1.2 OPTIMIZATION OF BUBBLE CHAMBER DESIGN PARAMETERS: MEASURING ACCURACIES FOR CHARGED PARTICLES

C.M. Fisher

Rutherford High Energy Laboratory,
Chilton, Berkshire, England.

1. Introduction

The hydrogen/deuterium bubble chamber has many general properties which make it an exceedingly valuable instrument for the study of elementary particle interactions. The characteristics of these interactions and the problems of analysis change considerably as the energy of the beam is increased. The problems presented in the study of strong interactions at 100 GeV for example are quite different from those presented at 2 or 3 GeV. The problems are also quite different it we wish to study weak interactions in a neutrino beam rather than strong interactions of pions, kaons, antiprotons, protons etc.

At Nimrod we are concerned particularly with the study of strong interactions at low energies where in general not more than one neutral particle is produced. Since at these energies ($\lesssim$ 7 GeV/c) the decay lengths of secondary hyperons are always small (a few centimetres) the ability of the chamber to provide a complete kinematic analysis, with good identification of resonances, depends essentially on how well angles and momenta of charged particle tracks in the energy range 0-8 GeV can be determined. In addition there is the ever present problem of statistics which becomes more relevant as the precision of the chamber is improved.

At CERN and certainly in the future at the 300 GeV accelerator particle interactions are in a higher energy range and a large fraction of the final states produced will involve several neutral particles, either neutral π-mesons or neutrons. It is therefore an essential requirement that an efficient gamma detections system be provided and that if possible sufficient path length of hydrogen be present to allow the detection of neutrons from their secondary elastic scatters. High energy accelerators are also a good source of neutrino and antineutrino beams so that any chamber required to work at such an accelerator must be designed with the characteristics of these interactions, in particular the exceedingly small cross sections, in mind.

The problem of gamma detection and neutrino physics at high energies are discussed elsewhere in this meeting (Leutz[1]), and the chamber best suited for these studies is readily shown to be extremely large.

In considering the possible developments of the technique there are essentially three possibilities:

(a) To increase the volume of the chambers by perhaps one or two orders of magnitude.

(b) To increase the magnetic field.

(c) To develop mixed systems which combine the bubble chamber with a system of triggered spark chambers.

These possibilities are not mutually exclusive but each offers different advantages. The decision as to which development should be undertaken at a particular accelerator depends on the energy and the variety of beams available and consequently on the kind of physics that can be studied.

In this paper the low energy optimisation is first described before extending the analysis to higher energies.

PART I

LOW ENERGY OPTIMISATION

2. General Considerations at Low Energies

In making the proposal for a new chamber at Nimrod we have considered the possibilities of overcoming the limitations on the study of strong interaction physics which exist in present chambers. These are:

(a) The lack of statistics particularly in channels with many final state particles.

(b) The difficulty of obtaining unbiased samples of events from the same reaction without the background of wrongly identified or ambiguous events from other channels.

(c) The resolution of effective masses is considerably worse than we believe can be achieved. This makes it difficult to separate resonances one from another or toobserve any narrow resonances which are not strongly produced. In this context I would quote from a recent theoretical paper on the Quark Model by Mitra and Ross[2].

"At present the sensitivity of experiments is such that many of the resonances will be unobserved. Many states have very small width and/or high inelasticity or are very broad such that they may not be observed for some time. It would be a mistake to assume that lack of observation of a resonance necessarily implies higher mass".

Condition (a) requires that the chamber be either large so that the number of interactions per picture is increased or that it be fast cycling. Because of possible ambiguities in correlating V^o's and gamma conversions with origins and pattern recognition problems of advanced F.S.D. and similar systems we have chosen that the chamber be fast cycling.

The limitations (b) and (c) can only be removed by increasing the precision of momentum measurements. The guiding principle in the choice of dimensions and magnetic field value has therefore been to minimise the error on a two body effective mass combination for particles in the Nimrod energy range. This is achieved by choosing the dimensions such that the momentum errors are dominated by the multiple coulomb scattering contribution and the angle errors are near minimum. The magnetic field value is the chosen so that the momentum and angle errors are matched in the effective mass calculation. This process is described briefly below.

3. The errors on Momentum Measurements on Charged Particle Tracks.

In the absence of ionization loss the trajectory of a charged particle moving in a magnetic field is a helix. The momentum is derived from the radius of curvature of the helix and the dip angle of the track through

$$p \cos\lambda = 0.3\ RH$$

p is in MeV/c, R in cms, λ the dip angle in degrees, and H is in the Kgauss. The error in p can be written as the sum of a contribution from multiple coulomb scattering and a contribution from the measurement precision

$$\left(\frac{\Delta p}{p}\right)^2 = \left(\frac{\Delta p}{p}\right)^2_{M.S.} + \left(\frac{\Delta p}{p}\right)^2_{\varepsilon} \qquad \ldots\ldots (1)$$

where

$$\left(\frac{\Delta p}{p}\right)^2_{M.S} = \frac{0.133\,[\ln 4.8p + \ln 145\,p/Mc]}{H^2 L \beta \cos^2\lambda} + \frac{5.10^{-2} L \tan^2\lambda}{p^2 \beta^2} \qquad \ldots\ldots (2)$$

Thus $\left(\frac{\Delta p}{p}\right)^2_{M.S}$ has two terms the first arising from the error in the radius of curvature and the second from the error in the dip angle.

The form of the contribution from the measurement error depends on the method of measurement. For an automatic system (F.S.D.) the number of points measured depends on the length of track and the master points are uniformly distributed. A conventional manual system usually has a fixed number of points per track, however, they can be distributed in an optimum fashion (¼N at beginning and end with ½N at centre). These considerations lead to the expressions:

$$\left(\frac{\Delta p}{p}\right)^2_{\varepsilon.\,FSD} \simeq \frac{1.44\, p^2 \varepsilon^2 10^{-4}}{H^2 L^5 \cos^3\lambda} + \frac{1.2\cdot 10^{-5}\, \varepsilon^2 \sin^2\lambda}{L^2 \cos\lambda} \qquad \ldots\ldots (3)$$

$$\left(\frac{\Delta p}{p}\right)^2_{\varepsilon\ conv:} \simeq \frac{3.55\, p^2 \varepsilon^2 10^{-6}}{H^2 L^4 \cos^2\lambda} + \frac{0.5\, 10^{-6} \varepsilon^2 \sin^2\lambda}{L^2} \qquad \ldots\ldots (4)$$

where ε is in microns chamber space, L is in cms.

For high energy tracks having small dip angles with respect to the median plane we can neglect the second terms on the above expressions with respect to the first. An inspection of the expressions (2) and (3) reveals that for high momenta and short track lengths ($\Delta p/p$)$_{F.S.D.}$ dominates the total errors (this is almost always the case in present day chambers) whilst for long tracks and lower momenta ($\Delta p/p$)$_{MS}$ dominates.

For any given momentum p it is possible to find a track length L for a given ε such that below p the Coulomb term dominates and the error is $\propto \frac{1}{H\sqrt{L}}$ whilst above p the error is essentially $\propto \frac{\varepsilon p}{H L^{5/2}}$. We find

$$L_{F.S.D} \simeq 2.8 \sqrt{p\varepsilon} \qquad \ldots\ldots (5)$$

p is in GeV/c, ε in microns, L in cms.

