

A COMPUTER PROGRAMME FOR DETERMINATION OF
CYCLOTRON FIELDS

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A general programme for an initial theoretical determination of realistic AVF cyclotron fields has been written in FORTRAN code for the CERN IBM 709 computer under monitor control. The programme is interesting because it is based on a completely different method of approach than other existing programmes, and permits a relatively quick survey of a large number of machines. The results are exact, apart from round-off, integration and interpolation errors, in the order of 10^{-6} .

The calculation is performed and all input and output is presented in cyclotron units, i.e.

Length	c/ω_0	Mass	m
Charge	q	Field	$m\omega_0/q$
Time	$1/\omega_0$	Energy	mc^2
Velocity	c	Momentum	mc

where m and q are the rest mass and charge of the accelerated particle, c the velocity of light, and ω_0 an arbitrary reference angular velocity. The angular velocity ω of the particle may be chosen as a function of the energy. For an isochronous cyclotron one chooses conveniently $\omega = 1$.

The computation is based on orbit coordinates (ρ, σ) in the median plane, ρ being the equivalent radius (closed orbit length divided by 2π) and σ the orbital arc length normalised to unity through one machine period. The curves $\sigma = 0$ and $\sigma = 1$ are the sector edges defining the machine spiral. In its most frequent application the programme determines the spiral which for a given azimuthal field variation $B(\rho, \sigma)$ yields a specified variation of the linear vertical betatron frequency Q_z versus equivalent radius or energy.

Orbit Theory

The magnet field for an N-sector machine is presented to the computer in the form

$$B = \gamma \omega F(\rho, \sigma) / \Theta \quad , \quad (1)$$

where $\Theta = 2\pi/N$ is the sector angle,

$$\gamma = (1 - (\omega\rho)^2)^{-\frac{1}{2}} \quad (2)$$

is the particle energy (or mass), and the specified function F must be periodic in σ with period 1 and have mean value Θ to give the correct mean field $\gamma\omega$ along the orbit ρ .

Considering any equilibrium orbit ρ the field, Eq. (1), as a function of arc length σ and the orbit energy, Eq. (2), will together determine the geometry of this orbit, but not its position in the plane. Introducing $dt = \Theta d\sigma/\omega$, $\bar{B} = -B\bar{k}$ with B given in Eq. (1), and the complex variable $z = x + iy$ on the median plane into the force equation

$$\gamma d^2\bar{r} / dt^2 = (d\bar{r}/dt) \times \bar{B}, \quad (3)$$

one obtains for determination of the closed orbits

$$\partial^2 z / \partial \sigma^2 = i F(\rho, \sigma) \partial z / \partial \sigma. \quad (4)$$

By a first integration:

$$\partial z / \partial \sigma = i\rho\Theta h(\rho, \sigma), \quad (5)$$

where

$$h(\rho, \sigma) = G(\rho, \sigma) \exp(i \delta(\rho)) \quad (6)$$

is the unit normal pointed outwards from the orbit ρ , and

$$G(\rho, \sigma) = \exp(i \int_0^\sigma F(\rho, \sigma) d\sigma). \quad (7)$$

The angle $\delta(\rho)$ is introduced as an integration constant. It is the angle between the x-axis and the normal $h(\rho, 0)$ at the sector edge $\sigma = 0$ where $G(\rho, 0) = 1$.

An integration of Eq. (5) finally gives

$$z(\rho, \sigma) = \rho S(\rho, \sigma) \exp(i \delta(\rho)) \quad (8)$$

with

$$\begin{aligned} S(\rho, \sigma) &= \Theta(H(\rho, \sigma) - E H(\rho, 1)), \\ E &= \frac{1}{2}(1 + i \cot(\pi/N)), \\ H(\rho, \sigma) &= i \int_0^\sigma G(\rho, \sigma) d\sigma. \end{aligned} \quad (9)$$

It is here assumed that $N > 1$, and the integration constant has been chosen such that the closed orbits $z(\rho, \sigma)$ for ρ const. are all centred at the machine centre $z = 0$. Notice that δ in Eq. (8) signifies a rotation of the orbit as a whole. Thus $\delta(\rho)$ defines the spiralling feature of the field and is needed in addition to Eq. (1) for a complete definition of the field.

