# On Removing the Wrong Jet Combinations in $2\times2$ -Body Final States

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#### Abstract

Several "2×2"-body final states are expected to play an important role at LEP II. They come from  $e^+e^- \to XY$  processes where X and Y decay into fermion pairs, leading dominantly to 4-jet configurations. The XY pair can be  $W^+W^-$ , HZ, hA,  $H^+H^-$ , and ZZ for instance. A new way of removing wrong jet combinations is demonstrated.

#### 1 Introduction

The main goals of LEP II are to measure the W mass and study its production in pairs, and to investigate the Higgs sector. The common feature of all these processes is that they give rise to two heavy bosons (of the order of a 100 GeV/ $c^2$  mass) decaying into much lighter fermions. The most frequent case is that these fermions are quarks and turn into jets, leading to a four-jet configuration. In the analyses projected at LEP II the events are usually forced into four jets. We will assume here that this has been done and will work at the four-parton level. What we will say in the following will concern various processes such as  $e^+e^- \to h^0A^0$ ,  $e^+e^- \to W^+W^-$ , and others. The two produced bosons will be noted X and Y in the following.

The study of all these processes is hampered by a background of combinatorial origin. For each four-jet event there are 3 possible pairings of jets. We propose a method to choose the right one, most effective in the cases where the masses of the two produced particles are close to each other.

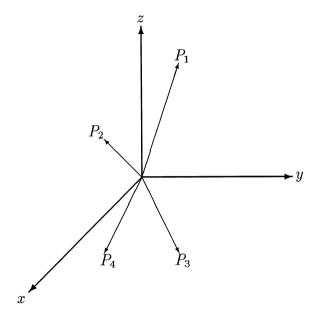
## 2 The $2\times 2$ -Body configuration

Such a configuration is defined by 4 four-vectors, that is, 16 variables. If we neglect jet masses, there are 4 relations ( $E_i = p_i, i = 1, 4$ ). The energy-momentum conservation leads to 4 relations. Two angles correspond to a general rotation of the reference frame around the beam axis and around the XY production axis. They are thus inessential. Two relations are imposed by the boson masses  $M_X$  and  $M_Y$ . There thus remains 4 relevant angles, which can be chosen to be the production angle  $\theta_p$  of the XY system, the angle  $\phi$  between the two decay planes of X and Y, and the two decay angles in these planes,  $\theta^*$  and  $\theta'^*$ . The distributions of these angles reflect the dynamics of the process, basically the spin of intermediate particles and the structure of the couplings. They will be investigated in detail in a forthcoming note, where it will be shown how the angular distributions can help disentangling the various processes. We concentrate here on the correlation between the two decay angles, and on their usage in removing the wrong combinations.

## 3 Decay Angles in the Case of Wrong Combinations

## 3.1 Analytical Calculation in the Case $M_X = M_Y$

Here we consider two mothers X and Y, where  $X \to 1+2$  and  $Y \to 3+4$ . Let us first study the simple case where X and Y have the same mass M and thus the same boost  $\beta = \sqrt{1-4M^2/s}$ . X and Y both decay in two massless particles of energy E = M/2 in their respective mother rest frame. We note  $\theta^*$  and  $\theta'^*$  the corresponding decay angles.



The momenta in the center of mass of each mother can be written as follows:

$$P_{1}^{*} = E \qquad \begin{pmatrix} 1 \\ \sin \theta^{*} \\ 0 \\ \cos \theta^{*} \end{pmatrix} \qquad P_{2}^{*} = E \qquad \begin{pmatrix} 1 \\ -\sin \theta^{*} \\ 0 \\ -\cos \theta^{*} \end{pmatrix}$$

$$P_{3}^{*} = E \qquad \begin{pmatrix} 1 \\ -\sin \theta'^{*} \cos \phi \\ -\sin \theta'^{*} \sin \phi \\ -\cos \theta'^{*} \end{pmatrix} \qquad P_{4}^{*} = E \qquad \begin{pmatrix} 1 \\ \sin \theta'^{*} \cos \phi \\ \sin \theta'^{*} \sin \phi \\ \cos \theta'^{*} \end{pmatrix}$$

$$(1)$$

The corresponding Lorentz transformations read:

$$\mathbf{\Gamma} = \mathbf{\Gamma}(\beta) = \begin{pmatrix} \gamma & 0 & 0 & \gamma \beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma \beta & 0 & 0 & \gamma \end{pmatrix} \text{ and } \mathbf{\Gamma'} = \mathbf{\Gamma}(-\beta)$$
 (2)