Thus from the point of view of momentum momentum measurements the best policy is to increase the dimensions of the chamber until the above values of track length are achieved. To reduce the error further it is then desirable to increase the magnetic field H rather than to further increase L since $\left(\frac{\Delta p}{p}\right)_{M.S} \propto \frac{1}{H\sqrt{L}}$.

Fig. 1 shows the track length as a function of ε for various momenta from (5). Clearly for ε in the region of 100 microns we require track lengths in the region of 50-70 cms for the Nimrod energy range.

4. The Errors on Angle Measurements

The errors on the azimuthal angle and the dip angle are given by

$$\langle\Delta\theta\rangle^2_{F.S.D} \simeq \frac{3.7\ 10^{-2} L}{p^2 \beta^2 \cos^2\lambda} + \frac{3.8\ 10^{-6} \varepsilon^2}{L^3 \cos^3\lambda} \qquad \ldots\ldots (6)$$

$$\langle\Delta\lambda\rangle^2_{FSD} \simeq \frac{1.2.\ 10^{-5} \varepsilon^2 \cos\lambda}{L^3} + \frac{5.\ 10^{-2} L}{p^2 \beta^2} \qquad \ldots\ldots (7)$$

$$\langle \Delta \theta \rangle^2_{\varepsilon\, conv} \approx \frac{2.8\ 10^{-2} L}{p^2 \beta^2 \cos\lambda} + \frac{1.44\ 10^{-7} \varepsilon^2}{L^2 \cos^2\lambda} \qquad \text{...... (8)}$$

$$\langle \Delta \lambda \rangle^2_{conv:} \approx \frac{5.10^{-7} \varepsilon^2 \cos^2\lambda}{L^2} + \frac{5.10^{-2} L}{p^2 \beta^2} \qquad \text{...... (9)}$$

These relations have minima at track lengths given by

$$L_{\substack{opt\\ F.S.D}} (\theta,\lambda) \approx 0.15 \left(\varepsilon p \beta\right)^{1/2} \text{ cms} \qquad \text{...... (10)}$$

$$L_{\substack{opt\\ conv:}} (\theta,\lambda) \approx 0.03 \left(\varepsilon p \beta\right)^{2/3} \text{ cms} \qquad \text{...... (11)}$$

p is in MeV/c, ε in microns.

Fig. 2 shows a plot of L_{opt} against p for various values of ε. Fig. 3 shows the variation of $\langle \Delta\lambda \rangle$ with L at different momenta and different values of ε. The following points should be noted:

(a) The values of $\Delta\lambda$ and $\Delta\emptyset$ are rather insensitive to variations of L about L_{opt}. (θ,λ) A factor of two in L only increases $\Delta\lambda$ by about 10%.

(b) Track lengths in the region of 50-70 cms yield near minimum angle errors for $\varepsilon \approx$ 50-100μ.

(c) At track lengths below L_{opt} the values of $\Delta\lambda$, $\Delta\emptyset$ are sensitive to ϵ.

5. The Dimensions of the Chamber

At low energies the diameter is chosen such that the track lengths from interactions in the central regions of the chamber are given by (5). This leads to a diameter in the region of 150 cms for p $\lesssim$ 5 GeV, and $\varepsilon \sim$ 100μ. The depth of the chamber is conditioned by the optical requirement of good resolution to give a small ε in the median plane. This depends on ones ability to predict the effects of thermal turbulence as discussed by D.B.Thomas(5).

The obvious advantage of a large depth is that low energy tracks can be constrained to spiral along the magnetic field axis and will have long track lengths. This could then provide useful particle identification at low energies. For example electrons may be identified by their bremmstrahlung energy loss and low ionisation over large track lengths. We have chosen a depth of 1 metre for the fiducial region of the chamber.

6. The Choice of Magnetic Field

Having chosen the diameter to ensure that the multiple scattering dominates the error on momenta this error is given by

$$\left(\frac{\Delta p}{p}\right) \approx \frac{1.6}{H\sqrt{L}} \approx \frac{23}{H}\% \quad ; L = 50\,\text{cms} \qquad \ldots\ldots (12)$$

Fig. 9 shows a plot of ($\Delta p/p$) against p for various H L and ε .

The error on a single two body effective mass calculation is given by

$$\Delta M = \frac{1}{M}\left[(\varepsilon_2\beta_1 - p_2\cos\phi)^2\Delta p_1^2 + (\varepsilon_1\beta_2 - p_1\cos\phi)^2\Delta p_2^2 + p_1^2 p_2^2 \sin^2\phi\,\Delta\phi^2\right]^{1/2} \qquad \ldots\ldots (13)$$

$\varepsilon_1 \varepsilon_2\ p_1\ p_2$ are energies and momenta and ϕ is the angle between the particles; $\beta = p/\varepsilon$ Resolutions obtainable on several known resonances have been calculated using this relation together with equations 2, 3, 6, 7 for Δp and $\Delta\phi$. Figs. 4 and 5 show the predicted resolutions for resonances decaying at 90° to the cms line of flight, as a function of the resonance momentum for various values of H and ε.

In these calculations all errors are computed assuming track lengths of 50 cms with 20° dip. The following points should be noted:

(a) For the determination of the effective mass of a resonance having a large Q value, such as K*(1400) in Fig. 5, a field $\simeq$ 70 Kgauss is required to give matched errors if L $\simeq$ 50 cms.

(b) The resolution is in the region of 1 to 3 MeV/c^2.

(c) The matched error condition approximately required ($\Delta p/p$) $\approx$ 0.3 - 0.4% which requires $H\sqrt{L} \simeq$ 500 Kgauss cms$^{\frac{1}{2}}$.

Fig. 6 shows the error on the two body effective mass values for 1000 four prong events from the 3.5 GeV/c K^-p experiment with errors reassigned to correspond to a mapping to the high field chamber. Errors calculated in the same way but using the field and track lengths relevant to the 1.5 metre British chamber are also shown. Fig. 7 shows the corresponding distribution for the error on the missing mass squared for the same events.

7. The Parameters of the High Field Chamber for Nimrod

The foregoing discussion leads to the following design conditions:

1. The dimensions of the chamber must be such that track lengths exceed $3\sqrt{p\varepsilon}$.

2. The dimensions and field must be such that if L is the actual track length then $H\sqrt{L} \geqslant 500$ Kgauss $\text{cms}^{\frac{1}{2}}$.

3. ε must be as small as possible.

Briefly the chamber parameters chosen are:-

Diameter	150 cms
Depth (useful)	100 cms
Field	70 Kgauss
Precision	$\varepsilon \leqslant 100\mu$ (Chamber space)
Rapid Cycling	2-5 expansions/sec

Axis Vertical with both median plane and on-axis beam entry ports.

The conditions (1) and (2) describe a spectrum of chambers, illustrated by Figs. 1 and 8 . The choice within these limits depends on both practical considerations and on physics considerations other than those discussed so far.

The practical problems of chamber design are discussed by Dr. D.B.Thomas(3).
Here I will mention briefly some other physics considerations:

Particles produced in the centre of the chamber with momenta less than

$$p_{const} \simeq 0.15\, R H \qquad \ldots\ldots (14)$$

where R is the chamber radius, are constrained to spiral along the magnetic field without leaving through the side walls of the chamber. Track lengths for such particles depend only on the depth of the chamber and the angle of dip.

