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Measurement of $\tau \rightarrow 3\pi\pi^0$ branching ratio and study of the ω production

Pierre BOURDON

LPNHE Ecole Polytechnique, IN2P3-CNRS

Using Aleph data until 1992 (about fifty thousand tau pairs), the branching ratio of the τ into $3\pi\pi^0$ is measured to be: $(4.25 \pm .13 \pm .13\%)$. The proportion of the decay width proceeding from an intermediate meson ω is estimated to be: $(43.8 \pm 1.9^{+2.5}_{-2.6}\%)$ from a fit of the $\pi^+\pi^-\pi^0$ invariant mass spectrum. The study of the angle between the ω decay plane and the remaining charged pion shows no evidence for a deviation from the standard 1^{st}-class hadronic current.

1 Theoretical and Experimental Overview

- From the theoretical point of view, the exact structure of the current generating the $3\pi\pi^0$ final state is still unknown. It is believed although that this channel incorporates an intermediate $\omega - \pi$ state and ρ resonance(s) in the two pion systems. The Monte-Carlo for τ decays library TAUOLA uses an hadronic current built as a sum of two currents. One gives rise to an $\omega - \pi$ state with the ω subsequently decaying into $\pi^+\pi^-\pi^0$ with a 89% branching ratio. It is completely determined if one assumes first class currents only . The other part as suggested in [1] is some extension of chiral dynamics to higher energies by form factors describing the ρ resonances.

- On the experimental side, Aleph last result, based on data until 1991, was presented at the Ohio workshop. It is $Br(\tau \to \geq 3h, \geq 1\pi^0) = 5.42 \pm .26 \pm .34\%$ but this figure includes the $3\pi 2\pi^0$ channel which is of the order of .5% according to [2] as well as $3\pi 3\pi^0$ which is about 4 times lower and 5-prong decays at the per mille level. The best value for $Br(\tau \to 3\pi\pi^0)$ quoted at the Ohio workshop was $4.8 \pm .4\%$ obtained by subtracting CLEO [2] value $Br(\tau \to 3\pi 2\pi^0) = .54 \pm .09\%$ from $Br(\tau \to 3\pi \geq 1\gamma) = 5.2 \pm .4\%$ given at the Dallas conference (note that CLEO has published this value for $3\pi 2\pi^0$ but nothing for $3\pi\pi^0$). This value still includes $3\pi 3\pi^0$ events. Here we want to measure a completely exclusive branching ratio from which the $3\pi 2\pi^0$, $3\pi 3\pi^0$, 5-prong and charged kaons channels are excluded. The latter are very small except for the $K\ K^0\pi^0$ which is simulated and hence accounted for.
  ARGUS [3] recently reporting on evidence for ρ presence in the $3\pi\pi^0$ channel improved also on their estimate of its ω proportion and find it to be $33 \pm 4 \pm 2\%$. This former estimate was published in [4] where they did already study the angle between the normal to the ω decay plane and the line of flight of the 4^{th} pion and also found no deviation from the standard 1^{st} class-current. Although their statistics was at that time (it was in 1987) only of around 150 ω-candidates, they were able to quote an upper limit for so-called 2^{nd}-class currents of 50% with a confidence level of 90%.

2 Selection of the $3\pi\pi^0$ candidates

- We start from class 15 events and use the EW run selection. Each event is divided in two hemispheres using the thrust axis and each hemisphere is studied separately. The main problem here will be the π^0 reconstruction and all issues related to tracking are rather secondary.

- Tracking:
  We make use of the TAUPIDX routine to tag electron candidates which should come from photon conversions. A more rudimentary tagging was also used (using dE/dx only below 4 GeV and Rt and Rl otherwise) and gave undistinguishable results for the efficiency and the contamination. We require to have 3 tracks among which none is positively identified as an electron and at least 2 of good quality with respect to their D0, Z0 and number of TPC hits. We may have 1 or 2 extra

tracks either identified as electrons or unidentified. If there is only one extra track (4-prong hemisphere) we take it as a converted photon of which one track was lost. If there are 2 extra tracks, we pair them using the PAIRFD routine and require to have a low invariant mass and small distance of approach. We also check that the retained 3 candidate pions have the right charge configuration, opening angle less than $\pi/6$ with the thrust axis and a sum making an angle of more than 18° with the beam axis. Note that no attempt is made to distinguish between charged pions and kaons so that the unsimulated channels with charged kaons and π^0s enter also in our branching ratio but they are expected below the per mille level.

- π^0:
  We require to have at least 2 photons, a photon being either a GAMPEX or a converted photon as explained above. For a converted photon we require it only to be within 30° of the thrust axis. For a GAMPEX photon we apply various cuts aimed at rejecting so-called fake photons. These cuts allow to reduce the 3π background ($BR(\tau \to \pi^+\pi^-\pi^\pm) = 5.6\%$) where the 3 pions create hadronic fake photons and also to clean up the $3\pi\pi^0$ sample:

  - fraction of energy in first 2 stacks larger than 65%
  - barycenter not in a tower adjacent to an ECAL crack
  - cut in the plane $d\%E$ against hadronic fake photons: d is the distance of the photon to the closest track of the same PECO, if any, and E its energy.
  - for photons under 3 GeV, cut based on distance to closest track of the PECO (if any), longitudinal shower profile and energy balance with neighbour clusters against track satellites and backscatterings.

  To limit the pollution from fake photons inside the overlap region, we require to have at most one photon in that region. After these cuts we ask to have exactly 2 remaining photons with an invariant mass between 80 and 200 MeV (see plot 1). One and only one extra photon might be tolerated if it is not a converted photon and if it is below 1.1 GeV since this is often a fake photon escaping the previous cuts. The candidate π^0 will be refitted with QPI0DO if it consists of 2 GAMPEX photons whereas the GAMPEX photon's energy will simply be rescaled to get the π^0 mass if it is coupled to a converted photon.

- Non-tau background rejection:
  From Monte-Carlo studies we find that the Bhabha background is now below 5 per mille and the two photons background below 1 per mille (only $\gamma\gamma \to \tau\tau$ contributes). As for the $q\bar{q}$ events, we suppress them by applying the following cuts on the recoil hemisphere:

  - at most 3 good tracks
  - at most 5 photons within 30° of these good tracks
  - at most 15° of opening between the good tracks direction and that of any good track or photon.

  After this we expect the $q\bar{q}$ contamination below 5 per mille too. The remnant of $q\bar{q}$ is from low angle events where many tracks and/or photons are not detected.

