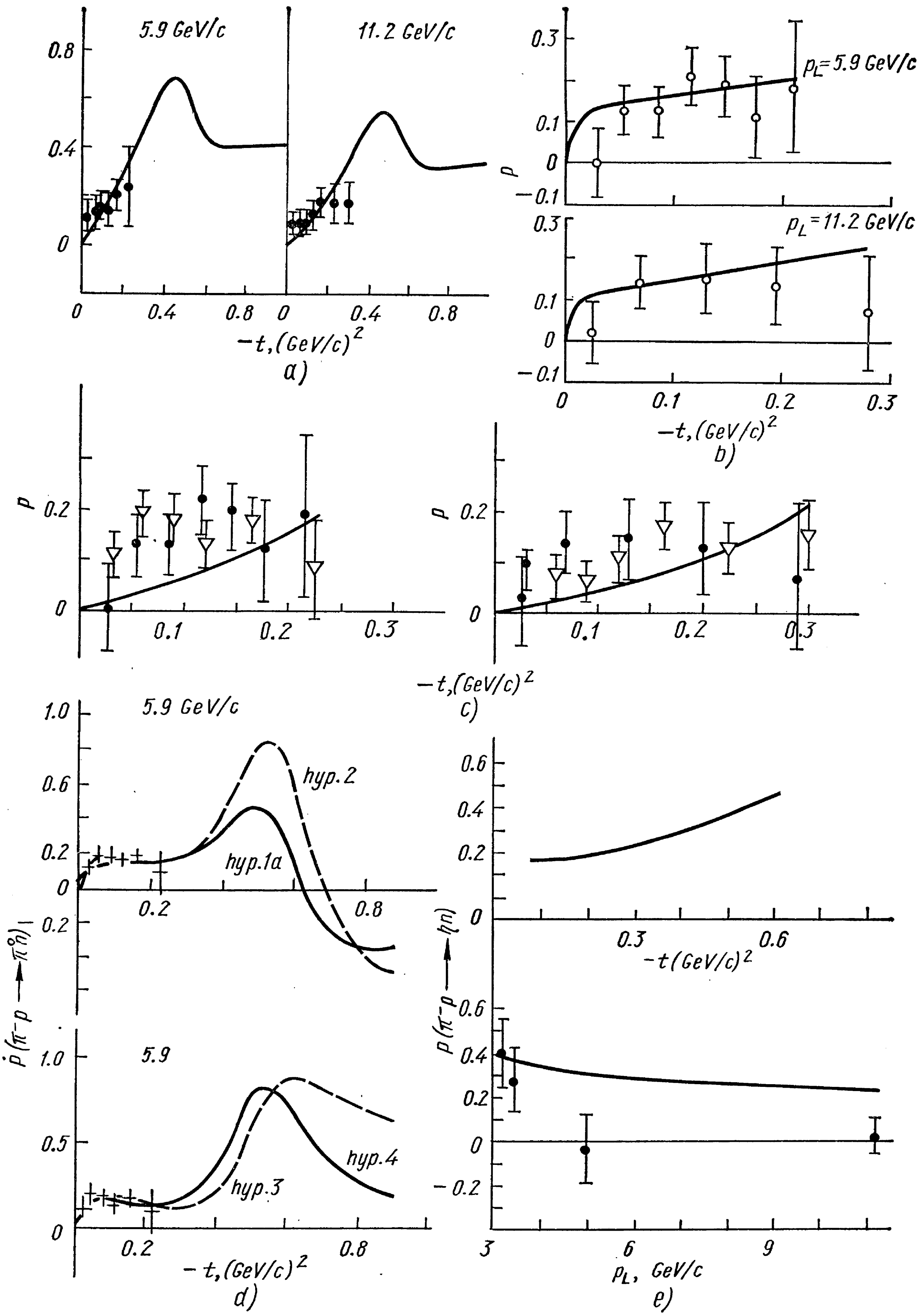


Fig. 26. Differential cross sections and polarizations for πN-charge exchange: *a)* $P_0(\pi^- p \rightarrow \pi^0 n)$ [23]; *b)* $P_0(\pi^- p \rightarrow \pi^0 n)$ [23]; *c)* $P_0(\pi^- p \rightarrow \pi^0 n)$ [24]; *d)* $P_0(\pi - p \rightarrow \pi^0 n)$ [25]; *e)* $P_0(\pi - p \rightarrow \eta n)$ [25];

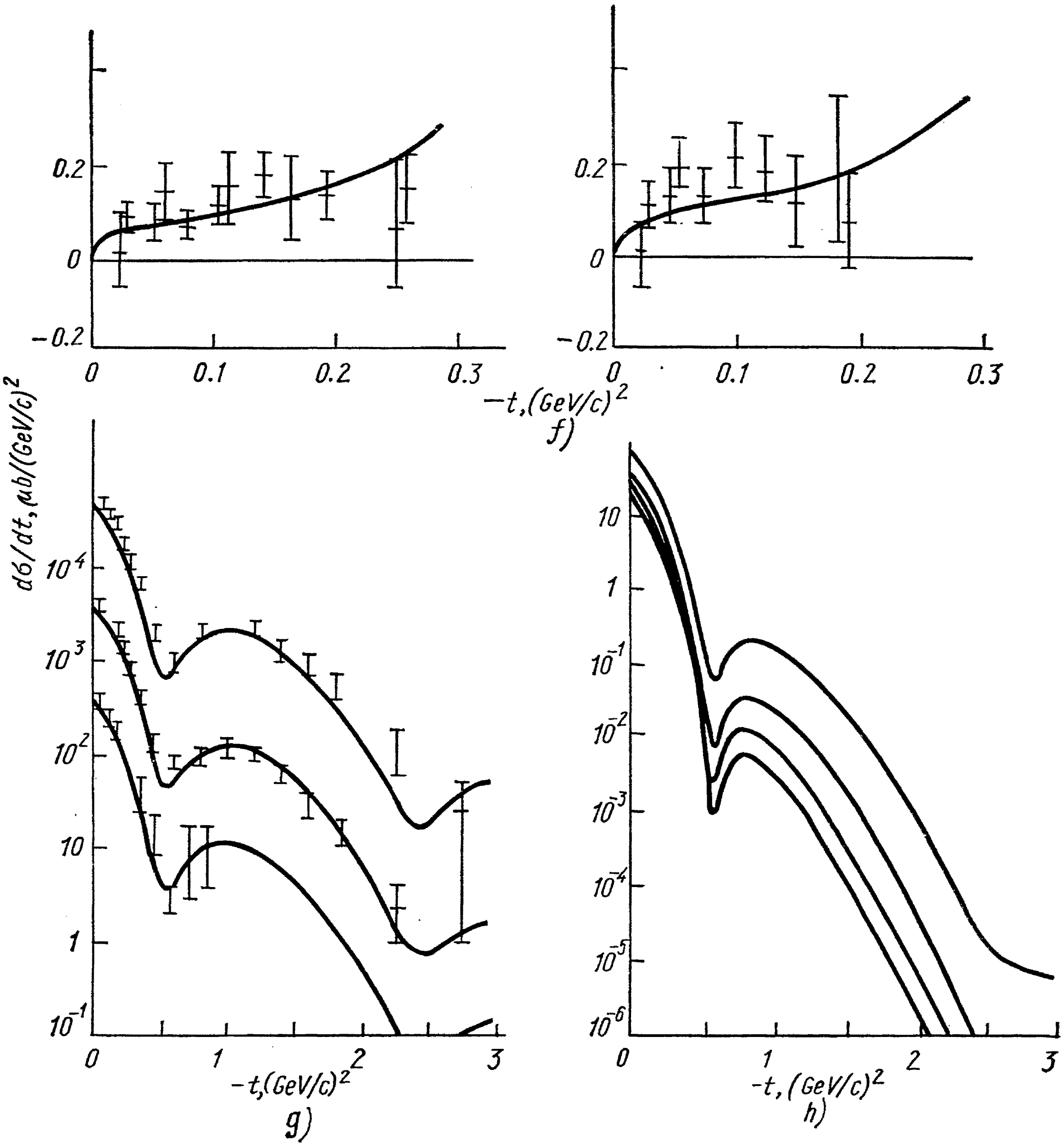


Fig. 26.

f) $P_0(\pi^- p \to \pi^0 n)$ [24];

g) $\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)$ [24]; *h)* $\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)$ predictions [24].

are investigated in the paper [29] (Darham, Genova, Hamburg, Milano, Saclay Collaboration). The processes $\pi^+ p \to \rho^0 \Delta^{++}$ (π-trajectory) and $\pi^- p \to \rho^0 n$ (π, A_2-trajectories) are simultaneously analyzed. The same process $\pi^+ p \to \rho^0 \Delta^{++}$ and the process $\pi^+ p \to \omega^0 \Delta^{++}$ (π-B-exchange degenerate trajectories) are analyzed in the paper [30] (Abrams, Maor).

In the paper [31] (Ming Ma, Smith, Sprafka, Williamson) the process $pp \to n\Delta^{++}$ is discussed in the framework of the (π, ρ, A_2)-pole model. A comparison with the one-pion exchange model is performed.

In the process of Y_1^{*+} (1385)-isobar production $\pi^+ p \to K^+ Y_1^{*+}$ and $K^- p \to \pi^- Y_1^{*+}$ there is a contribution of the two-meson Regge trajectories (K^* (892)

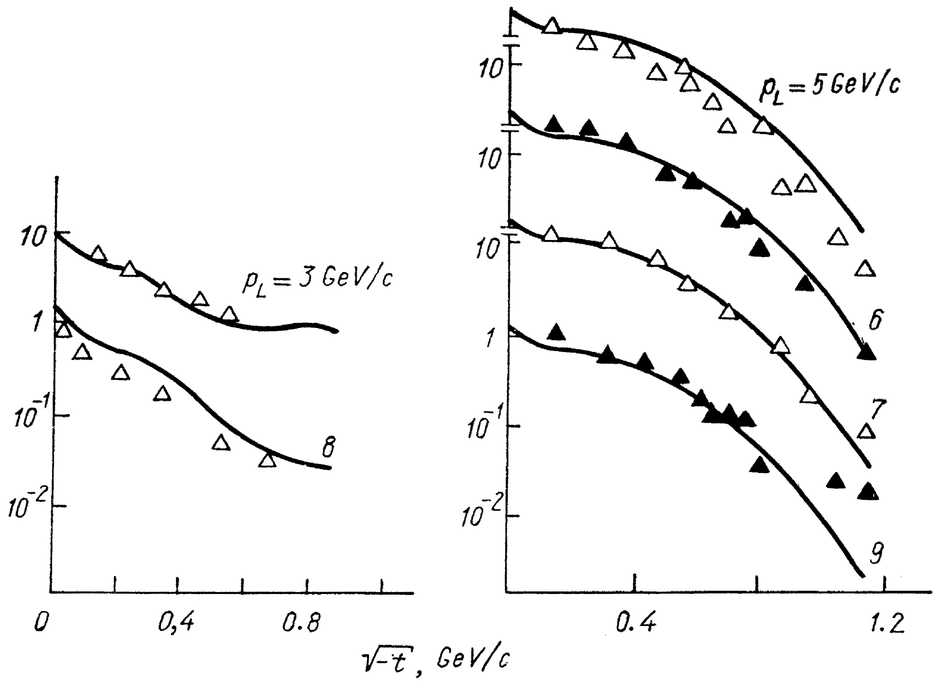


Fig. 27. NN-charge exchange differential cross sections [26].

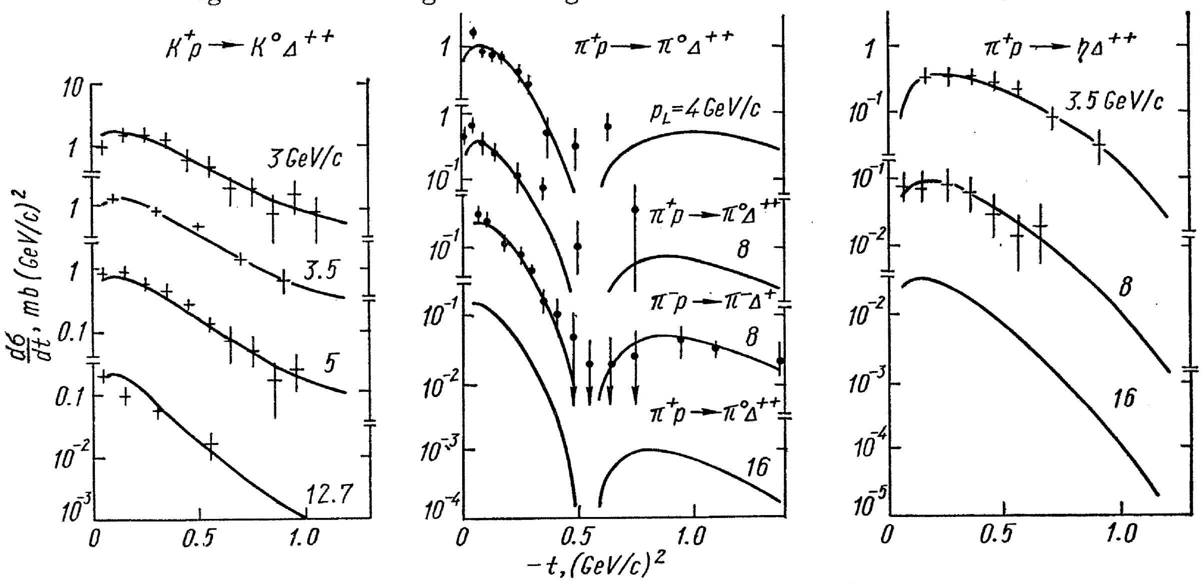


Fig. 28. Differential cross sections of $0^-\frac{1}{2}^+ \to 0^-\frac{3}{2}^+$ processes [27, 28].

