MESON ACCELERATION IN CYCLIC MACHINES(*)

R.P. Haddock
University of California, Los Angeles

Pion acceleration has been proposed[1] using the high-energy gradients of linacs. Whereas, high-energy proton accelerators are good sources of high-energy pions, the muon beams have not been extensively exploited[2]. One reason is pion contamination and another is low intensity. The muon, unlike the pion, interacts only weakly or by virtue of its electromagnetic properties; hence, pure high-energy muon beams of high intensity are required. A beam of 0.3 μA muons is required to investigate a total cross-section of 10^{-40} cm^2 at a rate of 1 count/hr using a 3 meter liquid hydrogen target. The intensity of μ^+ beams from a pion factory with a CERN-type muon channel are expected to be a fraction of a nano-ampere for energies between γ = 2 and 4. Because the weak interaction cross-sections are expected to be several orders of magnitude larger at energies in the GeV range, muon acceleration seems indicated.

Pion and muon mean lives are 0.0256 and 2.2 μs, respectively. Fig. 1 shows the

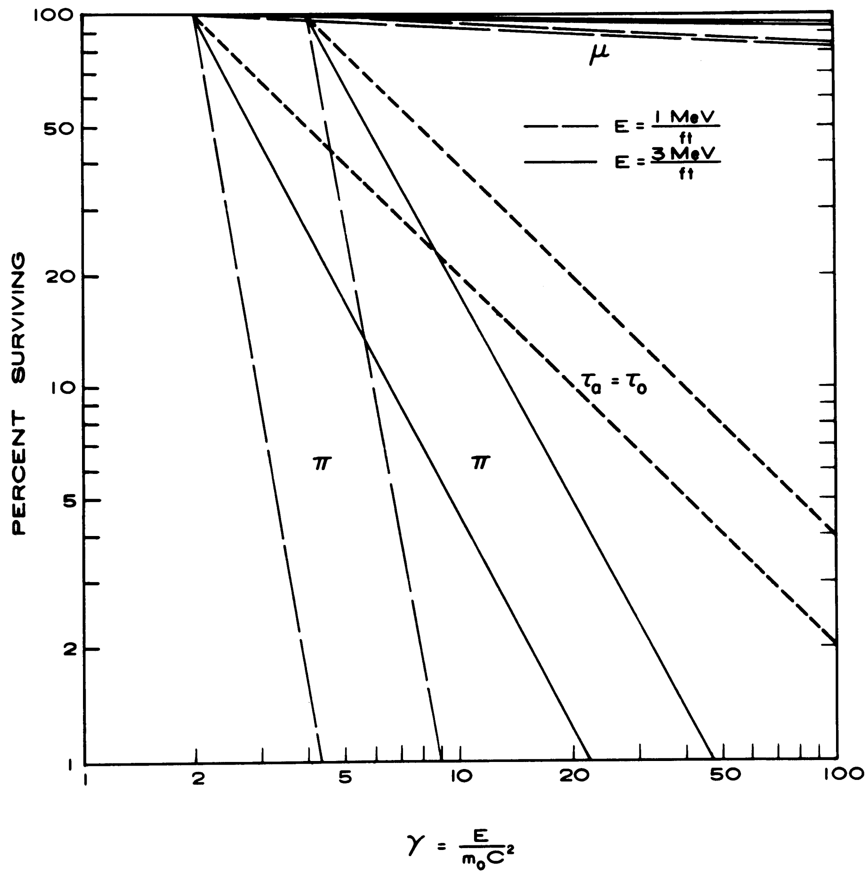

Fig. 1 Fraction of pions and muons surviving acceleration in a linac (solid and dashed curves) and of muons in a cyclotron (dotted curves).

(*) Work performed under the auspices of the U.S. Atomic Energy Commission.

fraction of particles surviving acceleration due to decay-in-flight. Curves are shown for injection energies of $\gamma_i = 2$ and 4 to illustrate this dependence. The two curves at each energy are for constant energy gradients of 1 MeV/ft and 3 MeV/ft (gradients available from linacs). Pions decay rapidly so that for a 3 MeV/ft gradient 10% survive acceleration from 419 to 1773 MeV ($\gamma = 4$ to 13.7); whereas nearly all muons survive for these energy gradients.