Betatron Oscillations

If X, Z denote the normal distances from the orbit in and normal to the median plane, then the betatron oscillations are described by the equations

$$\begin{aligned} d^2 X / d\sigma^2 + g_X X &= 0, \quad g_X = F^2 (1 - n), \\ d^2 Z / d\sigma^2 + g_Z Z &= 0, \quad g_Z = F^2 n, \end{aligned} \quad (10)$$

where F is the field-defining function in Eq. (1) and n the conventional field index, which we may here express as

$$n = -\frac{\omega\gamma\rho}{B^2} \frac{\partial B}{\partial X}. \quad (11)$$

The normal derivative $\partial B/\partial X$ is defined by

$$\frac{\partial B}{\partial X} = \frac{1}{\eta} \left(\frac{\partial B}{\partial \rho} - \frac{\tau}{\rho\theta} \frac{\partial B}{\partial \sigma} \right), \quad (12)$$

which with Eq. (1) gives the coefficients

$$\begin{aligned} g_X &= F^2 - g_Z, \\ g_Z &= \frac{1}{\eta} \left(\tau \frac{\partial F}{\partial \sigma} - \rho\theta \frac{\partial F}{\partial \rho} - \theta F\lambda \right), \end{aligned} \quad (13)$$

where $\lambda = (\rho/\gamma\omega) d(\gamma\omega)/d\rho$ is given from Eq. (2).

The scale factors η and τ are defined by the unit normal (6) and the coordinate function (8):

$$\eta + i\tau = h^* \partial z / \partial \rho = T(\rho, \sigma) + U(\rho, \sigma) d\delta/d\rho, \quad (14)$$

in which

$$\begin{aligned} T(\rho, \sigma) &= G^* [S(\rho, \sigma) - \rho\theta(K(\rho, \sigma) - E K(\rho, 1))], \\ U(\rho, \sigma) &= i\rho G^* S(\rho, \sigma), \\ K(\rho, \sigma) &= -\partial H(\rho, \sigma) / \partial \rho, \end{aligned} \quad (15)$$

and the asterisk denotes conjugate complex quantities. The coefficients (13) are hereby completely determined and the betatron frequencies Q_r , Q_z may be found by integration. It is important to notice that these frequencies do not depend directly on the spiral-defining angle $\delta(\rho)$, but rather on its derivative $d\delta/d\rho$ which determines the tangent of the edge $\sigma = 0$, i.e. the spiral angle at this point.

Programme Outline

The function $F(\rho, \sigma)$ must be presented to the computer in the following form (*):

$$F(\rho, \sigma) = \Theta + \sum_{i=1}^I R_i(\rho) \sum_{j=1}^J A_{ij} \cos(2\pi M_{ij}\sigma + B_{ij}), \quad (16)$$

which permits the ridge field and the azimuthal field variation to be chosen independently. Here $I \leq 5$, $J \leq 14$. The R_i are tabular or polynomial functions of ρ and the A_{ij} , M_{ij} , B_{ij} are constants. For each specified orbit ρ to be computed the programme immediately stores F , $\partial F/\partial \rho$ and $\partial F/\partial \sigma$ at each value of σ to be used in the Runge-Kutta integration processes to follow.

(*) It has been found that a tabular presentation versus σ of the second sums might be more convenient.

