ON ELECTROMAGNETIC CORRECTIONS TO THE DISPERSION RELATIONS FOR HIGH ENERGY FORWARD SCATTERING

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1. INTRODUCTION

In the recent experimental test of the pion-nucleon forward dispersion relations at high energies made by Lindenbaum's group [1], the achieved accuracy is so high that the experimental errors almost coincide with possible electromagnetic corrections to the nuclear amplitude. Moreover, the errors of the total cross-sections are even smaller than the electromagnetic corrections. A further improvement of experimental accuracy, though it seems to be difficult at present, would permit us to test causality on the level of both main hadron interactions : strong and electromagnetic ones. However, to do this it is necessary first to analyze the role of the ordinary electromagnetic interaction of hadrons in the dispersion relations for their scattering.

Two ways are possible for this analysis. First, one can try to calculate the electromagnetic corrections to nuclear amplitude, and test the well-known dispersion relations for nuclear interactions. This approach requires more information about electromagnetic interaction of hadrons than simple knowledge of its causal nature, and relies on approximate models.

Secondly, one can try to analyze general properties of the scattering amplitude which is derived directly from experiment and contains the contributions of both strong and electromagnetic interactions, and to derive dispersion relations for this amplitude. Let us discuss briefly both these approaches.

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2. DEFINITION OF THE SCATTERING AMPLITUDE

For this purpose, let us first consider the definition of the amplitude for forward scattering of charged hadrons using Feynman diagrams [2)] in which we temporarily assume that the photon has a small mass λ . These diagrams are of two types. First, diagrams in which hadrons exchange only photons :

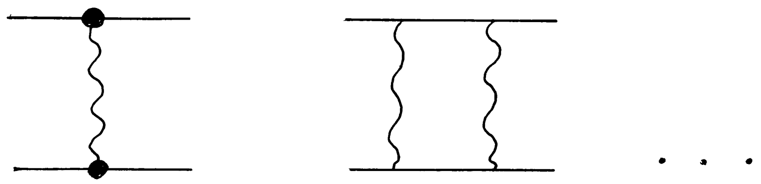

Fig. 1

We shall assume that the contribution of these diagrams at small angles is known. With an accuracy at least up to α^3 we can write it in the form :

$$e^{F_{\lambda C}} T_C \,, \tag{1}$$

where T_C is the Coulomb amplitude and

$$F_{\lambda C} = i \eta \ln \frac{\lambda^2}{-t} \,, \tag{2}$$

$$\eta = \frac{z_1 z_2 \alpha}{v_L} \tag{3}$$

where t is the four-momentum transfer squared, v_L is the laboratory velocity of the incident particle, $z_1 e$ and $z_2 e$ are the charges of the particles involved and α is the fine structure constant.

In the diagrams of the second class, hadrons exchange hadrons and photons :

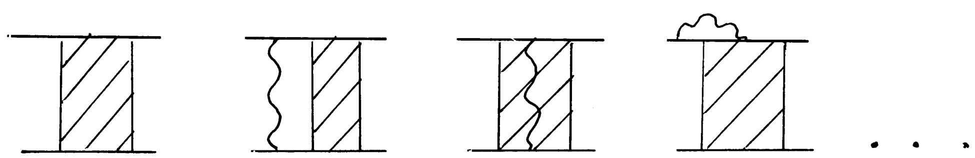

Fig. 2

Their contributions can be represented in the form

$$e^{F_\lambda} T \quad , \tag{4}$$

where F_λ is a known function [3] which contains all the infra-red divergences of these diagrams. At small t it is equal to

$$F_\lambda = i \eta \ln \lambda^2 + \Delta \; . \tag{5}$$

The real part of the $\ln \lambda$ term in F_λ , as well as the effect of emission of real soft photons is proportional to αt and is negligible for small t.

The amplitude T, which will be considered in what follows, is finite at $\lambda = 0$. It is also finite at $t = 0$ and depends on the choice of Δ . Therefore, the total amplitude is

$$e^{F_{\lambda c}} \left(T_c + e^{i\eta \ln(-t) + \Delta} T \right) , \tag{6}$$

so that at a given Δ the amplitude T

$$T = D + iA \tag{7}$$

can be derived from the differential and total cross-sections measured in experiment by means of the following relations

$$\frac{d\sigma}{d\Omega} = |T_c|^2 + 2T_c\left[D - (\eta \ln|t| + Jm\Delta)A\right] + \tag{8}$$
$$+ (1 + 2Re\Delta)(D^2 + A^2) ,$$