Then the four-momenta in the laboratory frame are given by:

$$P_{1,2} = \mathbf{\Gamma} P_{1,2}^* \text{ and } P_{3,4} = \mathbf{\Gamma}' P_{3,4}^*$$
 (3)

Let  $P_a = P_1 + P_3$  and  $P_b = P_2 + P_4$  be the momenta of the reconstructed mothers from a wrong combination. The (wrong) decay angle is defined by:

$$\cos \theta_{a,1}^* = \frac{\cos \theta_{a,1} - \beta_a}{1 - \beta_a \cos \theta_{a,1}} \tag{4}$$

where:

$$\cos \theta_{a,1} = \frac{\vec{P_1} \cdot \vec{P_a}}{\parallel \vec{P_1} \parallel \cdot \parallel \vec{P_a} \parallel}$$

$$= \frac{\gamma^2 (\beta + \cos \theta^*) (\cos \theta^* - \cos \theta'^*) + \sin \theta^* (\sin \theta^* - \sin \theta'^* \cos \phi)}{\gamma (1 + \beta \cos \theta^*) \sqrt{\gamma^2 (\cos \theta^* - \cos \theta'^*)^2 + (\sin \theta^* - \sin \theta'^* \cos \phi)^2 + (\sin \theta'^* \sin \phi)^2}}$$
(5)

and

$$\beta_a = \frac{\parallel \vec{P_a} \parallel}{E_a}$$

$$= \frac{\sqrt{\gamma^2 (\cos \theta^* - \cos \theta'^*)^2 + (\sin \theta^* - \sin \theta'^* \cos \phi)^2 + (\sin \theta'^* \sin \phi)^2}}{\gamma (2 + \beta (\cos \theta^* + \cos \theta'^*))}$$
(6)

Thus we obtain

$$\cos \theta_{a,1}^* = \frac{\beta(\cos \theta^* - \cos \theta'^*)}{\sqrt{\gamma^2(\cos \theta^* - \cos \theta'^*)^2 + (\sin \theta^* - \sin \theta'^* \cos \phi)^2 + (\sin \theta'^* \sin \phi)^2}}$$
(7)

If one performs the corresponding calculation to obtain  $\cos \theta_{b,2}^*$  one finds the same expressions with only  $\beta$  changed to  $-\beta$ . Thus we obtain:

$$\cos \theta_{a,1}^* = -\cos \theta_{b,2}^* \tag{8}$$

It is worth noticing that  $\theta^*$  is defined modulo  $\pi$ , so that only  $|\cos \theta^*|$  is meaningful, so:

$$\cos \theta_a^* = \pm \cos \theta_b^* \tag{9}$$

One also notes from the formula (7) that, in the wrong combinations,  $\theta^*$  is bounded:

$$-\beta \le \cos \theta_{a,b}^* \le \beta \tag{10}$$

This effect is purely kinematical. It is due to the equality of the masses. In the case of different masses there is still a correlation, but it is less strong, as shown in the next section.

### 3.2 Monte-Carlo Study, General Case

We wrote a simple Monte-Carlo program to simulate the effect described in the previous section. It illustrates relations (9) and (10). We generate two scalars X and Y, decaying isotropically into two jets. The production angle  $\theta_p$  of the XY pair follows a  $\sin^2 \theta_p$ 

distribution. Then the decay angles are evaluated for the right combination (1+2,3+4) and for the two wrong combinations (1+3,2+4) and (1+4,2+3). The  $e^+e^-$  center-of-mass energy is set to 176 GeV in all cases. By construction, the right combination always exhibits a uniform distribution in the  $(\cos \theta^*, \cos \theta'^*)$  plane (Fig. 1). For the wrong combinations, however, in the case  $M_X=M_Y=80$  GeV (Fig. 2), the correlation and the limitation predicted by Eqs. (9) and (10) are manifest. If one goes to slightly different masses of X and Y (78 and 82 GeV) the correlation is still clearly visible (Fig. 3). In the case of more different masses (60 and 100 GeV, Fig. 4, or 70 and 100 GeV, Fig. 5), the correlation between the two  $\cos \theta^*$ 's can still be used, though less efficiently, to reject wrong combinations. This two-dimensionnal distribution clearly contains a large sensitivity to the  $h^0$  and  $A^0$  masses in the case of  $e^+e^- \rightarrow h^0A^0$ .

The same distributions (Fig.6, right combinations and Fig. 7, wrong combinations) have been produced using the PYTHIA generator for  $e^+e^- \to W^+W^-[1]$ . It shows that the effect of the finite W width (or, equivalently, mass resolution) is to slightly spread the correlation pattern with respect to that of Fig. 2., but that a good discrimination between right and wrong jet pairings is preserved.