There are many advantages in having low energy particles so constrained. If angles of dip are small then very long track lengths can be obtained and can provide very useful information about particle identification. An obvious example would be the study of hyperon leptonic decays where the leptonic decay mode must be identified against a background of non leptonic decays 10^3 times more frequent. The energy loss characteristics or behaviour on coming to rest for a chamber with $p_{const} \gtrsim$ 600 MeV/c would yield unique identification over the whole lepton sepctrum both for hyperons produced by K^- at rest or by K^- or π^- beams chosen to give hyperon polarization.

This is an argument for going to larger dimensions and lower field since if we keep to the $H\sqrt{L}$ = const curve we get a larger HR product and higher momentum particles are constrained to stay in the chamber. At 70 Kgauss field 1.5 metres diameter we are already constraining 800 MeV/c particles which is about the same as the large volume proposals, and certainly sufficient for lower energy experiments.

A high field is of course also useful for measurements on hyperon and kaon tracks whose lengths are limited by decay. A consideration which could turn out to be a strong point in favour of choosing a high field is also the possibility of determining hyperon magnetic moments with this chamber.

It would be possible with a field of $\simeq$ 70 Kgauss to obtain the magnetic moments of Λ^0 and Σ^+ particles to better than 0.1 nuclear magnetons and the Ξ^- moment to about 0.7 magnetons with exposures of about half a million pictures.

I have omitted any detailed discussion of γ detection with this chamber.

Since the chamber is intended for operation with low energy beams there is no problem arising from a need to allow the γ rays from π^0's to separate spacially before conversion. The minimum opening angle for gammas at 1.5 GeV/c is about 10 degrees so that a distance of 40-50 cms will yield a spacial separation of greater than 10 cms. The geometry of the chamber is cylindrical and similar to the large volume projects and the higher magnetic field reduces the track length necessary to give good electron momentum measurements to a few centimetres. Thus complex plate systems of the kind envisaged by the large volume proposers can be scaled down without loss. The problems are associated with the optical design. Problems such as these are in the course of study.

PART II

HIGH ENERGY OPTIMISATION

8. Strong Interactions at High Energies

As the energy of the accelerated beam increases the characteristics of the elementary particle interactions change. Multiple production of neutral pions occurs making the inclusion of some form of γ detection a prerequisite and the study of neutrino interactions becomes an important consideration in chamber design. In this section we are principally concerned with the problems of making optimum measurements on charged tracks and it is readily seen that this consideration alone leads to a dimension of 4-5 metres when the particle energies are in the region of 20-30 GeV.

At very high energies the possibility of identifying an event as belonging to a particular reaction channel seems remote. The problems of sorting Λ^0 from Σ^0 events is not solvable without the inclusions of a γ detector. If we assume that at high energies the fitting procedure does not significantly improve the errors on the charged track variables we can ask if the errors on effective mass calculations using geometry values can be small anough to allow the identification of resonances. At high energies with interactions having many final state particles the phase space distributions become rather similar and for example the effect of adding or subtracting a π^0 to a channel already having five or six charged particles does not much alter the effective mass phase space distribution for combinations of two or three of the charged particles. The phase space distributions are always long and flat if the energy is high. Morrison(4) has shown that such phase space distributions make the study of resonances at high energies rather good. Of course if the number of missing neutral particles is greater than one the channel cannot be identified without the addition of a gamma converter. One can, however, study resonances composed of only charged particles or of charged particles plus all of the missing neutrals.

9. Measurement of Charged Particle Effective Masses at High Energies

As the energy of the particle increases the track length necessary to kill the contribution to the momentum error from the measurement error also increased. From equation 5, if $\varepsilon \sim 100\mu$ can be maintained independently of the chamber dimension then the track lengths necessary are given by $L \simeq 30\sqrt{p}$ cms. Thus at 10 GeV/c we require a metre of track length for each secondary particle from the interaction. If the point error gets as high as 400μ we would require 2 metres at 10 GeV/c etc. This is a measure of the track length in free hydrogen required to cause momentum errors to be dominated by multiple scattering. There is also of course the cut-off imposed at high energies on the track length available by the interaction length in hydrogen for strongly interacting particles of about 5 metres. Thus in a high multiplicity event we want to measure all secondaries well so that this limit becomes relevant. Table 1 shows the fraction of all events in which all secondaries do not interact within 50 cms, 100 cms and 200 cms of the production point. [Interaction length taken as constant and equal to 500 cms.]

MULTIPLICITY

Length	2	4	6	8
50 cms	81%	66%	53%	43%
100 cms	66%	44%	31%	8%
200 cms	41%	17%	7%	3%

Clearly unless we are prepared to throw away a large fraction of events, which will not in general be an unbiased sample because the cross sections are both a function of the particle type and energy, one should not assume a track length for measurement greater than about 100-150 cms. This implies that at a high momenta [for $\varepsilon \simeq 500\mu$ then high momenta are $\gtrsim$ 5 GeV] the error on the momentum measurement is proportional to the error on a point and to the momentum:

$$\left(\frac{\Delta p}{p}\right)_{\varepsilon} \simeq \frac{0.012\, p\, \varepsilon}{H\, L^{5/2} \cos^{3/2}\lambda} \qquad \ldots\ldots (15)$$

ε in μ, p in MeV/c, L in cms, H in Kgauss.

Once the chamber is large enough to give track lengths greater than 1.0 - 1.5 metres in free hydrogen then further increasing the size does not help much because the effective track lengths are cut by the interaction length. We are of course assuming here that a good estimate of a particles energy cannot be made by an analysis of the secondary interaction. This of course is possible however I would not consider it as a means of improving on a measurement of curvature if a long track length (i.e. $\gtrsim$ 50 cm) is available. It can, however, sometimes lead to the identification of a strange particle via hyperon or K_1^0 production. If we set the track length of our measurable particles as 150 cms then the error ($\Delta p/p$) is given by

$$\left(\frac{\Delta p}{p}\right) \simeq \frac{4.35\, p\, \varepsilon}{H} \qquad \ldots\ldots (16)$$

where p is in GeV/c, $\mathcal{E}$ in mms in chamber, H in Kgauss. This now neglects coulomb scattering.

Angle errors at high energies are always in the region where the measuring error contribution dominates. Then

$$\langle \Delta\lambda \rangle_{\mathcal{E}\,FSD} \approx \frac{3.5\,\mathcal{E}}{L^{3/2}} \qquad \langle \Delta\theta \rangle_{FSD} \simeq \frac{1.9\,\mathcal{E}}{L^{3/2}} \qquad \ldots\ldots (17)$$

$\mathcal{E}$ in mms L in cms. Thus the error on an angle between two tracks will be in the region of $\langle \Delta\phi \rangle \approx \frac{5.7\,\mathcal{E}}{L^{3/2}}$

and if L is limited to $\gtrsim$ 150 cms then $\langle \Delta\phi \rangle \approx 3\,\mathcal{E}$ mrads

The error on two body effective mass combination is given approximately at high energies by:

$$\left(\frac{\Delta M}{M}\right)^2 \approx \frac{Q^2}{M^2}\left\{\frac{1}{2}\left(\frac{\Delta p}{p}\right)^2 + \left(\frac{\Delta\phi}{\phi}\right)^2\right\} \qquad \ldots\ldots (18)$$

and $Q^2 \approx \phi^2 p^2$ so that

$$\left(\frac{\Delta M}{M}\right) \approx \frac{3\,Q\,\mathcal{E}\,p\,10^{-2}}{M}\left\{\frac{1}{H^2} + \frac{10^{-2}}{Q^2}\right\}^{1/2} \qquad \ldots\ldots (19)$$

