Energy Dependence of the Cronin Effect from Non-Linear QCD Evolution*

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The non-linear evolution of dense partonic systems has been suggested as one of the novel physics mechanisms relevant to the dynamics of hadron–nucleus and nucleus–nucleus collisions at collider energies. Here we study to what extent the description of Cronin enhancement in the framework of this non-linear evolution is consistent with the recent observation in $\sqrt{s} = 200$ GeV d–Au collisions at the Relativistic Heavy Ion Collider. We solve the Balitsky-Kovchegov (BK) evolution equation numerically for several initial conditions encoding Cronin enhancement. We find that the properly normalized nuclear gluon distribution is suppressed at all momenta relative to that of a single nucleon. Calculating the resulting spectrum of produced gluons in p–A and A–A collisions, we establish that the nonlinear QCD evolution is unable to generate a Cronin type enhancement, and that it quickly erases any such enhancement which may be present at lower energies.

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The observation that the ratio of particle yields in p–A and A–A, scaled by the number of collisions, exceeds unity in an intermediate transverse momentum range of a few GeV, is commonly referred to as the Cronin effect. This was first seen at lower fixed target energies [1] and was recently confirmed in $\sqrt{s} = 200$ GeV d–Au collisions at RHIC [2]. The current interest focuses mainly on comparing this Cronin *enhancement* in d–Au to the relative *suppression* of produced hadrons in Au–Au collisions at the same center of mass energy and in the same transverse momentum range [3]. The opposite trend of the two effects and their centrality dependence suggests that d–Au data may serve as an efficient benchmark measurement to distinguish between the two different physical mechanisms suggested for the relative suppression of hadron spectra in Au–Au collisions: initial state parton saturation[4] and final state jet quenching [5].

The physics of dense partonic systems and their nonlinear perturbative evolution to higher energy has motivated several attempts at understanding bulk properties of ultra-relativistic heavy ion collisions such as the multiplicity, rapidity distribution and centrality dependence of particle production [4, 6]. In particular it has been suggested that saturation effects can account for the suppression of the high-p_T hadronic spectra in Au–Au collisions at RHIC. On the other hand, it is known that saturation models based on multiple scattering (the so called Glauber-Mueller [7] or McLerran-Venugopalan [8] models) exhibit Cronin enhancement in p–A [9, 10, 11] and A–A [10, 12]. In these models, the saturation of low p_T gluons is the result of a redistribution of gluons in transverse phase space [13, 14] which does not change the total number of gluons, thus resulting in a compensating enhancement at momenta just above the saturation momentum Q_s. What is not fully understood is i) whether such Cronin enhancement encoded in the initial condition of a nuclear wave function persists in the non-linear perturbative QCD evolution to higher energy and ii) whether such Cronin enhancement can be generated by the non-linear evolution itself. This paper goes beyond earlier discussions [10, 11, 12] by providing the first complete (numerical) answer to these questions. We do not address other approaches to Cronin enhancement [15].

We start from the Balitsky-Kovchegov (BK) evolution equation [16, 17], which describes the evolution of the forward scattering amplitude $N(\mathbf{r}, y)$ of a QCD dipole of transverse size $|\mathbf{r}|$ with rapidity Y and $y = (\alpha_s\, N_c/\pi)\, Y$,

$$\frac{dN(|\mathbf{r}|, y)}{dy} = \frac{1}{2\pi} \int d^2\mathbf{z}\, \frac{(\mathbf{r}-\mathbf{z})\cdot\mathbf{z}}{(\mathbf{r}-\mathbf{z})^2\, \mathbf{z}^2} \times [N(|\mathbf{r}-\mathbf{z}|) + N(|\mathbf{z}|) - N(|\mathbf{r}|) - N(|\mathbf{r}-\mathbf{z}|)N(|\mathbf{z}|)]\,. \qquad (1)$$

The unintegrated gluon distribution is related to the inclusive gluon distribution $\phi(k) \propto \frac{d(xG(x,k^2))}{d^2k\, d^2b}$ and is given in terms of the dipole amplitude

$$\phi(k) = \int \frac{d^2r}{2\pi\, r^2} \exp\{i\,\mathbf{r}\cdot\mathbf{k}\} N(r)\,. \qquad (2)$$

In the following, we also use the modified gluon distribution

$$h(k) = k^2\, \nabla_k^2\, \phi(k)\,. \qquad (3)$$

The two definitions coincide for the leading order perturbative distribution $\phi(k) \propto \frac{1}{k^2}$, but are different in general, and especially at low momenta.