We now consider the feasibility of accelerating muons in cyclic machines. The elapsed proper time is $\tau = \int_{\gamma_i}^{\gamma_f} dt/\gamma$ for acceleration from γ_i to γ_f, where dt is a time interval in the lab. frame; and the fraction surviving is $N/N_0 = e^{-\tau/\tau_0}$, where τ_0 is the muon mean life. Acceleration is usually carried out with, at most, a slowly changing frequency f, and a nearly constant energy gain per turn, δE; so, $f dt = dE/\delta E$. The cyclotron frequency is $1/f = t = 2\pi m_0\gamma/eB = t_0\gamma$. We finally obtain the expression $\tau = 2\pi(m_0c)^2/e\delta E \int_{\gamma_i}^{\gamma_f} d\gamma/B$. There are two cases that arise assuming that the muons are relativistic at injection ($\beta_i = 0.968$ for injection at $\gamma_i = 4$).

a) The cyclotron frequency is nearly constant during acceleration; therefore, $B \sim \gamma$. Let $B = B_i(\gamma/\gamma_i)$; then, $\tau = 2\pi(m_0c)^2\gamma_0/eB_0\delta E \, \ell n(\gamma_f/\gamma_i) = \tau_a \, \ell n(\gamma_f/\gamma_i)$, and $N/N_0 = [\gamma_f/\gamma_i]^{-\tau_a/\tau_0}$.

A log-log plot of N versus γ is then a straight line of slope $-\tau_a/\tau_0$. The figure shows the case $\tau_a = \tau_0$ for the two injection energies; 10% of the muons survive acceleration from 317 to 4120 MeV ($\gamma = 4$ to 40). We will now set $\tau_a = \tau_0$ and derive conditions for muon accelerations for different classes of accelerators.

i) Synchrotrons have a constant radius of the synchronous orbit; therefore, $er_0B = P = m_0c\beta\gamma \sim m_0c\gamma$ and $B_i/\gamma_i = m_0c/er_0$. We obtain the condition $\delta E/2\pi r_0 = m_0c/\tau_0 =$ 48.3 KeV/ft. The energy gradient is constant as for a linac. The energy gain per turn, $\delta E = 2\pi e r_0^2 \Delta B/\Delta t = 2\pi r_0 \Delta E/c\Delta t$; so, $\Delta E/\Delta t = 47.6 \times 10^6$ MeV/s, a value several orders of magnitude greater than presently used. A similar result holds for the betatron where in addition the two-to-one rule must be obeyed.

ii) The magnetic field is $\bar{B} = \bar{B}_i(\gamma^2 - 1/\gamma_i^2 - 1)^{\alpha-1/2\alpha}$ for a scaling spiral sector FFAG accelerator; where $\bar{B}$ is the average field at an energy γ, and $\bar{B}_i$ is the average field at injection. The momentum compaction α is large compared to 1; so $\bar{B} - \bar{B}_i(\gamma^2 - 1/\gamma_i^2 - 1)^{1/2} \approx \bar{B}_i\gamma/\gamma_i$. We obtain the condition $\delta E\bar{B}_i/P_ic = 33.2$ G; and if $\gamma_i = 4$, $\delta EB_i = 13.6$ kG MeV. An energy gain per turn greater than 10 MeV is desirable for a large increase in γ. A similar result is obtained for the isochronous cyclotron, for which $\bar{B} = \bar{B}_i(\gamma/\gamma_i) = B_c\gamma$, where B_c is the field corresponding to $\gamma = 1$. The condition becomes $B_c\delta E = 3.5$ MeV kG.

b) The synchro-cyclotron and microtron are characterized by a significant change in cyclotron frequency during acceleration. We obtain $\tau = 2\pi(m_0c)^2(\gamma_f - \gamma_i)/eB\delta E = t_0(E_f - E_i)/\delta E$ for an accelerator with a uniform field; so, a plot of N/N_0 versus γ is not a straight line in this case. The ratio of ion frequencies $f/f_i = \gamma_i/\gamma$ is large for any significant increase in γ, and this fact rules out the synchro-cyclotron.

The energy gain per turn cannot be arbitrary for the uniform field microtron but must obey the relation $\delta E = m_0 c^2 n t_{RF}/t_0$, where t_{RF} is the oscillator period and n is the integral number of RF cycles gained per turn. We obtain the relation $f_{RF}(\gamma_f - \gamma_i)/B^2 = 404.6$ n Mc/s(kG)^2 as the condition that $N/N_0 = e^{-1}$. A L band RF system (2023 Mc/s) is required if B = 10 kG, $\gamma_f - \gamma_i = 20$, n = 1; delivering 7.06 MeV/turn (Fig. 2).

Roberts[3] has suggested modifying the conventional uniform field microtron for proton acceleration (auxinotron) by introducing sector-focusing, a linac for acceleration, and high-energy injection. Brannen and Froelich[4] have constructed an electron racetrack microtron (dromotron) along these lines. The microtron with these modifications seems to be the logical choice for muon acceleration; so, we will call this accelerator a muonitron.

The muonitron has the following features : a) it has a fixed magnetic field and RF frequency; b) it is free of any necessary connection between betatron frequencies and energy; c) it is phase stable (always above the transition energy); d) the synchrotron phase space will increase during acceleration for a constant energy gain per turn and synchronous phase angle; e) large orbit separation so the entire beam may be extracted; and f) the extracted beam will be of high quality and absolutely free of pions.