There is no point in having H much greater than 10Q with Q in GeV, because the angle error is independent of H. This leads to values of 20-30 Kgauss unless we require good resolution on resonances having masses greater than 2-3 GeV. The resolution is then of the order

$$\Delta M \approx 3\mathcal{E}p\ \mathrm{MeV}/c^2 \qquad \ldots\ldots (20)$$

where p is in GeV/c and $\mathcal{E}$ is in mms. For $\mathcal{E} \approx \frac{1}{2}$ mm and p $\lesssim$ 10 GeV/c

$$\Delta M \approx 15\ \mathrm{MeV}/c^2$$

which is much the same as we have now. This estimate is of course rather crude but I think serves to demonstrate that quite small errors on effective mass values can be obtained at high energies.

At the high energy limit given track lengths greater than about 1-2 metres and a field of greater than 20 Kgauss the effective mass errors are solely dependent on ε, and increase linearly with momentum.

At CERN it should be possible to achieve resolutions in the region of 10-20 MeV/c and at the 300 GeV accelerator one should have errors in the region of 100 MeV/c^2 from interactions producing secondaries of about 50-100 GeV/c.

To summarize: Purely from the point of view of minimising errors on effective mass calculations we require track lengths of at least a metre or more, a field of better than about 20 Kgauss and a small value of ε. These conditions are satisfied by the large volume chambers now proposed.