#### 4 Conclusion

The principle of a new method has been demonstrated in order to reject the wrong jet pairings in the case of  $2 \times 2$ -body production. It is based on the correlation between the calculated decay angles of the two two-body systems. The effect of a finite width has been shown using Monte-Carlo generations. A realistic detailed simulation of experimental conditions still needs to be carried out. It could be used in several  $2 \times 2$ -body reactions, as a number of examples will be under study at LEP II.

#### References

[1] H.-U. Bengtsson and T. Sjöstrand, Computer Physics Commun. 46 (1987) 43.

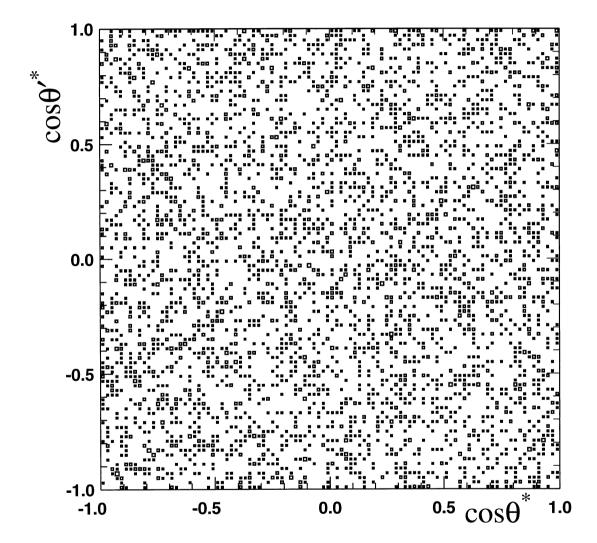


Figure 1: Decay angle in the center of mass of two jets in the case of the production of two spinless particles,  $80 \text{ GeV}/c^2$  each, each decaying into two quarks, versus the decay angle for the other two jets, for the right pairing of the four jets.

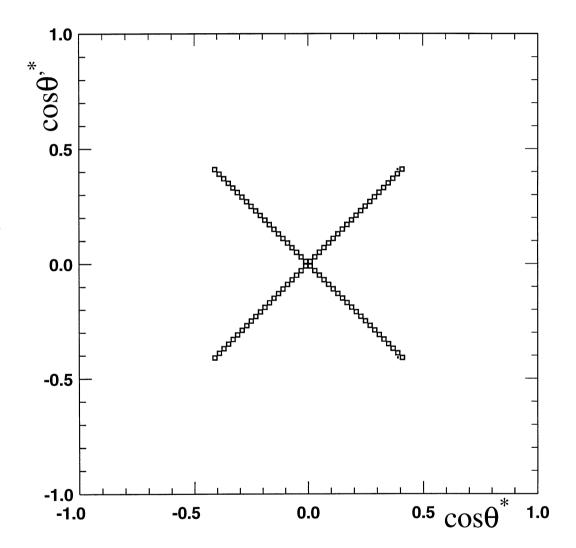


Figure 2: Same as Fig. 1, for the wrong pairings of the four jets.

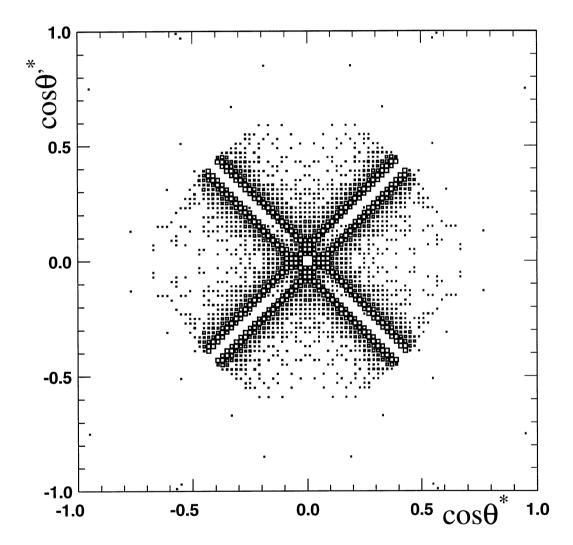


Figure 3: Same as Fig. 2, but the masses of the two spinless particles are now 78 and 82  ${\rm GeV}/c^2.$ 