We thus try to perform a completely exclusive measurement of the $3\pi\pi^0$ decay channel where all the contributions present in the quoted measurements are subtracted. Moreover we keep only fully reconstructed hemispheres where all 4-vectors of the charged tracks and of the π^0 are measured. The only source of extra width is from decays absent from the Monte-Carlo but these are whether unknown or negligible (like $K\pi\pi\pi^0$ or $KK\pi\pi^0$).

3 Monte-Carlo sample

We use 2×10^5 tau events from a KORL06 sample produced about 2 months ago. The main shortcoming is that one of the main backgrounds, namely $3\pi2\pi^0$, is poorly simulated. It is currently assumed to be phase-space distributed while CLEO [2] has shown that it contains around 81% of ω. The shape of the $\pi^+\pi^-\pi^0$ invariant mass of the background is consequently found flatter than it is in fact, possibly leading to an overestimation of the proportion of ω within the $3\pi\pi^0$ channel. This issue will be addressed in the discussion of the ω proportion estimation. Note that the proportion of ω put by hand in KORL06 is very close to the result of my fit so that the Monte-Carlo shape of the $\pi^+\pi^-\pi^0$ invariant mass fits the data very well.
As I remarked above, there is some imprecision in the simulation of the hadronic shower in GEANT so that GAMPEX photons close to the charged pions are found more abundant in the data than in the Monte-Carlo. This is why this distance versus energy cut is used and limits the otherwise rather large systematic error introduced by this discrepancy.

4 Branching ratio estimation

4.1 Event counting

After the selection we are left with 1443 events. The distribution of the total mass of the hemisphere is shown in figure 2 for all events and for a selection of 'ω-like' events. The tau background in this sample is estimated from the tau Monte-Carlo to be:

$$N_B = N_B^{MC} \times \frac{N_\tau^{data}}{N_\tau^{MC}}$$

N_B^{MC} is the number of background events surviving the selection scaled by the ratio of the estimated number of taus in the data by the number of taus in the Monte-Carlo. This yields $N_B = 188$ (13% contamination). We thus estimate a signal of $N_S = 1255$ ie a branching ratio of:

$$BR(\tau \to 3\pi\pi^0) = N_S/(\varepsilon \times N_\tau^{data})$$

The efficiency of the selection estimated from the Monte-Carlo is $\varepsilon = 28.0\%$.
We normalize to the number of produced taus which we evaluate the following way:
We estimate the produced number of $q\bar{q}$ with the following formula:

$$N_{q\bar{q}} = (N_{had} - 0.078 \times \mathcal{L})/(\varepsilon_h + 0.0032)$$

where:

- N_{had} is the number of hadronic Z^0s from the run summary
- $.078 \times \mathcal{L}$ estimates the $\gamma\gamma$ contamination
- $\varepsilon_h = .9749$ is the hadron selection efficiency
- .0032 corrects it for τ contamination

Once this number is obtained, we use the theoretically computed ratio of τ to hadrons at the relevant LEP energy because this is convenient and precise enough for our purpose. For an energy of $E = 91.270\ GeV$ we obtain a ratio of $N_{q\bar{q}}/N_\tau = 20.615$ and it is the same for all 3 years of data taking used. We thus find that the data correspond to 52684 produced $\tau\tau$ events i.e. that:

$$N_\tau^{data} = 105368$$

That number is a bit lower than that of the EW selection because we do not use exactly the whole EW set.
So finally we find:

$$BR(\tau \rightarrow 3\pi\pi^0) = 4.25 \pm .13\%$$

4.2 Systematic errors

We have studied the following systematic errors:

- Efficiency estimation from Monte-Carlo statistics:
  Since the 28% efficiency comes from 17936 analysed signal events we get a relative error $\Delta(\varepsilon)/\varepsilon$ corresponding to:

  $$\Delta(Br)/Br = 1.4\%$$

- Overall normalization to the number of produced taus:
  We estimate the error on N_τ^{data} to be less than 5 per mille.

  $$\Delta(Br)/Br = .5\%$$

- Background estimation from Monte-Carlo statistics:
  We use $N_B^{MC} = 689 \pm 26$ from the Monte-Carlo and then scale it to the data statistics. We thus get:

  $$\Delta(Br)/Br = .6\%$$

- Uncertainties on the dominant background branching fractions:
  The $3\pi 2\pi^0$ branching ratio is only known with a 12% relative precision and the 3π with a 3% relative precision. The former accounts for 40% of the background and the latter for 30% and the background is 13% of the data which yields:

  $$\Delta(Br)/Br = .8\%$$

- Non-tau background:
As explained in the selection paragraph, we expect here at most 5 per mille from either Bhabha or $q\bar{q}$ events that is:

$$\Delta(Br)/Br = 0.7\%$$

- Systematics from the selection cuts:
Any cut concerning the treatment of the charged tracks can hardly dramatically increase the systematic error. Everything that concerns the conversions bears only on 15% of the data. For the pion candidates, we can afford loose cuts for the track identification since we do not expect much background from 3 electrons and π^0 and the problem of reconstructing 2 close tracks of the same sign was studied in ALEPH [5] for the a1 decay and shown small enough to be overlooked at our level.
On the photon side, we do not perform a general branching ratio analysis and hence must take into account our efficiency to reconstruct a resolved π^0. The main effect here is the threshold in GAMPEX requiring a photon to have at least 280 MeV in its first 2 stacks. The systematic error from this is estimated to be 1.5%.
Then we cut hadronic fake photons and try to get rid of charged hadronic shower simulation problems by using the variable $x = distance + 2.5 \times energy$ (distance means distance from the photon to the closest charged track of the PECO if any). This variable was chosen because it takes into account the observed correlation between distance and energy in hadronic fake photons. Figure 3 shows the relative difference (data - MonteCarlo) / MonteCarlo (in percent) in the number of events satisfying the cut when varying its value. By cutting at x=7, we expect to eliminate nearly all the hadronic fake photons. Yet the disagreement we observe on this plot leads us to include for it a systematic error of 2%. We thus find a systematic error for the selection of:

$$\Delta(Br)/Br = 2.5\%$$

4.3 Conclusion

We find the branching ratio of the tau to 3 charged pions and one neutral pion to be:

$$BR(\tau \to 3\pi\pi^0) = 4.25 \pm .13_{stat} \pm .13_{syst}\%$$

The dominant error comes from the selection and could certainly be reduced by a more thorough study. The large error for the efficiency is of course simply a question of how much Monte-Carlo one wants to use but it is better now to await the next version of KORAL.

