Boost Invariance and Multiplicity Dependence
of the Charge Balance Function in π^+p and K$^+$p Collisions at $\sqrt{s} = 22$ GeV/c

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Boost invariance and multiplicity dependence of the charge balance function are studied in π^+p and K$^+$p collisions at 250 GeV/c incident beam momentum. Charge balance, as well as charge fluctuations, are found to be boost invariant over the whole rapidity region, but both depend on the size of the rapidity window. It is also found that the balance function becomes narrower with increasing multiplicity, consistent with the narrowing of the balance function when centrality and/or system size increase, as observed in current relativistic heavy ion experiments.

Charge balance and charge flow are measures of rapidity correlations between oppositely charged particles and have been used to study hadronization in hadron-hadron [1] as well as in lepton-hadron [2] and e$^+$e$^-$ [3] collisions. In the form of a charge balance function (BF) [4], they have recently gained new interest in the field of relativistic heavy ion collisions. A narrowing of the balance function is suggested as a signature of delayed hadronization, or of the formation of a Quark-Gluon Plasma (QGP) during the early stage of a collision. The integral of the balance function is related to the event-by-event charge fluctuations [5], which are expected to be suppressed in a QGP [6].

So far, two heavy ion experiments [7, 8] have measured the balance function at various centralities and for different colliding nuclei. A narrowing of the balance function is indeed observed with increasing centrality of the collision and with increasing size of the colliding nuclei. Coalescence [9], thermal [10] or blast wave [11] models with transverse flow and local clusters and/or resonances can explain these phenomena. The measured charge fluctuations, on the other hand, are consistent with those expected for a hadronic gas [12, 13, 14].

Before drawing any conclusions from the observed narrowing of the balance function, it is necessary to know how the BF behaves in hadron-hadron collisions, where no QGP is expected, and how the limited detector acceptance influences its width [4, 5, 9]. Since current heavy ion experiments [7, 8, 14] cover only a limited rapidity region, the measured BF's do not correspond to that for the full rapidity region. Whether the results from different heavy ion experiments are comparable or not depends on the influence of the acceptance.

A number of theoretical discussions [5, 6, 15, 16, 17] have been devoted to the influence of acceptance, or on how to find robust measures for the charge correlations and fluctuations. In particular, in ref. [5], based on the assumption of longitudinal boost invariance (rapidity independence), Jeon and Pratt proposed a relation between the balance function in a rapidity window $B(\delta y|Y_{\mathrm{w}})$ and in the full rapidity range $B(\delta y|Y = \infty)$ [5],

$$B(\delta y|Y_{\mathrm{w}}) = B(\delta y|\infty)\left(1 - \frac{\delta y}{Y_{\mathrm{w}}}\right), \qquad (1)$$

where Y_{w} is the size of the rapidity window and $B(\delta y|Y_{\mathrm{w}})$ can be measured by

$$B(\delta y|Y_{\mathrm{w}}) \; = \; \frac{1}{2}\left[\frac{\langle n_{+-}(\delta y)\rangle - \langle n_{++}(\delta y)\rangle}{\langle n_+\rangle}\right.$$
$$\left. + \frac{\langle n_{-+}(\delta y)\rangle - \langle n_{--}(\delta y)\rangle}{\langle n_-\rangle}\right]. \quad (2)$$

Here, $n_{+-}(\delta y)$, $n_{++}(\delta y)$ and $n_{--}(\delta y)$ are the numbers of pairs of opposite- and like-charged particles satisfying the criteria that they fall into the rapidity window Y_{w} and that their relative rapidity equals δy; n_+ and n_- are the numbers of positively and negatively charged particles, respectively, in the interval Y_{w}.

