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Acceptance for Bhabhas and the t-channel Problem

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1 Introduction

The usual way to correct the measured cross section of $Z^\circ \rightarrow \mu\mu$ or $Z^\circ \rightarrow \tau\tau$ for acceptance is to generate, with a Monte-Carlo program that corresponds closely to reality, (i.e. including events with 2 radiated photons) a lot of events, over all possible kinematical conditions i.e. all $\cos\theta^*$ c.m. and all energies of radiated photons.

This procedure is impossible for Bhabha scattering since without an explicit experimental limit the integrated cross section diverges. In order for the integrated cross section to be finite one has to exclude regions where either the e^+, or e^- is collinear with the beam or where the scattering energy goes to zero.

In practice one generates events in a region larger than the one of the experimental apparatus but small enough so as not to include the singular part of the cross section.

The detection efficiency for Bhabha events, contrary to the $\mu\mu$ or $\tau\tau$ case, is therefore defined as an acceptance of the apparatus within certain regions of $\cos\theta^*$ and acollinearity. The program BABAMC [1] is used to calculate the efficiency in this way.

The efficiency corrected events can then be used for the analysis.

- In principle this can be done by fitting the results with a full theoretical formula (s + t channel + interference) evaluated within the same $\cos\theta^*$ and acollinearity cut as the generation of Monte-Carlo events used for the efficiency correction.

  However, there remains the problem that the efficiency correction was done with a first order program BABAMC and therefore one must check carefully the contribution from events present in the data within the experimental cuts, predicted correctly by a second order program, but absent in BABAMC. Such events are for example double radiative events $e^+e^- \rightarrow e^+e^-\gamma\gamma$. This question will be addressed in a quantitative way in a later paper. However a crude correction for the main effects is discussed in section 5.2.

- Up to now in the analysis the strategy has been to evaluate the t-channel contribution within our experimental cut and to subtract it from the data, the data are then treated as $\mu\mu$ events (s-channel only) and corrected for the $\cos\theta^*$ and acollinearity cut using KORALZ [2] to obtain a total cross section.

2 t-channel evaluation with Greco

Up to recently the t-channel correction was done using the Greco [3] formula which suffers the following shortcoming: The natural cut in the Greco formula is on the energy of the photon radiated (which should be less than $\Delta E\gamma$). This is translated to a cut on acollinearity (which should be less than θ_{ACOL}) with the following formula (page 30 of reference [3]).

$$\begin{aligned}\frac{\Delta E\gamma}{\sqrt{s}} &= \left(\frac{1-\cos\theta_{ACOL}}{2}\right)^{1/2} + \cos\theta_{ACOL} - 1 \\ &= \sin\frac{\theta_{ACOL}}{2} - 2\sin^2\frac{\theta_{ACOL}}{2}\end{aligned}$$

This formula is approximately correct in the case of collinear radiation from incoming particles followed by scattering at 90° c.m. but is false at other c.m. angles (by a factor 2 at 30°). The correct formula for the case of collinear initial state radiation is:

$$\frac{\Delta E\gamma}{\sqrt{s}} = \frac{\sin\theta_{ACOL}}{\frac{\sin\theta_{ACOL}}{2} + \sin\theta + \cos\frac{\theta_{ACOL}}{2}}$$

There is no general formula that relates only $\Delta E\gamma$ and θ_{ACOL}. This is then the main reason for failure of the Greco formula and for the fact that its failure gets worse at larger value of $\cos\theta^*$. The relation between θ_{ACOL} and the final state energies is discussed for small angle Bhabha scattering in reference [4].

3 t-Channel Subtraction Using ALIBABA

A new solution to the t-channel problem is the ALIBABA program of W. Beenakker et al [5]. This provides a second order calculation of the electro-weak Bhabha scattering process.

The ALIBABA program has been run both for the 1989 and the 1990 center of mass energies, with a standard set of cuts, and parameters. These were:

- Angular range of outgoing electron: $15^\circ \le \theta^- \le 165^\circ$
- Minimum Energy of outgoing electron and positron : $E_1 > 1$GeV, $E_2 >1$GeV
- Z° mass $M_Z = 91.178$GeV
- Higgs mass $M_H = 100.0$GeV
- Top mass $M_t = 100.0$GeV
- ALIBABA in the full correction mode, i.e. not just leading log corrections.