5 ω proportion within $3\pi\pi^0$

5.1 Fit of the proportion

Preliminary remarks

We are looking for evidence of ω presence in the $3\pi\pi^0$ channel. Since ω decays predominantly into $\pi^+\pi^-\pi^0$, we will study the $\pi^+\pi^-\pi^0$ invariant mass spectrum to find a peak at the ω mass. Each candidate hemisphere makes 2 entries in that histogram since there are 2 pions of the charge of the tau. We assume throughout that the $3\pi\pi^0$ channel proceeds through 2 independent ways: that of an intermediate $\omega - \pi$ state and that of an intermediate $\rho - \pi - \pi$ state. Otherwise it wouldn' t even make sense to talk about an 'ω percentage'. This is correct insofar as the interference of the 2 currents can be neglected owing to the narrowness of the ω resonance. In that approximation, the spectrum of $\pi^+\pi^-\pi^0$ invariant mass is simply a linear combination of the spectra obtained for single 'ω-current' and single 'ρ-current' with weights giving the respective proportions of these processes. This is confirmed by the fact that the Monte-Carlo is indeed reproduced by such a simple superposition as is best seen at the KINGAL level. Figure 5 shows that indeed the agreement is excellent and that the fit procedure worked well. To obtain the $\pi^+\pi^-\pi^0$ shapes corresponding to the 2 currents, a dedicated production of events proceeding exclusively through each of them was made. The 2 samples thus obtained will further allow to estimate the selection efficiency for each process. Since these efficiencies are slightly different, they have to be used to correct the proportion obtained by the fit.

Performing the fit

The expected τ background contribution is subtracted from the $\pi^+\pi^-\pi^0$ spectrum. This makes the fit dependent not only on the dynamics of the $3\pi\pi^0$ channel used in TAUOLA but also, to a lesser extent (because the background is only 13% of the data), on the dynamics of TAUOLA in the main background channels, namely $3\pi 2\pi^0$ and $\pi^+\pi^-\pi^\pm$. Now we fit to the resulting spectrum the combination: $\alpha \times (spectrum\ from\ \omega - current) + (1 - \alpha) \times (spectrum\ from\ \ \rho - current)$. For this we use a binned Maximum Likelihood method. The tails of the distributions are cut and the binning is chosen to preserve sufficient statistics per bin while of course showing the narrow omega peak. The result of this fit is a 'raw' ω proportion:

$$\alpha_R = 46.0 \pm 1.8\%$$

The plot 4 shows the $\pi^+\pi^-\pi^0$ mass distributions for the data with the estimated background shaded, the subtracted data with the result of the fit and the shapes for both pure ω and pure ρ currents. This figure must be corrected with the different efficiencies (this slightly increases the error):

- $\omega - \pi$ has $\varepsilon_1 = 1635/6000 = 27.2 \pm .7\%$
- $\rho - \pi - \pi$ has $\varepsilon_2 = 2392/9600 = 24.9 \pm .5\%$

So that the real proportion is:

$$\alpha = \frac{1}{1 + (\frac{\varepsilon_1}{\varepsilon_2}) \times (\frac{1}{\alpha_R} - 1)} = 43.8 \pm 1.9\%$$

5.2 Systematic errors

They are very different from those of the branching ratio measurement. We distinguish 3 different sources:

1. Theoretical uncertainties:
   The effect of neglecting the interference is very small because of the excellent agreement shown on 5. One further relies on the dynamics of the Monte-Carlo but then the ω-current shape is well understood whereas the ρ-current shape, although largely unknown, shows such an agreement with the data that the fit is quite unsensitive to any reasonable variation of it.

2. Systematics from the fit proper:
   Since we use as theoretical shapes of $\pi^+\pi^-\pi^0$ spectra histograms obtained from finite Monte-Carlo samples, we have a systematic error from the Monte-Carlo statistics. This we estimate by randomly varying the histograms bin contents within their statistical uncertainty and each time redoing the fit. We obtain results for α_R showing a $\Delta(\alpha_R) \simeq 1\%$ as can be seen on figure 6. Of course we also check how our result α_R varies when the fit range is moved or the number of bins is changed. We vary these within reasonable bounds that is: the fit range must always include the ω mass zone and the bin number must be such that the ω peak is visible and the statistics sufficient. From this procedure we estimate a contribution of $\Delta(\alpha_R) \simeq 2\%$.

3. Background normalisation and branching fractions:
   The normalisation of the background could play. The (poorly simulated) background $\pi^+\pi^-\pi^0$ mass shape is more or less flat so that if too much of it is subtracted, the ω proportion will be overestimated. Yet varying the background amount within a 7% error has no observable influence on the result. We can also vary the composition of the background within the known uncertainties of the $3\pi2\pi^0$ and $\pi^+\pi^-\pi^\pm$ branching ratios. This gives us an estimate of $\Delta(\alpha_R) \simeq 1\%$.

4. Background simulation:
   As for the shapes themselves, we depend on the dynamics of the Monte-Carlo but also on possible distortions introduced by the variation of the selection acceptance over the $\pi^+\pi^-\pi^0$ mass range. We believe that at the present level of precision, we can assume that these possible acceptance variations are sufficiently well reproduced by the Monte-Carlo. As for the dynamics, as mentioned in the Monte-Carlo sample section, there is one grossly wrong $\pi^+\pi^-\pi^0$ shape, namely that of the $3\pi2\pi^0$ decay. Since only 1 π^0 was reconstructed while this channel contains 2, one expects to fail in the ω reconstruction about half of the time, yet the relative ω abundance in $3\pi2\pi^0$ is roughly twice that in $3\pi\pi^0$ so that one might expect the $\pi^+\pi^-\pi^0$ shape for $3\pi2\pi^0$ to show a peak comparable in significance (albeit probably larger) to that of the signal. The need for a better Monte-Carlo is obvious but meanwhile, we will satisfy ourselves with an upper limit to the influence of this defect. Remember that events coming from a $3\pi2\pi^0$ decay are expected only at the 5% level. Adapting the subtracted shape of the Monte-Carlo to better reflect the correct dynamics and redoing the fit gives an estimate of $\Delta(\alpha_R) \simeq^{+0.\%}_{-1.\%}$ (this error could only increase α_R !)