Conventionally, boost invariance refers to particle density being independent of rapidity, as originally assumed in [18] and applied in a simple solvable hydrodynamic model [19, 20]. While this may be correct in a very restricted region at mid-rapidity for the *rapidity density* itself [21, 22, 23, 24, 25], boost invariance of the

balance function only requires that the *charge correlation* between final state particles be the same in any longitudinally-Lorentz-transformed frame. Whether the BF is boost invariant over the *whole* rapidity region or only in the central region, cannot be simply deduced from the corresponding shape of the rapidity density distribution. This important issue has not yet been investigated in either its theoretical or experimental aspects.

In this letter, boost invariance and multiplicity dependence of the charge balance function is studied on π^+p and K$^+$p data at 250 GeV/c ($\sqrt{s}$ =22 GeV) of the NA22 experiment. This experiment was equipped with a rapid cycling bubble chamber as an active vertex detector, had excellent momentum resolution and 4π acceptance. The latter feature allows, for the first time, to study the properties of the balance function in full phase space.

Since no statistically significant differences are seen between the results for π^+ and K$^+$ induced reactions, the two data samples are combined for the purpose of this analysis. A total of 44 524 non-single-diffractive events is obtained after all necessary selections, as described in detail in [26]. In particular, possible contamination from secondary interactions is suppressed by a visual scan and the requirement that overall charge balance be satisfied within the whole event; γ conversions near the primary vertex are removed by electron identification.

In Fig. 1, the balance function is shown for five cms rapidity windows of width $Y_w = 3$, located at different positions, $[-3, 0]$ (open stars), $[-2, +1]$ (open crosses), $[-1.5, 1.5]$ (open circles), $[-1, 2]$ (open diamonds), and $[0, 3]$ (open triangles). In this and the following figures, errors are smaller than the size of the symbols. The five functions coincide within the experimental errors, except that a few points in $[-3, 0]$ are somewhat lower than the others. This is caused by very low multiplicities in the rapidity region $[-3, -2]$, where unidentified protons contribute and where the rapidity distribution is not completely symmetric to the region $[2, 3]$. The figure demonstrates that, despite a strong rapidity dependence of the particle density given in Fig. 2, the balance function is largely independent of the position of the rapidity window, i.e., the charge correlation is essentially the same in any longitudinally-Lorentz-transformed frame.

Since boost invariance of the BF is found to be valid over the whole rapidity region, it is now interesting to verify if the BF in a limited rapidity window can be deduced from that in the full rapidity region by Eq. (1), and vice versa. In Fig. 3, the balance function, $B(\delta y|Y_w)$ (solid points), for four rapidity windows (central in Fig. 3a, non-central in Fig. 3b), is compared to $B(\delta y|\infty)(1 - \frac{\delta y}{Y_w})$ (open points) obtained for the corresponding window from the BF in the full region. The data confirm that the relation Eq. (1) is indeed approximately satisfied, independently of size or position of the window. This result is especially useful for experiments with limited acceptance, in particular for the current heavy ion experiments.

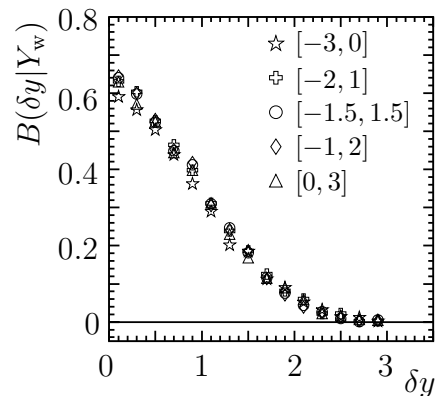Fig. 3 further illustrates that the BF becomes narrower with decreasing Y_w, in agreement with Eq. (1).