The ALIBABA program has a rather complicated set of requirements on the angles of the outgoing electron and positron. The electron angle was constrained to lie within the region $15^\circ \leq \theta^- \leq 165^\circ$. Since the data were analyzed in bins of $\cos\theta^*$, we wanted to provide the t-channel subtraction values in the same regions. ALIBABA allows only cuts on θ^+, so we binned our values in regions of this quantity. The correction necessitated by this will be discussed below. In addition to this difficulty ALIBABA restricts the angles θ^+ and θ^- such that the angle 90° must lie between them. This somewhat complicates the procedure required to extract the numbers in bins of $\cos\theta^+$, the contents of a bin must be determined by running the calculation twice, with different angular ranges, and subtracting the difference. The ALIBABA code can also be modified to remove this restriction. The angles must also be greater than 10° (we lie well within this limit), due to increasing inaccuracy in the leading log calculations as one goes to lower angles.

The calculation can be run to provide the full s-channel + t-channel + interference term, the s-channel alone, or the t-channel alone. What we call the t-channel subtraction, consisting of the t-channel and interference parts was determined from the difference of the first two.

The data are available on the Bhabha disk (LOCCIE193, on VM). The total cross section, s-channel cross section, and the t-channel + interference term subtraction were calculated in bins of $\cos\theta^+$. Below we discuss the corrections to these values, required by our experimental cuts; the data after corrections are in the file ALIBABA CORR, on the Bhabha disk.

4 ALIBABA and Greco

In this section we present a comparison between the Greco and the ALIBABA results for the Bhabha cross sections. In table 1 the Bhabha t-channel + interference cross sections are shown in bins of $\cos\theta$ for the peak point. Note that ALIBABA gives a larger result by about 9%, and that the difference seems to be independent of the angular region. A further comparison was made between ALIBABA and Greco to observe any dependence on c.m. energy. This is shown in table 2. Again the ALIBABA numbers are consistently larger. The variation changes as one passes to energies above the Z° resonance.

5 Corrections to the ALIBABA t-Channel Results

In order to make efficiency corrections to the data a Monte-Carlo program is required. The only suitable program that exists for Bhabha scattering is BABAMC. This program, though, is only capable of generating first-order radiative events, whereas a second order calculation is required to provide the necessary precision. Our procedure was then: use BABAMC to make the efficiency corrections, run ALIBABA to get the t-channel subtraction, and finally make corrections for the differences between the ALIBABA procedure and the BABAMC Monte-Carlo scheme. In this section we describe the corrections that were made and provide the resulting "recipe" for changing the ALIBABA t-channel numbers.

Angular Region		σ_{t+I} Greco	σ_{t+I} ALIBABA
0.90	0.80	303.0	331.9
0.80	0.70	96.6	105.9
0.70	0.60	46.1	48.1
0.60	0.50	26.3	28.3
0.50	0.40	16.7	18.7
0.40	0.30	11.2	11.5
0.30	0.20	8.0	8.8
0.20	0.10	5.8	6.4
0.10	0.00	4.5	5.2
0.00	-.10	3.4	3.8
-.10	-.20	2.6	2.3
-.20	-.30	2.1	2.6
-.30	-.40	1.7	1.6
-.40	-.50	1.3	1.4
-.50	-.60	1.1	0.5
-.60	-.70	0.9	1.9
-.70	-.80	0.8	0.6
-.80	-.90	0.6	1.0

Table 1: ALIBABA and Greco cross sections for t-channel + interference terms in bins of $\cos\theta^+$. Note that these values are those directly produced from the programs, no corrections are applied. Note also that the Grecco numbers are calculated in bins of $\cos\theta^+$, whereas the ALIBABA numbers are in bins of $\cos\theta^*$. Finally the values are calculated with $M_Z = 91.19$GeV, and $\sqrt{s} = 91.22$GeV.

Energy	Greco	ALIBABA	Greco	ALIBABA
	$(-0.9 \rightarrow +0.9)$		$(-0.9 \rightarrow +0.7)$	
88.22	734.2	766.6	214.1	215.7
89.22	781.2	831.0	237.2	238.6
90.22	799.5	829.6	248.3	249.4
91.22	532.9	574.0	133.3	138.9
92.22	246.9	297.1	5.4	19.9
93.22	225.0	268.6	0.0	9.3
94.22	248.9	288.3	12.6	19.0

