3-D PARALLEL SIMULATION OF CONTINUOUS BEAM-CLOUD INTERACTIONS

A. Ghalam,* T. Katsouleas, USC, Los Angeles, CA, USA
C. Huang, V. Decyk, W. Mori, UCLA, Los Angeles, CA, USA
G. Rumolo, GSI, Darmstadt, Germany

Abstract

A new 3D Particle-In-Cell (PIC) model for continuous modelling of beam and electron cloud interaction in a circular accelerator is presented. A simple model for lattice structure, mainly the quadruple and dipole magnets and chromaticity have been added to a plasma PIC code, QuickPIC, used extensively to model plasma Wakefield acceleration concept. The code utilizes parallel processing techniques with domain decomposition in both longitudinal and transverse domains to overcome the massive computational costs of continuously modelling the beam-cloud interaction. Through parallel modelling, we have been able to simulate beam propagation through up to 2000 turns of the LHC ring. Emittance and spot size growths have been studied for various circular accelerators and storage rings, (e.g. CERN-SPS, LHC) and the results compared with the previous single-kick models for electron cloud (e.g. HEAD-TAIL). The growth predicted by our code is generally less than that predicted by these models. It is also shown that the single kick approximation may not be accurate for beam- electron cloud modelling due to the highly nonlinear nature of the problem.

INTRODUCTION

The effects of electron clouds on beam dynamics in high-energy circular accelerators and storage rings with positively charged bunches are well known [1-5]. These effects have been experimentally observed and verified in many facilities around the world. In the case of the CERN proton synchrotron (PS) ring, electron cloud effects have been observed for LHC-type bunch trains, i.e., 72 bunches of $N_b = 1.1\times10^{11}$ protons per bunch spaced by 25 ns. The instability of the beam in the electron cloud is mainly observed in the horizontal plane as a single bunch (head-tail) instability [6]. In the Super Proton Synchrotron (SPS) machine at CERN with LHC-type bunch trains, the strong electron cloud build up manifests itself in a vertical plane instability [7]. Severe instability due to electron cloud build up has been observed in both 3 and 50 GeV proton storage rings in the Japan Proton Accelerator Research Complex (J-PARC) [8].

Due to the importance of the problem in many existing proton and positron rings as well as future high energy rings, a great deal of effort has been made to model the complex dynamics of beams in electron clouds. The first simulation model was developed at CERN (HEAD-TAIL) [9]. The model is based on the single kick approximation where the cloud is lumped at discrete points along the ring. Other simulation models based on the same principle were developed at KEK (PEHTS) and at SLAC [10]. While these models are good to the first order, they may not model the real physical situation accurately where the perturbation due to the pinched cloud is highly non-linear [11].

The second simulation model, QuickPIC, recently developed at USC/UCLA, is based on a PIC simulation method [12] in use for plasma wakefields. In this case, the beam and electron cloud particles are treated as two continuous species with Maxwell and Lorentz equations governing the field and the motion of particles, respectively. In this model, the electron cloud is spread all over the ring and the beam continuously interacts with the cloud particles as it evolves over the ring. While this approach is more realistic than the previous model, it is computationally much more intensive and requires high performance computation on parallel platforms.

This paper is organized as follows: In the first section, the PIC model QuickPIC is explained and the simplifying assumptions to make the code more efficient than the conventional PIC codes are discussed. The enhancement to QuickPIC to include the circular machine physics (betatron motion and chromaticity) and to obtain higher order accuracy in the pusher are also described in section two. In the third section the two codes are benchmarked against each other. The emittance growth of an LHC beam is chosen as a basis for comparison.

PIC MODEL FOR THE ELECTRON CLOUD

The PIC method we adopt was originally used for modelling beam-plasma interactions in plasma wakefield acceleration research [13,14]. The beam-electron cloud modelling is very much similar to beam-plasma interaction in the sense that the electron cloud can be considered as a low-density non-neutral plasma. In spite of a great similarity between these two problems, some modifications are needed to model beam- electron cloud problem. In the remainder of this section we explain the general PIC method briefly, introduce QuickPIC as a special PIC suitable for our model and then discuss the modification to QuickPIC model for this problem.