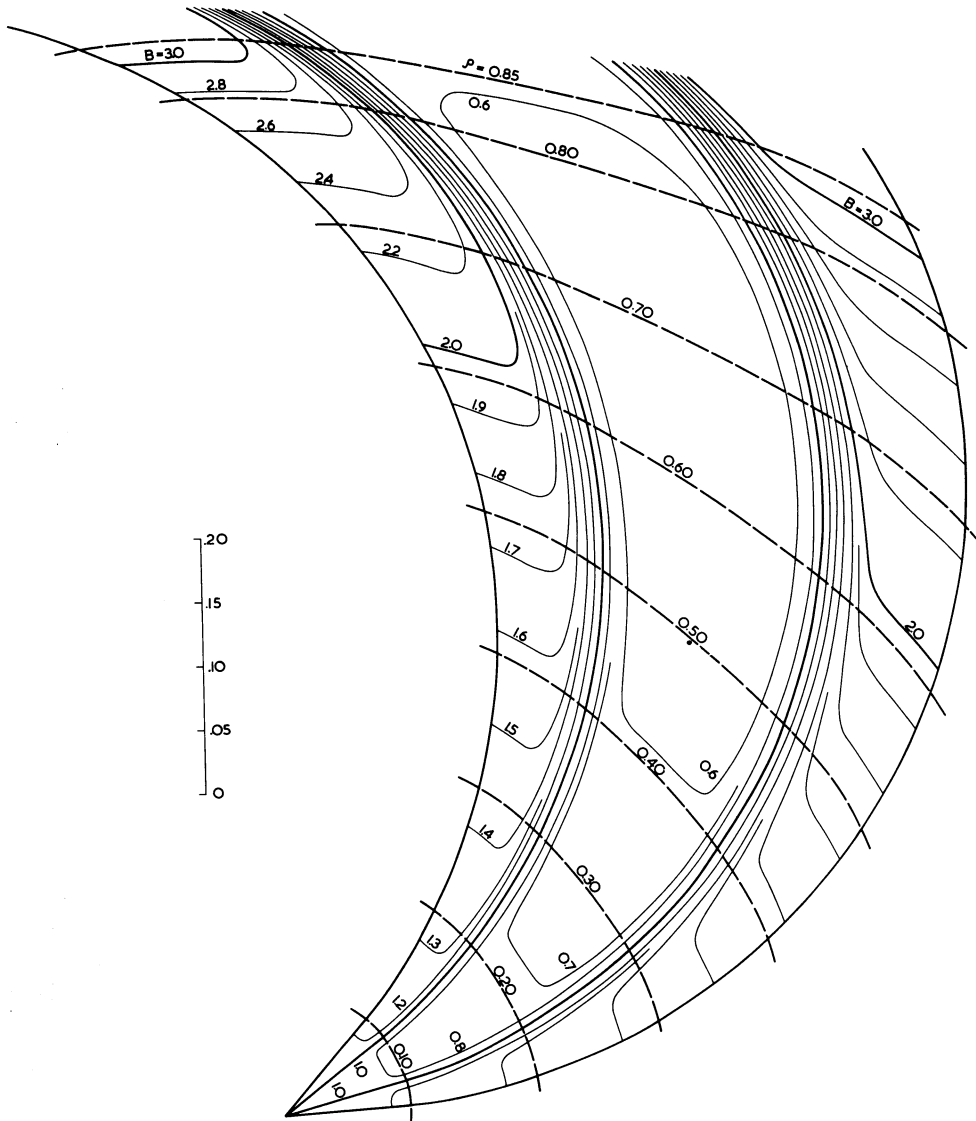


Fig. 1 Field map of 8-sector, 846.7 MeV isochronous cyclotron. $Q_r = 2$ and spiral angle = 60° at final orbit. Closed orbits ρ and field contours B shown in cyclotron units.

An initial integration is first performed whereby the functions G , H and K are determined from the system:

$$\begin{aligned} \frac{\partial G}{\partial \sigma} &= i F G , & \frac{\partial H}{\partial \sigma} &= i G , \\ \frac{\partial K}{\partial \sigma} &= g G , & \frac{\partial g}{\partial \sigma} &= \frac{\partial F}{\partial \rho} , \end{aligned} \tag{17}$$

using the initial conditions $G = 1$, $H = K = g = 0$ at $\sigma = 0$. The functions S , T and U defining the orbits z and $\eta + i\tau$ are then computed and stored at each σ necessary for the subsequent integration of the system (10), (13), (14) for the determination of the frequencies Q_r and Q_z .

The programme may be set for three different modes of operation: In "Mode 1" the spiral-defining function $\delta(\rho)$ is specified as input on tabular form. Its derivative $d\delta/d\rho$ is determined by a six-point Lagrange formula, whereafter the betatron frequencies

are computed from the Eq. (10) by the usual matrix method. Two types of output are available from this mode of operation. The first is a field map showing the sector edges $\sigma = 0$ and $\sigma = 1$, the closed orbits, and a selected set of constant field curves. An example is shown in Fig. 1. The map may be drawn automatically by a special magnetic tape output for an x-y plotter. Secondly, the field may be specified on a pre-chosen polar mesh within the sector $(0, \Theta)$. This output is used as input for our general cyclotron programmes where one can investigate non-linear effects, integration through resonances, etc.

"Mode 2" is a little used procedure in which the frequencies Q_r and Q_z are computed and listed at each ρ for a given F and a sequence of values of the derivative $d\delta/d\rho$.

In "Mode 3" the value of $d\delta/d\rho$ is determined and listed at each ρ by an iteration process such that a specified Q_z is obtained. The Q_z is input as a table in ρ . Notice that the iterative process only involves the final integration of the last of Eq. (10) since the T and U are unaffected by the value of $d\delta/d\rho$. Having determined the function $d\delta(\rho)/d\rho$ by computation at many radii throughout the machine, the computer automatically integrates this to obtain $\delta(\rho)$, whereafter a "Mode 1" operation is performed.