Table 2: ALIBABA and Greco cross sections for t-channel + interference terms as a function of c.m. energy, integrated between $(-0.9 \leq \cos\theta^* \leq +0.9)$ and $(-0.9 \leq \cos\theta^* \leq +0.7)$. The qualifications mentioned in the caption to figure 1 apply here too.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	319	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	305	4	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	278	5	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	270	4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	5	246	5	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	6	206	9	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	9	222	4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	5	207	4	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	10	218	5	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	4	206	7	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	5	191	5	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	6	196	5	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	8	266	13	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	8	300	10	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	8	340	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	12	427	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	22	618	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	1352	35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3: Table of events in bins of $\cos\theta^*, 0.9 \to -0.9$ (left to right) and $\cos\theta^+, -0.9 \to 0.9$ (top to bottom), for real data taken at the peak energy. Events migrating past the cuts at 0.9 were counted and divided by the total to produce the fractional correction factor.

5.1 Effects of Cuts on $\cos\theta^+$ vs $\cos\theta^*$

In the ALEPH analyses of lepton data a cut is made on effective cosine of the center of mass scattering angle, $\cos\theta^*$, (used because it is insensitive to the emission of collinear initial state radiation), rather than that of the lab scattering $\cos\theta^+$. The ALIBABA program implements only cuts on $\cos\theta^+$, so it was necessary to determine the effects of these different cuts on the final derived cross section. The angle θ^* is defined through:

$$\cos\theta^* = \frac{\cos\frac{1}{2}(\theta^- + \pi - \theta^+)}{\cos\frac{1}{2}(\theta^- - \pi + \theta^+)}.$$

In order to test the effects of these differences, and calculate a correction factor, the number of events passing a $\cos\theta^+$ cut was compared to the number of events passing the same $\cos\theta^*$ cut. A first test was done on the s-channel part, using 10^5 KORALZ events. The difference to the integrated cross section, between the use of $\cos\theta^+$ and $\cos\theta^*$ was found to be less than 0.015%. Then a test was done comparing Monte-Carlo events from BABAMC, ie. including the t-channel, with real data. The use of Monte-Carlo events was necessary in order to have enough statistics to provide a correction, but it was important to test that the first order simulation was consistent with data. Table 3 shows events in regions of $\cos\theta^+$ vs $\cos\theta^*$. From this we determined that, at the peak energy, for Monte-Carlo data, the number of events (as a fraction of the total number of events) passing the $\cos\theta^+$ cut, but failing the $\cos\theta^*$ cut, in the backward region $-0.9 < \cos\theta < 0.0$ was 0.00025 ± 0.00066 and in the forward region $0.0 < \cos\theta < 0.9$ was 0.0061 ± 0.0011. For real data the corresponding fractions were 0.0016 ± 0.0016 and 0.0074 ± 0.0019. The data and Monte-Carlo values are consistent; the first order Monte-Carlo is adequate for this correction, at the 0.2% level of accuracy.

This analysis was performed using Monte-Carlo data at all of the center of mass energies used in the experiment. It was found that normalizing the number of events failing the cuts to the t-channel fraction, rather than the total number of events, produced a more consistent result across the center of mass energy regions. The correction as a fraction of

Energy	Correction to t Cross Section
89.2 GeV	0.008 ± 0.001
90.2 GeV	0.012 ± 0.001
91.2 GeV	0.015 ± 0.002
92.2 GeV	0.014 ± 0.001
93.2 GeV	0.009 ± 0.001

Table 4: Correction for $\cos\theta^{+} vs \cos\theta^{*}$ cuts as a fraction of the t-channel cross section.

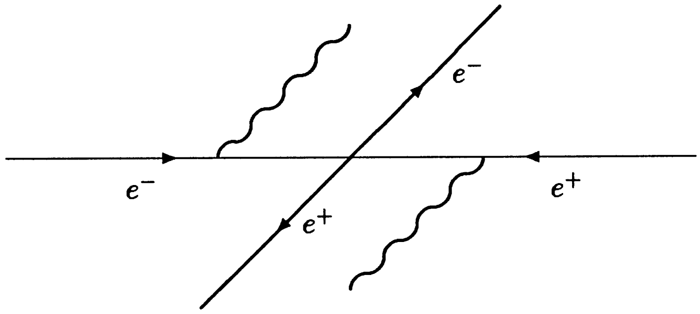

Figure 1: A double initial state bremstrahlung event.

the pure t-channel for the different beam energies is shown in table 4. To get an overall correction factor we choose 0.013 ± 0.004.