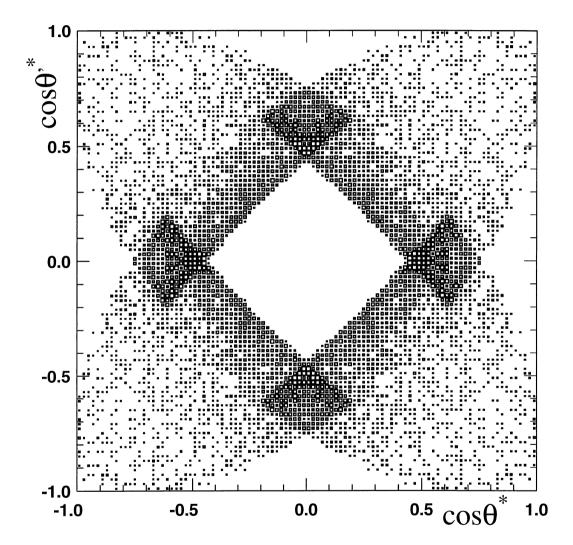


Figure 4: Same as Fig. 2, but the masses of the two spinless particles are here 60 and 100  ${\rm GeV}/c^2$ .

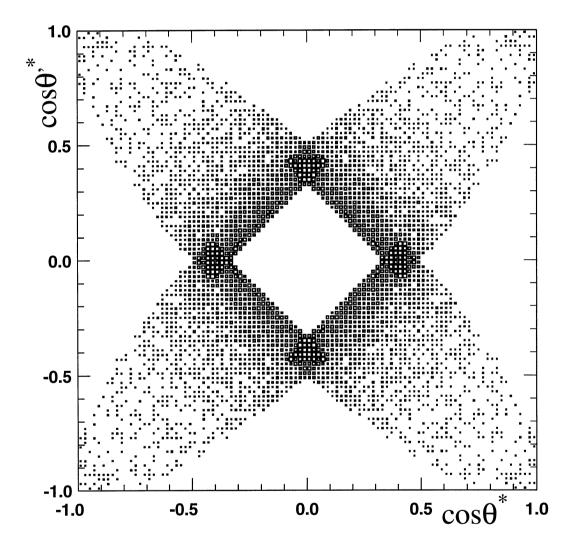


Figure 5: Same as Fig. 2, but the masses of the two spinless particles are here 70 and 100  ${\rm GeV}/c^2$ .

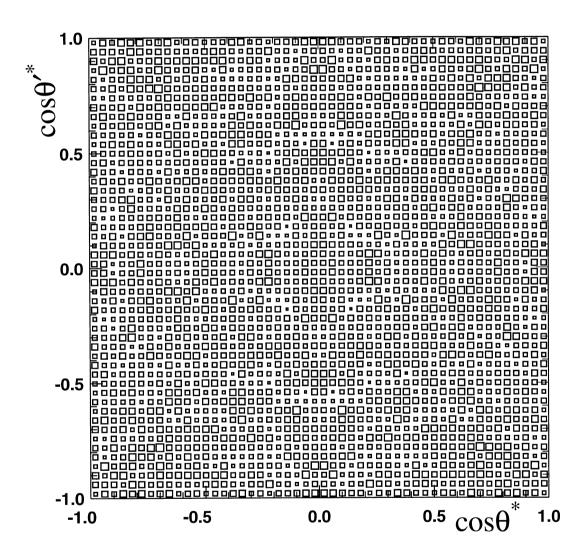


Figure 6: Decay angle in the center of mass of two jets in the case of WW production, versus the decay angle for the other two jets, for the right pairing of the four jets.

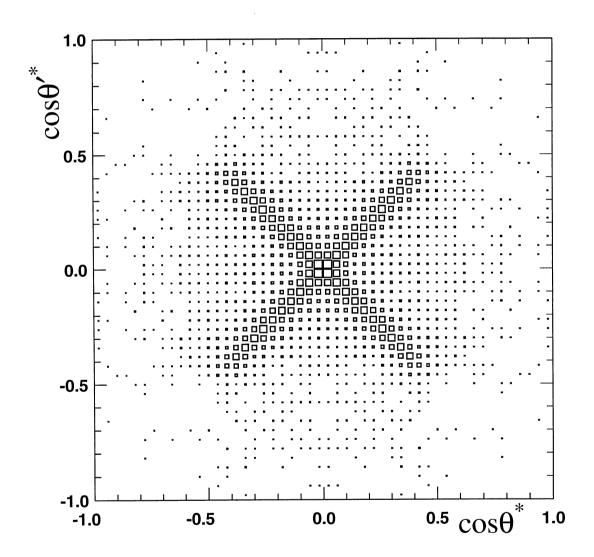


Figure 7: Same as figure 6, but for the wrong pairings of the four jets.