The muonitron also has some significant uncertainties or disadvantages. Linacs have a narrow velocity acceptance so that a cascade microtron may be required for injection at energies $\gamma_i < 4$. A L band linac has a small aperture (perhaps 2.5 cm diameter); so, high quality beams are required. Linacs at present consume large amounts of power in CW operation. Uncertainties exist due to the lack of experience with microtrons. For example, possibilities for strong focusing as well as sector-focusing have not been examined, and the necessary 300 turn acceleration has not been demonstrated. Finally, external injection

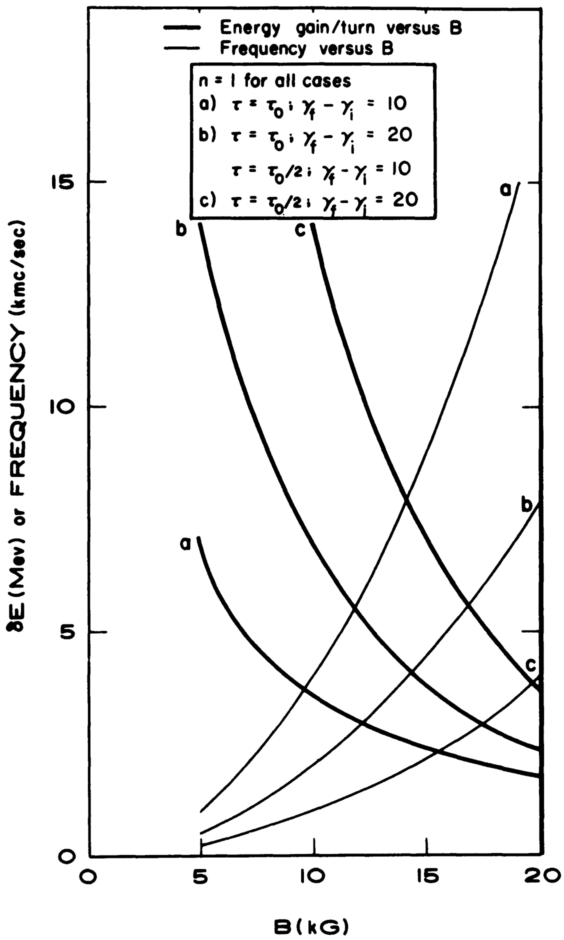

Fig. 2 Frequency and energy gain per turn required to accelerate muons in a microtron.

is required. Estimates of the admittance are necessarily rough without a firm design; so, we will assume that a linac with quadrupole focusing is the stop and estimate the admittance to be 20 π mrad cm with an allowable energy spread of 3%.

The CERN[5] muon channel generates about 10^5 μ^- per s at 300 ± 16 MeV with an estimated horizontal and vertical emittance of 80 π and 200 π mrad cm, respectively. A 600 MeV pion factory will generate 4 x 10^3 times as many μ^+'s with a 500 μA external beam, where a factor of 2 x 10^3 is due to the increased current, and a factor of 20 is due to the π^+/π^- ratio; then, an intensity of 3 x 10^6 μ^+ per s is possible within the linac admittance at injection, of which 37% survive acceleration. This estimate is a lower limit for a number of reasons, some of which are connected with the availability of an external beam and strong focusing. For example, Foote et al.[6] prepared a 427 ± 10 MeV/c positive pion beam of 2 x 10^6 π^+ per s with 19 inches of CH_2 in the 0.03 μA external beam of the Berkeley 184" synchro-cyclotron. A two-magnet achromatic system with field lens was used and the solid angle was a few millisteradians. We obtain 6.7 x 10^{10} π^+ per s when this system is scaled up to 500 μA. A muon channel with a 20% trapping efficiency will then generate 2 nA of muons. This is about 5 times larger than the extrapolated estimate from the CERN channel. We can imagine, at least, other means of making more intense pion beams so that it is not unreasonable to suppose that $\geq 10^7$ μ^+/s may be accelerated to a few GeV. We see that it may be possible to generate high energy muon beams several orders of magnitude greater than those from other proposed proton or electron machines for the same beam quality and freedom from contamination.

References

1. S. Devons (unpublished) 1959; T. Nishikawa, 1961 Brookhaven Conference on High Energy Accelerators (U.S. Government Printing Office, Washington 25, D.C.); R.L. Gluckstern, 1962 Brookhaven Conference on Linear Accelerators for High Energies.
2. G.E. Masek et al., Phys. Rev. 122, 937 (1961). A 2 ± 03 GeV/c muon beam was formed from the decay in flight of pions produced by the Bevatron. The maximum muon intensity was about 50 per s. An increase of about 50 may be achieved from the improved Bevatron.
3. A. Roberts, Annals of Physics 4, 115 (1958).
4. E. Brannen and H. Froelich, Jour. Applied Physics 32, 1179 (1961).
5. A. Citron et al., 1960 Berkeley Instrumentation Conference, Interscience Publishers, New York.
6. J.H. Foote et al., Phys. Rev. 122, 948 (1961).