5.2 Energy Cuts

. The first order BABAMC Monte-Carlo does not include double initial state radiative events, such as that shown in figure 1. It can be shown using KORALZ that such diagrams in the s-channel do not contribute to events in the apparatus (less than 10^{-4}). This is not true in the case of t-channel scattering. This missing part of the cross section was estimated using ALIBABA. In the analysis of the data a cut was made on $\Sigma E + \Sigma p > 1.2\sqrt{s}$. In the case of double initial state radiation the photons are not seen in the apparatus, E and p are the same, the effective cut for these events is $\Sigma E > 0.6\sqrt{s}$. ALIBABA applies a minimum energy cut on each of the outgoing particles (E_+, E_-) separately (set at 1GeV in this analysis). The difference between the nature of these two sorts of cuts is shown in figure 2. It shows the ALIBABA energy cuts along the E_+ and E_- axes, and the diagonal energy cut made on the data.

To check the effects of the energy cuts and make corrections for events missed in the Monte-Carlo, we approximated the cross section in the triangular region below the experimental energy cut, by the rectangular cuts made by ALIBABA. In table 5 the cross sections for different regions of E_+, E_- are shown. These values were determined at the peak energy, integrated over an angular range of $-0.90 < \cos\theta^{+} < +0.90$.

E_1	E_2	$\sigma(t)$ (pb)
1.0	1.0	563.2
1.0	30.0	541.4
30.0	1.0	525.6
30.0	30.0	508.4
1.0	1.0	563.2
1.0	15.0	554.6
15.0	1.0	554.7
15.0	15.0	548.8

Table 5: Cross section for various minimum energy cuts. To determine the correction the sum of two middle numbers in each section should be subtracted from the sum of the first and the last number. The fact that the cross sections are not the same under exchange of E_1 and E_2 is due to their different angular regions of integration ($-0.9 \leq \cos\theta^+ \leq 0.9$ and $15° \leq \theta^- \leq 165°$). The error on the cross section values is typically 1.5

With an energy cut of $E_1 = 30$GeV and $E_2 = 30$GeV as show in figure 2 the cross section in the excluded region was found to be 4.6 ± 1.5pb, from a total t-channel cross section of 563.2pb. With energy cuts of 15GeV against 15GeV, the region missed contained 2.7 ± 1.5pb of 563.2pb, giving the fraction missed as 0.82 ± 0.3 % and 0.48 ± 0.30% respectively. Since the region cut by 15, 15 contains over half of the cross section cut by 30, 30, we conclude that the cross section contained in the small triangular regions between 30GeV and the diagonal data cut, is very small. In any case attempting to refine these calculations further hits against the limits of ALIBABA accuracy. From this we conclude that a correction of 0.8 ± 0.3% of the t-channel must be applied to our t-channel subtraction values.

Using a simple program, that generates two hard collinear photons, with a leading log approximation bremstrahlung formula, a similar effect was found with a value of 0.9%.

5.3 The Recipe

Above we have described the corrections that were performed on the ALIBABA results to make them relevant to our experimental situation. The procedure for calculating the final t-channel subtraction numbers, was as follows. Take the raw ALIBABA t-channel + interference term value, and subtract $(0.008 \pm 0.003 + 0.013 \pm 0.004)\times$ pure t-channel cross section), to correct of the energy cuts and the $\cos\theta^+$ cut. These corrections were made and the data are available. The additional error from the correction procedure is small compared to the inherent systematic error from ALIBABA.

6 Sensitivity to Variation of the Parameters

We have tested the sensitiviy of our t-channel subtraction to changes in the values of parameters supplied to the program. In particular, to the variation of the $Z°$ mass (or

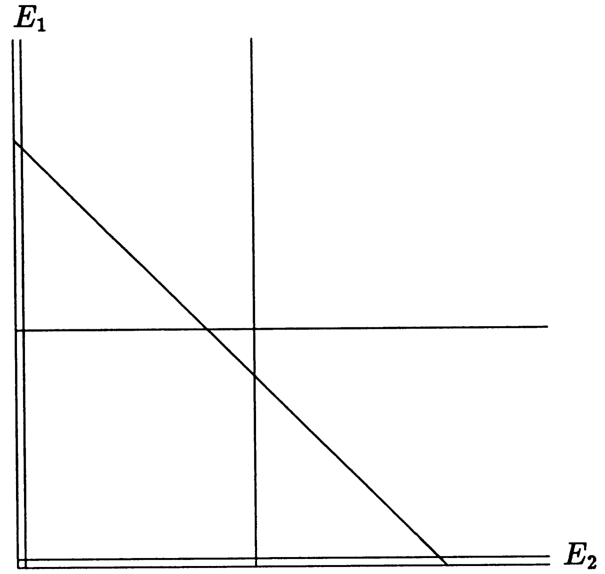

Figure 2: Energy regions for E1 vs E2 study. The horizontal and vertical lines represent the energy cuts as made in the ALIBABA program. The diagonal line is the energy cut actually made on the data. The effect of the ALIBABA cuts is approximated by determining the cross section contained in the appropriate regions set by horizontal and vertical