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* feizzarr@usc.edu

One way to solve Maxwell and Lorentz equations, governing the beam and electron cloud particles, is the PIC method [14]. It breaks up the problem into four distinct steps as shown in Fig.1: Given an initial configuration of particles, the electromagnetic field values known on a staggered grid that is defined throughout the simulation space, a PIC code first calculates the field at the particle positions by interpolating the fields on the grid to the particle positions. Then it uses these fields to calculate the new positions and new momenta of the particles. The updated position and momentum data are then used to find the updated current and charge density. In the final step, the currents and charge density are used to advance the electromagnetic fields in time via Maxwell's equation.

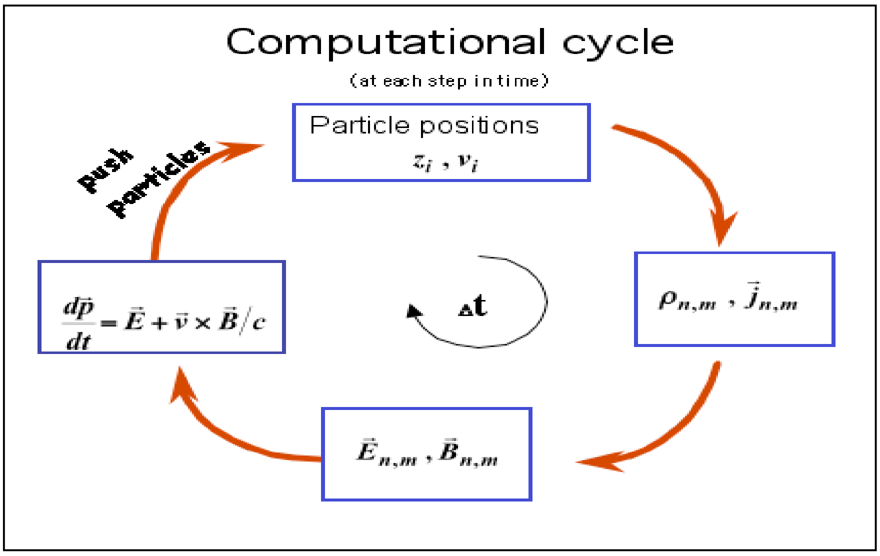

Fig. 1: General PIC code cycle

QuickPIC is a 3D Cartesian PIC code based on the quasi-static or frozen field approximation. It requires that the beam does not evolve significantly on the time scale that it takes the beam to pass by electrons. This is typically well satisfied in the electron cloud problem where the beam beta function, β, is much larger than the the bunch length, σ_z. Writing wave equations in terms of vector and scalar potentials and using quasi-static approximation to cancel ∂_z-$c\partial_t$ [13], the full set of equations describing the beam-cloud wakefield reduce to:

$$-\nabla_\perp^2 \varphi = 4\pi\rho$$
$$-\nabla_\perp^2 A = \frac{4\pi}{c} J \qquad (1)$$

where ρ is the sum of cloud and beam charge densities, J is the total axial current density of the beam and the electron cloud and it is assumed that the cloud motion is non-relativistic[*]. We can see from Eq. (1) that the full set of Maxwell's Equations reduce to 2D Poisson equations.

Defining the wake potential ψ as $\varphi - A_\parallel$, the total force exerted on the beam due to the cloud can be written as:

$$F_{b\perp} = -e\nabla_\perp \psi \qquad (2)$$

and the total force on the cloud due to the beam and the cloud itself can be expressed as:

$$F_{e\perp} = -e\nabla_\perp \varphi \qquad (3)$$

Figure 2 shows the QuickPIC cycle.