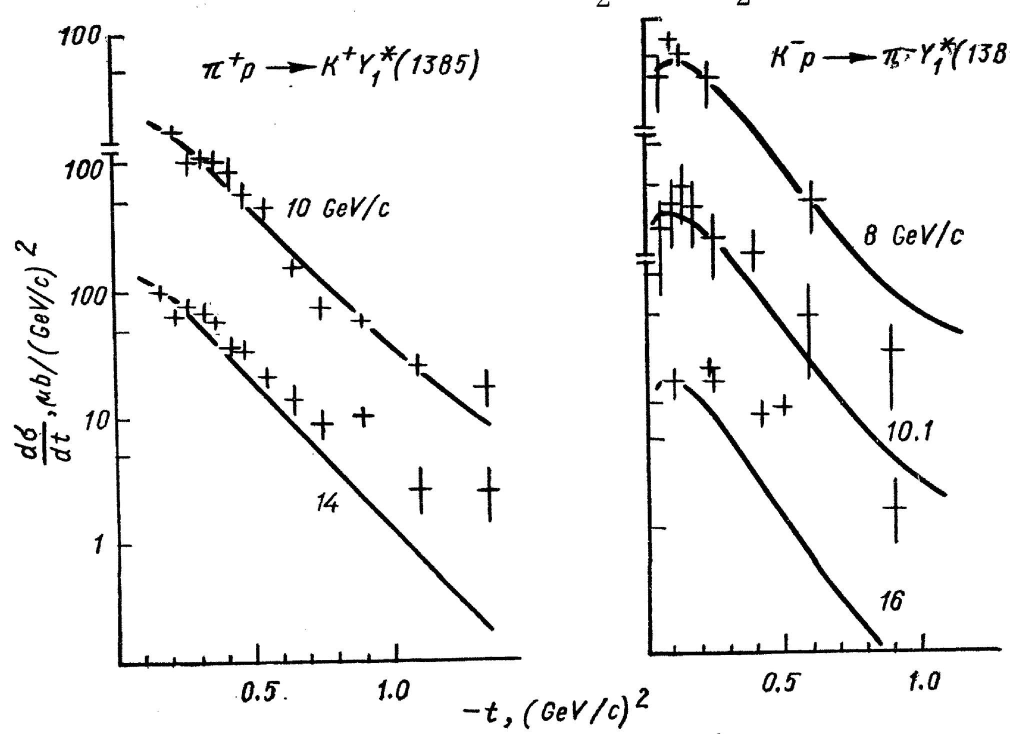


Fig. 29. Differential cross sections of Y_1^*-isobar production [32].

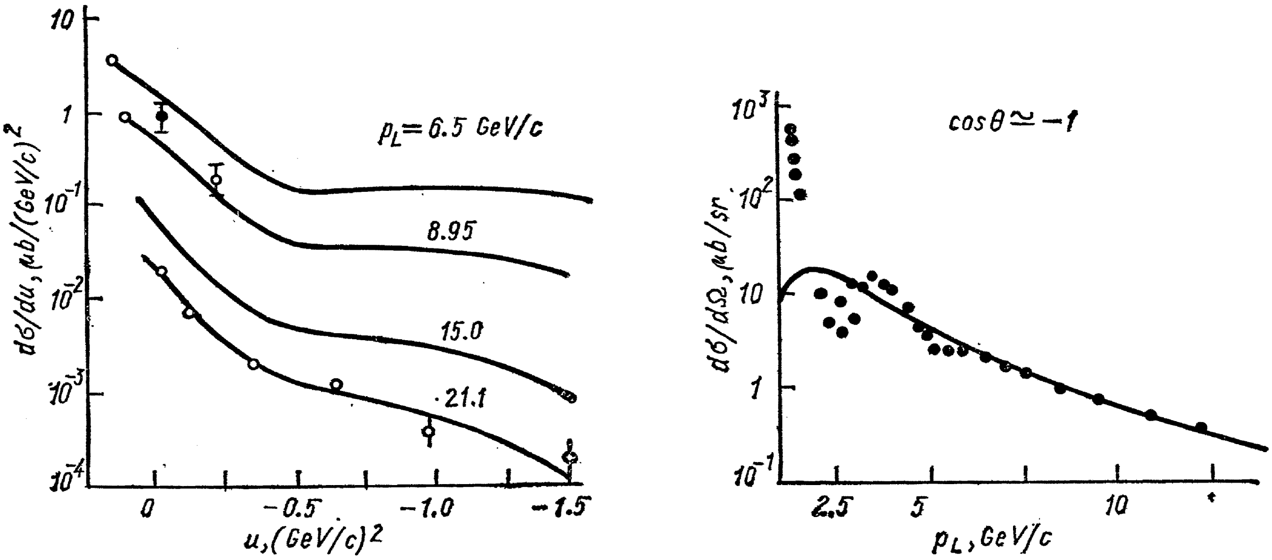


Fig. 30. $pp \to \pi^+ d^*$-process [33].

and K^{**} (1420)) (Renninger, Sarma [32]) (Fig. 29). A model with the $N_\alpha - N_\gamma$ exchange degeneracy breaking has been considered by Barger and Michael [33] for the reaction $pp \to \pi^+ d^*$ (Fig. 30). Analysis of the diffraction peak slopes $B\left(\frac{d\sigma}{dt} = Ae^{Bt}\right)$ for πN, KN and NN-elastic scattering has been done in the five-Regge-pole model (P, P', ρ, ω, A_2) [34].

Exchange degeneracy between vector and tensor trajectories and residues is assumed in this approach. The results obtained agree with experiments. The

Table II

Pomeranchuk Regge Pole Amplitude as a Function of Reference

Process	Model	Trajectory parameters α_0	Trajectory parameters α'	Residue Functions	Reference
$\pi^\pm p \to \pi^\pm p$	Regge poles only	1	0.36—0.37	$\beta_{++} \sim e^{6.5\,t} + 1.5\, e^{1.82\,t}$; $\beta_{+-} \sim e^{2.09\,t}$	4
» $K^\pm p \to K^\pm p$	Regge poles with cut contributions	1	?	?	35
$\pi^\pm p \to \pi^\pm p$	Regge poles only	0.988+0.005	?	?	36
»	Regge poles only	1	$0 \div 0.3$	$\beta_{++} \sim \alpha\,(\alpha+1)$; $\beta_{+-} \sim \alpha^2\,(\alpha+1)$	37
»	Regge poles with absorptive corrections	1	0.18	$\beta_{++} \sim \alpha\,(\alpha+1)$; $\beta_{+-} \sim \alpha\,(\alpha+1)$	38
» $K^\pm p \to K^\pm p$	Regge-eikonal	1	0	$\beta_{++} \sim (\mu^2 - t)^{-4}$; $\beta_{+-} = 0$	35, 39
$pp \to pp$	Regge-eikonal	1	1 ± 0.1	?	40
$\pi^- p \to \pi^0 n$	Regge poles with absorptive corrections	1	0	β_{++} has no zero; $\beta_{+-} = 0$	41
$\varphi p \to \varphi p$, $\gamma p \to \varphi p$	Regge poles only	1	0.5	?	34

prediction for higher energies is as follows: all the diffraction peaks should shrink at $p_L \sim 70$ *GeV/c*.

Some shortcomings of the Regge-pole model are:

(I) Description of the high-energy phenomena in terms of Regge-poles is restricted to small momentum transfers.

(II) A large number of parameters, arbitrariness in the choice of residue function and the necessity of introduction of hypothetic trajectories are characteristic for this approach.

Tables II and III demonstrate the ambiguity in the choice of the P and ρ-pole trajectories and residues as they are used by various authors.

Table III

ρ-Regge Pole Amplitude as a Function of Reference

Process	Model	Trajectory parameters		Residue Functions	Reference
		α_0	α'		
$\pi^{\pm} p \to \pi^{\pm} p$; $\pi^- p \to \pi^0 n$	Regge poles only	0.57—0.58	0.94—1.01	$\beta_{++} \sim (\alpha+1)$; $\beta_{+-} \sim \alpha(\alpha+1)$	37
$\pi^- p \to \pi^0 n$	Regge poles only	0.55	1.0	$\beta_{++} \sim \alpha$; $\beta_{\pm-} \sim 1 + t/0.23$	42
$\pi^{\pm} p \to \pi^{\pm} p$; $\pi^- p \to \pi^0 n$	Regge poles only	0.55	1.0	$\beta_{++} \sim (1+6t) \times \sin\left(\frac{1}{2}\pi\alpha\right) \times \Gamma(-\alpha)\, e^{2.55t}$	4
				$\beta_{+-} \sim \left(-1+2.5e^{1.29t}\right) \times \Gamma(1-\alpha) \sin\left(\frac{1}{2}\pi\alpha\right)$	43
$\pi^- p \to \pi^0 n$; $K^- p \to \bar{K}^0 n$	Regge poles only	0.58	0.92	$\beta_{++} \sim (1+\alpha)(1+2.9t)$; $\beta_{+-} \sim \alpha(\alpha+1)$	44
$\pi^- p \to \pi^0 n$	Regge poles only	0.58	0.78—0.87	$\beta_{++} \sim$ const; $\beta_{+-} \sim (1+1.7t)$	45
$\pi^{\pm} p \to \pi^{\pm} p$; $\pi^- p \to \pi^0 n$	Regge poles only	0.55	1.0	β_{++} has no zero; $\beta_{+-} \sim \frac{1}{\alpha}\left(1+\frac{t}{0.03}\right) \times \sin\frac{\pi\alpha}{2}$	46
$K^{\pm} p \to K^{\pm} p$	Regge poles only	0.57	0.9	$\beta_{++} \sim (\alpha+1)(1+6.6t)$; $\beta_{+-} \sim \alpha(\alpha+1)$	24
$\pi^- p \to \pi^0 n$; $K^- p \to \bar{K}^0 n$	Regge poles only	0,5	0.9	$\beta_{++} \sim 1/r(\alpha)$; $\beta_{+-} \sim 1/r(\alpha)$	
$\pi^{\pm} p \to \pi^{\pm} p$	Regge poles only	0.5	?		36
»; $K^{\pm} p \to K^{\pm} p$	Regge poles with cut contributions	0.4	?		35
$\pi^{\pm} p \to \pi^{\pm} p$; $\pi^- p \to \pi^0 n$	Regge-eikonal	0.55	0.8	$\beta_{++} \sim \alpha$; $\beta_{+-} \sim \alpha$	39
$\pi^- p \to \pi^0 n$	Regge poles with absorptive corrections	0.42—0.47	0.9—1.0	$\beta_{++} \sim \beta_{+-} \sim \frac{1}{t-m_\rho^2}$	41
$\pi^{\pm} p \to \pi^{\pm} p$; $\pi^- p \to \pi^0 n$	Regge poles with absorptive corrections	0.51—0.58	0.74—1.01	$\beta_{++} \sim \beta_{+-} \sim \alpha(\alpha+1)$	38
$\pi^- p \to \pi^0 n$	New interference model	0.58	0.9	$\beta_{++} \sim \alpha$; $\beta_{+-} \sim \alpha \times \left[\beta_1(t)e^{c_1 t} - \alpha\beta_2(t)e^{c_2 t}\right]$	47

(III) Cross-over effect difficulties.

For instance, the cross-over in the pp and $\bar{p}p$-elastic differential cross sections is usually ascribed to the vanishing residue function of the ω pole at $|t| \simeq 0{,}15\ (GeV/c)^2$. However, taking into account factorization of the residues, this leads to dips in the differential cross sections of the processes $\gamma p \to \pi^0 p$ and $K^+ p \to K^{*+} p$, which are not observed experimentally. Similar effects arise, when the factorization of the π-conspirator is considered.

(IV) The Regge-pole model underestimates total cross sections at high energies.

2. COMPLEX j-PLANE CUT CONTRIBUTIONS

Recently, in the analysis of experimental data on high energy two-body processes, the Regge-model with j-plane cut contributions has been often used.

The presence of the j-plane cuts has been indicated by a number of authors on the basis of the study of perturbation theory graphs, unitarity condition, and also in the framework of various potential models. A number of papers submitted to the conference contain attempts to investigate theoretically the cut contributions. Some applications of this approach to the description of high-energy experimental data are also given. Among them we note the reggeon diagram technique [49, 50, 52, 53] (Gribov, Ter-Martirosyan and co-workers) and the method of absorptive corrections, which is developed by the Michigan group [41, 54] (Henyey, Kane, Ross et al.) and the Cracow [55] group (Bialas, Zalewski).

