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Tagging the b quark charge at t=0

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Abstract

Different methods to tag the b quark charge at time t=0 are compared. It is shown that the jet charge with parallel momentum weighted charge using kappa values of zero for the lepton hemisphere and 0.5 for the opposite hemisphere has less bias.

1 Selection of the lepton events

The analysis has been carried out on 20000 lepton events from 1992 $b\bar{b}$ Monte carlo with a b lifetime of 1.5 ps satisfying the following selection:

- events are required to have at least one lepton with $p >$ 3GeV/c and p_T (lepton excluded) > 1.25 GeV/c;
- after after a clusterization with the Jade algorithm using Y_{cut} of 0.004, it is required that the event contains at least two jets;
- the lepton momentum is required to be less than 0.9 E_{jet};
- the charge multiplicity in the lepton jet should be ≥ 3;
- the events are splitted into two hemispheres with respect to the to the thrust axis and are required to have $cos\theta_{thrust} < 0.9$;

The events are classified in double tagged events where each hemisphere contain a high p and p_T lepton and single tagged events where only one hemisphere has a lepton. With this selection the sample composition is given in table 1 for single tagged events.

Table 1: Sample composition of the lepton events

event type	fraction %	
	e	μ
$b \to l$	82.7	73.5
$b \to \tau \to l$	1.2	1.1
$b \to c \to l$	7.7	7.9
$c \to l$	6.0	6.3
$K, \pi \to \mu$	-	6.3
photon conversions	1.0	-
Misid. hadron	1.1	3.9

2 What is the less biased estimator of the t=0 b charge:

The hemisphere charge with parallel momentum weighting is defined as follows:

$$Q_j = \frac{\sum_{i=1}^{n} P_{i\|}^{\kappa} Q_i}{\sum_{i=1}^{n} P_{i\|}^{\kappa}}$$

The sum is running over all the charged tracks of the hemisphere. With the kappa parameter equal to 0.5 or 1, Q_j is affected by the leading lepton charge. When using $\kappa = 0$ the above equation reduces to:

$$Q_j = \frac{\sum_{i=1}^{n} Q_i}{n}$$

where n is the number of charged particle in the hemisphere [1, 2].In this case the jet charge should not be affected by the mixing as B_d and B_s are neutral and should reflect the charge of the b quark through the fragmentation tracks.

We will also use the rapidity weighted jet charge with the Wisconsin group definition [3]:

$$Q_j = \frac{\sum_{i=1}^{n} y_i Q_i}{\sum_i y_i}$$

in this case the sum runs over the charges contained in the lepton jet. These jets are defined with a Y_{cut} of 0.02. For this method the two jets considered are the lepton jet and the most energetic jet in the opposite hemisphere. We define Q_{lj} as the product of the lepton charge Q_l by the jet charge Q_j. If Q_j reflects the quark charge at time t=0 then, in the lepton hemisphere Q_{lj} should be positive for unmixed events (average value $+Q_0$)and negative for mixed events (average value $-Q_0$). The signs will be reversed in the opposite hemisphere. However, the measurement of the jet charge Q_j in the lepton hemisphere can be biased by the leading lepton charge Q_l resulting in a bias in the measurement of Q_lj by a value ΔQ positive independently of the mixing.

The signed lepton jet charge, averaged on mixed and unmixed Monte Carlo events, depends on Q_0 and ΔQ as follows:

$$\begin{aligned} < Q_{lj}^{um} > &= Q_0 + \Delta Q \\ < Q_{lj}^{m} > &= -Q_0 + \Delta Q \end{aligned}$$

One can estimate Q_0 and the bias ΔQ from:

$$Q_0 = \frac{< Q_{lj}^{um} > - < Q_{lj}^{m} >}{2}$$
$$\Delta Q = \frac{< Q_{lj}^{um} > + < Q_{lj}^{m} >}{2}$$

We consider separatly the opposite hemisphere and the lepton hemisphere.

- Opposite hemisphere :
  Q_0 and ΔQ are listed in table 2 together with the separation power SP (=Q_0 / r.m.s.(Q_{lj})).

Table 2: separation power, charge at t=0 and bias in the hemisphere opposite to the lepton

choice	SP	Q_0	ΔQ
κ= 0.5	0.335 ± 0.019	-0.070 ± 0.004	0.000 ± 0.004
κ=1.0	0.290 ± 0.019	-0.099 ± 0.006	-0.002 ± 0.006
y	0.283 ± 0.019	-0.052 ± 0.004	0.002 ± 0.004

  One can see that $\kappa = 0.5$ has the best power separation and as expected there is no bias.