5.3 Conclusion

We have determined that the proportion of tau decays to $3\pi\pi^0$ proceeding through an intermediate ω meson is:

$$\frac{\Gamma(\tau \rightarrow \omega\pi) \times Br(\omega \rightarrow \pi^+\pi^-\pi^0\ )}{\Gamma(\tau \rightarrow 3\pi\pi^0)} = 43.8 \pm 1.9^{+2.5}_{-2.6}\%$$

This result can be translated into a branching ratio of the tau into $\omega - \pi$ using the obtained result for $Br(\tau \rightarrow 3\pi\pi^0)$ and $Br(\omega \rightarrow \pi^+\pi^-\pi^0\ ) = 88.8 \pm .6\%$. This yields:

$$Br(\tau \rightarrow \omega\pi\nu_\tau) = 2.10 \pm .18\%$$

6 χ angle distribution

6.1 The χ angle

The χ angle is the angle in the rest frame of the ω between the normal to its decay plane and the 4^{th} pion.
In the isospin conservation limit, the even G-parity $\omega - \pi$ system can only be obtained with a vector current. This implies that this system is in a pure p-wave state and of total angular momentum 1 and this in turn implies that the χ angle follows the distribution $\mid Y_1^1(\chi) \mid^2 = \sin^2\chi$

6.2 Building and fitting the distribution

Several troubles arise when building this distribution. It is already obvious that for the data, it will be spoiled by the background events. Inside the $3\pi\pi^0$ events, one would like to select 'ω' events only which can be done by cutting on the $\pi^+\pi^-\pi^0$ invariant mass or designing a more sophisticated cut using also the Dalitz densities but in any case losing some efficiency and conserving some of the 'ρ' events. Even with 'ω' events only, one must consider 2 combinations because there are 2 'like-signed' pions. Finally one needs a good reconstruction of all the pions (and in particular of the π^0) to compute this angle.
Notwithstanding these details, we proceed to obtain the distribution of $\cos\chi$ by using the side bands technique. We first select only those mass combinations that are in the region of the Dalitz plot where the omega decay products are expected. This mainly gets rid of the ρ-like events that have one mass combination below the ω peak while few of the real ω-like events are lost. Figure 7 shows the $\pi^+\pi^-\pi^0$ mass distribution of the data with the cut zone indicated by two arrows on top. The middle row shows this distribution for ω-like and ρ-like events separated by cutting on this mass band. The bottom row shows the same distributions where an event, to be considered ω-like, now must not only have mass inside the cut band but also be in the region of the Dalitz plot where an ω event should be. One sees that the peak on the 'ω' side is slightly attenuated while the distribution on the 'ρ' side is restored to a smoother shape.

We then take the cosine corresponding to all those combinations with mass inside the ω peak (we could equivalently take minus that cosine) and subtract from the resulting distribution that of all $\pi^+\pi^-\pi^0$ combinations with mass inside either of the side bands.

We hope to thus eliminate the contributions from both the non-ω events and the wrong combinations from the ω events. We normalize the sum of the subtracted spectra (left and right band) so that they represent as many entries as we estimate there are in the signal mass band below the ω peak. Figure 8 shows the data $\pi^+\pi^-\pi^0$ mass spectrum with the arrows showing the mass bands, the $cos\chi$ distributions for each band and the final $cos\chi$ shape fitted to the $\sin^2\chi$ function. The fit shows a very good agreement having a chi square of .4 per degree of freedom.

6.3 Interpretation

If we observe no deviation from the $\sin^2\chi$ distribution, we are able to put an upper limit on the possible contributions from non-standard currents like a second-class G-even axial current (that would be $b_1(1235)$ enhanced) for instance. Such currents would lead, according to their spin-parities to different χ distributions as shown in figure 9. This study was published at the beginning of ARGUS in [4]. With only ≈ 146 events, they observed no deviation from the standard current and were able to bound to at most 50% with a confidence level of 90% the contribution of 2^{nd}-class currents. We have here around 400 events in the peak because we use this cut on the Dalitz plot of the ω decay in conjonction with the mass cut to select 'ω' events. As mentioned, the fit of a pure $sin^2\chi$ function reproduces the data well and hence we see no evidence for non-standard contributions. To quantify this statement, we try to fit a combination of the standard $J^P = 1^-$ current and of the most similar non-standard current which is here the flat $J^P(l) = 1^+(0)$ current ie:

$$\alpha \times (spectrum\ from\ \ 1^- - current) \ + \ (1-\alpha) \times (spectrum\ from\ \ 1^+(0) - current)$$

We normalize both spectra to have the same sum (for $\cos(\theta)$ in $[-1;1]$) and total sum corresponding to the observed number of entries so that α measures the proportion of $J^P = 1^-$ current. The result is:

$$\alpha = 0.96 \pm .10$$

We thus conclude that there is no evidence for a deviation from the G-even hadronic vector current at a half a sigma level and that the 2^{nd}-class currents contribute less than 24% at a 95% confidence level.

References

[1] R. Fisher, J. Wess and R. Wagner, Z Phys C3 313 (1980)

[2] CLEO collaboration, PRL 71 12 (1993)

[3] ARGUS collaboration, PL B260 259 (1991)

[4] ARGUS collaboration, PL B185 223 (1987)

[5] Laurent Duflot, thesis, Orsay university (1993)