The Pomeranchuk, Gribov, Ter-Martirosyan [48] — approach is based on the following assumptions:

a) the asymptotic behaviour of the scattering amplitude at $|Z_t| \gg 1$ is completely determined by the Regge-poles and the cuts connected with poles in the complex j-plane,

b) there exists the right most pole, the Pomeranchuk pole,

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P \cdot t, \ \alpha_P(0) = 1, \ \alpha'_P > 0,$$

its quantum numbers being identical to those of the vacuum,

c) all elastic and inelastic amplitudes decrease as functions of the external masses $m^2 \to -\infty$ and no complex singularities in s_i and t_i appear in this limit.

The scattering amplitude in the framework of the reggeon diagram technique is determined by the sum of diagrams, where graphs containing the reggeon-reggeon interactions appear («enchanced» graphs).

In the papers [49] (Gribov, Levin, Migdal), submitted to the conference, the «enhanced» graph contributions to the ultra-high energy ($4\mu^2\alpha' \ln s \gg 1$) elastic scattering amplitude are analyzed. The investigation of the «enhanced» graphs containing only the vacuum reggeons shows that the constancy of the total cross sections in the ultra-high energy region is possible provided the 3-, 4- and 5-reggeon vertices vanish as ω when $\omega = j - 1 \to 0$. The estimation of the «enhanced» graphs with nonvacuum Regge-poles is the subject of the papers [50].

In the paper [51] by Budnev, Efremov, Ginzburg, Serbo, submitted to the conference, the asymptotic behaviour of the perturbation theory graphs is studied from the point of view of the j-plane singularities. In the $\lambda\varphi^3$-theory the sum of the leading logarithmic terms in all diagrams leads to the asymptotic Regge-behaviour for two-particle scattering amplitude. It is established that the amplitude is a rapidly decreasing function of $m^2 \to -\infty$. A careful treatment of the spin effects leads, however, to the appearence of the fixed cuts in the j-plane. The amplitudes do not decrease when $m^2 \to -\infty$.

In the papers by Ter-Martirosyan and co-workers [52, 53] the reggeon diagram technique is applied to the analysis of high energy experimantal data. In these papers, it is assumed that the «enhanced» graphs contributions are supressed and the amplitudes of reggeon-particle scattering are determined by the intermediate states with bounded masses. In this approximation a scattering amplitude of the eikonal-type is obtained:

$$f(s, t) = \text{const} \sum_{n=1}^{\infty} \frac{e^{at/n}}{n \cdot n!} C_n \cdot (-x)^{n-1}, \tag{2.11}$$

where $C_1 \equiv 1$; $C_2 = 1 + \frac{\sigma_{\text{inel}}}{\sigma_{\text{el}}} > 1$ and C_n, $n \geqslant 3$ are unknown constants. The authors connect the quantities with the «shower» production in the intermediate states of the unitarity condition for the amplitude of reggeon-particle scattering. Note that when $C_n = 1$ in the formula, Eq. (2. 11) coincides with the scattering amplitude in the eikonal and quasipotential approaches if a purely imaginary quasipotential with an energy — independent parameter is used.

In the papers of the Michigan [54] and Cracow [55] groups a similar approach is developed. The authors use the «Strong Cut Regge Absorption Model» (SCRAM) for the amplitudes of exchange processes. In this approach the amplitudes of exchange processes are presented in the form

$$T_{\text{exch}}(s, t) = T_R(s, t) + \frac{i\lambda}{4\pi} \int d\Omega T_R(s, t') \cdot T_{\text{el}}(s, t''), \tag{2.12}$$

where T_R denotes the Regge-pole amplitude, T_{el} — the elastic scattering amplitude and λ — an unknown parameter. The introduction of the parameter λ is argued by the necessity of taking into account the intermediate diffractive dissociation processes. The dip-bump structure of the differential cross section is explained as a distructive interference effect between the pole and correction terms. Regge-pole residues have no nonsense zeros in this approach. The magnitude of the parameter λ is of the order $\lambda \simeq 1{,}2 \div 2{,}0$ (see in this connection the paper by Drago et al. [56]).

In Fig. 32*a* the polarization parameter in $\pi^- p$- charge exchange is presented, which has been calculated in the framework of SCRAM. In the region of the

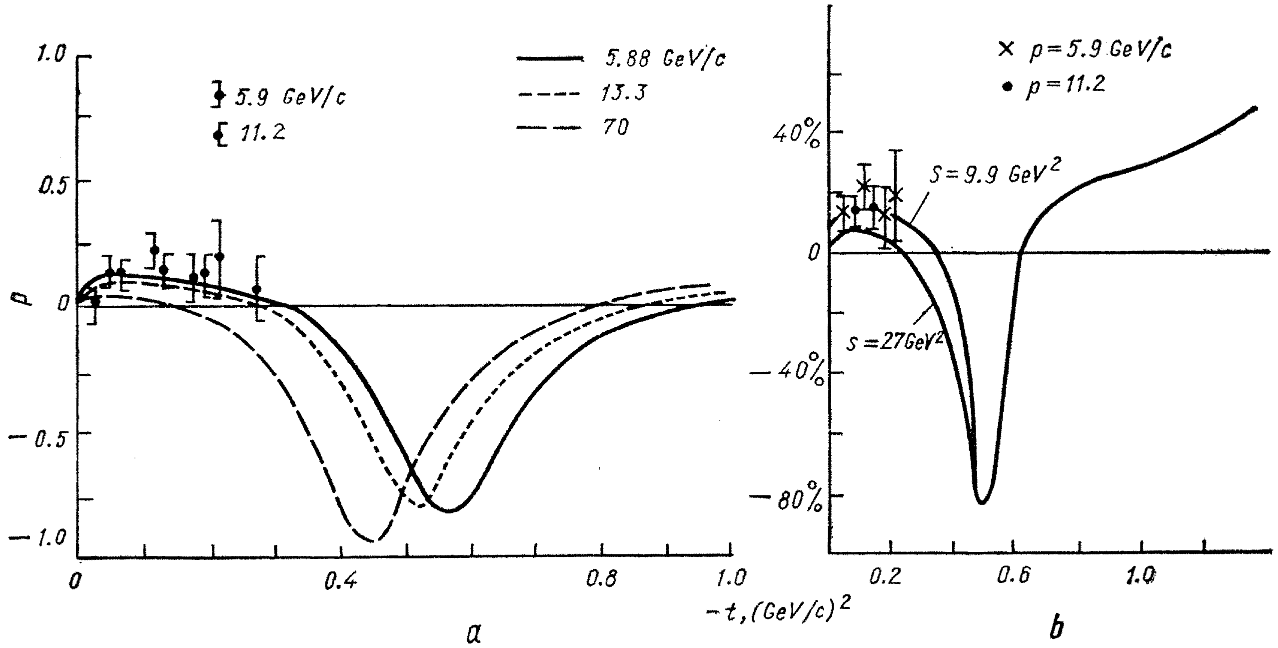


Fig. 31. *a*) Polarization parameter $P_0(\pi^- p \to \pi^0 n)$ in SCRAM [54]; *b*) Polarization parameter $P_0(\pi^- p \to \pi^0 n)$ in the Regge-eikonal model [39].

existing data the polarization parameter is in agreement with experiment. It has a sharp negative minimum near $|t| \simeq 0{,}5$ $(GeV/c)^2$ at $p_L = 13{,}3$ GeV/c. A similar behaviour of the polarization parameter P $(\pi^- p \to \pi^0 n)$ is predicted in the Regge-eikonal model [39]. Contrary to SCRAM in the Regge-eikonal model, the ρ-Regge-pole amplitude vanishes at $|t| \simeq 0{,}6$ $(GeV/c)^2$ (nonsense zero) (Fig. 31*b*).

In the submitted paper by Kancheli and Matinyan [57] an attempt is made to apply the reggeon technique to particle-nuclei scattering processes. It seems that from the theoretical point of view during the last two years no essential progress has been achieved in the proof of the validity of Regge approach in quantum field theory. The construction of the reggeon technique requires the knowledge of reggeon-reggeon interactions. This fact leads to a number of unknown functions even for the description of the elastic processes. It is necessary to analyze further the direct channel unitarity in all the approaches discussed.

Section III

Quasioptical approach and unitarity

1. PHENOMENOLOGICAL EIKONAL APPROACH

Recently, in the description of high energy particle scattering, much attention has been paid to an approach which makes use of the notion of an effective two-particle complex potential analogous with the optical picture of scattering (Blokhintsev et al. [1], Serber [2]). This approach is based on the validity of the eikonal representation for the high energy small angle scattering amplitude

$$T(s, t) = ip\sqrt{s}\int d^2\rho e^{i\vec{q}\vec{\rho}}(1 - e^{2i\varkappa}). \tag{3.1}$$

In quantum mechanics the eikonal phase $\varkappa$ is connected with the potential V by the following relation:

$$\varkappa = -\frac{1}{2p}\int\limits_{-\infty}^{+\infty} V(\sqrt{\rho^2 + z^2})\, dz. \tag{3.2}$$

In the «droplet» model (Byers, Yang; Chou, Yang [3, 4]), where hadrons are considered as extended objects of finite size, the eikonal function is a purely imaginary energy independent quantity, which is determined by the Fourier transform of the product of two electromagnetic form-factors,

$$\varkappa(\vec{\rho}, s) = iC\int d^2q e^{-i\vec{q}\vec{\rho}} F_a(-\vec{q}^2) F_b(-\vec{q}^2). \tag{3.3}$$

The constant C characterizes the hadron transparency in the scattering process. In this model the differential cross section at high energies tends to the finite limit

$$\lim_{s\to\infty} \frac{d\sigma}{dt} = f(t). \tag{3.4}$$

In the Regge-eikonal model (Arnold; Frautschi, Margolis [5, 6]) the phase $\varkappa$ is determined by the Fourier — transform of the Born approximation given by the Regge-pole contributions.

In the submitted papers [7] by Frautschi, Hamer and Ravndal a study of the Regge-eikonal model is given. Besides, a Pomeranchuk trajectory with nonzero slope contributions of the secondary Regge trajectories (P', ρ, ω, A_2) are taken into account. Their trajectories and residues are assumed to be completely exchange degenerate. The conclusions are as follows: (I) total cross sections are increasing functions of energy in the subasymptotic region; (ii) in the region of accessible energies the diffraction peak slopes for the pp, $\bar{p}p$ and πp-scattering shrink, antishrink and remain constant respectively (Fig. 33a, b). The ratio $\alpha = \mathrm{Re}\, T(s, 0) / \mathrm{Im}\, T(s, 0)$ is negative and tends to zero for $s \to \infty$ in the case of pp-scattering, while for $\bar{p}p$-scattering it turns out to be a small positive constant quantity (Fig. 32c).

2. QUASIPOTENTIAL APPROACH

The optical scattering idea finds its most natural generalization in the framework of the quasipotential approach in quantum field theory (Logunov, Tavkhelidze [8—10]).

According to this approach the two-particle scattering amplitude obeys a Lippmann — Schwinger type equation, which for scattering of equal mass spinless particles reads

$$T = V + \int \frac{d\vec{q}}{\sqrt{m^2 + \vec{q}^2}} \cdot \frac{VT}{q^2 + m^2 - E^2 - i0}. \tag{3.5}$$

V is the quasipotential, an energy dependent complex quantity. Its imaginary part is determined by the inelastic channel contributions. In the submitted papers [11, 12] by P. Bogolubov the positive definitness of the imaginary part of the quasipotential $\mathrm{Im}\, V > 0$ is proved with the help of the unitarity condition.

Analyzing the Orear behaviour of the scattering amplitude at large angles Alliluyev, Gershtein and Logunov [13] have suggested that the local quasipotential should show a smooth behaviour at high energies.

In the papers by Garsevanishvili, Matveev, Slepchenko and Tavkhelidze [14, 15], the case of a purely imaginary Gaussian-type quasipotential is studied in detail. For the high energy small angle scattering amplitude an eikonal representation of the type Eq. (3.1) is obtained. The eikonal function is related to the quasipotential in the following manner:

$$\chi_\rho(s) = \frac{1}{s} \int_{-\infty}^{+\infty} V(s, \sqrt{\rho^2 + z^2})\, dz. \tag{3.6}$$

In the papers [16, 17] corrections to the eikonal phase Eq. (3.6) of the order $1/p$ have been found. It turns out that the correction terms are essentially determined by the relativistic factor $1/\sqrt{m^2 + q^2}$ in Eq. (3.3).