- Lepton side hemisphere:
  The Q_{lj} distributions are displayed on fig.1 for $\kappa = 0$, fig. 2 for $\kappa = 1$ and fig.3 for rapidity weighting and in case of a B_s meson decay. Table 3 and table 4 give the results in terms of ΔQ and Q_0, respectively for the B_d and B_s primary leptonic decay. For the momentum weighting method the bias ΔQ is rapidly increasing with κ and is 20% smaller fort the B_s than for the B_d . It is small for $\kappa = 0$. In the rapidity weighting method, ΔQ is reduced compared to the momentum weighted method with $\kappa = 0.5$ and $\kappa = 1$, but remains comparable to Q_0, and is flavor dependent.

The next section deals with a study of how these different jet charge methods affects the mixing significance.

Table 3: Bias, charge at t=0 and r.m.s. of the signed lepton jet charge in the lepton hemisphere containing a B_d

B_d			
	ΔQ	Q_0	r.m.s.
κ=0	0.011 ± 0.003	0.038 ± 0.003	0.175
κ=0.5	0.186 ± 0.003	0.035 ± 0.003	0.191
κ=1	0.371 ± 0.005	0.031 ± 0.005	0.293
y	0.050 ± 0.003	0.041 ± 0.003	0.184

Table 4: Bias, charge at t=0 and r.m.s. of the signed lepton jet charge in the lepton hemisphere containing a B_s

B_s			
	ΔQ	Q_0	r.m.s.
κ=0	0.011 ± 0.004	0.046 ± 0.004	0.177
κ=0.5	0.143 ± 0.005	0.041 ± 0.005	0.192
κ=1	0.298 ± 0.007	0.031 ± 0.007	0.307
y	0.039 ± 0.004	0.052 ± 0.004	0.182

3 Optimization of the significance

In a time dependent mixing analysis the significance can be expressed as following [4]:

$$\frac{B}{\sigma_B} = f(1 - \eta_{um} - \eta_m)\sqrt{\frac{N}{2}} \exp\left[-\frac{1}{2}(\frac{\sigma}{\tau}\frac{\Delta m}{\Gamma})^2\right]$$

where N is the total number of events, f theB_s^0 purity, η_{um} and η_m the mistag rates for respectively the unmixed and mixed events, The exponential term correspond to the oscillation damping factor due to the time resolution σ, it depends on $\Delta m/\Gamma$ the oscillation frequency.

For a given method, one has to optimize the coefficient α=$(1 - \eta_{um} - \eta_m)\sqrt{\epsilon}$ where ϵ is the efficiency of the cut on the jet charge . The results given in the first line of table 5 and 6 correspond to the method which uses only the opposite hemisphere jet charge with $\kappa = 0.5$. The significance is improved by using the lepton hemisphere jet charge difference:

$$Q = Q^{s}_{lj} - Q^{o}_{lj}$$

Q is used to tag mixed and unmixed events. Values for α have been calculated. They are listed in table 5 for the case where there is no cut performed on the jet charge combination (ϵ=1) and table 6 when a cut $|Q| \geq 0.2$ is done. When just the jet charge in the opposite hemisphere of the lepton is used this cut is reduced to $|Q| \geq 0.08$. Please note that, except when explicitly specified all numbers given in the table below are for $b \to l$ transition only.

Table 5: α=$(1 - \eta_{um} - \eta_m)$, no jet charge cut applied

	B_d	B_s
Opp. jet only		
$\kappa = 0.5$	0.286 ± 0.017	0.260 ± 0.023
$Q^{s}_{\kappa=0} - Q^{o}_{\kappa=0.5}$	0.309 ± 0.016	0.313 ± 0.023
$Q^{s}_{\kappa=0.5} - Q^{o}_{\kappa=0.5}$	0.246 ± 0.017	0.267 ± 0.022
$Q^{s}_{\kappa=1} - Q^{o}_{\kappa=0.5}$	0.125 ± 0.014	0.129 ± 0.019
$Q^{o}_{y} - Q^{o}_{y}$	0.292 ± 0.017	0.326 ± 0.023

From the numbers listed in table 5 and 6 one can conclude that the mixing visibility is improved by using the two sides and cutting on the jet charge combination $|Q|$. Furthermore, using κ=0 in the lepton hemisphere and κ=0.5 in the opposite hemisphere looks as good as rapidity weighting. Nevertheless, the κ=0 case has the advantage of being bias free: the B_s and B_d values are equal. A comparison of the present mistag rates with those obtained for the Wisconsin group (Hongbo Hu private communication) for $b \to l$ transitions is given in table 7. The numbers differ by less than 1.5 σ. In both analysis one sees a significant difference (7% to 10%) in the B_d and B_s mistag rates when rapidity weighting is used. The influence of including $b \to c \to l$ on the α coefficient is given in table 8. These numbers correspond to a cut on the jet charge combination $|Q| \geq 0.2$