The wakes are stored and used to update the electron cloud particles in the slab and the slab is then pushed back a small step through the beam. After transiting the beam, the stored values of ψ are used to find the force on the beam (treated as a 3D PIC model) and it is pushed through a step (of the order of $\lambda_\beta/30$ where λ_β is the average betatron wavelength). A full description of the code can be found in [14].

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[*] The full quasi-static equations are more complex when the cloud motion is relativistic. Additional sources for the fields must be kept and the axial cloud motion must be included. This has been included[C.K. Huang, private communications] but does not alter the results in the regime of interest for electron cloud problems.

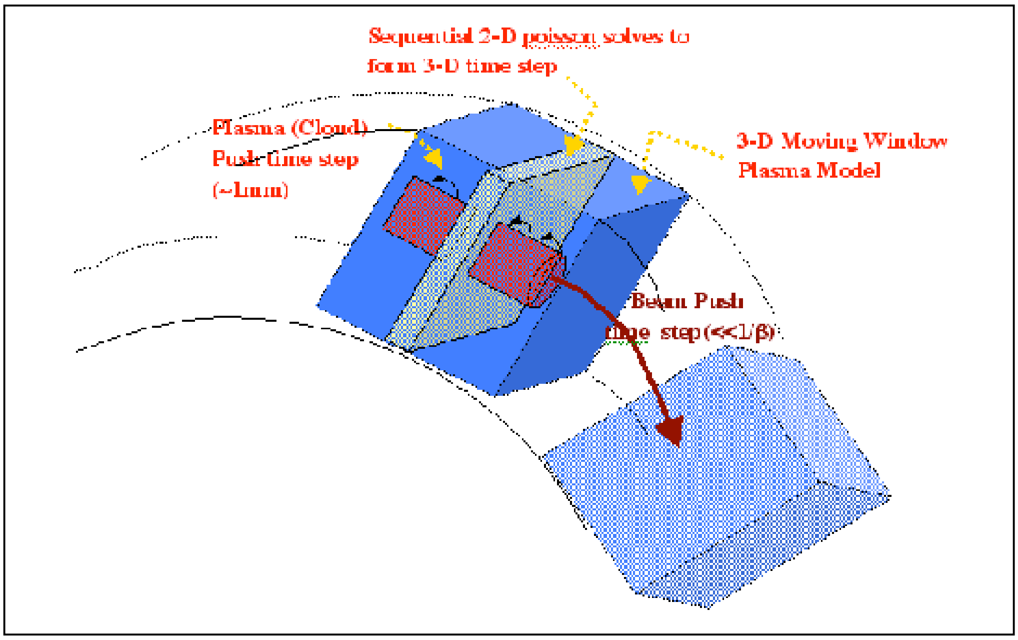

Fig. 2: QuickPIC cycle. A 2-D Poisson solver is used to calculate potentials and update positions and velocities in the plasma slab. After the slab is stepped through the beam. The stored potentials ψ and φ are used to push the 3-D beam.

MODIFICATIONS TO THE PIC CODE

In order to enable QuickPIC to simulate a bunch in a circular accelerator, the effect of the lattice structure has been added to the code to correctly model the bunch evolution. The effect of Quadruples and RF power fields on the beam have been introduced as a continuous external force which act on the beam together with the force due to the electron cloud particles. In other words at each 3D time step, the beam particles, position and velocity are updated by the force due to the electron cloud and the external forces. Similarly, the effect of dipole magnets on the cloud particles have been included in the code by adding an external force, which is turned on in the bending sections and off in the straight sections, acting on the cloud particles together with the force due to the beam and self consistent electron cloud particles. The full set of differential equations implemented in the code is in [12].