One of the important advantages of the quasipotential approach is the possibility of studying the large angle behaviour of the scattering amplitude. In particular, for the purely imaginary quasipotential $V(s, t) = isge^{at}$, the large angle scattering differential cross section reads

$$\frac{d\sigma}{d\Omega} \to \left| \frac{r_0^2}{6} \right|^2 q^2 e^{-2q\, \mathrm{Im}\, r_0}, \tag{3.7}$$

where $s \to \infty$, $\theta = \text{fixed}$, $q^2 = -t$,

$$r_0 = i \sqrt{2\pi i a \left(\frac{2i}{\pi} - \ln \gamma\right)}; \quad \gamma = \frac{sge^2}{p\,|t|} \left(\frac{\pi}{a}\right)^{3/2}. \tag{3.8}$$

It is interesting to note, that in the region of energies and momentum transfers, where the second term under the square root in Eq. (3.8) can be neglected, the differential cross section Eq. (3.7) weakly depends on energy (Fig. 33). In this case the energy dependence of the cross section $d\sigma/d\Omega$ enters only through the parameters a and g_0, which can be determined from the experimental data in the diffraction peak region (Fig. 34).

The behaviour of the total cross section σ_{tot} (s) in the model with a Gaussian quasipotential depends essentially on the energy dependence of the quantity g (s). In particular, if g (s) is energy independent, the total cross section tends to its asymptotic value from below [6, 14].

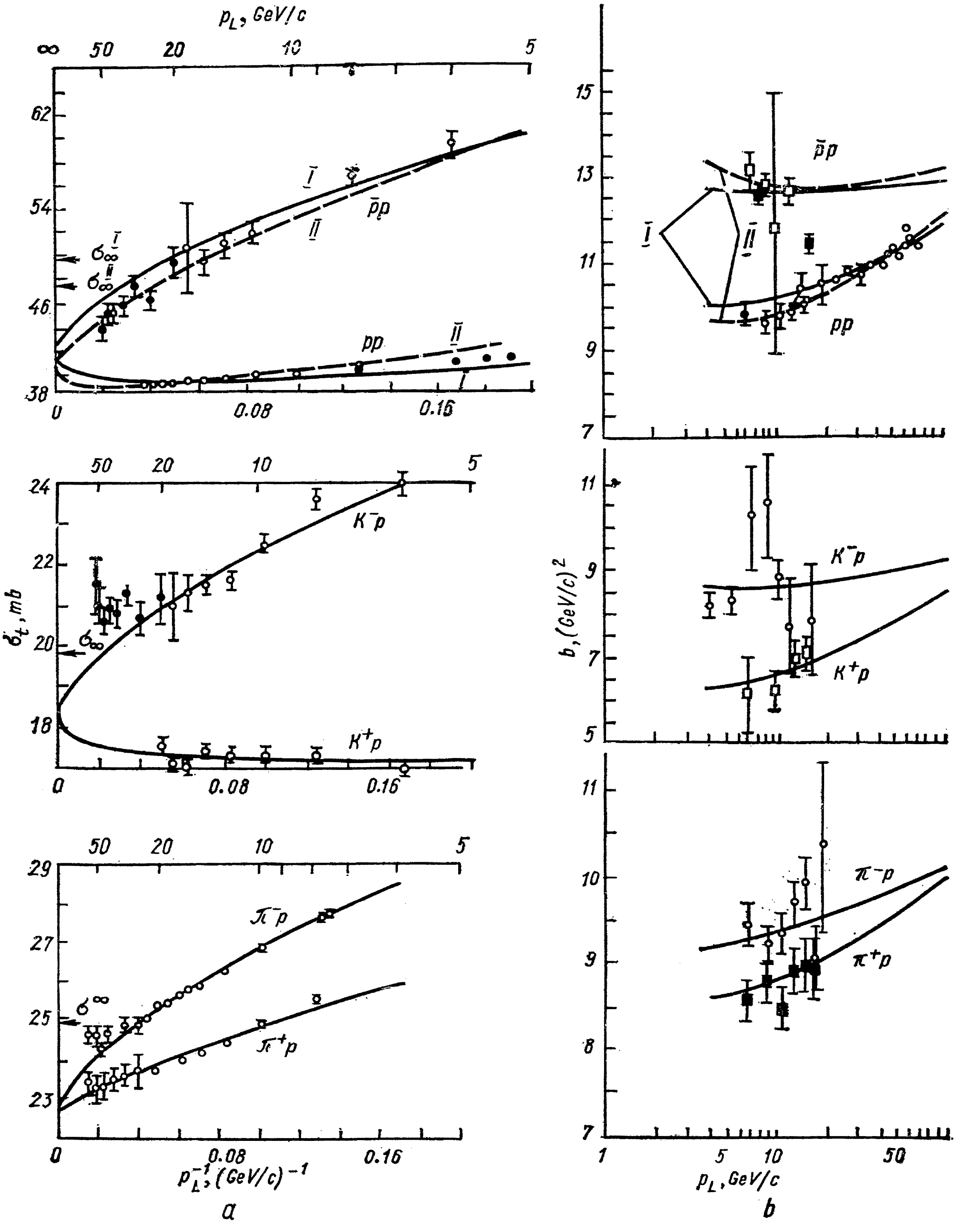


Fig. 32. *a*) total cross sections, *b*) diffraction peak slopes and *c*) ratio α = Re T (0) /Im T (0) model [7].

In the papers [19, 20] by Garsevanishvili, Matveev, Slepchenko and Tavkhelidze, use has been made of the multichannel generalization of the quasipotential equation for the description of isobar production processes in *pp*-collisions. It has been shown, that the angular distributions of the elastic scattering and isobar production processes in the region of large momentum transfers are in some sense universal. This fact is explained by the dominance of the elastic multiscattering effects. In the paper by Matveev and Slepchenko [21] the πN-charge exchange process is considered in the framework of the quasipotential approach. For the exchange quasipotential an integral representation of the type [22, 23]

$$V^{(-)}(s, t) = \int_0^1 dx x^{-1-t} f(x, s). \quad (3.9)$$

Such a representation is, for instance, valid for a superposition of Yukawa-type quasipotentials. Note, that the representation Eq. (3.9) includes singular as well as nonsingular quasipotentials.

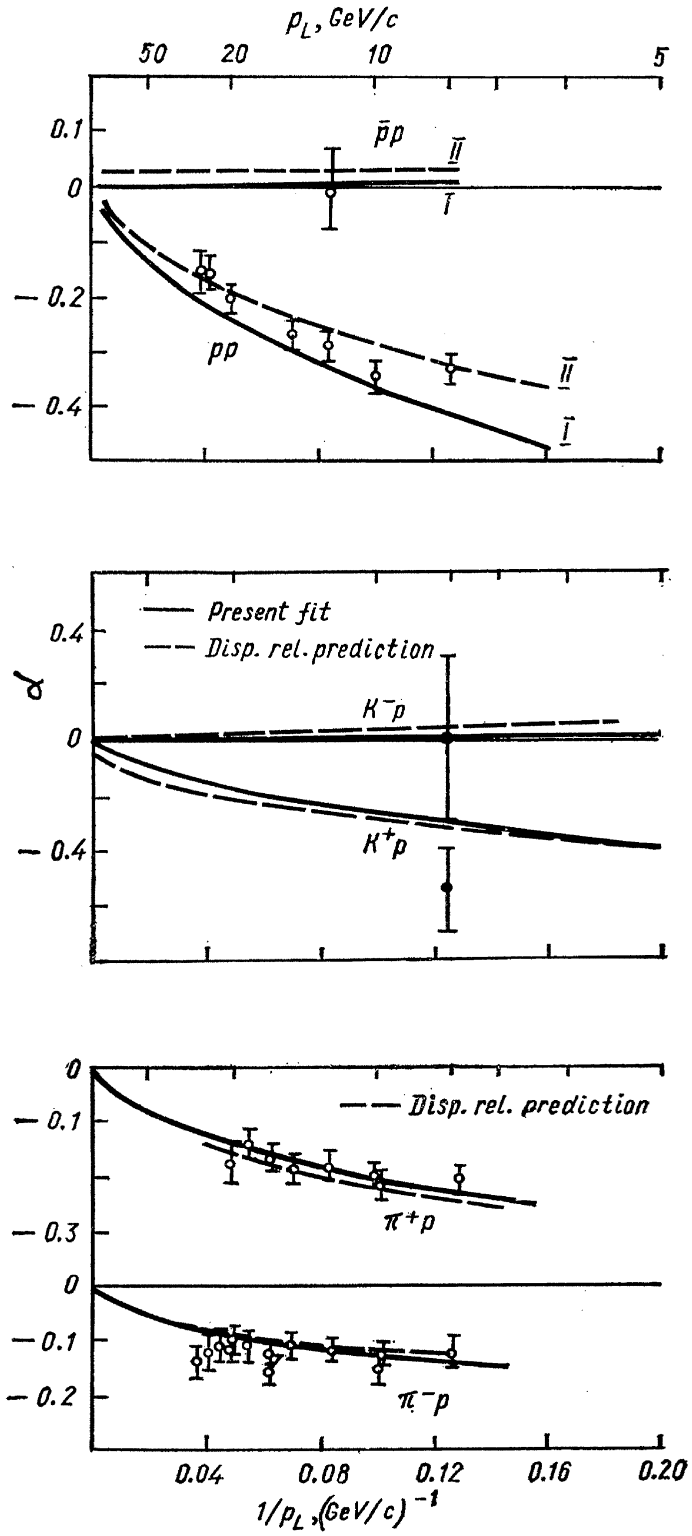
In a number of papers [24—25] (Savrin, Tyurin and Khrustalev) the high energy particle scattering problem is considered on the basis of the Schroedinger equation with complex potentials. It was shown, that in the case of smooth potentials the Schroedinger equation with relativistic kinematics reproduces rather well the main properties of the elastic scattering and may serve as a basis for a quantitative description. According to the smoothness hypothesis the authors consider potentials of the type

$$U(E, r^2) = g(E, r^2) e^{-\varphi(E, r^2)}, \quad (3.10)$$

where $g(E, r^2)$ and $\varphi(E, r^2)$ are smooth functions of r^2. It has been noticed by the authors that the potentials Eq. (3.10) correspond to the interaction with energy dependent radii

$$R^{-1} = \frac{d}{dr} \varphi(E, r^2), \quad (3.11)$$

as opposed to Yukawa potential, which have constant radii.



C

for πN, KN and NN-scattering in the eikonal

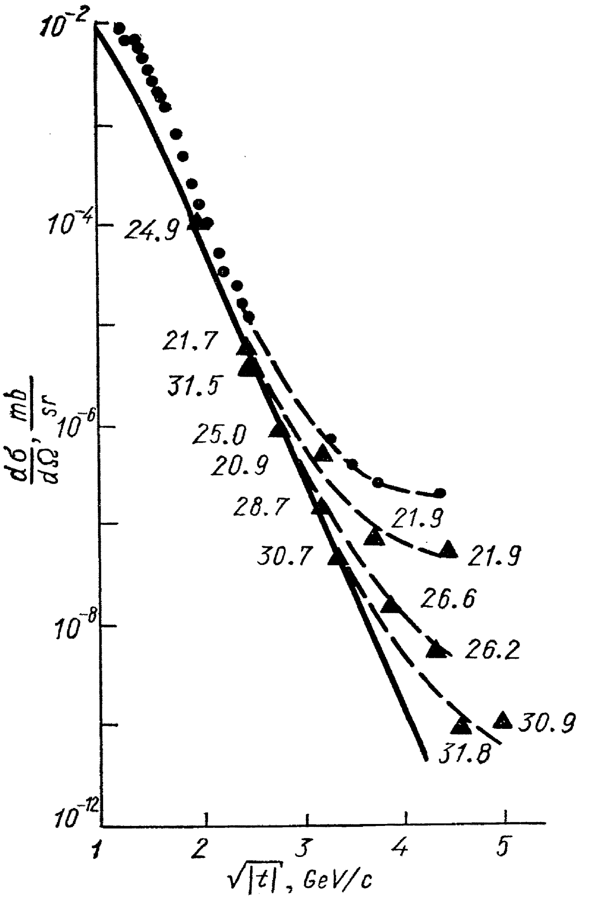


Fig. 33. Differential cross section for pp-scattering at large angles in the quasipotential approach [18, 19].

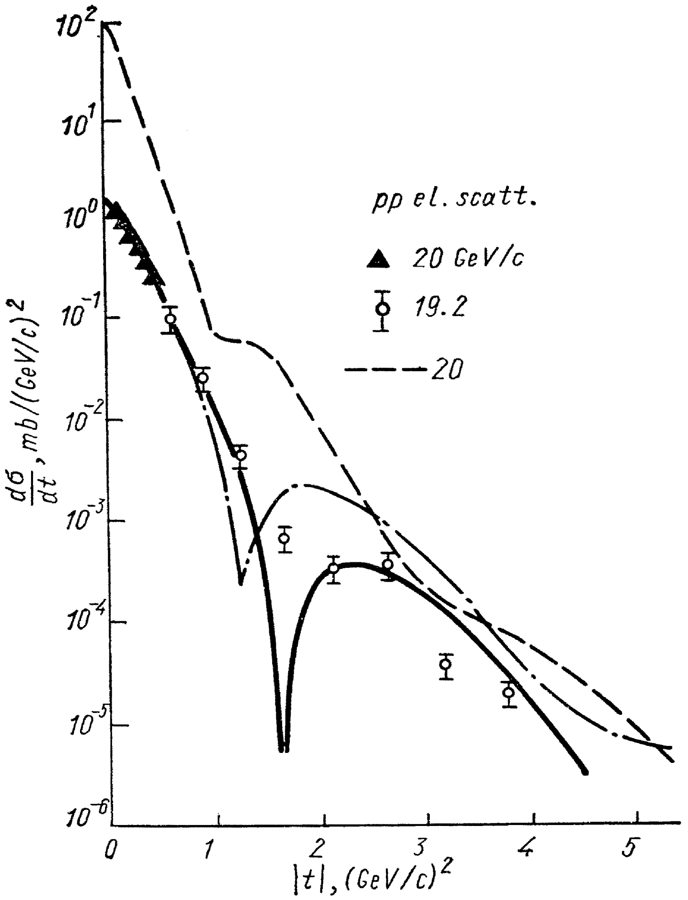


Fig. 34. Differential cross section for $pp \to pN^*$ (1518) at large momentum transfer [19, 20].

