Precision Higgs Masses with FeynHiggs 2.2

Thomas Hahn, Wolfgang Hollik

Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 Munich, Germany

Sven Heinemeyer

CERN TH division, Dept. of Physics, 1211 Geneva 23, Switzerland

Georg Weiglein

Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK

FeynHiggs is a program for computing MSSM Higgs-boson masses and related observables, such as mixing angles, branching ratios, and couplings, including state-of-the-art higher-order contributions. The centerpiece is a Fortran library for use with Fortran and C/C++. Alternatively, FeynHiggs has a command-line, Mathematica, and Web interface. The command-line interface can process, besides its native format, files in SUSY Les Houches Accord format. FeynHiggs is an open-source program and easy to install.

1. Introduction

One of main goals of future colliders is to find a Higgs boson. In order to establish the mechanism of electroweak symmetry breaking it will in addition be necessary to measure the properties of the Higgs boson, hopefully allowing to distinguish between different models. While the LHC will almost certainly take the prize of finding a Higgs [1], if it exists, it will take the International Linear Collider (ILC) to nail down many of the properties [2, 3, 4] to the desired level of accuracy.

Unlike in the Standard Model (SM), where the Higgs mass is only rather loosely constrained by higher-order effects, the Higgs couplings in the Minimal Supersymmetric Standard Model (MSSM) [5] are directly related, through supersymmetry, to the gauge couplings. This implies that the lightest Higgs-boson mass M_h can be predicted in terms of the other model parameters. The mass measurement at the ILC is estimated to $\delta M_h^{\mathrm{exp}} \approx 0.05$ GeV [2, 3, 4], thus M_h will become a precision observable.

Together, these two issues mandate precise calculations of observables on the theory side in a variety of models, but in particular in supersymmetric ones, where M_h is a prediction. The FeynHiggs [6, 7] package provides masses, couplings, branching ratios, etc. in the real, complex, and non-minimal flavour-violating MSSM including state-of-the-art radiative corrections.

2. The MSSM Higgs sector

The MSSM contains two Higgs doublets,

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 + \mathrm{i}\chi_1) \\ \phi_1^- \end{pmatrix}, \qquad H_2 = \mathrm{e}^{\mathrm{i}\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + \mathrm{i}\chi_2) \end{pmatrix}, \tag{1}$$

where a possible CP-violating phase ξ has been indicated. The Higgs potential is given by

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\varepsilon_{\alpha\beta} H_1^\alpha H_2^\beta + \mathrm{h.c.}) + \frac{g_1^2 + g_2^2}{8}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \frac{g_2^2}{2}|H_1\bar{H}_2|^2. \tag{2}$$

The only non-trivial CP-violating phase (besides ξ) is contained here in m_{12}. At tree level all CP phases can be rotated away, giving five physical states of distinct CP parity: h^0, H^0 (CP-even), A^0 (CP-odd), and $H^\pm$.

The quartic Higgs couplings are completely determined by the gauge couplings g_1 and g_2 and this leads to the well-known tree-level prediction $M_h < M_Z$, which stands in conflict with measurements since LEP. Fortunately (for the MSSM), significant quantum loop contributions push the upper bound on M_h up to about 140 GeV [8, 9] for a top mass of 178 GeV. But the quantum effects lead also to qualitative changes. In the presence of CP-violating phases, all three neutral Higgs bosons mix and CP is no longer conserved [10],

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \\ A^0 \end{pmatrix}. \tag{3}$$

The three mass eigenstates are denoted as h_i, $i = 1, 2, 3$, and ordered as $m_{h_1} \leqslant m_{h_2} \leqslant m_{h_3}$.

3. FeynHiggs

3.1. Download and Installation

Installing FeynHiggs is simple and fast. Version 2.2 requires no prerequisites (e.g. LoopTools) as before.

- Get the latest FeynHiggs tar file from `http://www.feynhiggs.de`.
- Unpack, configure, and build:
  ```
  tar xfz FeynHiggs-2.2.N.tar.gz
  cd FeynHiggs-2.2.N
  ./configure
  make
  ```
  To build also the Mathematica part, replace "`make`" by "`make all`".
- (Optional:) Type "`make install`" to install the files and "`make clean`" to remove intermediate files.