Illustrative Example

As an example consider the $N = 8$ sector isochronous cyclotron meson factory defined by

$$F(\rho, \sigma) = R(\rho) S(\sigma), \quad (18)$$

where

$$S(\sigma) = \sum_{j=1}^8 A_j \cos(2\pi(2j-1)\sigma) \quad (19)$$

and the harmonic amplitudes A_j are determined^(*) such that the 15 first derivatives of S vanish at $\sigma = 0$ and $\sigma = 0.5$:

$$\begin{array}{ll} A_1 = 1.23409090 & A_2 = 0.31994943 \\ A_3 = 0.11518194 & A_4 = 0.03739665 \\ A_5 = 0.00969542 & A_6 = 0.00183060 \\ A_7 = 0.00022128 & A_8 = 0.00001279 \end{array}$$

This ensures completely flat and symmetric hills and valleys as illustrated in Fig. 2.

A "Mode 3" computation was performed using a function $Q_z(\rho)$ as shown in Fig. 3. It rises sharply to the value 0.2 where it remains for all $\rho > 0.2$. The function $R(\rho)$ in Eq. (18) was chosen such that the sector edge $\sigma = 0$, here situated at mid-ridge, should be approximately a circle of a certain radius r . $R(\rho)$ was in fact

(*) I am indebted to Mr. Werner Joho, ETH, Zürich, who has written a special programme for providing Fourier polynomials of the type here mentioned.

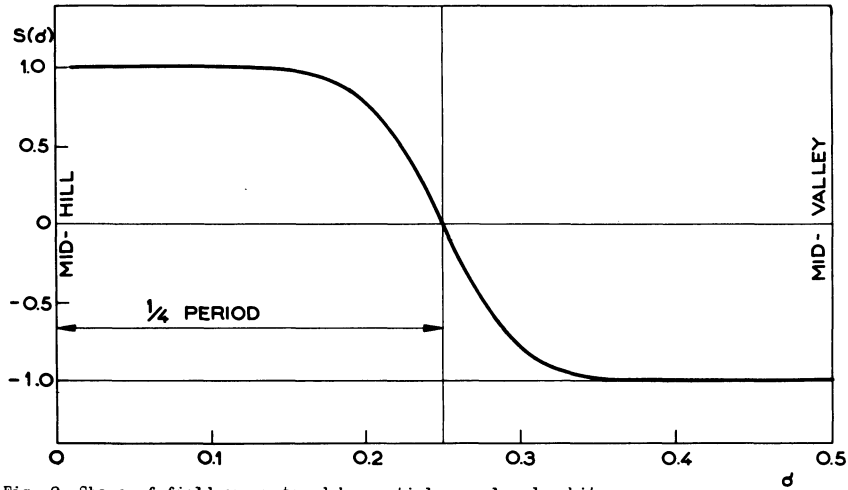


Fig. 2 Shape of field encountered by particle on closed orbit.

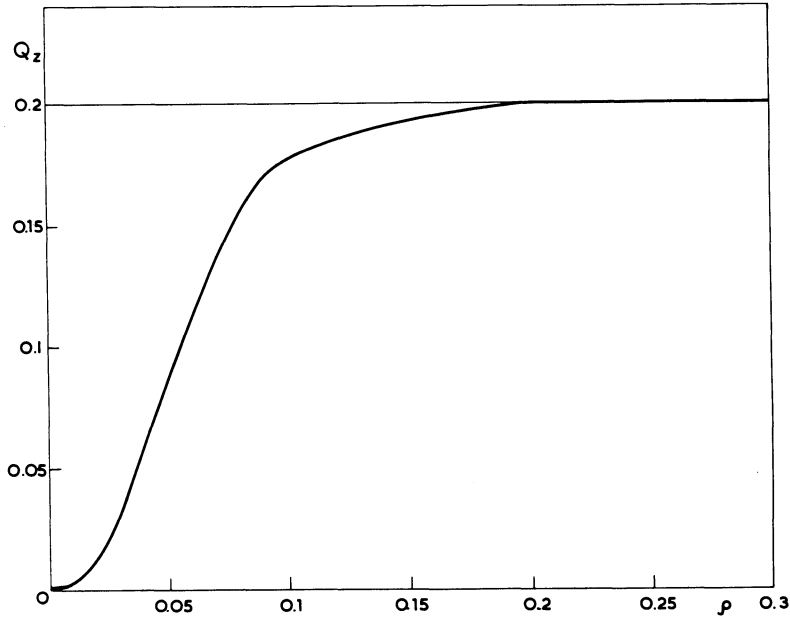


Fig. 3 Rise of Q_z versus ρ towards its constant value 0.2.