A number of papers [26—35] submitted to this conference is devoted to the further theoretical development of the quasipotential approach. In the framework of the quasipotential approach Garsevanishvili, Kadyshevsky, Mir-Kasimov and Skachkov [26] have obtained a generalization of the eikonal approximation for elastic scattering by means of an expansion on the group of the horospherical shifts on the hyperboloid

$$T(s, t) = ip\sqrt{s}\int_0^\infty \rho d\rho J_0(\rho\sqrt{-t})\left\{e^{i\int_{-\infty}^{+\infty}\ln\left[1-\frac{V(z,\rho)}{2p}\right]dz}-1\right\}. \qquad (3.12)$$

In the paper by Kuleshov, Matveev, Sissakyan [27] the eikonal representation for the scattering of Dirac particles on smooth quasipotentials is obtained.

In the papers [28—30] relativistic analogues to exactly solvable quantum mechanical problems have been considered: the relativistic harmonic oscillator [28] by Donkov, Kadyshevsky, Matveev and Mir-Kasimov and the relativistic Coulomb problem by Itzykson, Kadyshevsky and Todorov [29]. In the contribution by I. Todorov [29] another quasipotential equation is considered in which the relativistic kinematical root is replaced by the external energy parameter $w = \sqrt{s}$. This is still consistent with the elastic unitarity condition for the relativistic amplitude. For the Yukawa potential $V(p, q) = -\frac{g^2}{\mu^2 + (\vec{p}-\vec{q})^2}$

this equation is local and leads in the limit $s \to \infty$ (or $\mu \to 0$, $t \to 0$) to the relativistic eikonal formula

$$T(p, q) = \frac{|\vec{p}|w}{4\pi i} \int d^2y e^{i(\vec{p}-\vec{q})\vec{y}} [e^{\frac{2ig^2}{Wp} K_0(\mu y)} - 1].$$

The relativistic two-body problem has been treated by Fronsdal, Huff and Lundberg [30] in a way which is similar to the quasipotential approach. An original approach to derive the two-particle equations is developed in the submitted papers by P. Bogolubov [31]. Using the Markov — Yukawa conditions for the two-particle wave function the author has obtained quasipotential-type equations for the scattering amplitude of particles with arbitrary spins and masses. In the paper by Jenkovsky, Shelest, Struminsky and Zinoviev [32] a unitarization procedure for the Veneziano amplitude is developed on the basis of the quasipotential equation. Correction to the linear Regge trajectories of the type $s^{\alpha(t)+\delta(t)}$,

$$\delta(t) = \frac{\sqrt{\pi}\alpha(t)}{\Gamma\left(\alpha(t)+\frac{3}{2}\right)} \int_0^\infty \frac{dk\,(\alpha' k^2)^{\alpha(t)}}{\sqrt{k^2+m^2}\left(k^2 - \frac{t-4m^2}{4} - i\varepsilon\right)} \tag{3.13}$$

have been found.

Related problems are treated in the papers by Cocho [33]; Campbell and Yaes [34].

In the paper presented by Faustov [35] the properties of the bound state matrix elements of the local operators are considered. In the case of the vector current operator this matrix element determines relativistic form-factors of the bound system (electromagnetic or weak). The essential feature of this approach is the consistent account of the recoil of the particle system as a whole with the help of the relativistic transformation of the wave function to the centre of mass frame. It can be expressed in terms of the so called Wigner rotation.

3. UNITARITY CONDITION AND STATISTICAL APPROACH

The unitarity condition allows one to take into account the contributions of inelastic channels to the two-particle scattering amplitude in a natural way. It is convinient to use the «impact parameter» representation for the elastic scattering amplitude

$$f(s, t) = p\sqrt{s} \int_0^\infty b\,db\,J_0(bq)\,a(s, b); \quad q = \sqrt{-t} = 2p\sin\theta/2, \tag{3.14}$$

which is obtained by replacing the partial wave sum by an integral

$$l + 1/2 \to pb,$$
$$P_l(\cos\theta) \to J_0(bq),$$
$$a_l(s) \to a(s, b).$$

The amplitude $a(s, b)$ obeys the following unitarity condition

$$\mathrm{Im}\, a(s, b) = \frac{1}{2}|a(s, b)|^2 + \rho(s,b), \tag{3.15}$$

where $\rho(s, b)$ is the Van Hove [36] «overlap function» in the impact parameter representation, which corresponds to the inelastic channel contributions.

The solution of the unitarity condition Eq. (3.15) has been considered by Amati, Cini and Stanghellini [37]. In general, it does not lead to the typical minima and maxima in the angular distributions.

In the papers by Khrustalev, Savrin and Tyurin [38, 39], the following general solution of Eq. (3.15) is suggested:

$$a(s, b) = \frac{1}{i}\left(e^{2i\delta(s,b)} - 1\right) + ie^{2i\delta(s,b)}\left(1 - \sqrt{1 - 2\rho(s, b)}\right), \tag{3.17}$$

where $\delta(s, b)$ is an arbitrary real function. This approach rejects the assumption of a purely imaginary amplitude. The authors have considered the behaviour of the scattering amplitude assuming that the functions $\delta(s, b)$ and $\rho(s, b)$ are of the form

$$\begin{aligned} \delta(s, b) &= d(s, b^2)\, e^{-\psi(s,b^2)}, \\ \rho(s, b) &= g(s, b^2)\, e^{-\varphi(s,b^2)}, \end{aligned} \tag{3.18}$$

where d, g, ψ and φ are smooth functions of b^2.

The introduction of the nonvanishing function $\delta(s, b^2)$ allows one to obtain the diffraction structure in the angular distribution and at the same time does not lead to the dips, which appear in the eikonal approach with a purely imaginary quasipotential. For large angle scattering an Orear type formula is obtained in this approach.

The paper [39] deals with the behaviour of the total cross section at high energies in this model. Provided the functions d and g do not depend on b^2, the following formula is obtained:

$$\sigma_{tot} = 8\pi\gamma\left\{1 - \sqrt{1 - g(s)} + \ln\frac{1 + \sqrt{1 - g(s)}}{2} + \int_0^1 \frac{dx}{x}\sqrt{1 - g(s)\,x}\,\sin^2\frac{d(s)\cdot x}{2}\right\}, \tag{3.19}$$

where γ is the inverse of the diffraction peak width.

It is worth noting that the behaviour of the total cross section in the subasymptotic region depends essentially on the form of the functions $g(s)$ and $\gamma(s)$ in this region.

The direct channel unitarity condition allows one to describe the high energy scattering as a random process (Logunov, Khrustalev) [40, 41]. Assuming that particles in the intermediate state are not correlated and each particle transfers a definite transverse momentum, it is convenient to introduce a random function which gives the number of particles with a definite transverse momentum. Such an approach allows one to explain the main features of the high energy scattering and, in particular, a smooth complex quasipotential appears naturally in this scheme.

4. VALIDITY OF THE EIKONAL APPROXIMATION IN QUANTUM FIELD THEORY

The successful application of the eikonal representation for the description of high energy particle scattering raises the question of its validity in quantum field theory.

A number of papers submitted to this conference [42—45, 50] are devoted to the study of this problem. All these papers are based on the investigation of the asymptotic behaviour of some classes of perturbation theory graphs.

In the papers by Cheng and Wu [42], the so-called «impact picture» is formulated by studying the asymptotic behaviour of diagrams in quantum electrodynamics and in «scalar-nucleon» electrodynamics.

In the papers of the Dubna group [27, 43, 44] (Barbashov, Kuleshov, Matveev, Pervushin, Sissakian, Tavkhelidze) and also in the papers by Andreev, Andreev and Batalin [45], functional integration methods have been used [46].

In the last paper by Barbashov et al. [43] a straight line particle paths approximation (SLPA) is put forward to study the asymptotic behaviour of the elastic and inelastic scattering amplitudes. The essence of the method is expressed as follows: in the high energy region the main contribution to the functional integrals are obtained from the particle paths which are close to the classical ones. This method is closely connected to the so-called «$k_i \cdot k_j = 0$» approximation of Fradkin and Barbashov [47].

For the study of the asymptotic behaviour of the elastic scattering amplitude, the Dubna group considers a model of scalar «nucleons» interacting with a neutral vector field

$$L_{\text{int}} = g : A_\mu \, (\psi^* \partial_\mu \psi - \partial_\mu \psi^* \psi) : + g^2 : A_\mu^2 \psi^* \psi : \tag{3.20}$$

Neglecting radiative corrections and closed nucleon loops the following eikonal representation for the elastic scattering amplitude has been obtained [43, 45] in the framework of SLPA (t = fixed, $s \to \infty$):

$$f^{(0)}(s, t) = is \int d^2\rho e^{i\vec{q}\vec{\rho}} (e^{2i\chi_\rho^{(0)}} - 1); \quad t = -\vec{q}^2, \tag{3.21}$$

where

$$\chi_\rho^{(0)} = \frac{1}{S} \int_{-\infty}^{+\infty} V(s, \sqrt{\rho^2 + z^2})\, dz = \frac{g^2}{8\pi^2} \int \frac{d^2 q' e^{i\vec{q}'\vec{\rho}}}{\vec{q}'^2 + \mu^2}. \tag{3.22}$$

The result obtained corresponds to a sum of ladder type diagrams, where the contributions from the leading logarithmic terms cancel [48, 49]. The amplitude is represented by a sum of quasipotential-type graphs. Taking into account the radiative corrections to the scattering amplitude in the framework of SLPA one obtains the following result [43, 50]:

$$f(s, t) = H(t) f^{(0)}(s, t); \tag{3.23}$$

$f^{(0)}(s, t)$ is determined by the formula Eq. (3.21). For small $|t| \ll m^2$, $H(t)$ depends exponentially on t,

$$H(t) = e^{-a\vec{q}^2}; \quad a \sim \frac{g^2}{m^2}\left(\ln \frac{m^2}{\mu^2} + {}^1\!/_2\right). \tag{3.24}$$

The radiative corrections lead naturally to a smooth quasipotential. Indeed, writing Eq. (3.23) in the eikonal form (3.21) the following expression is obtained for the eikonal phase:

$$e^{2i\chi(\vec{\rho})} = \int \frac{d^2 b}{4\pi a} e^{-b^2/4a} \cdot e^{2i\chi^{(0)}(\vec{\rho}+\vec{b})}. \tag{3.25}$$

It is easy to see, that $\chi(\vec{\rho})$ is a complex quantity with positive definite imaginary part, $|e^{2i\chi}| < 1$, in accordance with unitarity.

Expanding the exponential under the integral in Eq. (3.25) in powers of $\chi^{(0)}$ one obtains

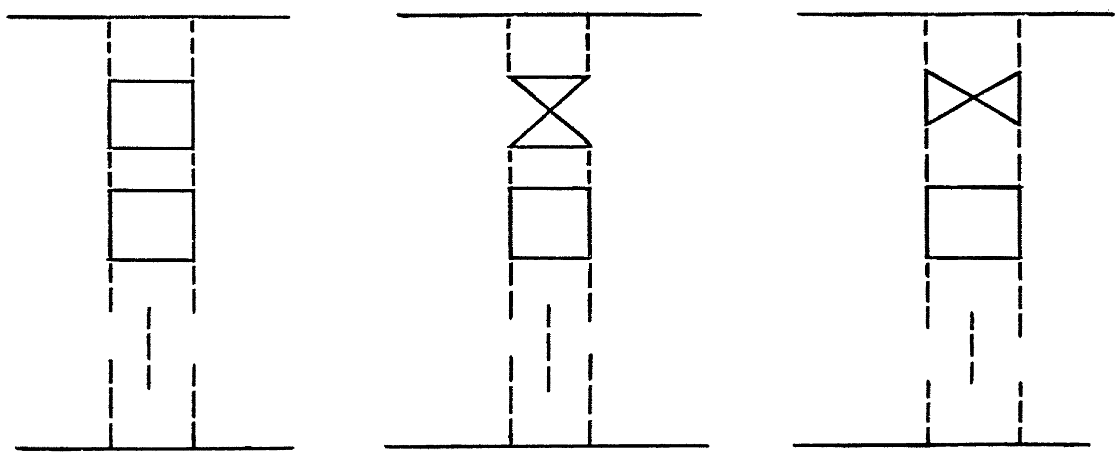
$$\chi(\vec{\rho}) = \frac{q^2}{8\pi^2} \int \frac{d^2 q e^{-i\vec{q}\vec{\rho}}}{q^2 + \mu^2} e^{-a\vec{q}^2} +$$

$$+ i \left(\frac{g^2}{8\pi^2}\right)^2 \int \frac{d^2 q d^2 q' e^{i\vec{\rho}(\vec{q}+\vec{q}')}}{(q^2 + \mu^2)(q'^2 + \mu^2)} [e^{-a(\vec{q}+\vec{q}')^2} - e^{-aq^2 - aq'^2}] + \cdots \tag{3.26}$$

The first term in Eq. (3.26) is purely real and corresponds to the scattering on a Yukawa quasipotential. Its center of forces is randomly distributed according to a Gaussian law. The second term in Eq. (3.26) contributes to the imaginary part of the quasipotential.