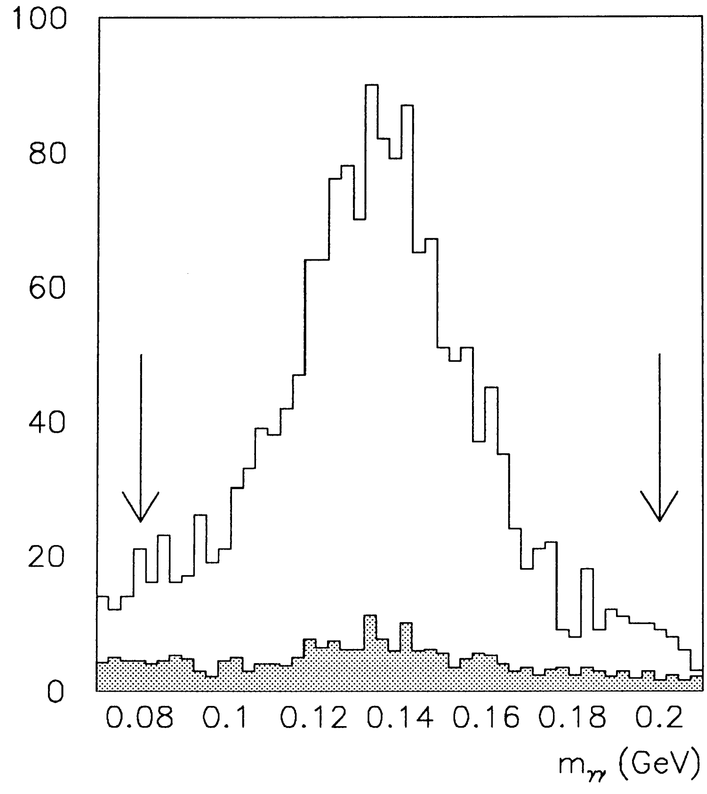
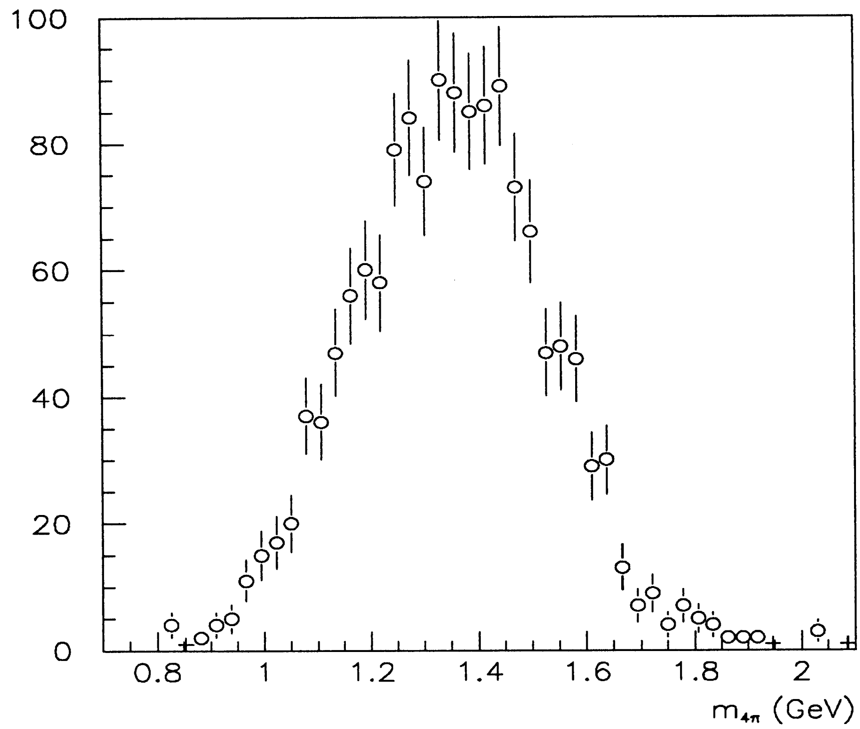


Figure 1: $\gamma\gamma$ invariant mass for data with estimated tau background contribution shaded and cut zone.



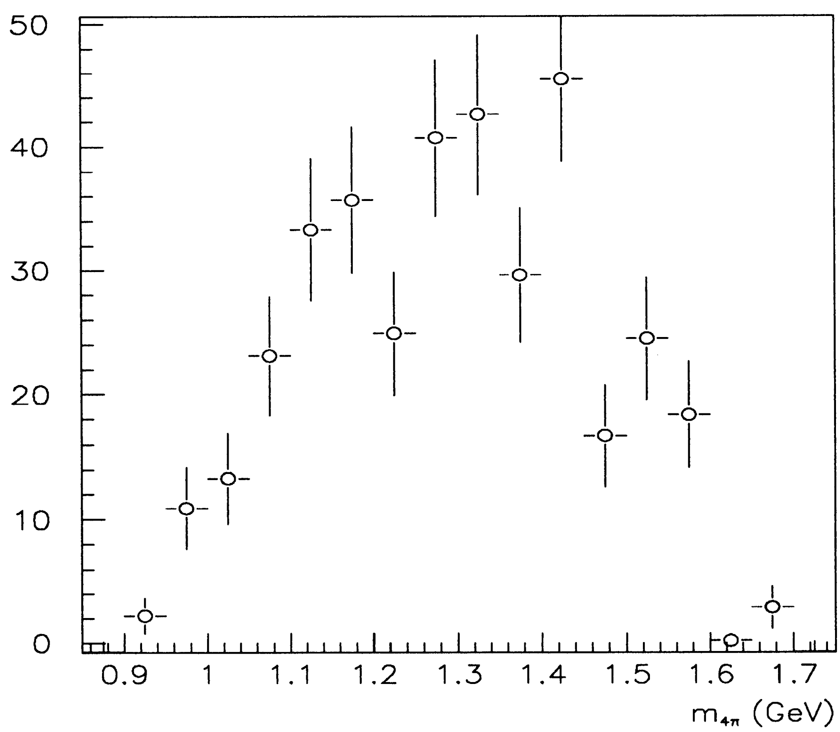


Figure 2: Total mass of hemispheres for all data (above) and for 'ω' selection (below)

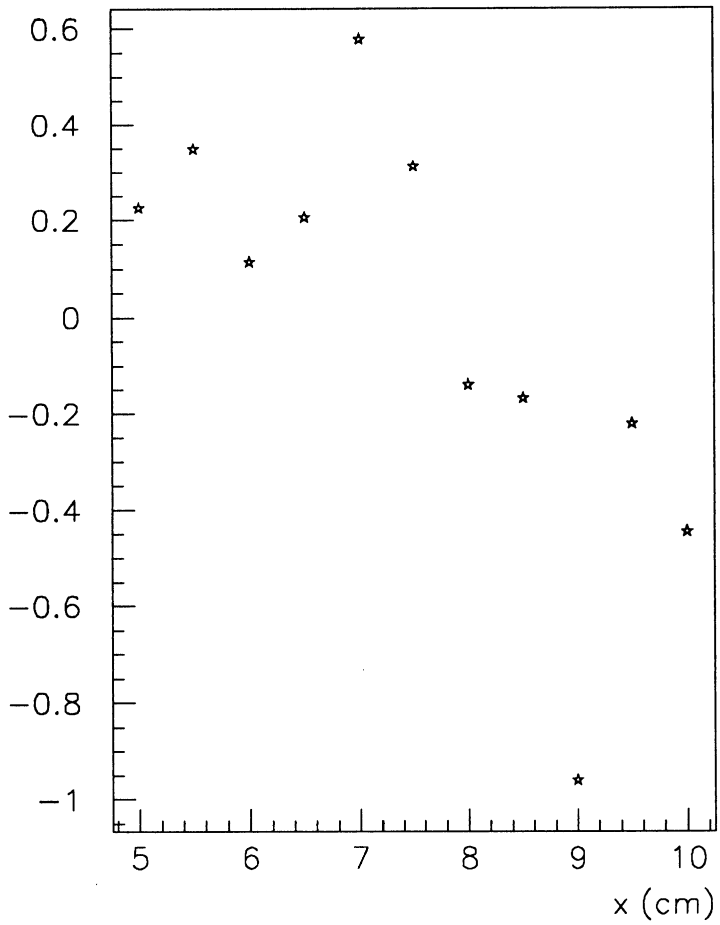


Figure 3: Relative difference in percent between data and Monte-Carlo in the number of events passing the cut on $x = distance + 2.5 \times energy$ as a function of x