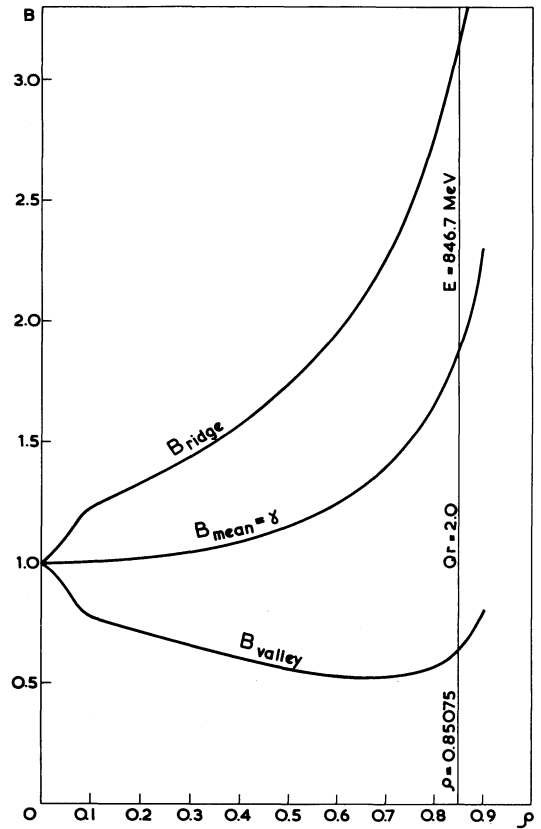


Fig. 4 Maximum, mean and minimum fields versus equivalent radius for cyclotron mapped in Fig. 1.

calculated from the approximate formula

$$Q_z^2 = 1 - \gamma^2 + \frac{N^2}{N^2-1} f^2 (1 + 2 \tan^2 \epsilon) \quad (20)$$

with $N = 8$, $f^2 = \langle B^2 \rangle / \langle B \rangle^2 - 1 = R(\rho)^2 \langle S(\sigma)^2 \rangle = 0.82 R(\rho)^2$, and for a circular edge $\sin \epsilon \approx \rho/2r$, ϵ being the conventional spiral angle. The edge radius r was chosen at the value 0.48825, which according to interpolation on previously obtained results would give a reasonable spiral angle of exactly 60° when Q_r reaches the value 2 convenient for resonant extraction.

The result is the meson factory mapped on Fig. 1. Its final orbit characteristics are as follows

Equivalent radius ρ	0.85075	or	4.33 m
Max orbit excursion	0.8591	or	4.37 m
Energy γ	1.9025	or	846.7 MeV (protons)
Max field	3.1611	or	19.47 kG
Min field	0.6443	or	3.97 kG
Radial frequency Q_r	2		
Vertical frequency Q_z	0.2		
Spiral angle	60°		

The metric units are obtained by choosing the reference angular velocity $\omega_0 = 5.9 \times 10^7$ rad/s corresponding to an orbital frequency of 9.4 MHz. The ridge and valley fields are described as functions of ρ in Fig. 4.

DISCUSSION

REISER : Are your plans now to convert the synchro-cyclotron into an 850 MeV machine, or are your investigations presented here independent of this problem?

VOGT-NILSEN : This is completely independent.

KHOE : What is the next step when you want to find the steel configuration?

VOGT-NILSEN : The next step would be to hand maps like the one I showed over to our magnet people for comments. One has certainly to do more computer runs to try to conform to their wishes, and then perhaps finally find a suitable design.

LAPOSTOLLE : How much computer time is necessary to get such map.

VOGT-NILSEN : For this one I used about an hour and a half, but this was the first one tried, and I had to search a little to find the method. I would say half-an-hour on the next case.

VERSTER : Does this programme also allow you to specify the tolerances on the flutter so the magnet group could know how accurately they should try to realise the field?

VOGT-NILSEN : Not directly. I have a second output from this programme, which is used as input for general orbit programmes; on these we could introduce small errors to see what they do.