3.2. Modes of Operation

FeynHiggs operates in one of four basic modes:

- Library Mode: The FeynHiggs routines are invoked from a Fortran or C/C++ program linked against the FeynHiggs library.
- Command-line Mode: Parameter files in FeynHiggs' native format or in SUSY Les Houches Accord (SLHA) [11] format are processed at the command line with the standalone executable `FeynHiggs`.
- WWW Mode: The user interactively chooses parameters at the FeynHiggs User Control Center (FHUCC) and obtains the results on-line at `http://www.feynhiggs.de/fhucc` .
- Mathematica Mode: The FeynHiggs routines can be used in Mathematica via the MathLink executable `MFeynHiggs`.

3.3. Application Programming Interface

The FeynHiggs library `libFH.a` is a static Fortran 77 library. Its global symbols are prefixed with a unique identifier to minimize symbol collisions. The library contains only subroutines (no functions), so that no include files are needed (except for the couplings) and the invocation from C/C++ is hassle-free. Detailed debugging output can be turned on at run time. All routines are described in detail in the API guide and on man-pages, so only a brief overview is needed here:

- `FHSetFlags` sets the flags of the calculation.

- `FHSetPara` sets the MSSM input parameters directly.

- `FHSetSLHA` extracts the input parameters from an SLHA data structure.

- `FHSetDebug` sets the debugging level.

- `FHGetPara` retrieves (some of) the derived parameters.

- `FHHiggsCorr` evaluates the Higgs masses and mixings, $M_{h_1,h_2,h_3,H^\pm}$, α_{eff} (the effective mixing angle in the CP-conserving case), U_{ij}, featuring:

  - In the neutral Higgs sector, the following propagator matrix is diagonalized,

$$
\begin{pmatrix}
q^2 - M_h^2 + \hat{\Sigma}_{hh}^{①②③} & \hat{\Sigma}_{hH}^{①②③} & \hat{\Sigma}_{hA}^{②③} \\
\hat{\Sigma}_{Hh}^{①②③} & q^2 - M_H^2 + \hat{\Sigma}_{HH}^{①②③} & \hat{\Sigma}_{HA}^{②③} \\
\hat{\Sigma}_{Ah}^{②③} & \hat{\Sigma}_{AH}^{②③} & q^2 - M_A^2 + \hat{\Sigma}_{AA}^{②③}
\end{pmatrix},
\tag{4}
$$

    where the self-energies include the following terms as indicated,

    ① the most up-to-date leading $\mathcal{O}(\alpha_s\alpha_t, \alpha_t^2)$ [8, 12, 13] and subleading $\mathcal{O}(\alpha_s\alpha_b, \alpha_t\alpha_b, \alpha_b^2)$ [14, 15] two-loop corrections in the rMSSM (complex effects are taken into account only partially in the two-loop part at present),

    ② full one-loop evaluation (all phases included),

    ③ complete q^2 dependence.

  - Full one-loop corrections for the charged Higgs sector [16].

  - Mixed $\overline{\text{DR}}$/OS renormalization for the one-loop result [17].

  - "Δm_b" corrections = leading $\mathcal{O}(\alpha_s\alpha_b)$ and $\mathcal{O}(\alpha_t\alpha_b)$ terms for Higgs masses, couplings, etc. [18].

  - Non-minimal flavour-violating effects (e.g. $\tilde{c}$–$\tilde{t}$ mixing) [19].

- `FHUncertainties` estimates the uncertainties of the Higgs masses and mixings. The total uncertainty is the sum of deviations from the central value, $\Delta X = \sum_{i=1}^{3} |X_i - X|$ with $X = \{M_{h_1,h_2,h_3,H^\pm}, \alpha_{\text{eff}}, U_{ij}\}$, where

  - X_1 is obtained by varying the renormalization scale (entering via the $\overline{\text{DR}}$ renormalization) within $\frac{1}{2}m_t \leqslant \mu \leqslant 2m_t$,

  - X_2 is obtained by using m_t^{pole} instead of the running m_t in the two-loop corrections,

  - X_3 is obtained by using an unresummed bottom Yukawa coupling, y_b, i.e. an y_b including the leading $\mathcal{O}(\alpha_s\alpha_b)$ corrections, but not resummed to all orders.