As mentioned earlier in this section, in a general PIC model, particle positions and velocities are advanced by the forces due to EM fields. There are several schemes to update the position and velocity of the particles. The simplest scheme, which is used in QuickPIC, is leap frog. If $x_i(t)$, $v_i(t)$, $a_i(t)$ represent particle i^{th} position, velocity and acceleration at time t then particle i^{th} position and velocities are updated as follows:

$$\begin{aligned} v_i(t+\Delta t/2) &= v_i(t-\Delta t/2) + a_i(t)\Delta t \\ x_i(t+\Delta t) &= x_i(t) + v_i(t+\Delta t/2)\Delta t \end{aligned} \tag{4}$$

Although this is an efficient and fast scheme for typical beam-plasma problems, it needs a modification in the case of beam-electron cloud modelling. This is because the method introduces a small numerical shift in the transverse oscillation frequency. This shift may be large enough to compete with and/or obscure small physical tune shifts present in the cloud problem. For a harmonic oscillator of frequency ω_0, the leap frog Eqs. (4) lead to a frequency:

$$\omega = \frac{\sin^{-1}(\omega_0 \Delta t/2)}{\Delta t/2} \tag{5}$$

Therefore, there is always a difference between the calculated (ω) and the nominal (ω_0) value of the oscillation frequency (or tune) and this difference becomes smaller as the time step approaches zero.

While choosing a small value for the time step can result in the required accuracy for the tune, this method may not be efficient because as we decrease the time step, the simulation time would increase. Another efficient way to minimize the tune shift effect is to set the nominal oscillation frequency to $\dfrac{\sin(\omega_0 \Delta t/2)}{\Delta t/2}$ as Eq. (5) suggests. Applying this value to Eq. (4), we obtain the modified leap frog equations as follows:

$$v_i\left(t+\Delta t/2\right)=v_i\left(t-\Delta t/2\right)-\frac{\sin^2\left(\omega_0\,\Delta t/2\right)}{\left(\Delta t/2\right)^2}x_i\left(t\right)\Delta t \qquad (6)$$

$$x_i\left(t+\Delta t\right)=x_i\left(t\right)+v_i\left(t+\Delta t/2\right)\Delta t$$

Using the above equations to update the position and velocities of the particles eliminates numerical shift on the nominal tune. While it introduces a small error for oscillation at other frequencies, these will generally be a correction to what is already a small correction.

MODEL COMPARISON

In this section, the two methods for beam-electron cloud simulations has been compared and benchmarked against each other. The physical parameters for these simulations are consistent with the LHC type beam in the CERN-LHC ring. These parameters are summarized in Table 1. As a basis for comparison, the horizontal emittance of the beam is studied over 1700 turns of beam evolution in the ring with the two codes.

Table 1. LHC parameters used in the simulations

Horizontal Spot Size (mm) (rms)	0.884
Vertical Spot Size (mm) (rms)	0.884
Bunch Length (m) (rms)	0.115
Horizontal Box Size (mm)	18
Vertical Box Size (mm)	18
Bunch Population	1.1×10^{11}
Average Horizontal Beta Function (m)	66
Average Vertical Beta Function (m)	77.5
Momentum Spread	4.68×10^{-4}
Beam Momentum (GeV/c)	479.6
Circumference (km)	26.659
Horizontal Betatron Tune	64.28
Vertical Betatron Tune	59.31
Synchrotron Tune	0.0059
Horizontal and Vertical Chromaticity	2,2
Electron Cloud Density (cm^{-3})	6×10^{5}

Figure 3 shows the horizontal spot size growth of the beam over 1700 turns of beam evolution obtained by HEAD-TAIL and QuickPIC codes. As can be seen the growth predicted by HEAD-TAIL is much more severe than the growth obtained by QuickPIC.