$$A(1 + Re\Delta) = \lim_{t_{min}\to 0} \left\{ \frac{p}{4\pi}\left[\sigma(t_{min}) - \sigma_c(t_{min})\right] - \right. \tag{9}$$
$$\left. - (\eta \ln|t_{min}| + Jm\Delta) D \right\} .$$

where, for simplicity, we limit ourselves to the first order in α and where $\sigma(t_{min})$ is the total cross-section of the process in which the elastic scattering takes place only for $|t| > |t_{min}|$ and $\sigma_C(t_{min})$ is the known analogous Coulomb cross-section. Let us note that we can determine from experiment only the combinations $A(1+\mathrm{Re}\,\Delta)$ and $D - \mathrm{Im}\,\Delta\, A$, so that we have to know (to choose) the function Δ to be able to get D and A.

The function Δ in these formulae should be chosen in such a way that it is crossing symmetric, that it obeys a dispersion relation in energy and increases at high energies not faster than, let us say, a logarithm. Usually Δ is taken to correspond to the Bethe phase [4]

$$\Delta = \Delta_1 = 2i\eta \ln \frac{a}{1.06} \ , \tag{10}$$

where a is the radius of nuclear interaction. For Δ one can also take the function which corresponds to the usual expression for F_λ in Eq. (4), given by the Feynman integral [3]

$$F_\lambda = -\sum_{i<j} z_i \xi_i z_j \xi_j \frac{i\alpha}{8\pi^3} \int \frac{d^4k}{k^2-\lambda^2} \left(\frac{2\xi_i p_i - k}{2\xi_i p_i k - k^2} - \frac{2\xi_j p_j + k}{2\xi_j p_j k + k^2} \right)^2 , \tag{11}$$

where the sum is over all charged particles in the initial and final states, $z_i e$ and p_i are the charge and the momentum of a particle, and $\xi_i = -1$ (+1) for incoming (outgoing) particles. In this case at high energies the function Δ is equal to

$$\Delta_2 = -\eta \left(i \ln s + \frac{\pi}{2} \right) , \tag{12}$$

where s is the total c.m.s. energy squared. The corresponding amplitudes T_2 and T_1 are connected by the relation

$$T_2 = \exp\left\{ \eta \left(2i \ln \frac{a\sqrt{s}}{1.06} + \frac{\pi}{2} \right) \right\} T_1 \ . \tag{13}$$

This relation makes it possible to get an idea about the magnitude of possible electromagnetic corrections to the amplitude T. In the range of laboratory pion energies 10–60 GeV, the real parts of both amplitudes D_1 and D_2 differ from each other by 20–40%. Such a big effect is due to the fact that at high energies the real part is much smaller than the imaginary part, and, therefore, a small electromagnetic correction to the imaginary part gives relatively big contribution to the real part. However, the total experimental error of measuring the small real part [1] is at present also about 20–40%. The imaginary parts A_1 and A_2 of both amplitudes, or the corresponding total cross-sections $\sigma = (4\pi/p)A$ differ by 0.6%, but the experimental error of measuring the imaginary part is only 0.3%.

Let us note that if the total cross-section σ_2, corresponding to the amplitude T_2, satisfied the Pomeranchuk theorem, then for the total cross-sections $\sigma_{1\pm}$ for $\pi^{\pm}p$ scattering, corresponding to the Bethe phase, we would get

$$\lim_{s\to\infty} \frac{\sigma_{1-}}{\sigma_{1+}} = e^{\alpha\pi} = 1.023$$

This effect, as well as a relatively big difference between the amplitudes T_1 and T_2, are due to the $\ln s$ term in Δ_2, Eq. (12). Therefore, if we want to obtain from experiment the pure nuclear amplitude T_n, then first of all it is necessary to know whether the corresponding function Δ_n contains a $\ln s$ term, or not.

3. ON CALCULATIONS OF THE PHASE SHIFT BETWEEN NUCLEAR AND COULOMB AMPLITUDES

Is it possible to calculate the function $\Delta = \Delta_n$ for the pure nuclear amplitude $T = T_n$? If we can tolerate an error of 10% in the real part of the nuclear amplitude D_n, then for $D_n/A_n = 0.1$ the absolute error in calculating $\mathrm{Im}\,\Delta_n$ must not exceed $\pm\alpha$.