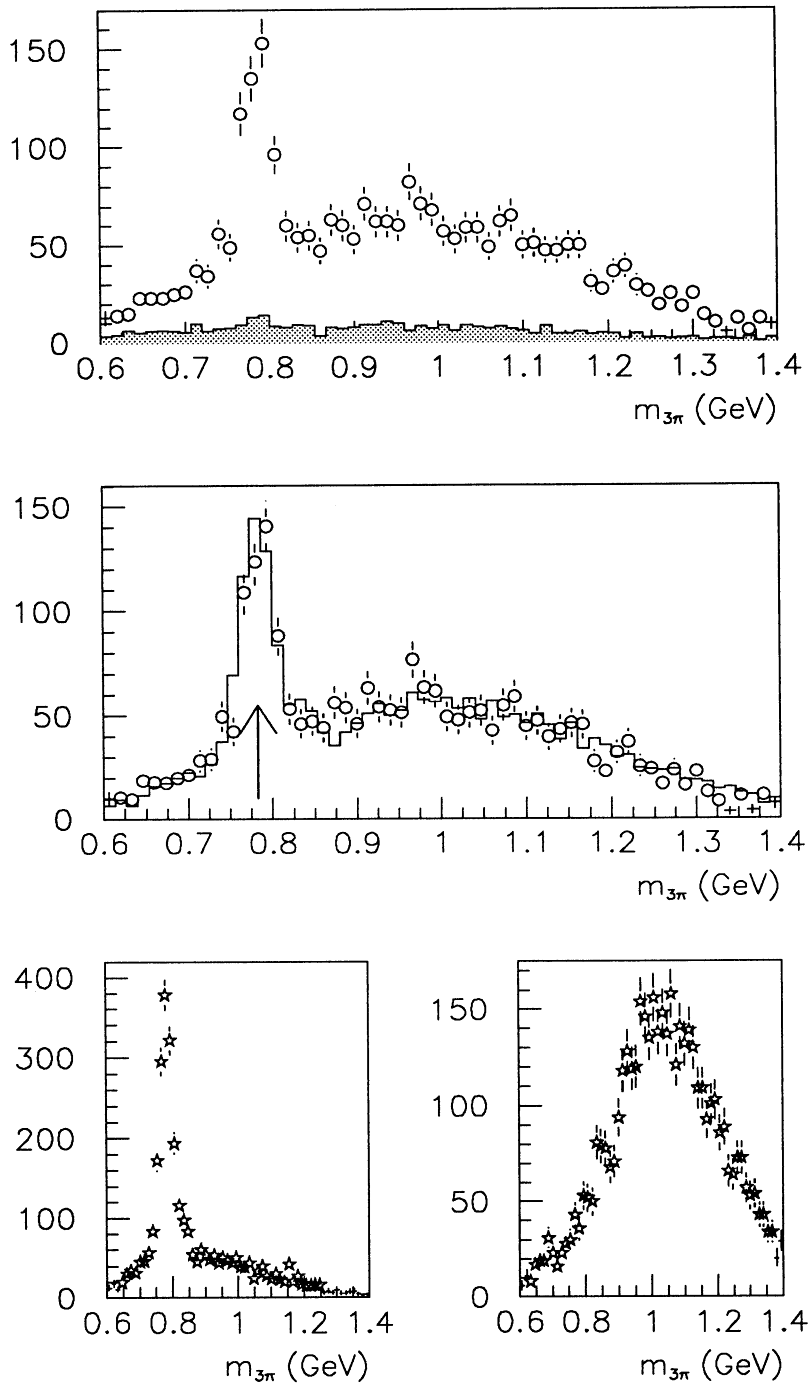


Figure 4: top: Data $\pi^+\pi^-\pi^0$ mass spectrum with estimated background shaded. middle: Subtracted data with fit result (the arrow shows the ω mass). bottom: Expected $\pi^+\pi^-\pi^0$ spectra for a Monte-Carlo of ω-events (left) and ρ-events (right).