Table 6: α values when a cut on the jet charge combination is applied

	B_d		B_s	
	ϵ	α	ϵ	α
Opp. jet only				
$\kappa = 0.5$	0.69	0.30 ± 0.02	0.66	0.29 ± 0.02
$Q^s_{\kappa=0} - Q^o_{\kappa=0.5}$	0.49	0.34 ± 0.02	0.48	0.34 ± 0.03
$Q^s_{\kappa=0.5} - Q^o_{\kappa=0.5}$	0.63	0.21 ± 0.02	0.56	0.22 ± 0.03
$Q^s_{\kappa=1} - Q^o_{\kappa=0.5}$	0.79	0.09 ± 0.01	0.72	0.10 ± 0.02
$Q^o_y - Q^o_y$	0.47	0.30 ± 0.02	0.45	0.34 ± 0.03

Table 7: Mistag rates: rapidity weighting with cut

	present work	Wisconsin
B_d unmixed	$(17.0 \pm 0.7)\%$	$(15.9 \pm 0.6)\%$
B_d mixed	$(38.7 \pm 2.4)\%$	$(37.3 \pm 2.3)\%$
$(1 - \eta_{um} - \eta_m)$	$(44.3 \pm 2.5)\%$	$(46.8 \pm 2.4)\%$
B_s unmixed	$(17.8 \pm 1.7)\%$	$(15.3 \pm 1.3)\%$
B_s mixed	$(31.1 \pm 1.6)\%$	$(27.7 \pm 2.1)\%$
$(1 - \eta_{um} - \eta_m)$	$(51.1 \pm 3.1)\%$	$(57.0 \pm 2.5)\%$

While, for the B_s the significance is about the same with rapidity weighting and momentum weighting with $\kappa = 0$ in the lepton hemisphere, for B_d momentum weighting is significantly better.

For momentum weighting with κ=0 in the lepton hemisphere and κ=0.5 in the opposite hemisphere, the α values have been calculated for different cuts in the jet charge combination $|Q|$. The correponding values, listed in table 9 show that the the cut choice between 0.12 and 0.20 is not important.

Table 8: α values including $b \to c \to l$transitions when a cut on the jet charge is applied,

	B_d	B_s
$Q^s_{\kappa=0} - Q^o_{\kappa=0.5}$	0.268 ± 0.010	0.317 ± 0.019
$Q^o_y - Q^o_y$	0.238 ± 0.010	0.333 ± 0.013

Table 9: α values when a cut on the jet charge combination is applied

	B_d		B_s	
cut on $\|Q\|$	ϵ	α	ϵ	α
0.0	1.0	0.29 ± 0.02	1.	0.33 ± 0.02
0.04	0.89	0.32 ± 0.02	0.88	0.34 ± 0.03
0.08	0.78	0.33 ± 0.02	0.78	0.33 ± 0.02
0.12	0.69	0.34 ± 0.02	0.68	0.35 ± 0.02
0.16	0.59	0.34 ± 0.02	0.57	0.34 ± 0.02
0.20	0.49	0.34 ± 0.02	0.48	0.34 ± 0.02

4 Conclusion

Different methods used to tag the b quark charge at t=0 have been compared. It has been shown that in the opposite hemisphere to the the best jet charge estimator is obtained with momentum weighting and a κ value of 0.5 while in the lepton hemisphere, the less biased estimator is also obtained with momentum weighting but with a κ value of 0. To optimize the significance in an analysis, the best tagging is obtained by combining the jet charge of the two hemispheres and eliminating a region around zero on this jet charge combination. Compared to rapidity weighting, the jet charge combination $Q^s_{\kappa=0} - Q^o_{\kappa=0.5}$ provides a comparable significance for B_s and is slightly better for B_d.