Using the second order Runge-Kutta algorithm [18], we solve the BK equation (1) numerically with 8000 equally spaced intervals in $\ln k$-space between -15 and 35 and a step $\Delta\, y = 0.0025$. The accuracy of this algorithm is better than 2 % in the entire range of k discussed below. We evolve two initial conditions given by the McLerran-Venugopalan [8] (MV) and Golec-Biernat–Wüsthoff [19]

*We dedicate this work to the memory of Ian Kogan.

(GBW) model respectively:

$$N_{MV}^{Q_s} \;=\; 1-\exp\left[-\frac{Q_s^2 r^2}{4}\ln\left(\frac{1}{r^2\Lambda_{QCD}^2}+e\right)\right]\,, \quad (4)$$

$$N_{GBW}^{Q_s} \;=\; 1-\exp\left[-\frac{Q_s^2 r^2}{4}\right]\,, \quad (5)$$

where $\Lambda_{\rm QCD} = 0.2$ GeV. For momenta $k \geq O(1$ GeV$)$, the sensitivity on the infrared cut off e is negligible. The amplitudes N_{MV} and N_{GBW} are similar for momenta of order Q_s, but differ strongly in their high k behavior; $\phi_{GBW}(k)$ decays exponentially while ϕ_{MV} has a power-like tail $1/k^2$.

Fig.1 shows the evolution of $h(k,y)$ and $\phi(k,y)$ for different initial conditions. The solutions for $h(k,y)$ quickly approach a universal soliton-like shape and do not change further except uniformly moving in k on the logarithmic plot. The position of the maximum is the evolved value of the saturation momentum $Q_s(y)$. The solutions for different initial conditions and different rapidities scale as a function of the scaling variable $\rho = k/Q_s(y)$. The shape of the initial condition affects only the value of the saturation momentum $Q_s(y)$, but not the shape of the evolved function $h(\rho,y)$. The y-dependence of $h(\rho,y)$ is very weak: the function evolves fast towards a scaling form $h(\rho)$. As the rapidity changes between $y=4$ and $y=10$, the ratio $h(\rho,y_1)/h(\rho,y_2)$ varies by at most 40% over three orders of magnitude of the scaling variable ρ. Similar behavior was found for ϕ (results not shown). This is consistent with previous numerical works [20, 21].

To get a quantitative idea of the behavior of the scaling solution, we fitted the numerical solution of $\phi(\rho)$ to two analytical expressions: $s_1(\rho) = a\rho^{2(1-\lambda)}$ and $s_2(\rho) = a\ln(b\rho)\rho^{2(1-\lambda)}$ for $\rho > 5$. The functional form s_1 with $\lambda = 0.37$ and $\ln Q_s \propto y$ describes the scaling behavior of solutions of the linear BFKL equation [22]. It was argued in Ref. [23] that s_2 with the same value of λ and $\ln Q_s \propto \frac{2\chi(\lambda)}{1-\lambda}y - \frac{3}{2(1-\lambda)}\ln y$ accounts for the effects of nonlinearities in Eq. (1). We find that s_1 does not give an acceptable fit to $\phi(\rho)$ in any extended range of ρ. For values of ρ between 1 and 10^3 the value of λ varies between 0.39 and 0.46. This is in contrast to the BFKL equation, where we find numerically that s_1 with $\lambda = 0.37$ does indeed approximate the solution over several orders of magnitude with very good accuracy (results not shown). On the other hand, for $5 < \rho < 1000$, s_2 gives a good fit with $\lambda = 0.32$. If, following [23] we fix $\lambda = 0.37$, the fit is still good.

