

SESSION II

NUCLEAR STRUCTURE INFORMATION FROM  
HIGH ENERGY ELECTRON SCATTERING

Speaker :

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I. INTRODUCTION

The purpose of this paper is to summarize the types of nuclear information that can be gained by high energy electron scattering studies and to indicate what new information will become available with the next generation of high intensity electron accelerators.

Essentially all the studies of nuclear structure using electron scattering techniques have been made with beams from electron linacs. The characteristics of electron linacs have so far prevented the utilization of coincidence techniques in the study of electron-induced inelastic reactions. All the results we will discuss involve the detection only of momentum-analyzed electrons scattered from the primary electron beam by various target nuclei. These scattering experiments employ electrons of primary energies from about 45 MeV up to 600 or 700 MeV. They may be separated into three groups depending on the nature of the nuclear transitions induced during the scattering process. The groups are characterized by :

- i) elastic scattering;
- ii) scattering leaving the nucleus in a discrete excited state, and
- iii) continuum scattering in which the nucleus undergoes disintegration.

## II. ELASTIC SCATTERING

Elastic scattering through angles less than about  $135^\circ$  is dominated by the interaction of the electron with the charge of the target nucleus. The elastic cross-sections are, in general, ten to many thousand of times larger than cross-sections for excitation of low-lying nuclear states and absolute values can be determined with typical uncertainties of about 5 to 8%. For values of the momentum transfer  $q$  in the range from 0.2 to  $1.8 \text{ (fermi)}^{-1}$  the observed cross-sections depend strongly on the spatial structure of the nuclear ground state charge distribution and can be used to determine within narrow limits the functional forms of the distributions. The uncertainties are smallest for the regions just within, and at the edge of the nucleus.

For nuclei of  $A$  greater than about thirty, absolute cross-section studies <sup>1)</sup> of the elastic scattering of 183 MeV electrons have shown that the nuclear charge distributions are fitted better by a Fermi distribution than by the Ford "Family II" model. The Fermi distribution is given by :

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r-c)/Z.]}$$

where  $c$  is the radius at which the charge density falls to one half its maximum value,  $Z.$  is the surface thickness parameter, and  $r$  is the radial variable measured from the centre of the nucleus. The quantity  $c$  may be determined to within uncertainties of one to three percent, and  $Z.$  to within five to ten percent. The quantity  $c/A^{1/3}$  has an approximately constant value 1.07 Fermi varying no more than  $\pm 2\%$ . This reflects the approximate uniform density of nuclear matter within the heavier nuclei. The parameters are determined by comparison of the measurements with predictions based on phase shift analyses <sup>2)</sup> of scattering, assuming one or more functional forms for the nuclear charge distributions. Elastic charge scattering cross-sections are the quantities most precisely determined in electron scattering measurements and are interpreted with what is the most detailed theory.

It should be noted that the most recent study <sup>1)</sup> of the  $\rho(r)$  for the heavy nuclei Bi-209 and Pb-208 shows that  $\rho(r)$  for these nuclei cannot be approximated by any choice of the parameters of the Fermi model nor of the Ford "Family II". The Fermi model predictions fit the scattering data better than those of the Ford model but theoretical and experimental cross-sections differ by 35% at the highest momentum transfers. Work presently under way <sup>3)</sup> is expected to resolve this situation.

There are a number of Born approximation treatments of elastic scattering which are somewhat more ambiguous and hence far less useful than the more complex phase shift analyses, especially for high  $Z$  targets. Nevertheless, they can be used to determine radius and edge parameters which are in fairly close agreement with those found from the more complex theoretical analyses <sup>4)</sup>. The predicted cross-sections differ from the observed ones in that the former have zeros at the diffraction minima whereas the latter, while having minima, fail always to have true zeros. Theory and experiment are in adequate agreement for values of  $q$  away from those determining the diffraction dips. The Born elastic scattering results are useful in the analysis of the inelastic scattering as described in Part III. Within the framework of this theory, the cross-sections for charge scattering (elastic as well as inelastic) depend only on  $q$  as does the square of the charge form factor,  $F_e^2$ , defined by  $\sigma_{\text{observed}} = \sigma_{\text{point}} \times F_e^2$ .