- `FHCouplings` computes the Higgs couplings, decay widths, and BRs in the MSSM and also for an SM Higgs boson with mass M_{h_i} (denoted as $h_{1,2,3}^{\text{SM}}$) for comparison:

$$
\begin{aligned}
&h_{1,2,3} \to f\bar{f}, \gamma\gamma, ZZ^*, WW^*, gg, & &H^\pm \to f\bar{f}', & &h_{1,2,3}^{\text{SM}} \to f\bar{f}, \gamma\gamma, ZZ^*, WW^*, gg. \\
&\quad h_i Z^*, h_i h_j, H^+ H^-, & &\quad h_i W^{\pm *}, & & \\
&\quad \tilde{f}_i \tilde{f}_j, & &\quad \tilde{f}_i \tilde{f}_j', & & \\
&\quad \tilde{\chi}_i^\pm \tilde{\chi}_j^\pm, \tilde{\chi}_i^0 \tilde{\chi}_j^0, & &\quad \tilde{\chi}_i^0 \tilde{\chi}_j^\pm, & &
\end{aligned}
\tag{5}
$$

- `FHHiggsProd` calculates approximately (by means of effective couplings) the following inclusive Higgs production cross-sections: $bb \to h + X$, $gg \to h + X$, $qq \to qqh + X$, $qq, gg \to tth + X$, $qq \to Wh + X$, $qq \to Zh + X$ [20].

- `FHConstraints` evaluates several electroweak precision observables, to be used as additional constraints:

  - $\Delta\rho$ at $\mathcal{O}(\alpha, \alpha\alpha_s)$ [21, 22]. Too large values of $\Delta\rho$ indicate experimentally disfavoured $\tilde{t}/\tilde{b}$ masses.

  - $(g_\mu - 2)_{\text{SUSY}}$ including full one-loop and leading/subleading two-loop SUSY corrections [23, 24].

  - (Preliminary:) The electric dipole moments of Th, N, and Hg. This part is not yet fully tested.

4. Command-line Modes

The FeynHiggs command-line frontend, `FeynHiggs`, reads input files both in its own and in SLHA format. The FeynHiggs format simply lists the parameters and their values, for example:

```
MT         178
MB         4.7
MW         80.450
MZ         91.1875
TB         50
MA0        200
MSusy      975
...
```

More sophisticated variants are possible, e.g. "TB 5 50 2.5" declares a loop over TB $= \tan\beta$ from 5 to 50 in steps of 2.5. This input file, e.g. `fh.in`, is run through FeynHiggs by

```
FeynHiggs fh.in
```

Optionally, the flags can be given behind the filename as a string of digits, as in

```
FeynHiggs fh.in 40020211
```

The output is listed on stdout in a human-readable form, for example

```
-------------------- HIGGS MASSES --------------------
| Mh0       =      117.186672
| MHH       =      194.268239
| MA0       =      200.000000
| MHp       =      212.662071
| SAeff     =       -0.36496659
| UHiggs    =        0.99589960     0.09046538     0.00000000 \
|                   -0.09046538     0.99589960     0.00000000 \
|                    0.00000000     0.00000000     1.00000000


---------------- ESTIMATED UNCERTAINTIES ---------------
| DeltaMh0  =        0.919435
| DeltaMHH  =        0.728304
| DeltaMA0  =        0.000000
| DeltaMHp  =        1.929728
...
```

The listing can become quite lengthy, and although FeynHiggs automatically spawns a pager for easier viewing, one would sometimes like to mask off the details. Such lines contain a % character, thus

```
FeynHiggs fh.in | grep -v %
```

turns off the details.