The first calculation of Δ_n was performed by Bethe [4], who showed that for non-relativistic scattering on nuclei in semi-classical W.K.B. approximation $\Delta_n = \Delta_1$, where Δ_1 is the Bethe phase, (10). Recent works by Rix and Thaler [5], Islam [6], Locher [7] and Yennie [8] were devoted to the relativistic generalization of the Bethe result. First of all, Rix and Thaler [5] have shown that one obtains Δ_n which, like the Bethe phase, does not contain $\ln s$ if :

i) of all the diagrams describing the radiative corrections to the nuclear amplitude one treats only the two diagrams of Fig. 3

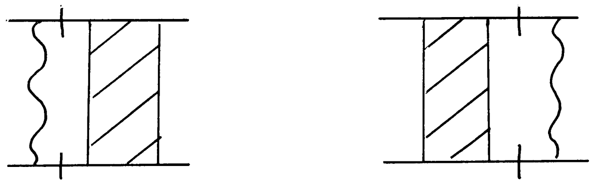

Fig. 3

and:

ii) one takes into account only a part of their contribution given by its discontinuity in going across the cut in the energy plane when the crossed lines in Fig. 3 are real particles.

Locher [7] and Yennie [8] have obtained the same result, taking into account the electromagnetic form factors in the photon vertices of these diagrams. As Locher has demonstrated, the absence of $\ln s$ term in this part of Δ_n is due to the structure of interaction of any kind - electromagnetic or nuclear (the nuclear structure means here the dependence of the nuclear amplitude on t). However, if one takes into account the structure of both kinds in the diagrams of Fig. 3, one gets the Bethe formula with the radius $a\sqrt{2}$ instead of a. In general Locher has shown that different reasonable assumptions about the form of the electromagnetic form factors of pions and protons, even in this simple model, give different values for $\mathrm{Im}\,\Delta_n$ which differ from each other by terms close to 2α.

Because the analysis of other diagrams for radiative corrections is very difficult, Rix and Thaler [5] have formulated their result in a general form. Namely, they have shown that the Bethe result corresponds to the additivity assumption about the partial wave phase shifts of the total amplitude

$$\delta_\ell = (\delta_\ell)_{Coulomb} + (\delta_\ell)_{nuclear} . \tag{14}$$

The verification of this assumption has been done in the semi-classical approximation with the non-relativistic Coulomb potential and a simplest nuclear potential. Islam [6] performed similar calculations, using the relativistic impact parameter approximation with a relativistic generalization of the Coulomb potential. This problem can be also treated in the quasi-potential approach [9]. However, the construction of the whole electromagnetic potential in this approach describing, even up to α , all the electromagnetic corrections again makes it necessary to consider the Feynman diagrams.

Thus, from these calculations, it follows that :

i) $\ln s$ in Δ_n probably is absent, and

ii) the existing models are not reliable for deriving D_n/A_n with an absolute error $\pm 2\alpha = \pm 0.015$ or less.

However, for the test of causality of hadron interactions, we need only to know the causal properties of strong and electromagnetic interactions, which do not depend on models. But in this case we encounter the problem of obtaining dispersion relations for the electromagnetic interaction which are treated in the next Section.

4. DISPERSION RELATIONS WITH THE ELECTROMAGNETIC INTERACTION TAKEN INTO ACCOUNT

For any choice of the function Δ satisfying the above-mentioned general properties, for the amplitude T derived from experiments on π^+ and π^- scattering on protons by means of Eqs. (8) and (9), and including the radiative corrections, we can write down the following forward dispersion relations

$$D^{(+)}(\omega) = D^{(+)}(\omega_0) + \frac{(\omega^2-\omega_0^2)\,2af^2}{(\omega_0^2-a^2)(\omega^2-a^2)} + \frac{\omega^2-\omega_0^2}{\pi} P\int_{-a}^{\infty} \frac{d\omega'\,2\omega' A^{(+)}(\omega')}{(\omega'^2-\omega_0^2)(\omega'^2-\omega^2)}, \tag{15}$$

$$D^{(-)}(\omega) = D^{(-)}(\omega_0) - \frac{(\omega-\omega_0)\,2(a^2+\omega\omega_0)f^2}{(\omega_0^2-a^2)(\omega^2-a^2)} + \frac{\omega-\omega_0}{\pi} P\int_{-a}^{\infty} \frac{d\omega'\,2(\omega'^2+\omega\omega_0)\,A^{(-)}(\omega')}{(\omega'^2-\omega_0^2)(\omega'^2-\omega^2)}, \tag{16}$$

where $T^{(+)}$ and $T^{(-)}$ are the symmetric and antisymmetric combinations of π^+p and π^-p scattering amplitudes, ω is the pion laboratory energy and

$$a = (m^2 + M_p^2 - M_n^2)/2M_p , \tag{17}$$

where m, M_p and M_n are the masses of the charged pion, proton and neutron, respectively.