FIG. 1: The balance function for five different positions of a rapidity window of size $Y_w = 3$: $[-3, 0]$ (open stars), $[-2, 1]$ (open crosses), $[-1.5, 1.5]$ (open circles), $[-1, 2]$ (open diamonds) and $[0, 3]$ (open triangles).
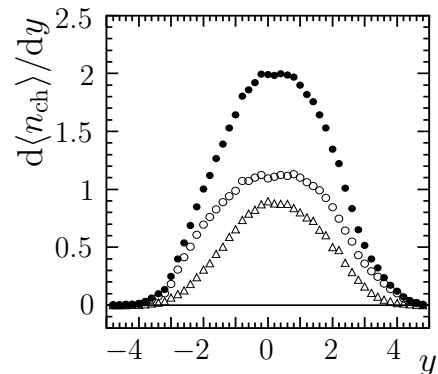

FIG. 2: Center-of-mass rapidity distribution of positively (open circles), negatively (open triangle), and all charged (solid circles) particles.
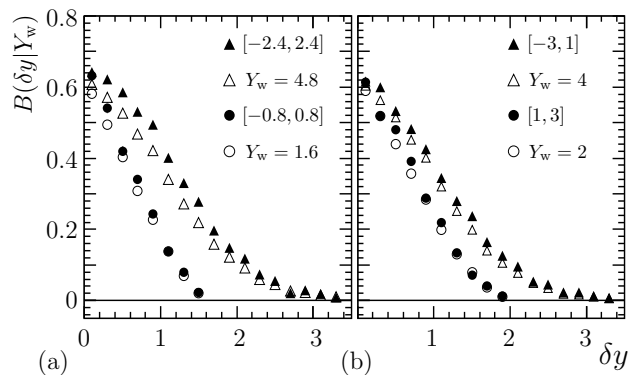

FIG. 3: The balance functions $B(\delta y|Y_w)$ (solid symbols) (a) for two central rapidity windows, $[-2.4, 2.4]$ (triangles) and $[-0.8, 0.8]$ (circles) and (b) two asymmetric rapidity windows $[-3, 1]$ (triangles), and $[1, 3]$ (circles), compared with corresponding $B(\delta y|\infty) \cdot (1 - \frac{\delta y}{Y_w})$ (open symbols).
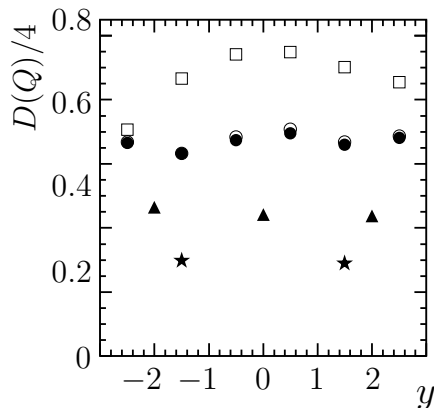
FIG. 4: $D(Q)/4$ versus the position of a rapidity window of size $Y_{\rm w} = 1$ (circles), 2 (triangles), and 3 (stars). Open circles and open squares are $D(Q)/4$ under the same transverse momentum and azimuthal angle cuts as STAR ($p_{\rm t} > 0.1$ GeV/c) and PHENIX ($p_{\rm t} > 0.2$ GeV/c and $\Delta\phi = \pi/2$) with a rapidity window of size $Y_{\rm w} = 1.0$.

Since the charge fluctuation $D(Q)$ [5] is approximately related to the BF by

$$\frac{D(Q)}{4} = 1 - \int_0^{Y_{\rm w}} B(\delta y | Y_{\rm w}){\rm d}\delta y + \mathcal{O}\left(\frac{\langle Q \rangle}{\langle n_{\rm ch}\rangle}\right), \quad (3)$$

where $Q = n_+ - n_-$ and $n_{\rm ch} = n_+ + n_-$, it is interesting to see how the charge fluctuation changes with position and size of the rapidity window. For this purpose, $D(Q)/4$ is presented in Fig. 4 for different positions and sizes of a rapidity window, $Y_{\rm w} = 1.0$ (circles), 2.0 (triangles), and 3.0 (stars). The results confirm that for a given window size its value is independent of the position of that window [27], in agreement with the boost invariance of the balance function. The data also show that $D(Q)$ is sensitive to the size of the observed window. So it is necessary to give the exact size of the rapidity region when the fluctuation is treated quantitatively [6].