To convert the human-readable into a machine-readable form, the `table` utility is used. For example, the following line produces a file `fh.out` with two columns, TB and Mh0,

```
FeynHiggs fh.in | table TB Mh0 > fh.out
```

The SLHA mode works similarly, only that the output is not listed on screen, but saved in a file (input filename plus ".fh"), again in SLHA format. This way, FeynHiggs can act as a filter in a chain of commands operating on an SLHA file. FeynHiggs tries to read each input file in SLHA format first and if that fails, falls back into its native format. FeynHiggs' SLHA interface uses the SLHA Library [25].
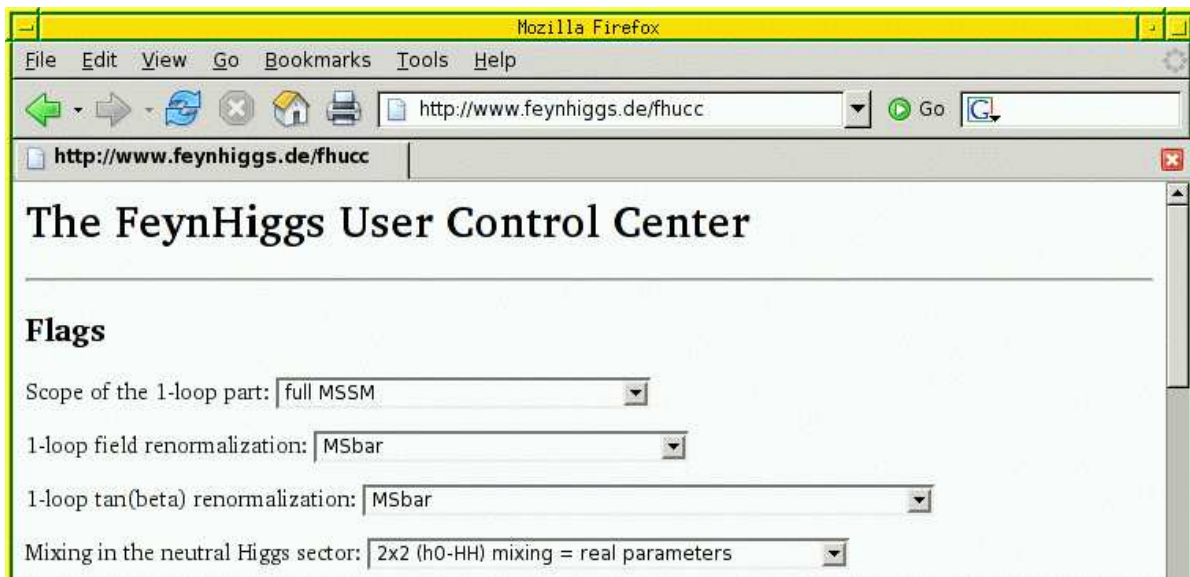
Figure 1: The FeynHiggs User Control Center.

5. Interactive Modes

FeynHiggs can be used interactively in WWW Mode or in Mathematica Mode. In WWW Mode, point your browser to the FeynHiggs User Control Center at `http://www.feynhiggs.de/fhucc`. The Web interface allows to select one of the Les Houches benchmark scenarios, or choose each parameter directly. Fig. 1 shows a screen shot.

A much more powerful interactive environment is provided by the Mathematica interface of FeynHiggs. The MathLink executable `MFeynHiggs` must first be loaded with

```
Install["MFeynHiggs"]
```

and makes all FeynHiggs routines (see Sect. 3.3) available as Mathematica functions. Standard Mathematica functions, such as `ContourPlot` and `FindMinimum`, then make some sophisticated analyses possible.

6. Summary

The FeynHiggs package computes Higgs masses, mixing angles, branching ratios, couplings, etc. in the MSSM including state-of-the-art radiative corrections. The heart of the program is a static Fortran library which can be accessed either directly (in Fortran or C/C++) or through various frontends (command-line, Mathematica, WWW). FeynHiggs is freely available from `http://www.feynhiggs.de` and is straightforward to compile and install.

References

[1] ATLAS Collaboration, *Detector and Physics Performance Technical Design Report*, CERN/LHCC/99-15 (1999), see: `atlasinfo.cern.ch/Atlas/GROUPS/PHYSICS/TDR/access.html`; CMS Collaboration, see: `cmsinfo.cern.ch/Welcome.html/CMSdocuments/CMSplots/`.