These relations differ from the usual ones by the lower limit of integration which now lies lower than the threshold. Relations (15) and (16) contain a contribution from the non-observable region $-a < \omega < m$. However, this contribution gives a small correction to $D^{(-)}$ of order α and can be estimated if one uses what is known about π^-p electromagnetic processes at low energies. More important at high energies is the fact that it is necessary to make a subtraction in the dispersion relation for $D^{(-)}$. This follows from the above-mentioned possibility for the imaginary parts of the amplitudes T for π^+p and π^-p scattering not to satisfy the Pomeranchuk theorem.

If one uses some assumptions about the radiative corrections to T at high energies, then relation (16) can be written in another form, containing, evidently, the parameters describing these corrections. For instance, if the phase of the amplitude T_1, corresponding to the Bethe phase, at high energies differs from the phase of the nuclear amplitude T_n only by a constant δ,

$$T_1(\pi^{\pm}p) = \exp(\mp i\delta + r)\,T_n(\pi^{\pm}p), \tag{18}$$

where r is a real electromagnetic correction, then

$$D_1^{(-)}(\omega) = \omega\frac{\sin\delta}{4\pi}\sigma_\infty + \frac{\omega 2f^2}{\omega^2-a^2} + \frac{\omega}{\pi}P\int_{-a}^{\infty}\frac{d\omega' 2A_1^{(-)}(\omega')}{\omega'^2-\omega^2}, \tag{19}$$

where σ_∞ is the average of the total cross-sections for π^+p and π^-p scattering at infinity.

The proof of the analyticity required for these dispersion relations at present is based only on the lowest order perturbation theory [10]. The analytic properties of the considered Feynman diagrams with photons differ from those corresponding to hadrons only near the threshold. But the near-threshold singularities can be obtained in all orders of perturbation theory [10]. As a result, instead of a pole $(s-M^2)^{-1}$ we obtain an infra-red singularity $\phi(t)(s-M^2)^{-1+\beta(t)}$, where $\phi(t)$ and $\beta(t)$ are known. At $t = 0$ this singularity reduces to the pole and the imaginary parts in Eqs. (15) and (16) are integrable near the lower limit in the usual sense. The analogy between Feynman diagrams with hadrons and photons makes it reasonable to hope that their analytic properties are similar in all orders after extracting from the photon diagrams the infra-red divergences and singularities. Therefore, we can hope that the dispersion relations for the strong and electromagnetic interactions will be proved as rigorously as has been done for the strong interaction only. Thus, it seems to me that there is no theoretical limitation to the accuracy of experimental tests of dispersion relations.

I am very grateful to Drs. D. Beder, V. Bolotov, S. Denisov, C. Geobel, A. Diddens, H. Epstein and C. Lovelace for discussions. I also wish to thank Professors L. Van Hove and J. Prentki for the hospitality of the Theoretical Study Division of CERN.

REFERENCES

1) K.J. Foley, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.D. Platner, C.A. Quarles and E.H. Willen, Phys.Rev.Letters 19, 193, 330 (1967).

2) L.D. Soloviev, Zhur.Eksp.i Teoret.Fiz. 49, 292 (1965).

3) D.R. Yennie, S.C. Frautschi and H. Suura, Ann.Phys.(N.Y.) 13, 379 (1961);

K.E. Eriksson, Nuovo Cimento 19, 1010 (1961).

4) H.A. Bethe, Ann.Phys.(N.Y.) 3, 190 (1958).

5) J. Rix and R.M. Thaler, Phys.Rev. 152, 1357 (1967).

6) M.M. Islam, Phys.Rev. 162, 1426 (1967).

7) M.P. Locher, Nuclear Phys. B2, 525 (1967).

8) D.R. Yennie, private communication to Lindenbaum, Ref. 1).

9) A.A. Logunov and A.N. Tavkhelidze, Nuovo Cimento 29, 300 (1963).

10) L.D. Soloviev, Zhur.Eksp.i Teoret.Fiz. 44, 306 (1963);

L.D. Soloviev and Yu.Ya. Yushin, Zhur.Eksp.i Teoret.Fiz. 45, 1202 (1963);

L.D. Soloviev, Nuclear Phys. 64, 657 (1965).