As has been demonstrated in [27], $D(Q)$ also depends on the acceptance in transverse momentum and azimuthal angle. In Fig. 4, $D(Q)/4$ is also presented under the same transverse momentum and azimuthal angle cuts as used in STAR ($p_{\rm t} > 0.1$ GeV/c) and PHENIX ($p_{\rm t} > 0.2$ GeV/c and $\Delta\phi = \pi/2$) with a rapidity window of size $Y_{\rm w} = 1.0$, by open circles and open squares, respectively. The transverse momentum cut used by STAR has little influence on the result, while the combined transverse momentum and azimuthal cut used by PHENIX destroys the boost invariance of $D(Q)$. These results show that a limited acceptance in transverse momentum and azimuthal angle can destroy the boost invariance of charge fluctuations. Furthermore, it has been verified that the effect is the larger the larger the percentage of particles lost.

In Fig. 5, the full-rapidity BF, $B(\delta y | \infty)$, is presented for all charged particles (full circles) and for the three multiplicity intervals, $0 < n_{\rm ch} < 6$ (open circles), $6 \leq$

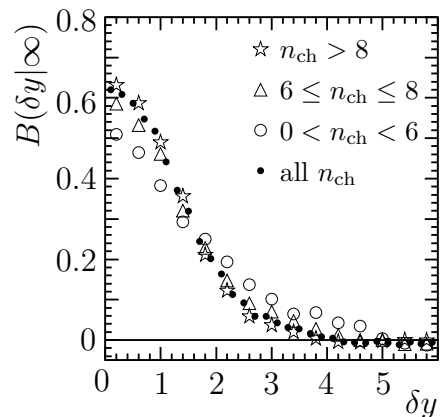FIG. 5: The balance function for all charged particles (full circles) and for three multiplicity intervals, $0 < n_{\rm ch} < 6$ (open circles), $6 \leq n_{\rm ch} \leq 8$ (open triangles) and $n_{\rm ch} > 8$ (open stars).

TABLE I: The width of the balance function in three multiplicity intervals and for all charged particles

Multiplicity	$\langle \delta y \rangle$
$n_{\rm ch} > 8$	0.957 ± 0.011
$6 \leq n_{\rm ch} \leq 8$	1.096 ± 0.014
$0 < n_{\rm ch} < 6$	1.359 ± 0.026
all $n_{\rm ch}$	0.991 ± 0.008

$n_{\rm ch} \leq 8$ (open triangles), and $n_{\rm ch} > 8$ (open stars). The width of the balance function, defined as

$$\langle \delta y \rangle = \frac{\sum_i B(\delta y_i | \infty)\delta y_i}{\sum_i B(\delta y_i | \infty)}, \quad (4)$$

for the corresponding multiplicity intervals and for all charged particles is listed in Table I. The width decreases with increasing multiplicity. This is, at least qualitatively, consistent with the narrowing of the balance function with increasing centrality observed in current heavy ion experiments [7, 8]. So, before a narrowing of the BF with increasing centrality and increasing mass number of the colliding nuclei can be interpreted as due to the formation of a QGP, the multiplicity effect observed here should be properly accounted for.

The results of this paper can be summarized as follows:

1. In contrast to the strong dependence of the particle density on rapidity, the BF is invariant under a longitudinal boost over the whole rapidity region. This property allows to determine the BF in full rapidity, $B(\delta y | \infty)$, from a measurement with limited rapidity acceptance.

2. The balance function becomes narrower with decreasing size of the window. Therefore, only the full-rapidity BF can be used in comparing data from different experiments.

3. The balance function becomes narrower with increasing multiplicity, an effect also observed in heavy ion interactions when the centrality of the collision increases.

4. The charge fluctuations are boost invariant but depend on the size of the rapidity window.

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