In order to resolve this difference in the results, QuickPIC is modified to operate in the single kick regime. Since QuickPIC is a "continuous kick model," changing the code to single-kick model is straightforward. We turn off the force due to the electron cloud on the beam everywhere along the ring except for the kick point. At the kick point the total force exerted on the beam particle is the sum of the external forces due to magnets and RF power and the force due to the electron cloud. Since the cloud is lumped at the kick point, the cloud density should be scaled at this point so that the average cloud density along the entire ring is constant. In this simulation there are 2048 time steps for a full beam evolution over the ring and the kick point corresponds to one time step. Therefore the cloud density should be scaled by a factor of 2048 at the kick point. However, the cloud density is not scaled; instead the force obtained by the electron cloud is scaled by this factor. Scaling the density at the kick point causes a huge space charge effect and this effect is not included in the HEAD-TAIL code (in this particular simulation). Furthermore, since there is no cloud effect on the beam dynamics throughout the whole ring except for the kick point and since the force exerted on the beam is due to the external forces at all the other points, it is more efficient to use matrix transformation to update the beam particles' positions and velocities at these points. So, the beam particles coordinates are transformed right after the kick point to a point right before the kick point with the transformation matrix.

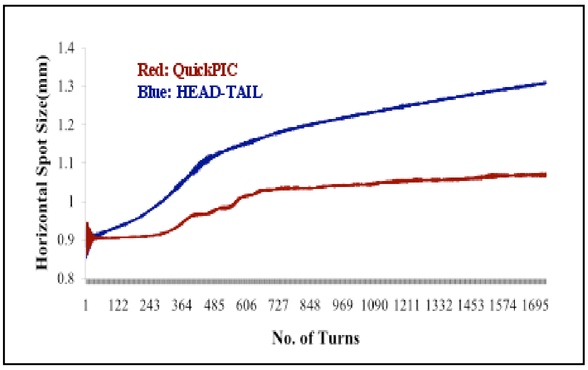

Fig. 3. Horizontal spot size of the beam predicted by the two codes.

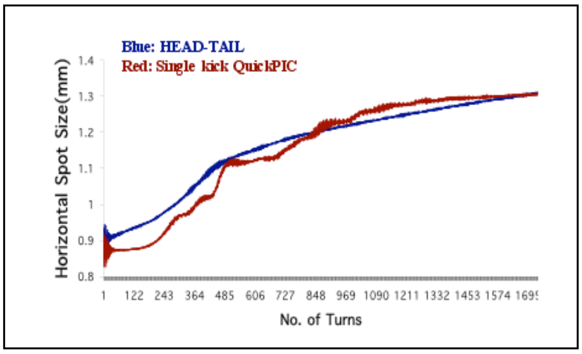

Fig. 4. Horizontal spot size growth of the beam predicted by HEAD-TAIL and single kick QuickPIC

The benchmarking between two codes is an ongoing process and progress has been made to make the physical situations and parameters used in the codes as equal as possible. Figure 4 shows one example of code comparison when the results are in agreement; however, it is worth mentioning that the boundary conditions used in QuickPIC is conducting while the boundary in HEAD-TAIL is open. However, we argue that the single kick may not be accurate for electron cloud modeling. To verify this statement, we increase the number of kick points in the "single kick QuickPIC" and repeat the simulation. Figure 5 shows the simulation results for different number of kick points. If the single kick approximation were valid, the results should be identical with different numbers of kicks. It is seen from this figure that the spot size growth is changing with the number of kick points. That is because the single kick approach is applicable when there is a linear perturbation. In the case of electron cloud we are dealing with a highly nonlinear electron cloud perturbations.

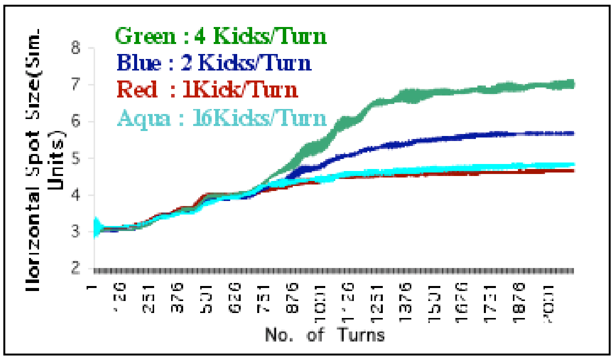

Fig. 5. Horizontal spot size growth with different number of kick points.

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