In Born approximation, cross-sections for the elastic scattering by the magnetic moment distribution in the nucleus are not simply  $q$  dependent but have, in addition, an explicit dependence on  $\tan^2 \vartheta/2$ , where  $\vartheta$  is the scattering angle. In principle, the cross-sections for magnetic elastic scattering can be determined by programming a series of measurements at constant  $q$ . In practice, a combination of the greatly decreased scattering cross-section for large angles [ $\sigma_{\text{point}}$  varies as  $(\cos^2 \vartheta/2)/\sin^4 \vartheta/2$ ], the smallness of  $F_m^2$ , and the small values of the nuclear magnetic moments (of order one nuclear

magneton rather than  $Z$  times this) have so far prevented measurements of the distribution of magnetization from being carried out for elements heavier than  ${}^7\text{Li}$ . Magnetic moment distributions have been studied in the proton <sup>5)</sup>, the neutron <sup>5)</sup>, the deuteron <sup>6)</sup>,  ${}^6\text{Li}$  and  ${}^7\text{Li}$  <sup>7)</sup>. High current linear electron accelerators are expected to allow successful studies of the elastic  $F_m^2$  for heavier nuclei.

### III. INELASTIC SCATTERING : DISCRETE STATES

Studies of discrete nuclear transitions excited by inelastic scattering of high energy electrons are beset by experimental and theoretical difficulties considerably more severe than those associated with elastic scattering. The inelastic peaks as observed experimentally ride on the bremsstrahlung tail of the elastic scattering peak <sup>8)</sup> and, except for the strongest inelastic transitions, the necessary background subtractions arising in the process of making radiative corrections correspond to "signal-to-noise" ratios in the range from 2 to 0.05. Many known transitions are simply buried in this background. Excepting the lightest nuclei whose levels may be MeV apart, one finds that the uncertainties in inelastic cross-sections are rarely less than 10% and are typically 20 to 30%. Phase shift predictions for the cross-sections are far more complicated than for elastic scattering and, moreover, it is in general not as appropriate to substitute a semi-classical transition charge density for the square of the nuclear transition matrix element as in the elastic case, where such a substitution correctly leads to the static nuclear charge density. That is to say, a quantum-mechanical description of the nuclear transition appears to be necessary and it is difficult to insert in an already difficult theory. Only one inelastic phase shift analysis <sup>9)</sup> has so far been published, the bulk of the analysis of the experiment data <sup>10), 11)</sup> having been accomplished with the aid of an ambiguous Born approximation method employing a semi-classical description of the nuclear transition.

More detailed Born treatments <sup>12)</sup> have recently become available for lighter nuclei for which the Born approximation is expected to be adequate.

For heavier nuclei the Born results allow one to determine the multipolarity of an observed transition and the decay rate of the corresponding de-excitation gamma ray. It is surprising that results from such analyses are almost always in agreement with determinations by other methods. (For example, see Ref. <sup>8)</sup>).

As in the case of elastic scattering the electron nuclear-charge interaction gives rise to the largest cross-sections so that electric transitions are observed predominantly. An explicit angle dependence in the expressions for magnetic transition cross-sections allows, in principle, the identification and separation of such transitions from the more easily excited electric ones. The same difficulties as for elastic magnetic scattering are encountered in these inelastic studies. Bishop and others at Orsay and Barber and his group at Stanford have succeeded in observing magnetic transitions in some nuclei lighter than <sup>32</sup>S. For these transitions the momentum transfers were so low that no structure information was available. In the light nuclei and for one or two isolated transitions among heavier nuclei ( $A \gtrsim 40$ ) electric transitions of about single particle speed have been observed. In general, however, most of the induced nuclear excitations are those corresponding to "collective" states, i.e., those states which  $\gamma$  decay to the ground state with rates ten to forty times single particle speed.

Information from the limited inelastic phase shift studies and also from the corrected Born approximation analysis indicate that there should be some diffraction structure apparent in the  $q$  dependence (or angular dependence) of the observed inelastic form factors. This prediction appears to be valid in spite of the known uncertainties in the different treatments. No diffraction structure is apparent (outside of uncertainties in the measurements) in any of the inelastic  $F^2$  measured in any nuclei, to date. Prominent structure is evident in inelastic nucleon scattering cross-sections for excitation of the same

states employing reactions such as  $(p,p')$ ,  $(d,d')$  and  $(\alpha,\alpha')$ . The contrasting situation in electron excitation is not at present understood.