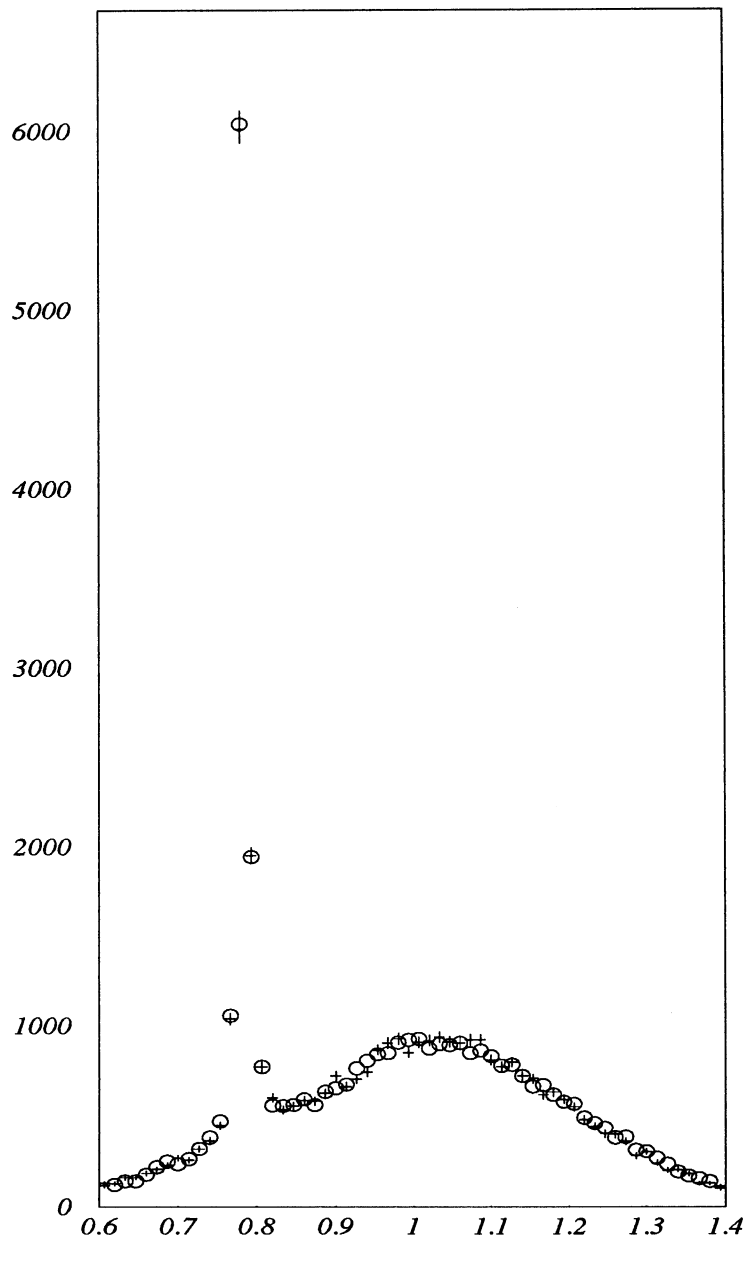


Figure 5: Superposition of the $\pi^+\pi^-\pi^0$ mass shape with a linear combination of pure ρ and pure ω spectra (circles: linear fit, crosses: full spectrum). The disagreement measures the effect of the interference.

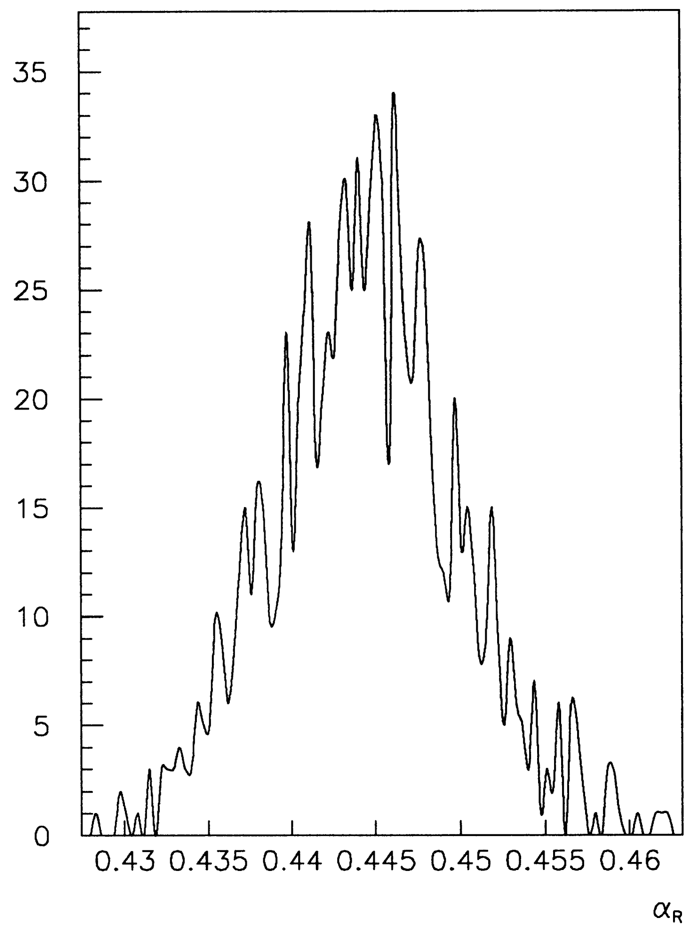


Figure 6: α_R distribution when shapes varied

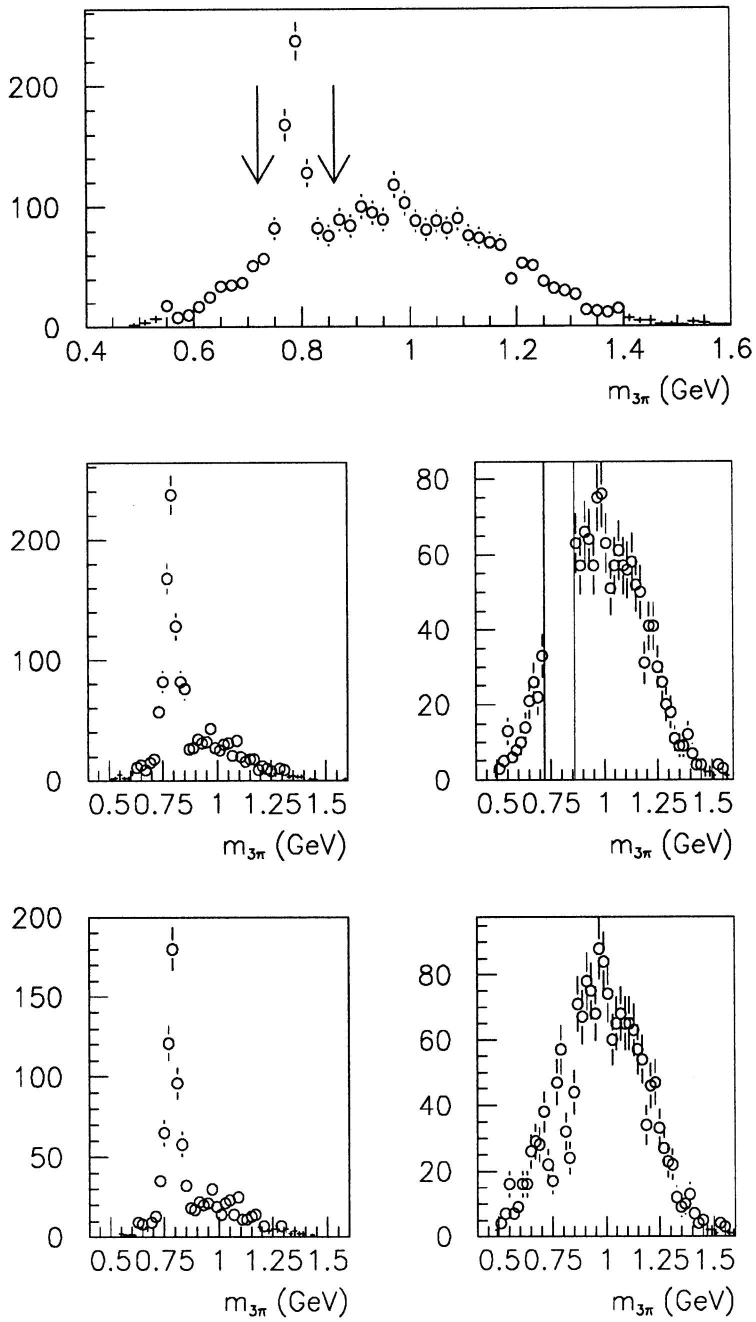


Figure 7: $\pi^+\pi^-\pi^0$ mass distribution of the data after selection above, splitting in 2 types using mass cut below and splitting using mass and Dalitz density bottom