References

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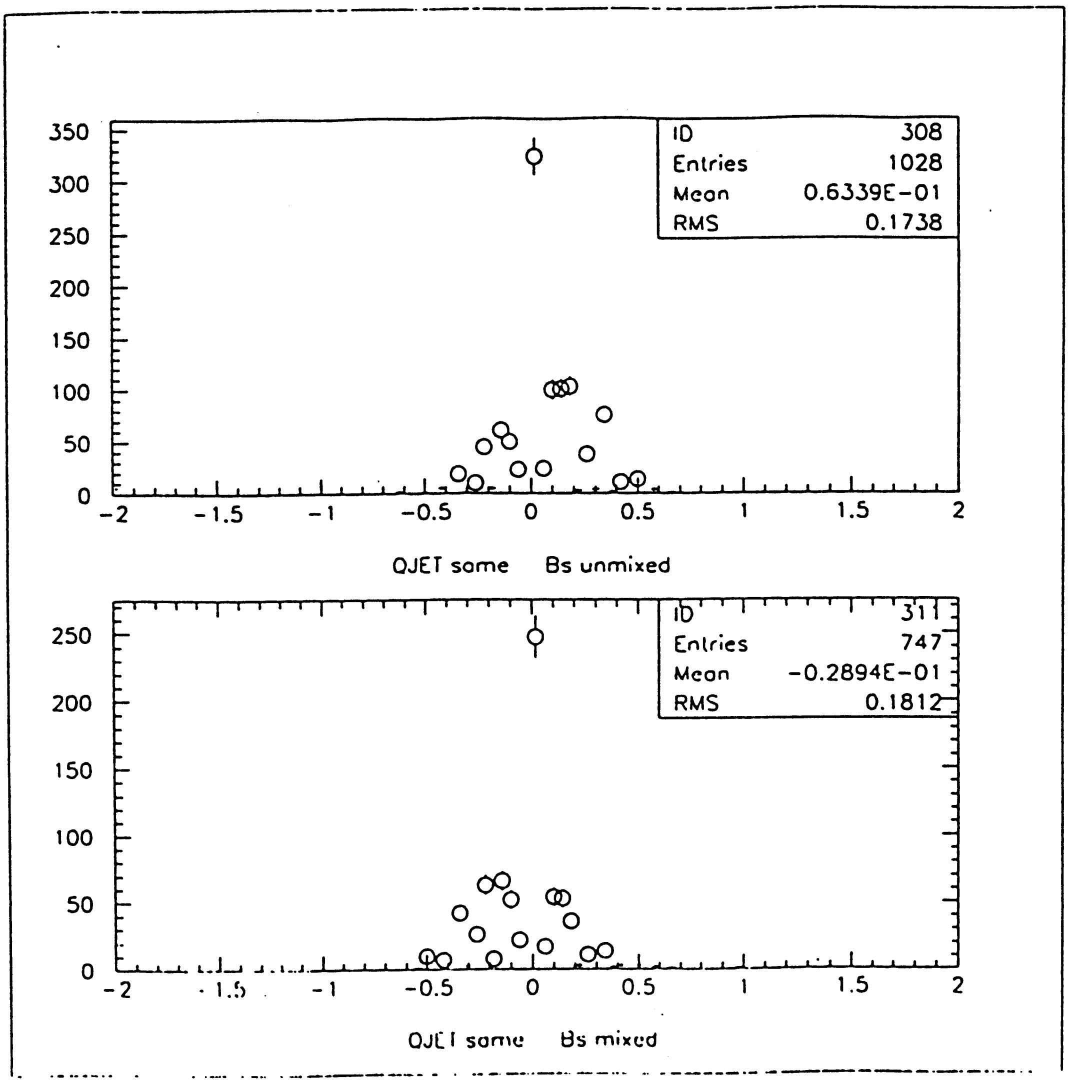

Figure 1: The signed lepton jet charge obtained for momentum weighting and $\kappa = 0$ for unmixed B_s (top) and mixed B_s (bottom)