#### IV. INELASTIC SCATTERING TO THE CONTINUUM

The inelastic scattering of electrons from nuclei in which energies larger than the nucleon separation energy are communicated to the nucleus is more difficult to study experimentally than most of the processes we have thus far discussed. Such inelastic scattering is often referred to as "quasi-elastic" as it can be qualitatively understood as the scattering from the quasi-free nucleons in the target nucleus. It differs from scattering from a free nucleon through the nucleon's momentum distribution that arises from its initial binding. The spectrum of electrons scattered quasi-elastically is characterized by a broad peak approximately centered at the kinematic position associated with scattering from a free nucleon. For large momentum transfers the area under the peak is roughly equal to  $(Z\sigma_p + N\sigma_n)$  where  $Z$  and  $N$  are the numbers of protons and neutrons, respectively, and  $\sigma_p$  and  $\sigma_n$  are the free electron-proton and electron-neutron scattering cross-sections. The principal experimental difficulty encountered in studying these spectra arises from radiation of photons by the electrons before, during, and after the actual scattering. These radiative processes degrade the observed spectra badly and corrections are very difficult to make. The corrections to the data approach 100% in the low-energy wings of the quasi-elastic peaks and although there exist theoretical studies of the radiative processes<sup>13),14)</sup> and of the unfolding techniques<sup>14)</sup> necessary to correct the data, there is general agreement that the necessary correction formulae are uncertain to within as much as 20%. The principal part of the experimentally determined quasi-elastic peaks may be corrected with moderate resulting uncertainties at the cost of rather extended programmes of data taking<sup>14)</sup>. No such programme has yet been completed.

The shape and magnitude of the quasi-elastic peaks contain information on nucleon momentum distributions and on the mean nucleon binding energy. There have been remarkably few attempts to extract this information experimentally or to provide a firm theoretical basis for interpretation of measurements.

Of all theoretical studies of inelastic continuum scattering the most reliable are the recent sum rules developed by Van Hove and McVoy<sup>15)</sup>. The sums extend over the quasi-elastic peak but do not include meson production processes. The results are related to the properties of the nuclear ground state including both Pauli correlations and, of more interest, those correlations consequent on the two-nucleon force for small nucleon-nucleon separation. The alterations of the sum rule predictions resulting from these latter correlations is unhappily so small as to be undetectable with present experimental techniques: recent studies by Bishop and Isabelle<sup>16)</sup> at Orsay have shown that evidence for the Pauli correlations is all that can be extracted from sums measured to within 5% uncertainties.

For high  $Z$  targets the wing of the quasi-elastic peak (which corresponds to greater energy loss by the incident electron than the centre of the peak) receives its dominant contributions from two-particle correlations in the nucleus. A theoretical study<sup>17)</sup> of large-energy-loss electron scattering from a dilute Fermi sea of nucleons has indicated that experimental investigations employing heavy nuclei as targets may be able to extract meaningful information concerning these two-nucleon correlations. The interpretation of all earlier measurements of these elusive correlations by other techniques have been troubled by such serious theoretical difficulties that no unambiguous results have emerged<sup>18)</sup>.

When the incident electron has lost more than 136 MeV to the target nucleus meson production becomes important and, indeed, dominates the inelastic scattering for such energy losses. Studies of electroproduction of pions from free protons have been employed by Panofsky



and Allton<sup>19)</sup> and by Hand<sup>20)</sup> to study the neutron's structure form factor but similar studies in heavier nuclei cannot be used to extract information of equivalent interest as the smearing of the kinematics by the nucleon's momentum distributions clouds the interpretation of the measurements.

## V. CONCLUSION

Electron scattering has as perhaps its most important property the ability to determine spatial variations of nuclear structure for elastic and inelastic scattering and for electric and magnetic nuclear transitions. It can also shed light on momentum distributions including those that arise from the nucleon-nucleon force. In all these processes the nature of the electron's interaction with the nucleus can be determined with sufficiently small uncertainties so that the reliability of theories employed to interpret experimental measurements may become extremely high. On the experimental side, although there are difficulties arising from radiative smearing and from low cross-sections, there is no reason to believe that these cannot be overcome increasingly well as experimental as well as theoretical techniques improve.

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