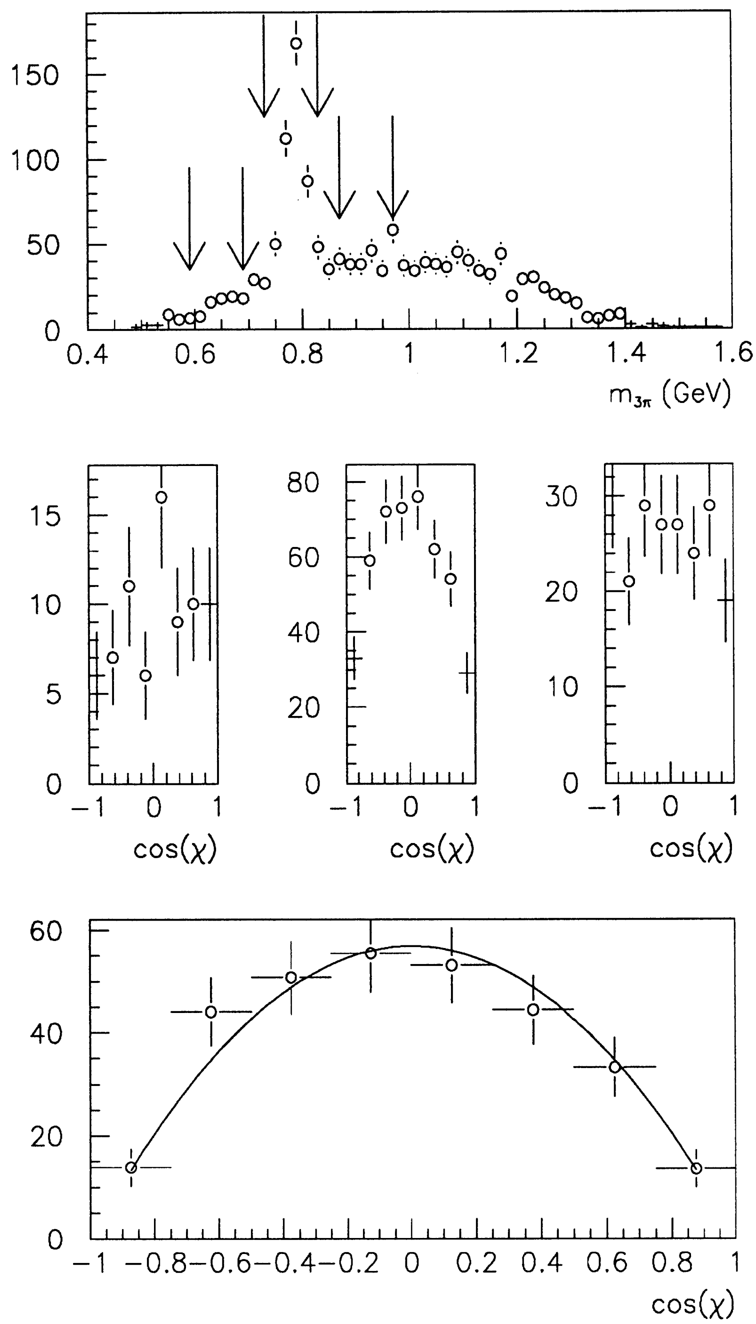


Figure 8: $\cos\chi$ distribution of the data in the 3 bands and fit by $1-\cos^2\chi$ of the subtracted center band

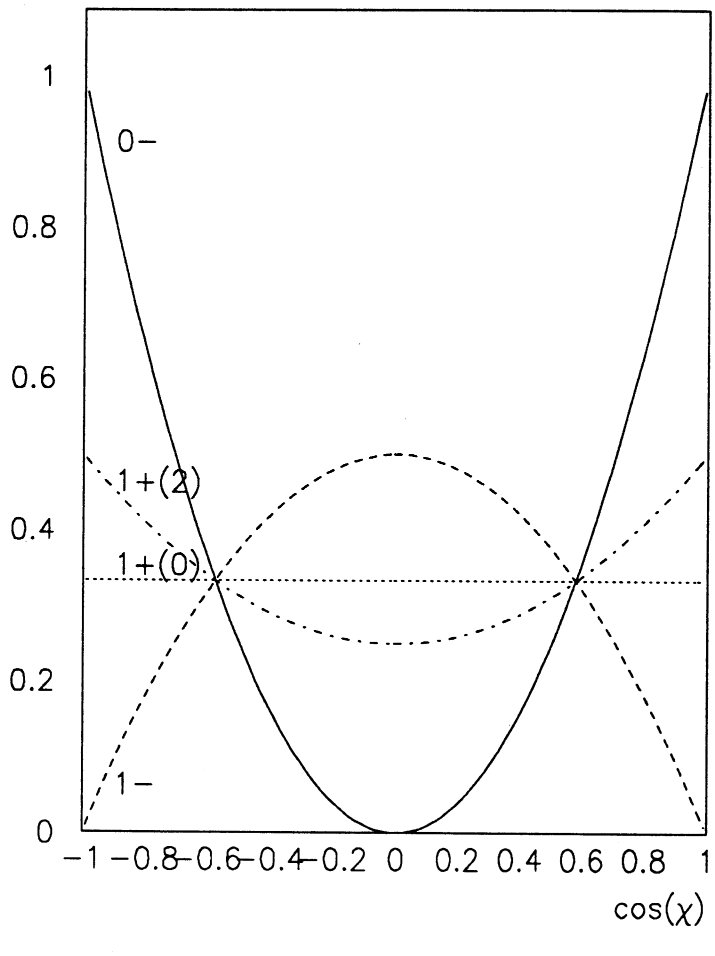


Figure 9: $\cos\chi$ distributions of various $J^P(l)$