The question of validity of the eikonal approximation is more complicated, when diagrams containing closed nucleon loops are included [42, 45, 51, 52].

There are no complete and clear results on this subject yet. It is known that the consideration of classes of diagrams containing the nucleon closed loops



leads asymptotically to terms of the type $is\,(\ln s)^n f_n(t)$, where $n \geqslant 1$. The summation of such terms gives, in general, increasing total cross sections [42, 51].

A possible way to avoid this contradiction has been suggested by Cheng and Wu [42]. They have used an eikonal representation for the unitarization of the scattering amplitude.

In the paper by Cheng and Wu [42] the following predictions based on the above-mentioned observations are given for high energy hadron scattering:

$$\sigma_{tot} \to 2\pi R^2 + O(\ln|S|), \tag{3.27a}$$

$$\frac{\mathrm{Re}\,T(s,0)}{\mathrm{Im}\,T(s,0)} \to \frac{\pi}{\ln|S|} + O((\ln|S|)^{-1}), \tag{3.27b}$$

$$\sigma_{el}/\sigma_{tot} \to {}^1/_2 + O((\ln|S|)^{-1}), \tag{3.27c}$$

where

$$S = \left| -\frac{(-s)^a}{\ln^2(-s)} + \frac{(-u)^a}{\ln^2(-u)} \right|^{1/a}, \tag{3.28}$$

$$a > 0,\quad R = R_0 \ln|S|, \tag{3.29}$$

and R_0 is an energy independent constant.

In the paper by Andreev and Khoruszy [53] a problem of the validity of the eikonal approximation is considered for the case of scattering on the weakly bound deutron-type system.

5. COMPOSITE MODELS

Some papers submitted to this conference use the quark model to describe high-energy scattering.

In the papers by Kobushkin, Kukhtin and Shelest [54] the consequences of the factorization of the quark-quark [55] amplitude are studied, taking into account spin complications. A number of relations has been obtained between the differential cross sections of elastic and quasielastic processes at small and large angles. Note, for instance, the relation

$$\frac{d\sigma}{d\Omega}(pn)/\frac{d\sigma}{d\Omega}(pp)\,|_{\theta=90^\circ} \simeq 1.05, \tag{3.30}$$

which is in good agreement with experimental data.

In tbe paper by Tomozawa [56] a possible violation of the Pomeranchuk theorem is discussed in the framework of the quark model.

In the paper by Zalewski [57] the quark model is used to describe the process

$$0^- + \frac{1^+}{2} \rightarrow J^P + \frac{3^+}{2}.$$

In the paper by Kielanowski and Kupczynski [58] relations between transversity amplitudes of various quasi-two-body processes are obtained on the basis of the relativistic quark model.

One can justify the heuristic assumptions of Bogolubov's quark model (an infinite potential well, etc.) by assuming that quarks are permanent stable motions of permament excitations within the relativistic fluid droplet model of elementary particles (Vigier) [59].

Section IV

Analysis of the Serpukhov data and new models for the asymptotic behaviour

Analyzing the Serpukhov data a number of authors have conjectured on the growth of total cross sections at ultra-high energies. In this connection the question of possible bounds on the asymptotic behaviour of the total cross sections and the equality of the particle — antiparticle total cross sections at high energies arises. These problems have been investigated in the papers by Martin [1] and Eden [2] and, more completely, in the paper by Logunov, Mestvirishvili and Volkov [3]. A review of these papers will be given by Prof. Nguen Van Hieu. Here, we shall only list some of the results, which are necessary for the discussion of the models on the asymptotic behaviour, recently suggested.

Using analyticity inside the Martin ellipse and polynomial boundness of the elastic scattering amplitude as well as unitarity, Logunov, Mestvirishvili and Volkov have shown with the help of the Fragmen — Lindelöff theorem, that in the case of increasing total cross sections, $\sigma^{\pm}_{\text{tot}}(s)$, the following asymptotic relation holds:

$$\frac{\sigma^{+}_{\text{tot}}(s)}{\sigma^{-}_{\text{tot}}(s)} \xrightarrow[s\to\infty]{} 1.$$

If the total cross sections tend to constant limits two cases can be distinguished:

(I) $\frac{\operatorname{Re} T^{\pm}(s)}{\operatorname{Im} T^{\pm}(s)\ln s} \xrightarrow[s\to\infty]{} 0$ and the Pomeranchuk theorem is valid, $\sigma^{+}_{\text{tot}}(\infty) = \sigma^{-}_{\text{tot}}(\infty)$,

(II) $\frac{\operatorname{Re} T^{\pm}(s)}{\operatorname{Im} T^{\pm}(s)\ln s} \rightarrow \text{const} \neq 0$ and the cross sections are in general not equal. In that case the effective radius of the interaction increases logarithmically and the diffraction peak shrinks in a maximally allowable way. A detailed discussion of the various models suggested for high energy scattering is given in the paper by Barger and Phillips [4, 5].

There is a group of papers, where a description of the Serpukhov data using the Pomeranchuk theorem in its usual form is given (case 1). In this group we find the papers [4—9], where an analysis of the high energy πN, KN, NN scattering data is put forward with the help of the following models:

a) phenomenological Regge model with cuts (Barger, Phillips [4, 5]);

b) eikonal approximation (Kölbig and Margolis [6, 7]; Capella and Kaplan [8]);

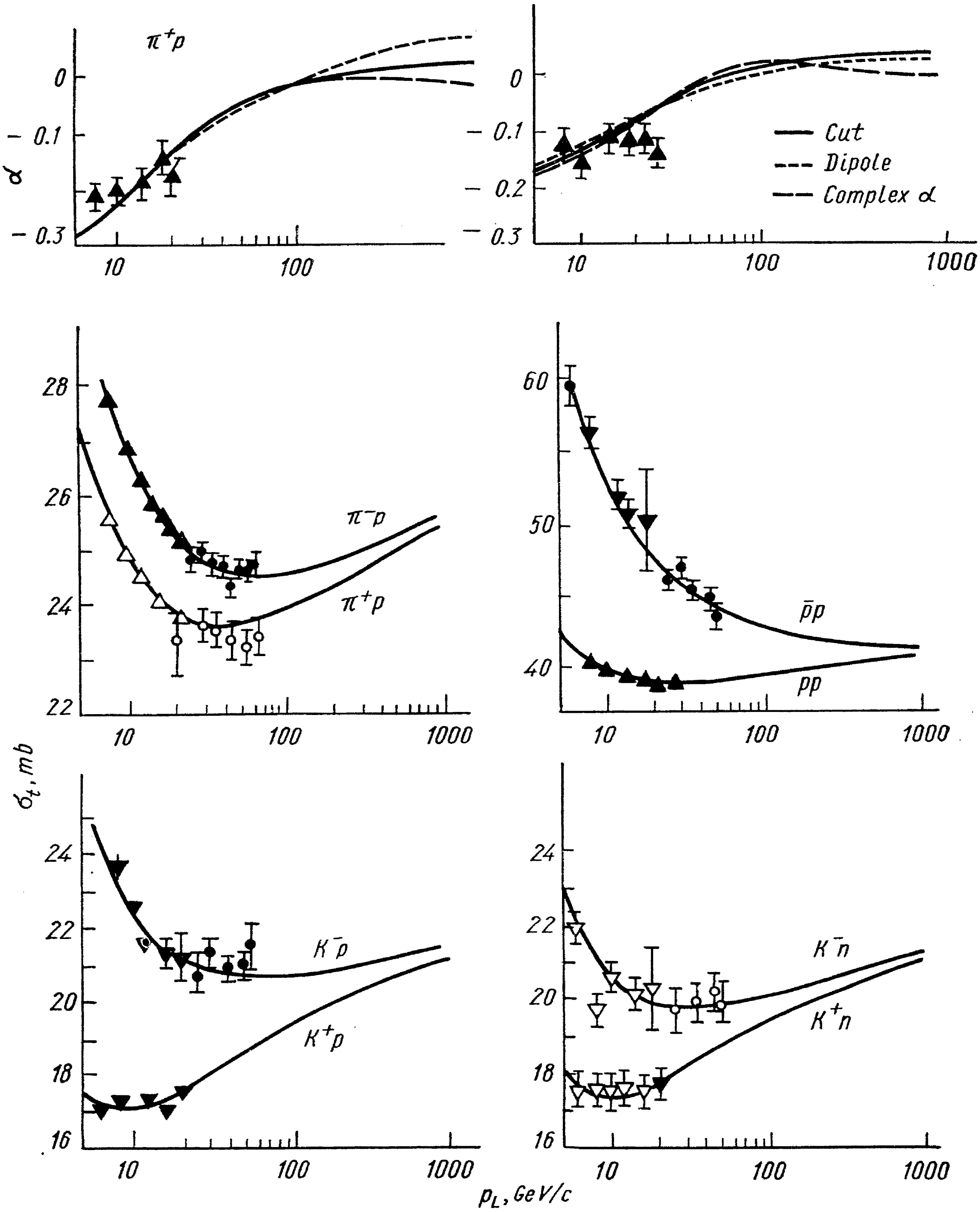


Fig. 35. Total cross sections and $\alpha = \mathrm{Re}\, T(0)/\mathrm{Im}\, T(0)$ for πN, KN and NN-scattering in the Regge-cut model [4, 5].

c) optical model with taking into account the so-called intermediate state showers (Lendyel, Ter-Martirosyan [9]).

In all of these papers the total cross sections have minima in some energy region, and then reach slowly their common asymptotic limits $\sigma^{-}_{\rm tot}(\infty) = \sigma^{+}_{\rm tot}(\infty)$ (Fig. 35).

For instance, at $p_L \sim 10$ GeV/c the cut contributions in the models a) and b) amount to about 50% of the Pomeranchuk pole contribution for elastic scattering. This changes essentially the values of the Regge residues, previously used. To explain the high energy behaviour of πN-total cross sections in the framework of MRPA a model has been suggested by Giesecke [10], where a negative residue for the P'-pole has been assumed. The following values for the intercepts $\alpha_P(0) = 0.988$, $\alpha_{P'}(0) < 0$ and $\alpha_{P''}(0) < 0$ are used. This leads to the vanishing total cross section at ultra-high energy (see figs. 36—37).

In the paper by Rarita [11] a πN-scattering model introducing an additional N-pole with a negative residue is suggested. The parameter $\alpha_N(0)$ is allowed to vary in the interval

$$0.63 < \alpha_N(0) \leqslant 1.$$

When $\alpha_N(0)$ increases, the asymptotic value of the total cross section increases (Fig. 38).