equivalently, a change in the beam energy) and to variations in the total and electronic widths of the Z°: $\Gamma_{tot}, \Gamma_{ee}$.

Near the Z° pole the interference part of the cross section varies rapidly, but well away from the peak this term becomes more stable. Consequently uncertainty in the knowledge of the beam energy and Z° mass has a large effect on the precision of the determination of the t-channel subtraction, near the peak. In table 6 the variation of the cross section is shown as for different values of the beam energy, at points around the peak, and furthest from the peak. As expected the sensitivity at the off peak point is negligible, but near the peak a 10MeV change in the center of mass energy or Z° mass leads to a change of 3.8pb/10MeV in the cross section integrated over the region -0.9 to $+0.9$.

The cross section is also sensitive to variations in the total and electronic widths (Γ_{tot}, Γ_{ee} of the Z°. This effect could not be checked directly in ALIBABA as the widths are calculated in the program from Standard Model constraints. Nonetheless they were estimated analytically by looking at the formula for the t-channel term from Greco: The term in t^2 is of course essentially independent of Γ_{ee} and Γ_{tot}, the interference term is linear in Γ_{ee} and is a more complicated function in Γ_{tot}. The internally used value of Γ_{ee} was 83.4MeV, whereas measurement gives 84.2MeV. The change in the interference component is then about 1%. The value for Γ_{tot} used in ALIBABA is 2.485GeV, whereas the measured value is 2.534GeV. A 1% change in Γ_{tot} gives a 1.2% change in the interference term at $\pm$1GeV. On peak these then make a negligible difference to the total cross section. At points off the peak this leads to an effect of up to 3pb (integrated over the interval -0.9 to 0.9) for each of these variations. The ALIBABA cross sections divided between their different components, for all energies, are shown in table 7.

Energy	σ_{t+I} (to 0.9)	σ_{t+I} (to 0.7)
91.241	567.0	137.6
91.231	569.9	137.3
91.221	576.7	139.5
91.211	577.9	141.7
91.201	582.1	142.7
89.240	811.2	
89.230	810.6	
89.220	810.5	
89.210	811.8	
89.200	817.2	

Table 6: t-channel plus interference cross sections (in picobarn) for variation of the beam energy around the mean value taken at that point. First set of values is integrated over the angular range -0.9 to 0.9, the second only to 0.7. The dependence on the forward region is clear. The typical error is 1.3 pb on each of the cross sections. We leave out the integration to 0.7 for the 89GeV point as the differences are negligible.

Energy	σ_{tot}	σ_s	σ_t	σ_I
88.2	958.5	191.9	601.6	165.0
89.2	1186.1	355.1	588.7	242.3
90.2	1589.5	759.9	575.7	253.9
91.2	1827.6	1253.6	564.9	9.1
92.2	1188.1	891.0	552.0	-254.9
93.2	784.9	516.3	540.4	-271.8
94.2	623.8	335.5	526.9	-238.6

Table 7: Bhabha cross sections (in picobarn) for the different center of mass energies integrated in the angular range -0.9 to 0.9. Columns show the total cross section, and the cross section divided between the various sources: s-channel, t-channel, and interference terms.

7 Conclusion

We have calculated the t-channel and interference subtractions needed for analysis of wide angle Bhabha data. The values calculated have been corrected for differences between our experimental situation and the requirements of the ALIBABA program. The results of this program have a claimed accuracy of approximately 0.5% of the total cross section. Folding in the errors on the correction factor, and we determine a total systematic error on our t-channel subtraction numbers, of 2%. In addition the sensitivity of the values determined, to uncertainties in experimentally measured quantities is about 1% of the cross section at the peak and much lower at off peak points. We still plan to conduct a further test, with a comparison of ALIBABA results with results from the EXPOSTAR program.

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