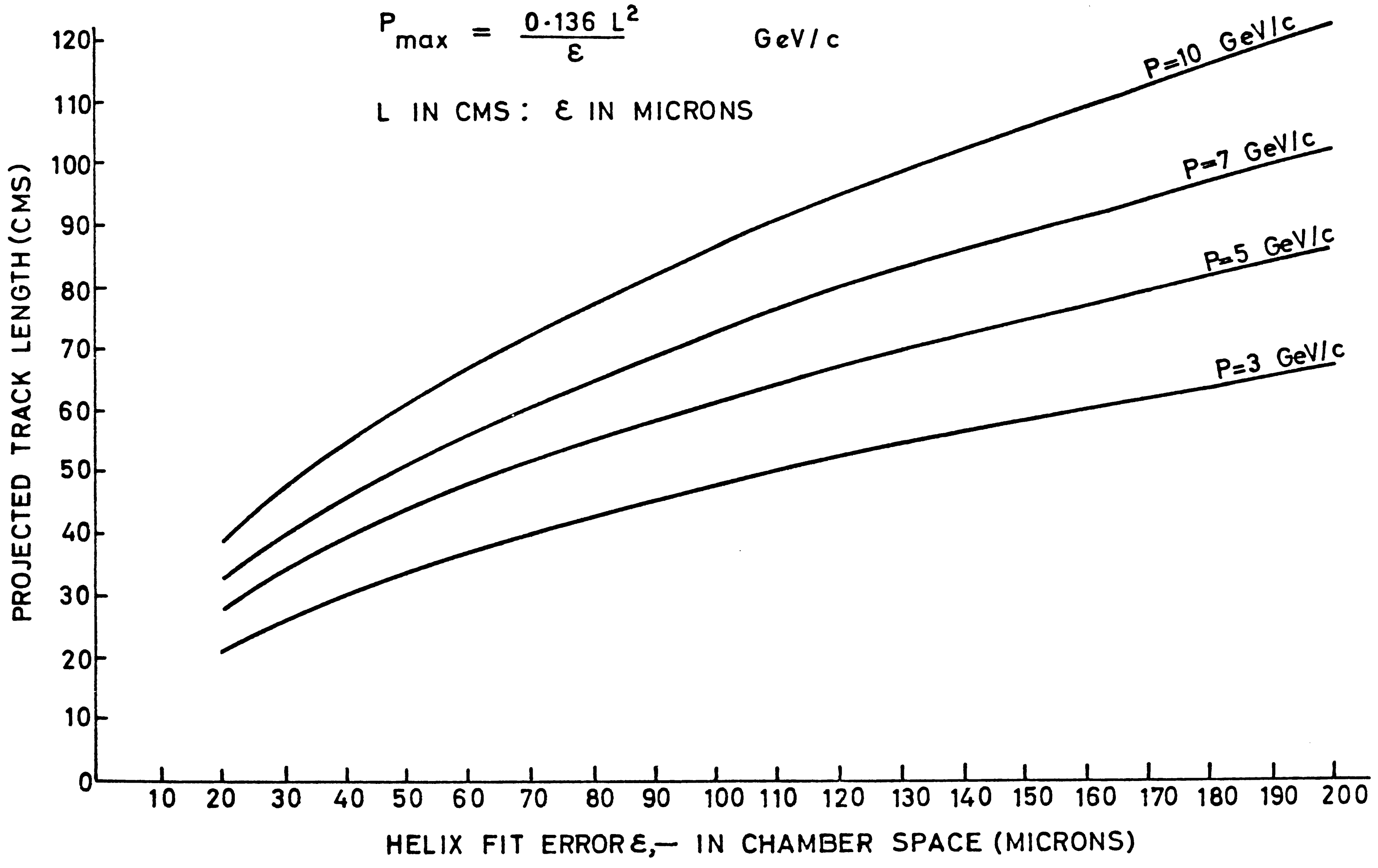

FIG. 1. PROJECTED TRACK LENGTH VS MEASURING ERROR FOR VARIOUS P_{max}

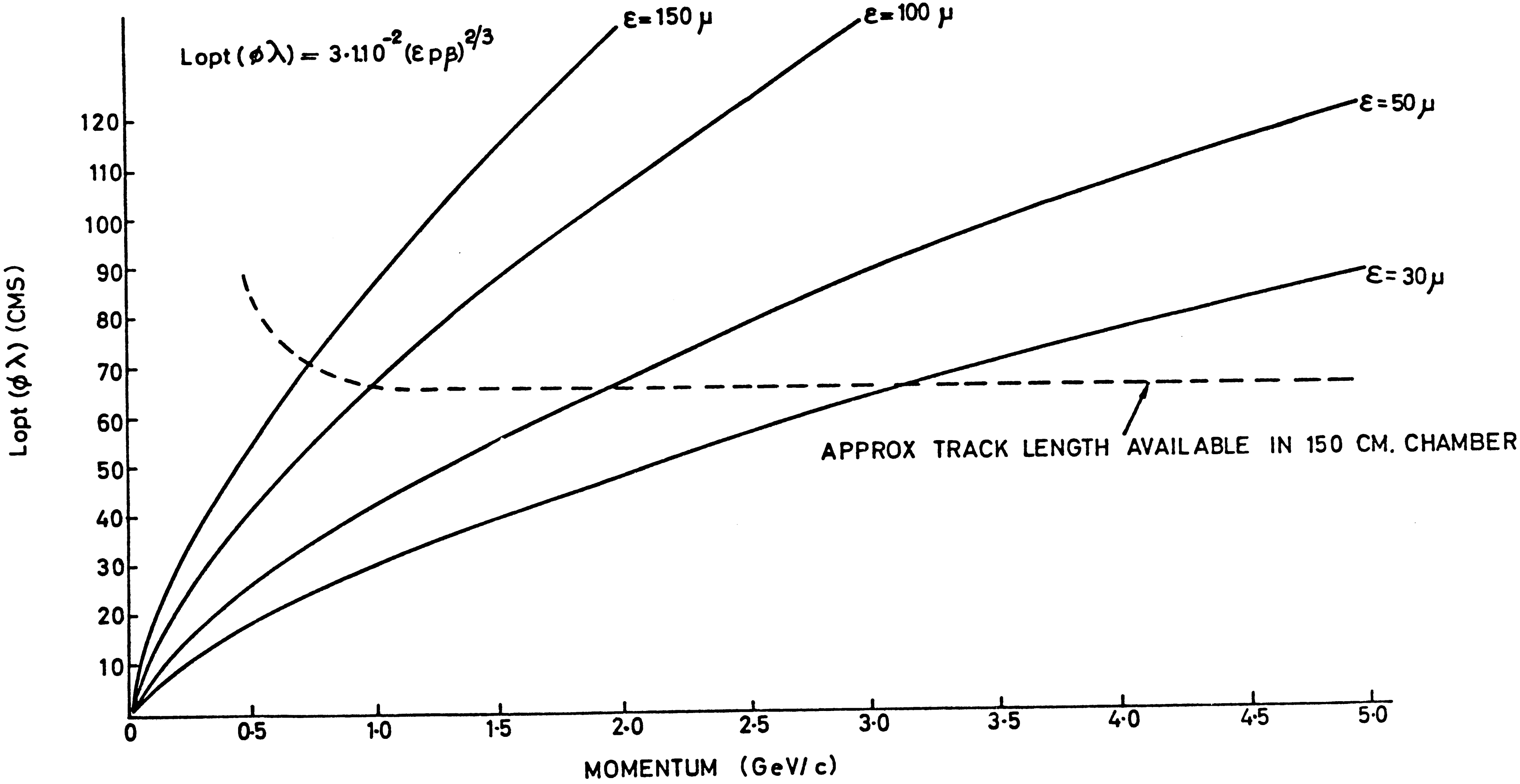

FIG. 2. OPTIMUM TRACK LENGTH FOR ANGLE MEASUREMENTS AGAINST MOMENTUM FOR VARIOUS VALUES OF ε

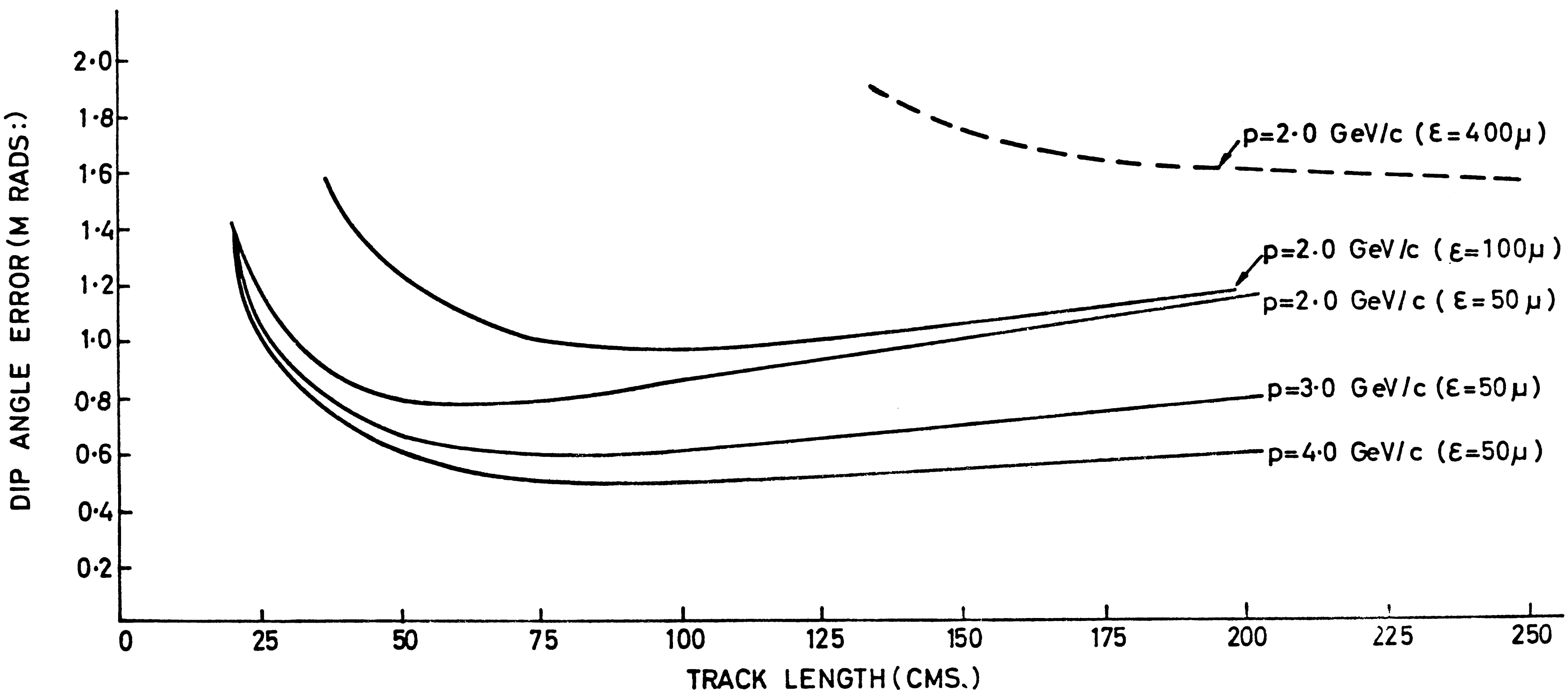