To study the effect of the evolution on the Cronin enhancement, we consider two initial conditions, $N_{MV}^q(r)$ and $N_{MV}^Q(r)$ with $q^2 = 0.1\,\mathrm{GeV}^2$ and $Q^2 = 2\,\mathrm{GeV}^2$. Since $q \sim \Lambda_{QCD}$ and Q is of order of the estimated saturation momentum for a gold nucleus [6], this choice mimics the gluon distributions of a proton and of a nucleus respectively. At large transverse momenta the ratio

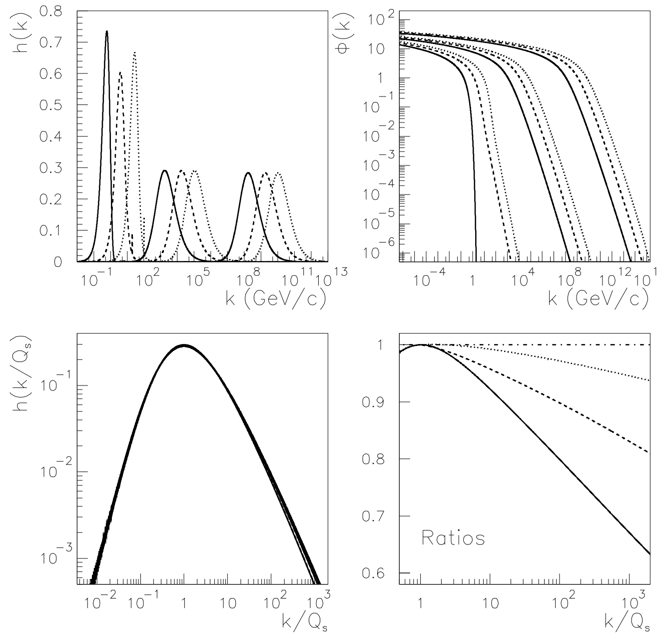

FIG. 1: Solutions of the BK equation. *Upper-left*: $h(k)$ evolved (left to right) from $y=0$ to 5 and 10 for different initial conditions: GBW with $Q_s^2 = 0.36$ GeV^2 (solid lines), MV with $Q_s^2 = 4$ GeV^2 (dashed lines) and MV with $Q_s^2 = 100$ GeV^2 (dotted lines). *Upper-right*: The same as upper left for $\phi(k)$. *Lower-left*: the scaled function $h(\rho)$ versus $\rho = k/Q_s$ for $y = 4, 6, 8, 10$ and the same initial conditions and conventions (lines cannot be distinguished). *Lower-right*: Ratio of $h(y,\rho)/h(y,\rho=1)$ over $h(y=10,\rho)/h(y=10,\rho=1)$ for $y=4$ (solid line), 6 (dashed line), 8 (dotted line) and 10 (dashed-dotted line), and initial condition MV with $Q_s^2 = 4$ GeV^2.

of the corresponding Fourier transforms is given by the ratio of the saturation momenta,

$$\frac{h^Q(k,y=0)}{h^q(k,y=0)} = \frac{Q^2}{q^2} = A^{1/3}\,. \quad (6)$$

This relation also holds for ϕ. As discussed in [10, 11], these initial conditions exhibit Cronin enhancement, namely $\frac{h^Q(k,y=0)}{A^{1/3}h^q(k,y=0)} > 1$ for $k \sim Q$. We solve the BK equation with these two initial conditions and construct the ratio $R(k,y) = h^Q(k,y)/A^{1/3}h^q(k,y)$ and the corresponding ratio for ϕ, see Fig. 2. The initial Cronin enhancement at rapidity $y=0$ is seen to be wiped out very quickly by the evolution. Within less than half a unit of rapidity y the ratios show uniform suppression for all values of transverse momentum. The observed behaviour persists if different amounts of Cronin enhancement are included in the initial condition.

As seen in the lower panel of Fig. 2, the Cronin enhancement also disappears rapidly with rapidity when gluon distributions are evolved according to the linear BFKL equation. Qualitative differences between the BFKL and BK dynamics are only visible at momenta