Some authors suggest the existence of complex-conjugated pairs of trajectories (Chew, Snider [12], Oehme [13]). Their contribution to the total cross section in

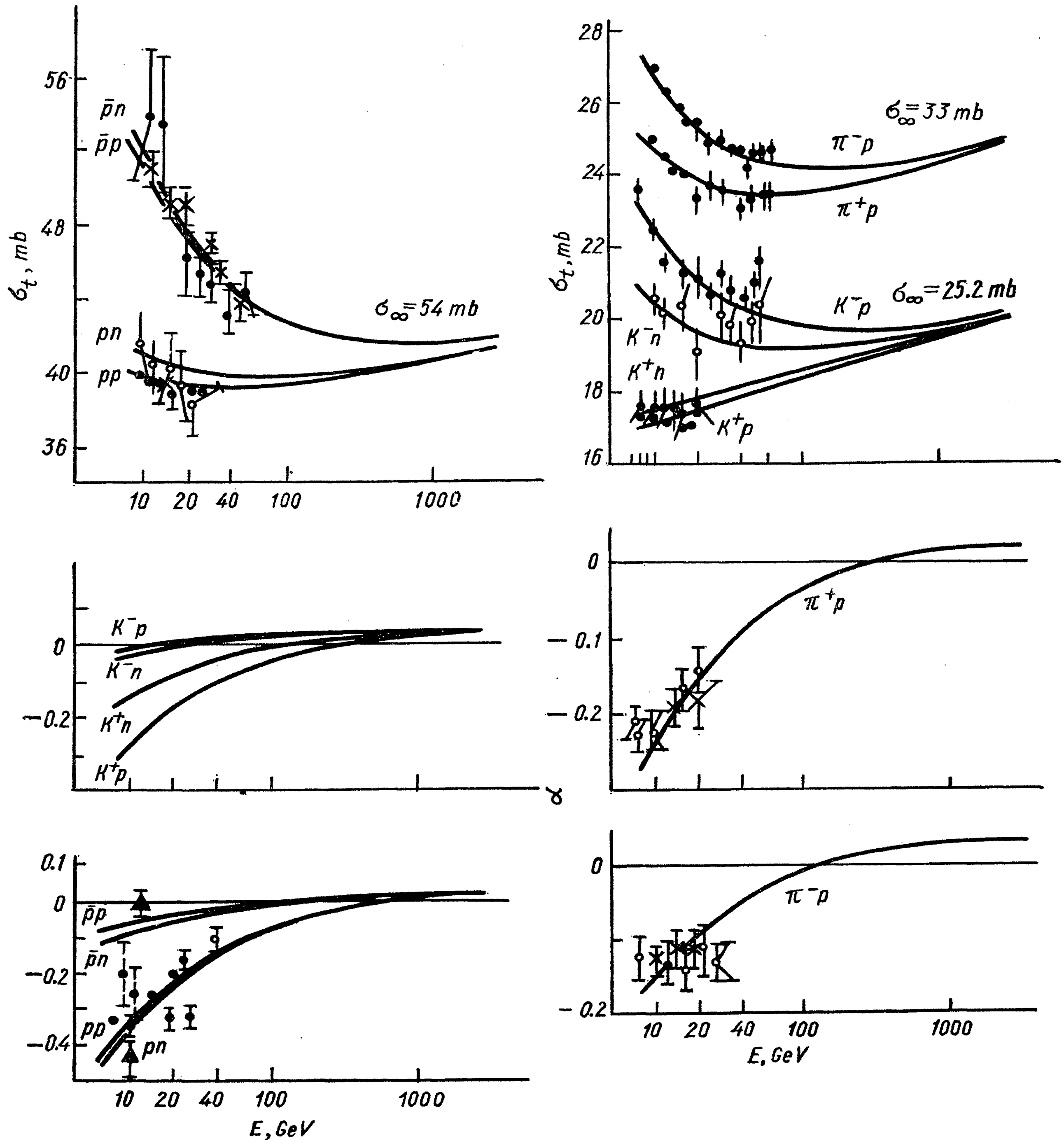


Fig. 36. Total cross sections and $\alpha = \mathrm{Re}\,T(0)/\mathrm{Im}\,T(0)$ for πN, KN and NN-scattering in the Regge-cut model [9].

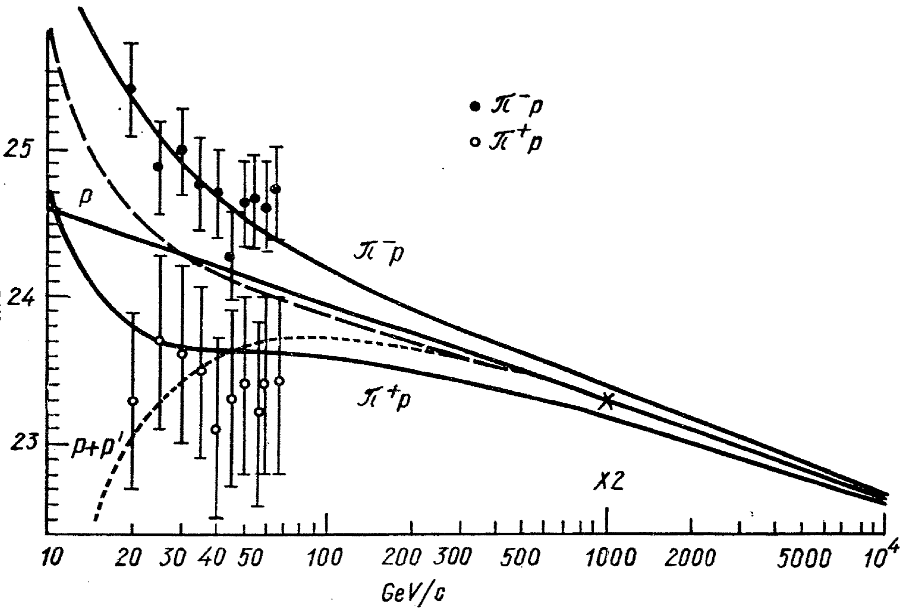


Fig. 37. Total cross sections for πN-scattering ($\alpha_P(0) = 0.988$) [10].

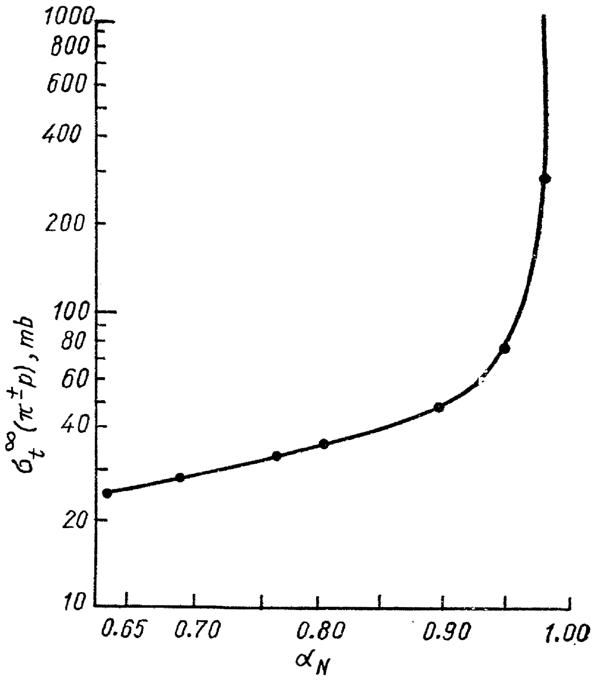


Fig. 38. Total cross sections for πN scattering. The dependence of $\sigma_{tot}(\infty)$ on $\alpha_N(0)$ in the interval $0.63 < \alpha_N < 1$ [11].

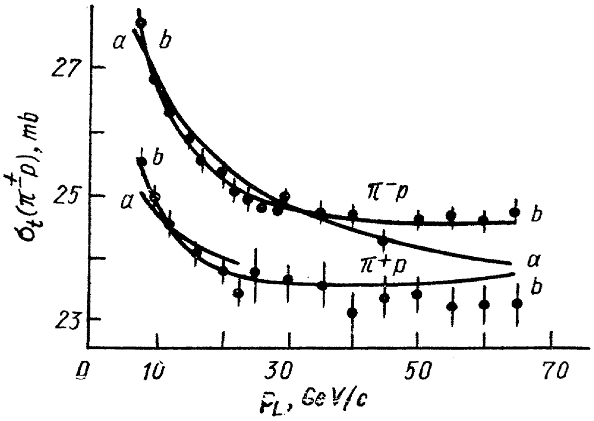


Fig. 39. Total cross sections for πN and KN scattering in the model with complex trajectories [12, 13].

the case of real residues is equal to

$$\sigma_\alpha = \frac{\gamma}{2} \nu^{\alpha_R - 1} \left\{ e^{\frac{1}{2}\pi\alpha_I} \sin\left(\frac{\pi}{2}\alpha_R - \alpha_I \ln\frac{\nu}{\nu_0}\right) + e^{-\frac{1}{2}\pi\alpha_I} \sin\left(\frac{\pi}{2}\alpha_R + \alpha_I \ln\frac{\nu}{\nu_0}\right)\right\};$$

$$\alpha = \alpha_R + i\alpha_I.$$

This leads to oscillating total cross sections (Fig. 39).

The Pomeranchuk trajectory cannot be complex, since this would lead to negative total cross sections in some region. The authors suggest that the P' trajectory be complex. There is a second group of papers, where the possibility of infinitely rising cross sections are considered provided the condition (case 1*b*) holds:

$$\sigma_{tot}^{+}(s)/\sigma_{tot}^{-}(s) \to 1; \quad s \to \infty.$$

To this group belong the papers by Finkelstein [14], Barger and Phillips [15], where it is assumed that the leading singularities in the *j*-plane are Regge-dipoles with $\alpha_{dipole}(0) = 1$. The contributions of such Regge dipoles with positive and negative signature to the πN-scattering amplitudes are as follows:

$$T_{dipole}^{(+)} = \frac{1}{2}(T_{\pi^- p} + T_{\pi^+ p}) = i\gamma^{(+)}\nu\left[\ln\frac{\nu}{\nu_0} - b^{(+)} - i\pi/2\right],$$

$$T_{dipole}^{(-)} = \frac{1}{2}(T_{\pi^- p} - T_{\pi^+ p}) = \gamma^{(-)}\nu\left[\ln\frac{\nu}{\nu_0} - b^{(-)} - i\pi/2\right].$$

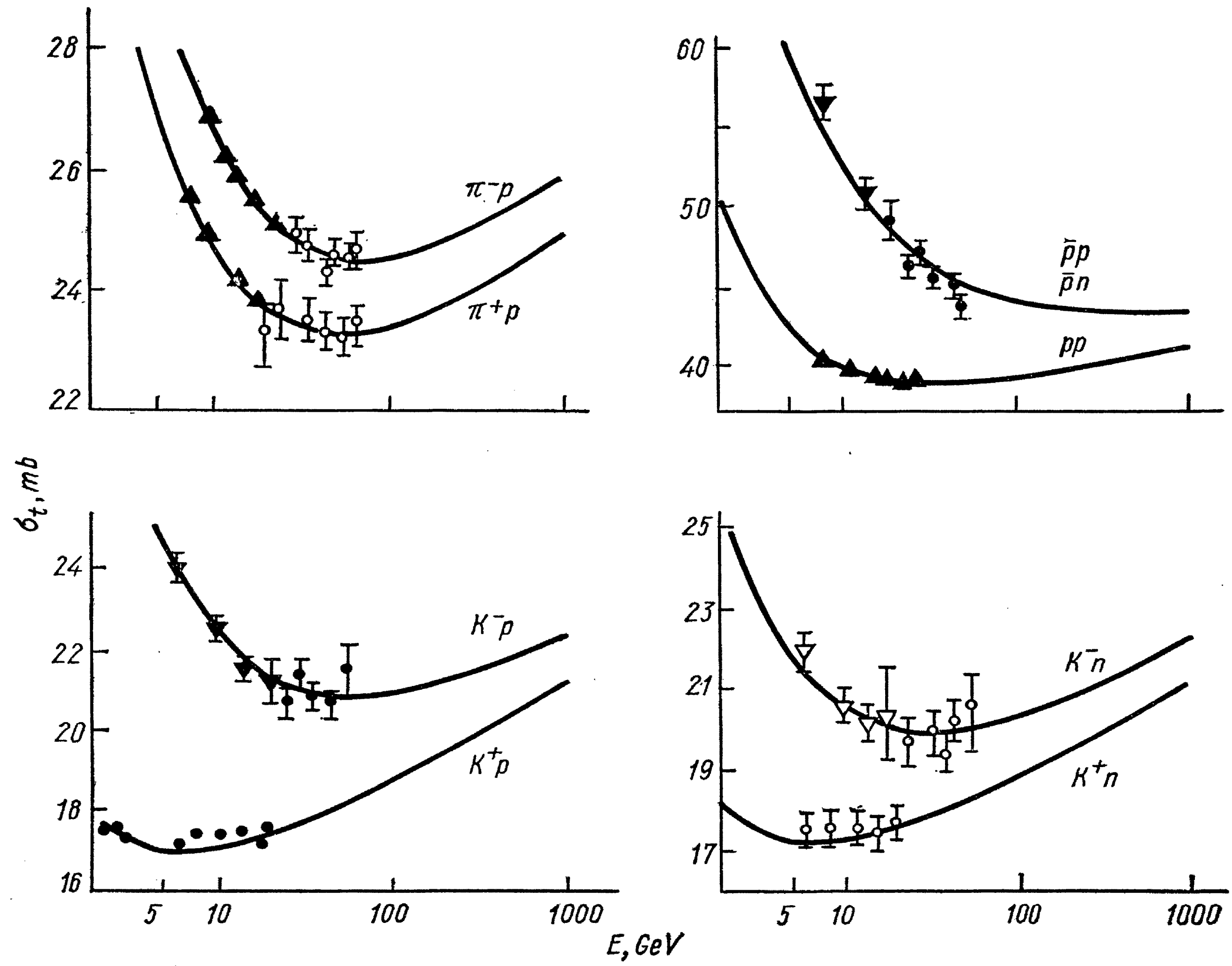


Fig. 40. Total cross sections for πN, $N\bar{N}$, NN and KN-scattering in the Regge-dipole model [14, 15].

Note, that the difference of the cross sections, σ_{π^-p} and σ_{π^+p}, in such a model does not vanish in the limit $s \to \infty$,

$$\sigma_{\pi^-p} - \sigma_{\pi^+p} = \pi\gamma^{(-)}.$$

The essential difference of this model from previous ones is the prediction of the interception of α_+ and α_- ($\alpha_\pm = \mathrm{Re}\, T^\pm/\mathrm{Im}\, T^\pm$) at $p_L \sim 100$ GeV/c.