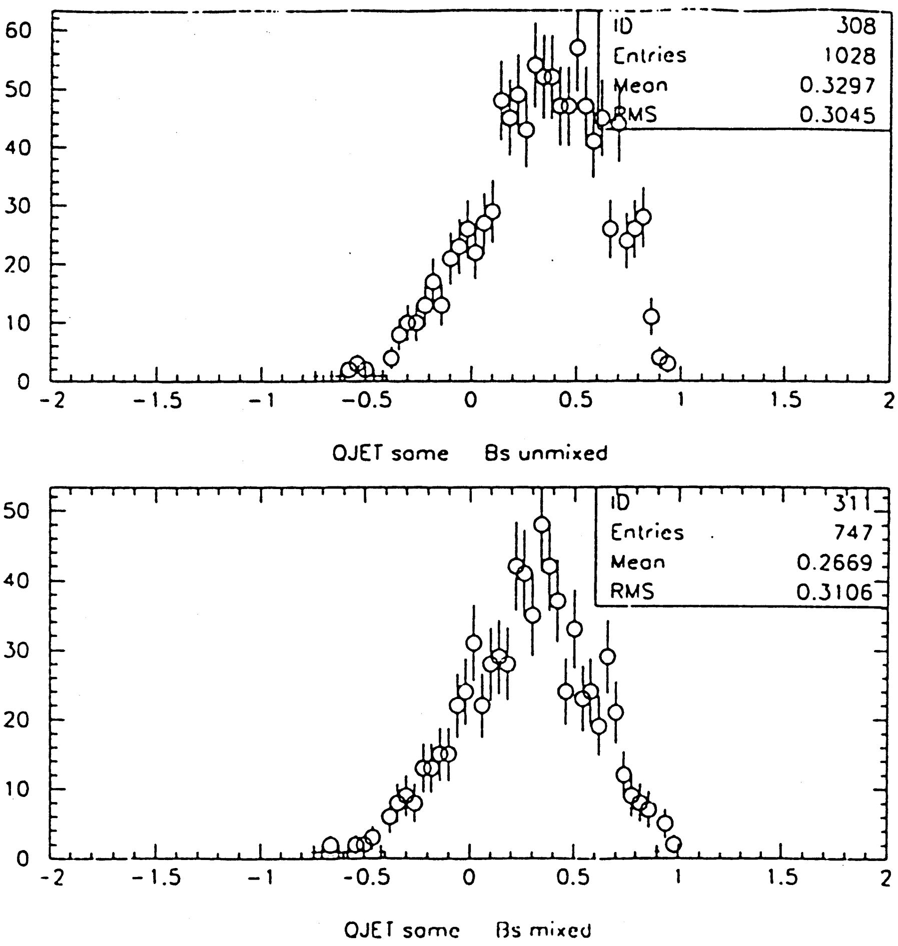

Figure 2: The signed lepton jet charge obtained for momentum weighting and κ= 1 for unmixed B_s (top) and mixed B_s (bottom)

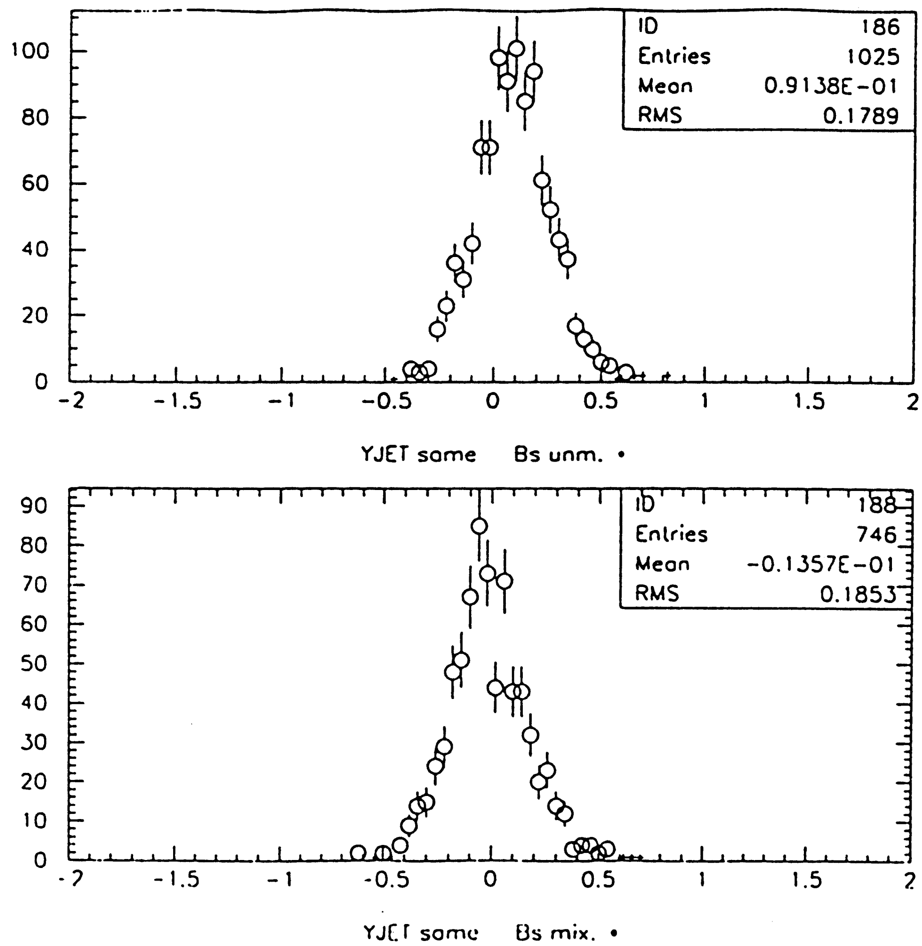

Figure 3: The signed lepton jet charge obtained for rapidity weighting for unmixed B_s (top) and mixed B_s (bottom)