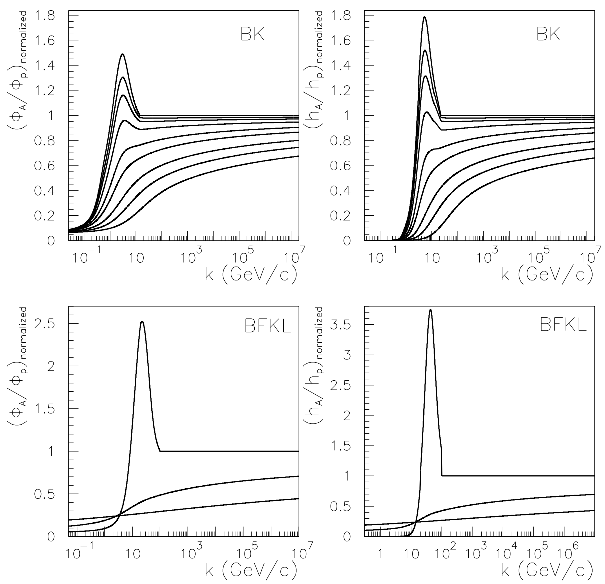

FIG. 2: Ratio of distributions ϕ and h in nucleus and proton, normalized to 1 at $k \to \infty$. *Upper plots*: BK evolution, with MV as initial condition with $Q_s^2 = 0.1$ GeV^2 for p and 2 GeV^2 for A. Lines from top to bottom correspond to $y = 0$, 0.05, 0.1, 0.2, 0.4, 0.6, 1, 1.4 and 2. *Lower plots*: BFKL evolution, with MV as initial condition with $Q_s^2 = 4$ GeV^2 for p and 100 GeV^2 for A. Lines from top to bottom correspond to $y = 0$, 1 and 4.

$k < Q_s$, where saturation effects are important. For larger momenta k, the ratios are very similar for linear and non-linear QCD evolution. We thus conclude that the wiping out of the initial enhancement is primarily driven by the linear BFKL dynamics which is contained in the BK equation as well.

For the evolved gluon distributions determined above, we have calculated the yield of produced gluons in p–A and A–A collisions at central rapidity according to the factorized expressions [24]

$$\frac{dN_{pA}}{dyd^2p\,d^2b} \;\propto\; \frac{1}{p^2}\int d^2k\, h^q(y,k)\, h^Q(y,p-k)\,, \qquad (7)$$

$$\frac{dN_{AA}}{dy\,d^2p\,d^2b} \;\propto\; \frac{A^{2/3}}{p^2}\int d^2k\, h^Q(y,k)\, h^Q(y,p-k)\,. \qquad (8)$$

From these spectra, we calculate the p- and y-dependent ratios

$$R_{pA} \;=\; \frac{\frac{dN_{pA}}{dyd^2p\,d^2b}}{A^{1/3}\frac{dN_{pp}}{dyd^2p\,d^2b}}\,, \qquad R_{AA} = \frac{\frac{dN_{AA}}{dyd^2p\,d^2b}}{A^{4/3}\frac{dN_{pp}}{dyd^2p\,d^2b}}\,.$$

As seen in Fig. 3, the non-linear BK evolution quickly wipes out any initial Cronin enhancement not only on the level of single parton distribution functions but also on the level of particle spectra. We have checked that this behaviour is generic by evolving different initial conditions corresponding to different initial amounts of enhancement. We note that in calculations of the gluon production in p–A in the eikonal approximation [13, 25, 26], the gluon distribution h rather than ϕ enters the right hand side of (7), but no similar statement exists of nucleus-nucleus collisions. We have checked that the results using ϕ are very close to those shown in Fig. 3. More generally, the expressions (7) and (8) are based on rather strong approximations discussed in Ref. [10]. However, our conclusion about the disappearance of Cronin enhancement during QCD evolution is likely to persist in more refined ways of calculating particle spectra, since it is rooted directly in the rapidity dependence of gluon distributions.