FIG. 3. VARIATION OF ERROR IN DIP MEASUREMENT WITH TRACK LENGTH FOR DIFFERENT MOMENTA

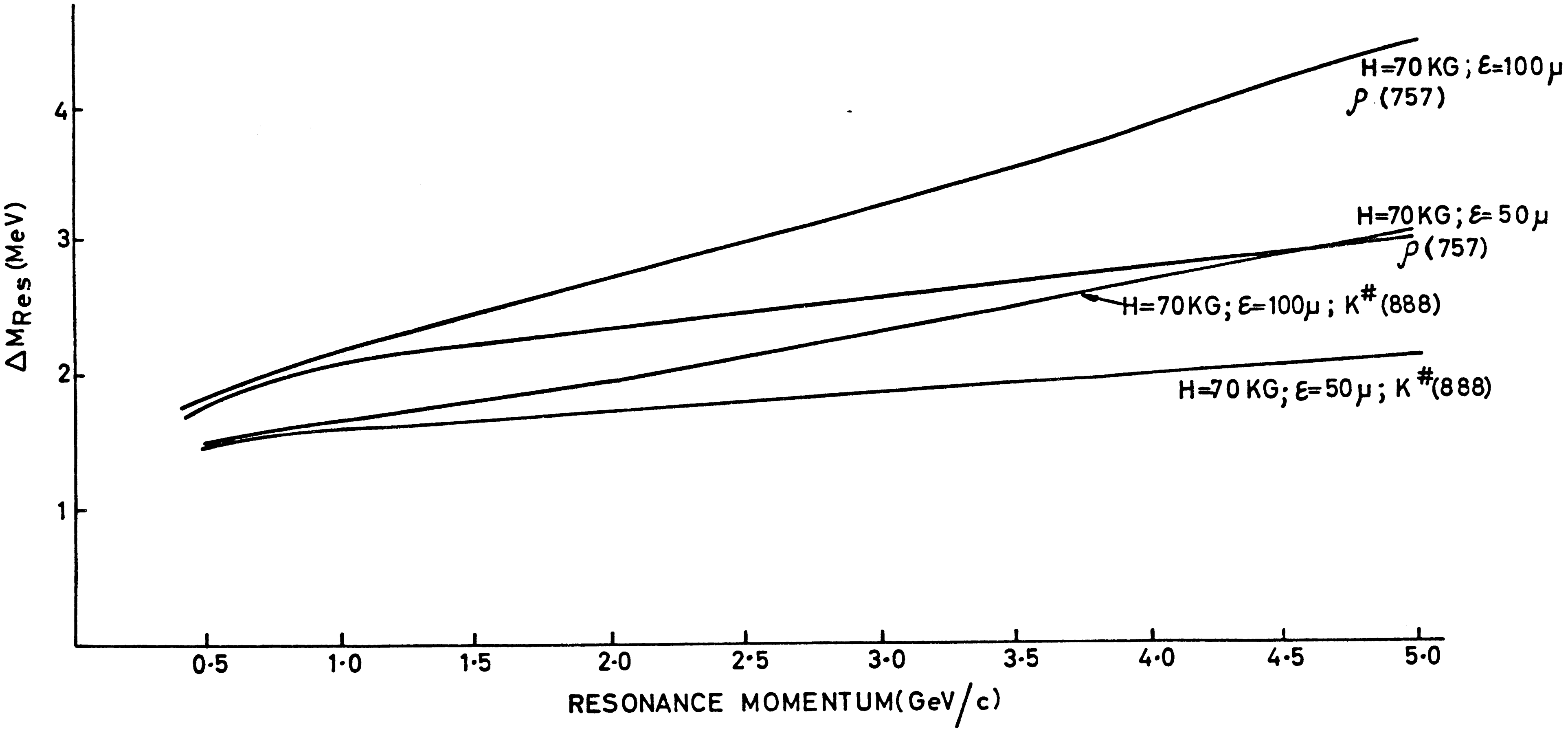

FIG. 4. UNFITTED ERROR ON TWO BODY EFFECTIVE MASS VALUES

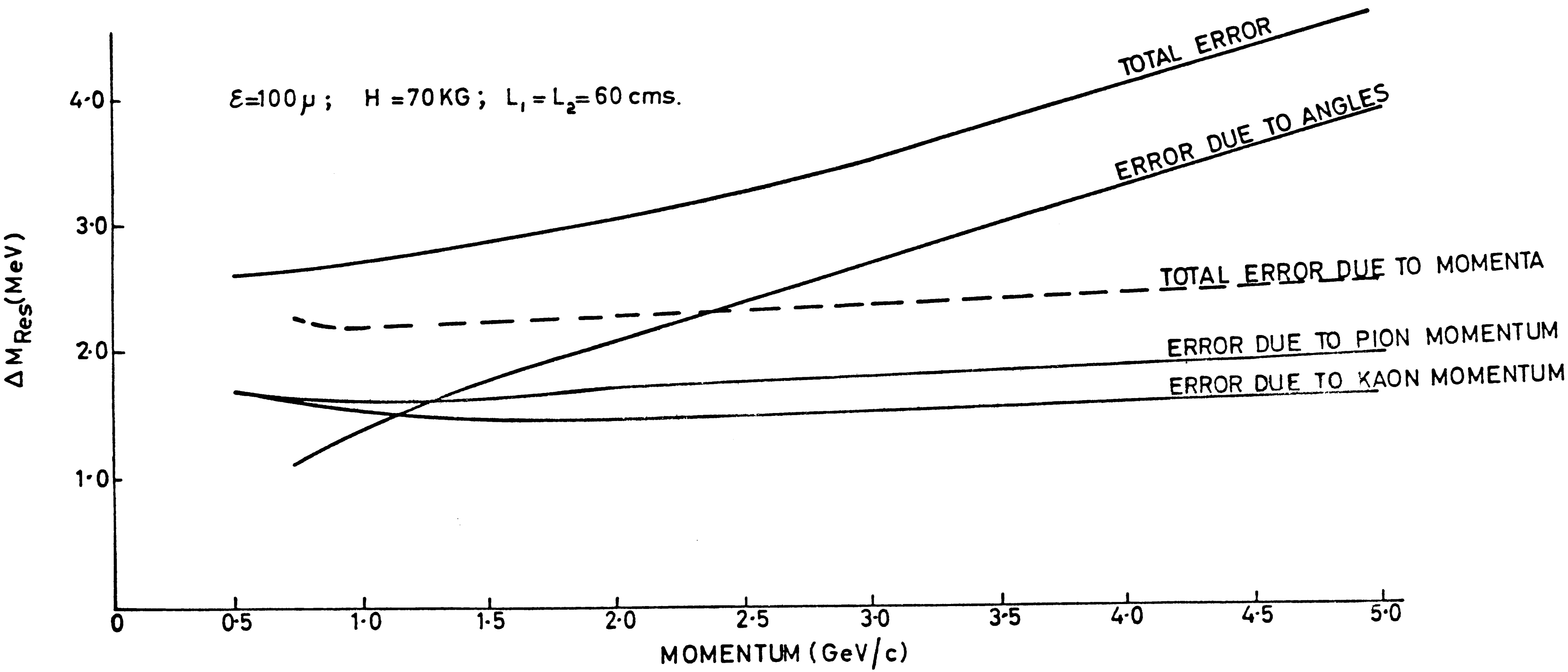

FIG. 5. UNFITTED ERRORS ON THE MASS OF RESONANCE $K^{\#}(1400)$
(DECAY AT 90° IN CENTRE OF MASS SYSTEM)