[2] J. Aguilar-Saavedra et al., TESLA TDR Part 3, hep-ph/0106315, see: `tesla.desy.de/tdr/`.

[3] T. Abe et al. [American Linear Collider Working Group Collaboration], hep-ex/0106056.

[4] K. Abe et al. [ACFA Linear Collider Working Group Collaboration], hep-ph/0109166.

[5] H.P. Nilles, *Phys. Rep.* **110** (1984) 1,
   H.E. Haber, G.L. Kane, *Phys. Rep.* **117** (1985) 75;
   R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1.

[6] S. Heinemeyer, W. Hollik, G. Weiglein, *Comp. Phys. Commun.* **124** (2000) 76 [hep-ph/9812320].
   The FeynHiggs Web site is at `http://www.feynhiggs.de`.

[7] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, G. Weiglein, in preparation.

[8] S. Heinemeyer, W. Hollik, G. Weiglein, *Eur. Phys. J.* **C9** (1999) 343 [hep-ph/9812472].

[9] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein, *Eur. Phys. J.* **C28** (2003) 133 [hep-ph/0212020].

[10] A. Pilaftsis, *Phys. Rev.* **D58** (1998) 096010 [hep-ph/9803297],
   —, *Phys. Lett.* **B435** (1998) 88 [hep-ph/9805373],
   D. Demir, *Phys. Rev.* **D60** (1999) 055006 [hep-ph/9901389],
   S. Choi, M. Drees, J. Lee, *Phys. Lett.* **B481** (2000) 57 [hep-ph/0002287],
   A. Pilaftsis, C. Wagner, *Nucl. Phys.* **B553** (1999) 3 [hep-ph/9902371],
   M. Carena, J. Ellis, A. Pilaftsis, C. Wagner, *Nucl. Phys.* **B586** (2000) 92 [hep-ph/0003180],
   T. Ibrahim and P. Nath, *Phys. Rev.* **D63** (2001) 035009 [hep-ph/0008237],
   —, *Phys. Rev.* **D66** (2002) 015005 [hep-ph/0204092],
   S. Heinemeyer, *Eur. Phys. J.* **C22** (2001) 521 [hep-ph/0108059],
   —, hep-ph/0407244.

[11] P. Skands et al., *JHEP* **0407** (2004) 036 [hep-ph/0311123].

[12] G. Degrassi, P. Slavich, F. Zwirner, *Nucl. Phys.* **B611** (2001) 403 [hep-ph/0105096].

[13] A. Brignole, G. Degrassi, P. Slavich, F. Zwirner, *Nucl. Phys.* **B631** (2002) 195 [hep-ph/0112177].

[14] A. Brignole, G. Degrassi, P. Slavich, F. Zwirner, *Nucl. Phys.* **B643** (2002) 79 [hep-ph/0206101].

[15] A. Dedes, G. Degrassi, P. Slavich, *Nucl. Phys.* **B672** (2003) 144 [hep-ph/0305127].

[16] M. Frank, S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/0212037.

[17] M. Frank, S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/0202166.

[18] M. Carena, D. Garcia, U. Nierste, C. Wagner, *Nucl. Phys.* **B577** (2000) 88 [hep-ph/9912516].

[19] S. Heinemeyer, W. Hollik, F. Merz, S. Peñaranda, *Eur. Phys. J.* **C37** (2004) 481 [hep-ph/0403228].

[20] The SM cross-sections are taken from the Web site `http://maltoni.home.cern.ch/maltoni/TeV4LHC` by F. Maltoni and S. Willenbrock.

[21] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. Weiglein, *Phys. Rev. Lett.* **78** (1997) 3626 [hep-ph/9612363], *Phys. Rev.* **D57** (1998) 4179 [hep-ph/9710438].

[22] S. Heinemeyer, W. Hollik, G. Weiglein, hep-ph/0412214.

[23] S. Heinemeyer, D. Stöckinger, G. Weiglein, *Nucl. Phys.* **B690** (2004) 62 [hep-ph/0312264].

[24] S. Heinemeyer, D. Stöckinger, G. Weiglein, *Nucl. Phys.* **B699** (2004) 103 [hep-ph/0405255].

[25] T. Hahn, hep-ph/0408283.