The diffraction peak of elastic scattering shrinks in the maximally allowable way,

$$b^{-1}(s) \sim \frac{C}{(\ln s)^2}.$$

The results are given in Fig. 40.

Finally, there is a third group of papers where the violation of the Pomeranchuk theorem is discussed $\sigma^+_{\mathrm{tot}}(\infty) \neq \sigma^-_{\mathrm{tot}}(\infty)$ (case II). This is considered in the paper by Arnowitt, and Rotelli [16], where apart from the known trajectories two additional weakly interacting trajectories P_0 and P_1 with positive signature and the quantum numbers of the ω- and ρ-mesons, respectively, are included. Their intercepts are assumed to be $\alpha_{P_0}(0) = \alpha_{P_1}(0) = 1$. All these models describe the experimental data satisfactorily, but give different predictions for the higher energy.

In the paper by Aznauryan and Soloviev [17] consequences of the constancy of π^-p, π^+p-scattering total cross sections are analyzed with the help of dispersion relations. It is shown that the measurement of the forward charge exchange differential cross section with an accuracy of 10% at $p_L \geqslant 50$ GeV/c allows one to distinguish the case of the finite difference of the total cross sections from the predictions of other models (See also papers by Babayev and Margvelashvili [18]).

Section V

Discussion of some new tendencies in the theory of strong interactions at high energies

One of the best established properties of two-body and quasi-two-body processes at high energy is their diffractive character at small angles. The diffraction peak slopes, which are connected with the effective range of interaction are, in general, energy dependent. The bound on the interaction radius found by Khrustalev, Logunov, Mestvirishvili and Nguen Van Hieu [1, 2] in the framework of quantum field theory,

$$R^2 \leqslant \frac{\mathrm{const}}{\mu^2} \ln^2 s, \tag{5.1}$$

does not contradict existing experimental data. In the Regge approach the energy dependence of the diffraction peak slope is determined by the relation between the pole and cut parameters in the complex j-plane. The maximal growth of R^2 is determined by the Pomeranchuk-pole contribution

$$R^2 \sim \alpha_P \ln s. \tag{5.2}$$

From the quasi-optical point of view diffraction scattering at high energies can be considered as an interaction of two extended objects and is described by smooth (nonsingular) complex quasipotential.

In recent years a tendency has appeared to consider hadrons as composite systems with internal degrees of freedom in processes of strong interactions at high energy.

In particular, the «parton» model of Feynman [3] has been widely discussed. The «parton» model is based on an assumption that at extremely high energies hadrons can be considered as composite systems containing an arbitrary number of bare particles («partons»). Further, using qualitative kinematical consideration it is found that the main contribution to the wave function of physical particles is given by the «parton» configurations, which move synchronously with a particle having a finite values of the transverse momentum and nonzero longitudinal momentum $x_i = p_{zi}/W$. (p_{z_i} — is the longitudinal momentum of i-th «parton», W is the total energy of the system).

The high energy scattering of two particles is related with the overlapping of the so called «wee» partons (slow parton) clouds, which belong to the various colliding particles. One of the fundamental quantities in this approach is the «wee» parton spectrum,

$$\rho(x)\,dx \to \frac{1}{x}\,dx; \quad 1 \gg x \geqslant \frac{1\,GeV}{W}, \tag{5.3}$$

which is obtained provided the total cross sections at high energy tend to constant limits. The knowledge of the parton spectrum allows one to obtain a number of consequences for the multiparticle production processes at high energies. In particular, it follows from Eq. (5.3), that the average multiplicity of secondaries in inelastic processes increases logarithmically with increasing energy.

Another point of view is developed in the papers by Wu and Yang [4], Chou and Yang, Byers and Yang. In this approach hadrons are considered as extended systems, interacting in an optical manner and decaying into «fragments» («droplet» model).

In the paper by Benecke, Chou, Yang and Yen [5] a hypothesis of limiting fragmentation is formulated, according to which fragmentation achieves saturation in the limit of extremely high energies. A number of physical quantities (e. g. inelasticity coefficient, average transverse momenta of secondaries in inelastic collisions etc.) should tend to finite limits. In particular, partial cross sections of particles production processes with a given momentum $\vec{p}$ in the rest system of one of the colliding particles should have a finite limit,

$$\lim_{E\to\infty} dN(\vec{p};\ E) = \rho(\vec{p})\,d\vec{p}, \tag{5.4}$$

where $\rho(\vec{p}) > 0$.

It is assumed that the integral of Eq. (5.4) is logarithmically divergent at the upper limit of integration. This corresponds to the increasing multiplicity of the secondaries at high energies. In the «droplet» model the fast exponential decrease (with respect to t) of the elastic scattering differential cross sections is connected with the statistical character of the rearrangement mechanism of the momentum transfer.

In recent years various statistical approaches (Van Hove [6], Hagedorn [7], Feinberg, Chernavsky, Sissakian [8], Khrustalev, Logunov [9]) have been developed to describe high energy hadron interactions. Van Hove's approach is based on the unitarity condition and on the assumption that the intermediate particles do not correlate.

In the submitted papers by Khrustalev and Logunov the central limiting theorems of probability theory are used to estimate the high energy scattering amplitudes. They suggest that the number of particles in the intermediate state carrying a given transverse momentum be considered as a random quantity.

For the imaginary part of the scattering amplitude the following representation is obtained:

$$\mathrm{Im}\,T(s,\theta) = \int \xi d\xi \rho(\xi)\, J_0(\xi \varphi^{1/2}(s,\theta)), \tag{5.5}$$

where

$$\rho(\xi) = \sum_{n=1}^{\infty} \frac{C(n)}{n} e^{-n\xi^2/2}. \tag{5.6}$$

This amplitude reproduces rather well the structure of the angular distribution, provided some natural assumptions for the behaviour of $\varphi(s, \theta)$ and $C(n)$ are made.

The representation Eq. (5.5) can also be used to obtain smooth local quasipotentials in quantum field theory.

In the submitted paper by Matveev and Tavkhelidze [10] a model for high energy hadron interaction is studied in which hadrons are considered as strongly degenerated states of a composite system with an infinite number of degrees of freedom. It is assumed that the hadron states can be described by the coherent wave functions of the four-dimensional relativistic oscillator,

$$e^{-ipx} \rightarrow e^{-ip(x+\rho D)} | 0 \rangle, \tag{5.5}$$

where

$$D_\mu = a_\mu^+ + a_\mu; \quad [a_\mu^+, a_\nu] = -g_{\mu\nu} \tag{5.6}$$

and ρ is a constant having dimension of length.

In this scheme elastic scattering is described as exchange of longitudinal «oscillations». Diffraction exponentials appear as a consequence of the overlapping of the transverse oscillation clouds. The total differential cross section for particle production depends weakly on momentum transfers in this model. This is similar to the behaviour of the deep inelastic lepton-hadron interaction cross sections (Matveev, Muradyan, Tavkhelidze [11]). Note, also, that the method of coherent states plays an important role in the problem of factorization of dual amplitudes [12].

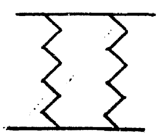
DISCUSSION

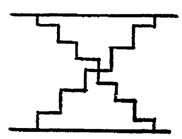
N o v o z h i l o v:

In the straight line paths approximation (SLPA-model) described by the rapporteur one starts from the relativistic Lagrangian, and approximations do not include the neglect of retardation effects. But at first sight somehow these effects have been dropped out during the calculation because they are absent in results. May I ask Prof. Tavkhelidze to clarify this point?

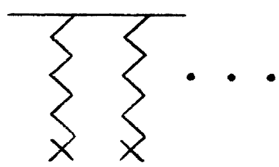
T a v k h e l i d z e:

First of all, I want to note that the final results contain the relativistic kinematics. What about dynamics? The concealing of the so — called main logarithmic terms really takes place in the summing of Feynman diagrams in the high energy limit. For example, it is possible to illustrate this point by considering the fourth order diagrams in the scalar model:





At high energies and fixed momentum transfer the asymptotical behaviour of the first diagram is $s^{-1} \ln s$, and the second one — $u^{-1} \ln u$. In this limit $s \approx -u$ and the asymptotical behaviour of the sum of the diagrams is s^{-1}. Thus in this approximation the sum of the above diagrams asymptotically tends to the sum of quasipotential graphs



and one may really say that at $s \to \infty$ and t — fixed the disappearance of some of the retardation effects takes place.

T o d o r o v:

I would like to make the following remark. The quasipotential equation reproduced in Prof. Tavkhelidze talk is not the only one consistent with elastic unitarity and relativistic kinematical root $\left(2\sqrt{m^2 + \vec{p}^2}\right)$ by the total-energy-parameter ω (see I. T. Todorov, Trieste preprint IC (70/59). In this way we arrive at a simpler (local) equation which can be treated rigorously if the potential is replaced by its Born approximation. As a result we obtain in a simple way the relativistic eikonal formula of Abarbanel and Itzykson (Phys. Rev. Letters **23**, 53 (1969)) and Barbashov et al. (Dubna preprint E2—4692 (1969)), as well as the relativistic Balmer formula proposed by Brezin, Itzykson and Zinn — Justin (Phys. Rev. **D1** (1970)). This approach provides also a straight forward method for calculating higher-order corrections, both to the relativistic eikonal formula and to the energy levels of the hydrogen atom and positronium.

K a d y s h e v s k y:

I would like to point out some heuristic features of the quasipotential approach which has been described by Prof. Tavkhelidze in his report.

The quasipotential method is the 3-dimensional formalism for the description of the relativistic two-body system. As in the non-relativistic theory one uses here the center of mass system in order to pick up the relative motion of the particles. It turns out that in the quasipotential picture the relative momenta belong to the mass shell hyperboloid, i. e. they are the vectors of the 3-dimensional Lobachevsky space. We can put in correspondence to these non-euclidian relative momenta the non-euclidian relative coordinates. It can be done using the relativistic version of the Fourier transformation which is connected with Lorentz group in the same way as the usual Faurier transformation is connected with Galilei group.

It turns out that in the new coordinate space the quasipotential equation takes the form of the finite-difference equation with the step $\hbar/mc$. When $c \to \infty$ one obtains from it the usual differential Schrödinger equation. So in some sense the quasipotential theory can be considered as a consistent model of the theory with a «fundamental length». Let me emphasize that when the distances between particles become of the order $\hbar/mc$ we should use the difference equation instead of the Schrödinger one.

We have considered in this way the eikonal approximation when $t \ll s$, $m^2 \ll s$. In such a case the quasipotential equation can be reduced to one-dimentional difference equation which can be solved exactly like the one-dimensional differential equation which appears in the nonrelativistic eikonalization procedure.

T e r - M a r t i r o s y a n:

I would like to make a general remark on the two experimental reports and the theoretical talk to which we have listened today. Prof. Allaby and Prof. Morrison in their reports presented a great amount of material which seems to me a nightmarish multitude of facts disconnected from each other.

From the theoretical report we have learned a large number of models and ideas, also disconnected from each other. I disagree with such a point of view in high energy physics. I want to note that there is now the theoretically selfconsistent and orthodox scheme which deals with the complex orbital momenta and which describes all the known facts at high energies. This scheme takes into account Regge poles and all appropriate branch points. The complex momenta scheme has predicted the largest part of the phenomena observed in Serpukhov and permits a quantitative explanation of them.

S h i r k o v:

Please, compare briefly the properties of the elastic and inelastic unitary in the s-channel as well as the unitarity in the t-channel for the following models:

1. Regge phenomenology;
2. eikonal approximation;
3. method of the absorption corrections;
4. quasipotential approach.

T a v k h e l i d z e:

In the approaches listed by Prof. Shirkov the exact s-channel unitarity condition is provided by the sign definiteness of the imaginary part of the eikonal phase or the Regge pole contribution or the quasipotential, etc. However since the series defining the scattering amplitude in these approaches is sign alternative, the violation of the unitarity condition can take place when the series is cut off (usually in fitting the experimental data one uses only a few terms of the corresponding series). In the set of works submitted to the conference (the works of the Michigan and ITEP groups) the additional parameter is introduced in a phenomenological way. For an arbitrary choice of this parameter it is possible for the approximate amplitude not satisfy the s-channel unitarity condition. Probably such a situation is connected with the fact that these models take into account explicitly only the t-channel unitarity condition.

B o g o l u b o v:

It seems to me very adequate that in this talk Prof. A. Tavkhelidze paid the principal attention to the different approaches and new ideas.

Surely the models considered have within themselves very attractive features but nevertheless they describe only a part of reality, only a part of the experimental data.

Therefore in my opinion their main interest consists in their heuristical possibilities. We must not forget that our main aim is to obtain more a comprehensive and general theory of strong interactions based upon fundamental principles.

B l o k h i n t s e v:

At present it is impossible to treat the strong interactions starting only with the general principles and using only the universal constants. So we come to models. From the mathematical point of view the model means that we have a set of arbitrary constants or possibly functions. In this situation we must prefer the model which uses the least number of such constants or functions. Keeping in mind the interpretation of the elastic hadron scattering at high energies and the total cross section behaviour, then we must prefer the optical approach (eikonal approximation or quasipotential model) where we just use the least number of constants.

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