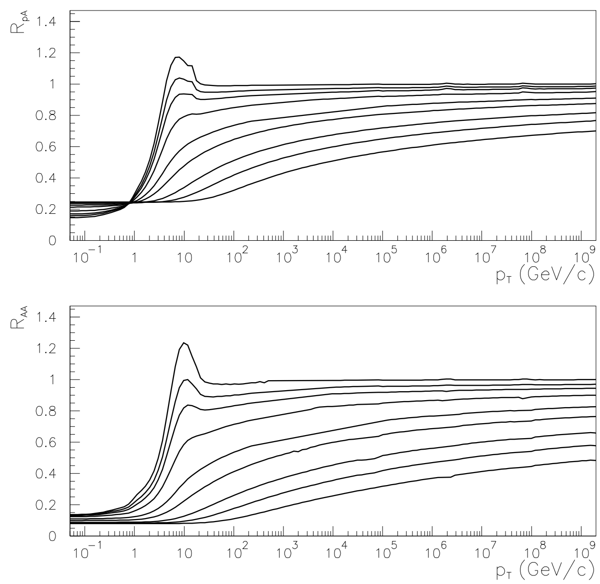

FIG. 3: Ratios R_{pA} and R_{AA} of gluon yields in p–A (upper plot) and A–A (lower plot) for BK evolution, with MV as initial condition with $Q_s^2 = 0.1$ GeV^2 for p and 2 GeV^2 for A. Lines from top to bottom correspond to $y = 0$, 0.05, 0.1, 0.2, 0.4, 0.6, 1, 1.4 and 2.

We now comment on a recent formal argument [11] which - in contrast to our numerical findings - suggests that Cronin enhancement survives the non-linear evolution. It is based on the observation that at very short distances $r \to 0$, the dipole amplitude $N(r)$ is not affected by evolution. Thus, the integral of the gluon distribution function ϕ over the transverse momentum is expected to be rapidity independent,

$$\int d^2k\,\phi(k) = \frac{1}{r^2}\,N(r)|_{r=0}\,. \qquad (9)$$

One thus obtains the sum rule

$$\int d^2k\,\phi_A(k,y) = A^{1/3}\int d^2k\,\phi_p(k,y) \qquad (10)$$

valid for any rapidity, since it is satisfied by the initial condition ϕ_{MV}. Since the nonlinear evolution leads to the depletion of the gluon distribution $\phi_A(k)$ relative to $A^{1/3}\phi_p(k)$ at low momenta, it must follow that in some range of momenta this effect is compensated by enhancement of ϕ_A. Similar arguments can be made about the distribution h and also about the gluon yield in p–A and A–A collisions.

However, this argument breaks down since the quantity defined in Eq. (9) is infinite. As such Eq. (10) relates only the divergent parts of the integrals which are dominated by ultraviolet, and carries no information about possible behaviour at finite momentum. To be more specific, we use the scaling property $\phi(k, y) = \phi(k/Q_s(y))$ of the solution of the BK equation established above. It is known that the ratio of the saturation momenta for any two solutions is preserved by the BK evolution [20, 21, 22]. For our two solutions representing a nucleus and a nucleon, this implies $\frac{Q_s^A(y)}{Q_s^p(y)} = A^{1/6}$. We now rewrite the sum rule (10) by regulating the divergent integrals with a large but finite UV cutoff $aQ_s^A(y)$,

$$\int_0^{a^2(Q_s^A)^2} d^2k\phi_A(k, y) = (Q_s^A)^2 \int_0^{a^2} d^2\rho\phi(\rho) \tag{11}$$
$$= A^{1/3}(Q_s^p)^2 \int_0^{a^2} d^2\rho\phi(\rho) = A^{1/3} \int_0^{a^2(Q_s^p)^2} d^2k\phi_p(k, y)\,.$$

In the formal limit $a \to \infty$, we recover Eq. (10). However, since $Q_s^A \gg Q_s^p$, the regularized sum rule (11) is easily satisfied even if the nuclear distribution is suppressed relative to that of a single nucleon uniformly at all momenta. Thus, the sum rule (10) carries no information about either presence or absence of the Cronin enhancement.

In summary, we have found that the nonlinear QCD evolution to high energy is very efficient in erasing any Cronin type enhancement which may be present in the initial conditions. For realistic initial conditions this disappearance occurs within half a unit of rapidity. We note that in our units the evolution from 130 GeV to 200 GeV corresponds to $\delta y \simeq 0.1$ for $\alpha_s = 0.2$, and thus is not sufficient to completely eliminate an initial enhancement at central rapidity. For forward rapidity, δy is greater. The evolution to the LHC energy corresponds to $\delta y \sim 1$. Thus the BK evolution suggests the reduction of the Cronin effect in d–Au for forward rapidities at RHIC and it predicts the disappearance of the Cronin effect for p–A collisions at LHC.

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