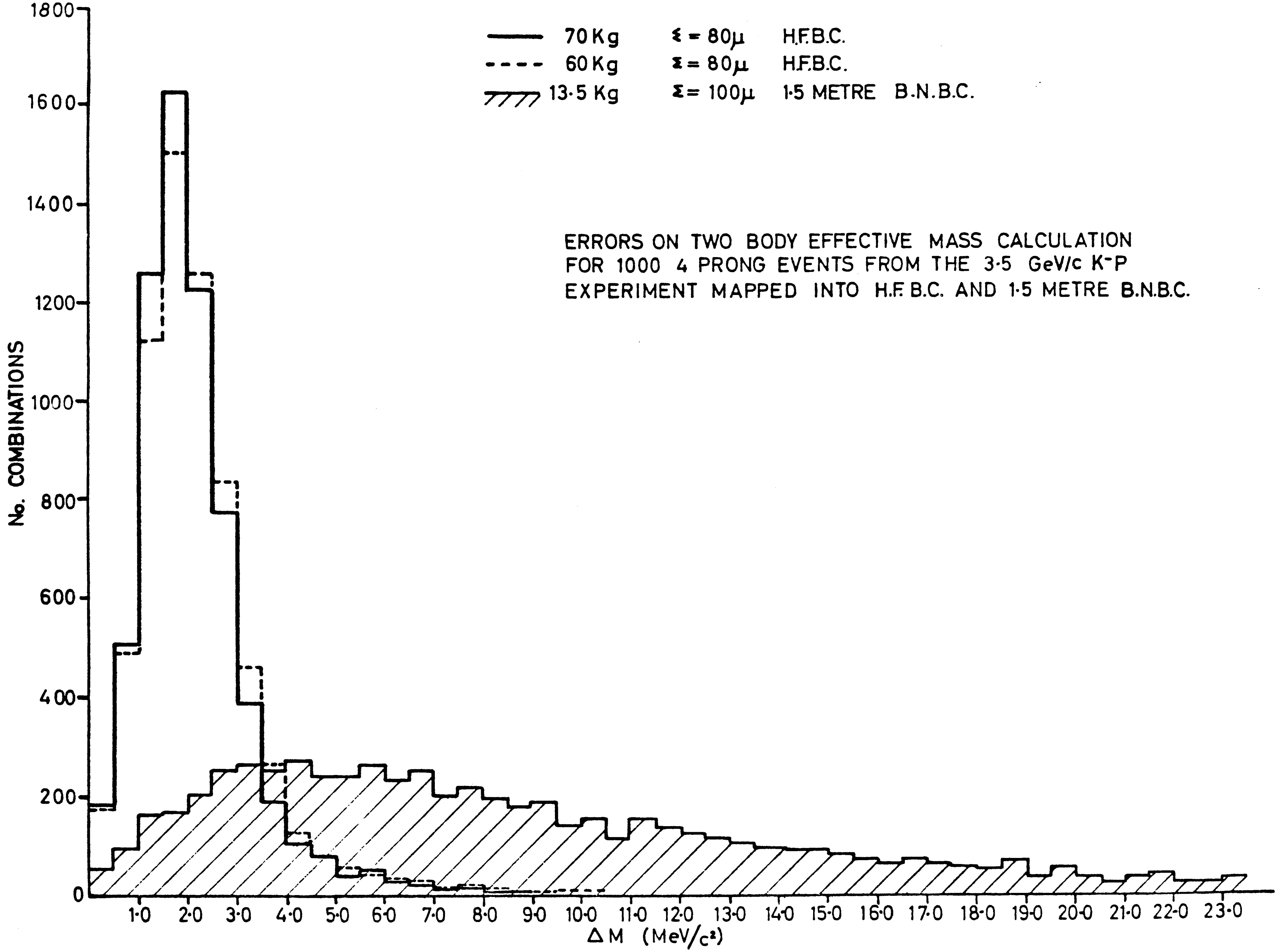

FIG. 6. UNFITTED ERRORS ON TWO PION EFFECTIVE MASS COMBINATIONS

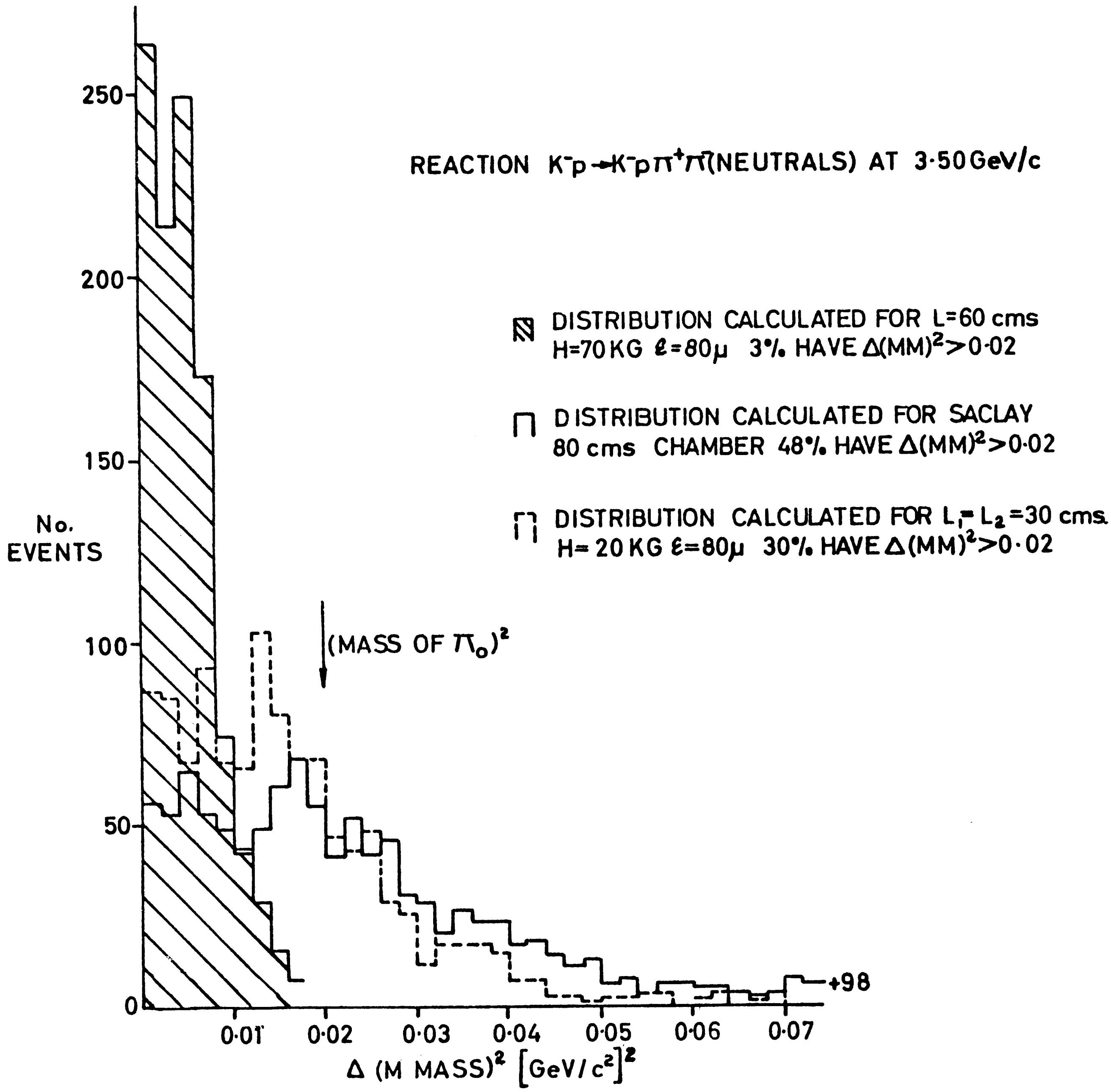

FIG. 7. HISTOGRAM OF Δ(M. MASS)^2 SHOWING IMPROVED RESOLUTION USING HIGH FIELD CHAMBER

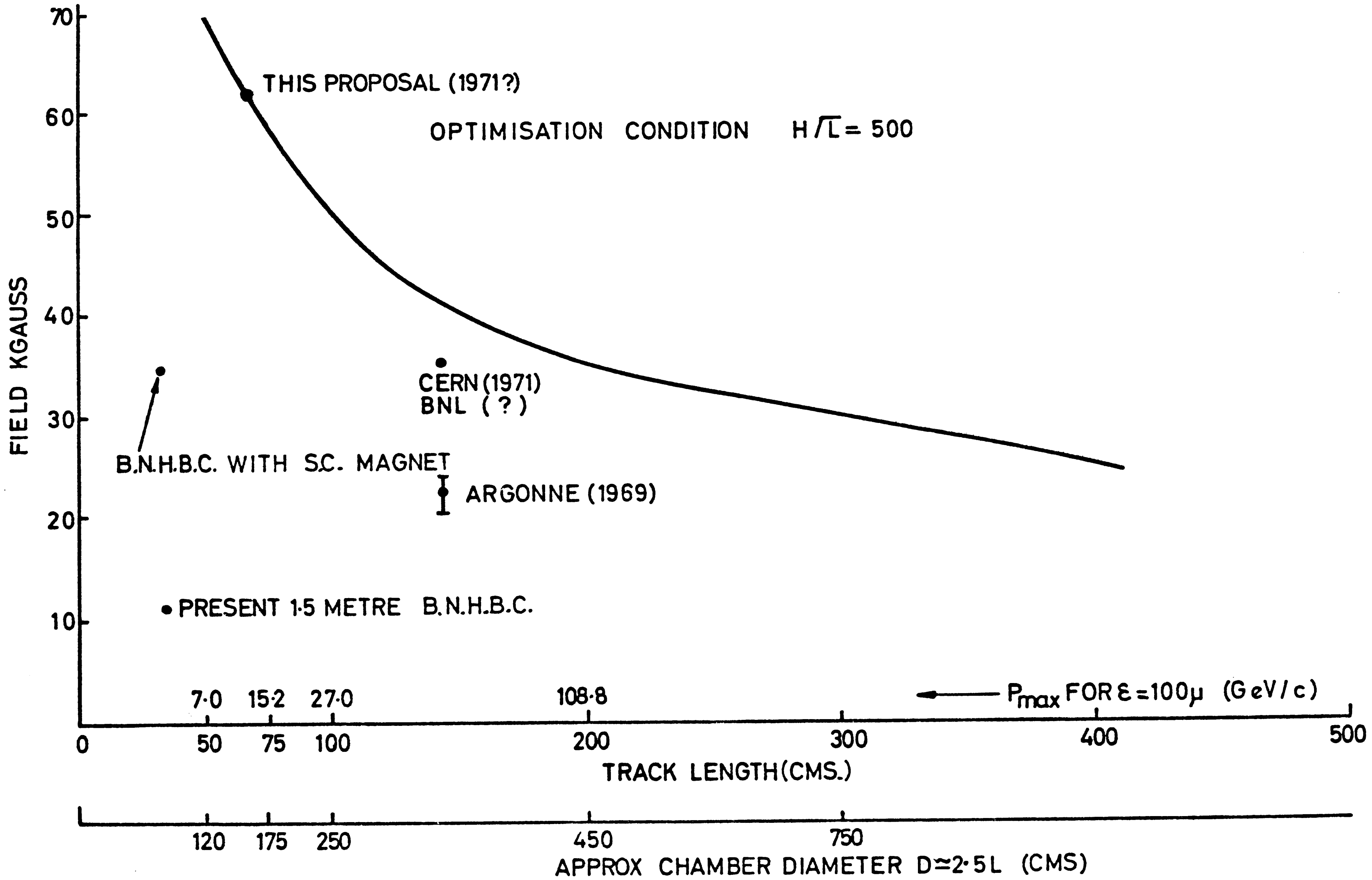

FIG. 8. . CONDITION FOR MATCHED ERRORS

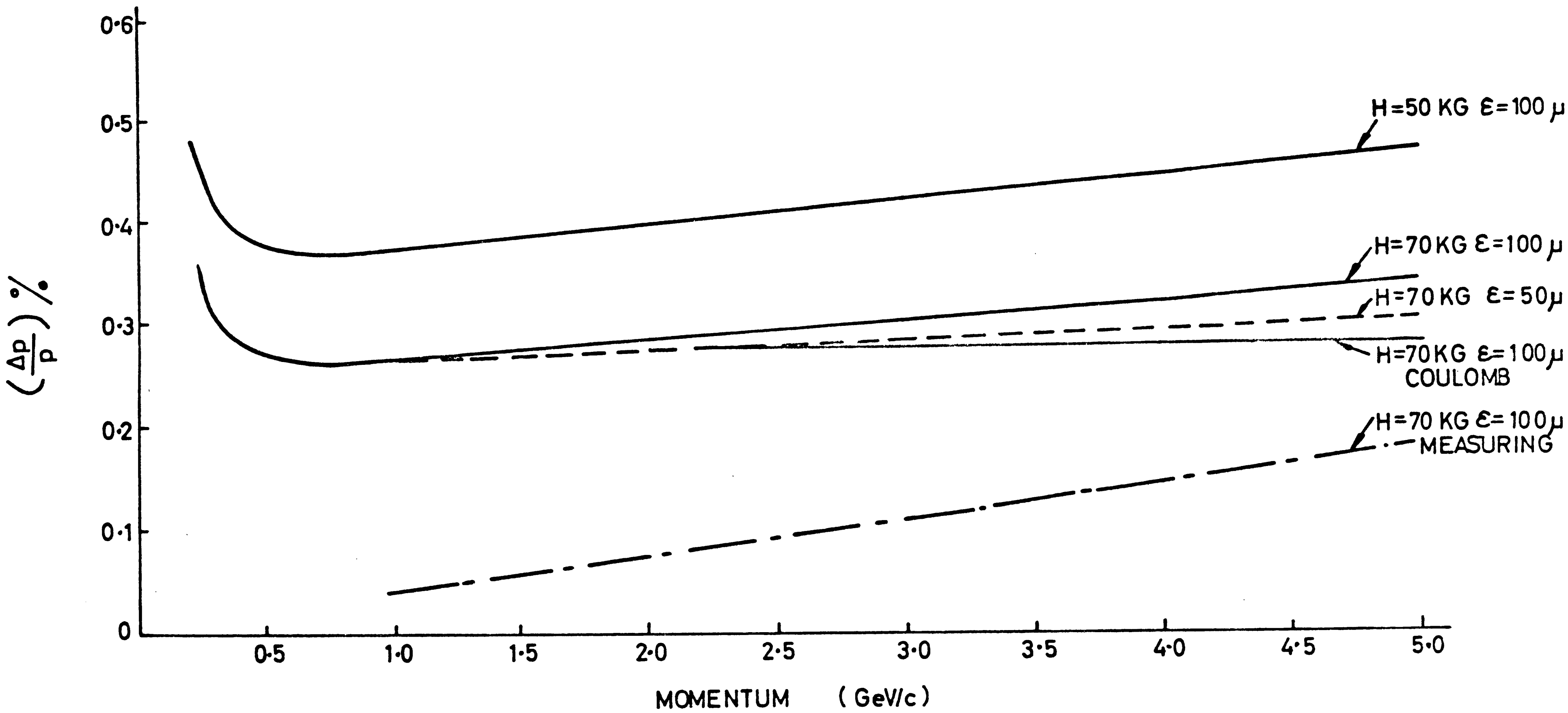

FIG. 9. PERCENTAGE ERROR ON A MOMENTUM MEASUREMENT AGAINST MOMENTUM FOR L = 60 CMS.