ALEPH 88-60
MINIV 88-01
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Minivertex detector alignment

Introduction

The Aleph minivertex detector will provide spatial measurements for charged tracks with a resolution of 10 μm in r-phi and 20 μm in z. It's impossible to measure by optical means the position of the silicon crystals up to this precision once the detector is installed. Moreover no mechanical solution can allow such a precision to be obtained by the construction procedure.

A method has to be developed to improve the relative alignment between different crystals within the detector and the alignment of the whole minivertex with respect to ITC and TPC. Such a method, which is an algorithm that uses track measurements, is presented here. An application of this method to the alignment between crystals using cosmic rays and minivertex data only has been explored and it's illustrated here. The results are given and interpreted showing satisfying performances of the method. A more realistic knowledge of the alignment starting conditions is needed in order to define our ultimate software alignment capability once in the pit.

The alignment procedure

In order to get full use of the minivertex detector measurement of track coordinates we must have knowledge about the position of all the crystals that form the two barrels of the detector itself. Those positions have to be known up to a degree of precision better than the spatial resolution given by the crystals, i.e. about 10 μm in r-phi and 20 μm in z. This will be achieved in three steps:

1) Position measurement after detector assembly over beam pipe
   OR alignment by construction

2) Alignment with cosmic μ
   using OR not using ITC + TPC coordinates

3) Alignment with LEP events:
   Z^o -> jets
   Z^o -> $\mu^+\mu^-$

First step: preliminary alignment

The first step gives different alignment conditions depending on the assembly procedure and detector support structure we will use. If the minivertex is assembled over the beam pipe and then inserted within the ITC the relative positions of the crystals should be measured with an Olivetti Inspector machine. This leads to an accuracy of about 1 μm that can be altered by mechanical stretching of the support during the following assembly. These distorsions can be estimated from rigidity tests of the support to be below 20 μm.

It's highly probable that we shall adopt a different assembly procedure. In this case the support structure allows for the insertion and extraction of minivertex detector in thirds after the whole ALEPH apparatus has been set up. Very preliminary tests performed on a mock-up of the mechanical support give a reproducibility of the position of these thirds within 200 μm. The stability of different crystals within the same third will be far below this figure.

Second step: alignment before LEP start

The second step can be usefully applied if the minivertex detector is installed within ALEPH before LEP collisions start, and no subsequent movement is forseen, even if a delayed insertion could be envisaged, thus preserving the crystals and electronics from possible high radiation doses due to out of tune beams in the first period of LEP operations.

Once the preliminary condition is fulfilled, one can use cosmic muons crossing the detector to improve the knowledge of each crystal's alignment constants, as demonstrated in the following, and maybe get them already within our precision specifications.

This can proceed two ways: one can either start using minivertex data only to align crystals one to each other (see "an application"), and then the whole detector with respect to the other tracking devices in a similar way to what is forseen for ITC [Forty]; or directly align single crystals to muon tracks given by a fit from ITC and TPC measurements.

It's completely non-trivial to guess which way leads to faster and/or more precise alignment, since this strongly depends on errors over minivertex, ITC and TPC alignment starting points, not well known at this moment. Simulation can solve this doubt once these conditions are given realistic figures. However the minivertex alignment software can be structured in such a way that a minimal effort is needed to switch from one solution to the other, thus providing easy adaptability to different conditions.

Third step: alignment after LEP start

After LEP start continuing to use cosmic rays for alignment purposes is not forbidden, but machine events obviously free us from statistics problems. The only difference is that single tracks coming from Z^o (or anything else) events can cross the detector in two points at most, one for each barrel, while a cosmic muon can hit four minivertex crystals. Since three points at least are needed for any alignment task, the track fit from ITC + TPC must be used in this case to align

minivertex crystals. However, back to back muons from $e^+e^- \rightarrow \mu^+\mu^-$ may be selected to give us a sample of events containing three or four collinear hits in the minivertex. This still allows for internal alignment using minivertex data only, which is a very interesting cross-check, given the high intrinsic precision of the crystals' measurement.

The alignment algorithm

The task one is confronted with is the following: for each crystal in the minivertex we know where its **nominal** position is, but its **true** position is to be determined, which differs from the nominal one by a "small" displacement.

In order to perform this operation we can use a certain number of tracks. Those tracks' impact points on that crystal can be predicted from measurements performed by some other detector and can be measured by the crystal itself. By minimizing the weighted sum of squares of the difference between the predicted and measured impact point we obtain the **reconstructed** position of the crystal, which is hopefully close to the true one.

The reference frame

We start by defining a reference frame tied to the nominal position of each silicon crystal in such a way that the origin is the geometrical centre of the crystal, the X axis is orthogonal to the crystal and directed outside, the Z axis is parallel to the ALEPH Z axis and the Y axis is defined in the right way to form a left handed orthogonal system (fig.1).

The true position of the crystal is then defined by 6 (a priori unknown) parameters. In this reference frame we have chosen these to be the translation along and the rotations around the three axes (fig. 2). This definition of the alignment parameters has the advantage that they are small quantities, and this is essential in the following to neglect higher than first order terms and get linear equations, easily solvable with standard tools.

The calculation of the alignment

Now every track can be approximated to a straight line near the plane X=0 (the crystal plane). We can choose a convenient parametrization for it:

i) $Y = A_1X + B_1$

$Z = A_2X + B_2$

The parameters A_1, A_2, B_1, B_2 are definite functions of the full helix fit to the track or any other kind of track fit however done, without the contribution of the measurement from our crystal. The point measured in the crystal for the same track is given in its own reference frame (i.e. tied to the true position). This reference frame is defined by two orthogonal axes, the U axis

and the V axis (fig. 3). Then the **measured** quantities are the coordinates u_m and v_m (see again fig. 3)

If the crystal was in its nominal position we could extrapolate the track defined by equations i) and get the **presumed** hit coordinate u_p and v_p in a trivial way:

ii) U axis = Z axis -> $u_p = B_2$

V axis = Y axis -> $v_p = B_1$

But we assume that the crystal is slightly misaligned, and we get the presumed hit coordinates as function of the (known) track parameters and the (unknown) alignment parameters. By applyng the transformation from the nominal to the true reference system we have very complicated expressions that, once reduced to first order terms in the alignment parameters, give us the following result:

iii) $u_p = B_2 + A_2 \Delta X + A_2 B_2 \psi - A_2 B_1 \theta - B_1 \phi - \Delta Z$

$v_p = B_1 + A_1 \Delta X - A_1 B_1 \theta + A_1 B_2 \psi + B_2 \phi - \Delta Y$

Given a certain number of tracks it' s possible to define a chi square, i.e.:

iv) $$\chi^2 = \sum_{TRACKS} \left[\left(\frac{u_p - u_m}{\sigma_u} \right)^2 + \left(\frac{v_p - v_m}{\sigma_v} \right)^2 \right]$$

where σ_u and σ_v are errors coming from composition of measurement (in the crystal) and extrapolation (from fit) errors. By analytically minimizing our chi square with respect to the alignment parameters we get a set of six linear equations that can be written in form of the matrix equation

v) $\mathbf{M} \bullet \underline{P} = \underline{K}$

where $\underline{P}$ is the column vector containing our alignment parameters as elements, **M** is a symmetric 6x6 matrix and $\underline{K}$ is again a 6 element column vector.

M's elements are functions of A_i, B_i, σ_u and σ_v, while $\underline{K}$'s elements are functions, in addition, of the (measured) coordinates u_m and v_m. The system is explicitely expanded in Appendix I, and can be solved with any standard tool for linear (symmetric) systems. If some weighting factor W dependent on each different track is known, and not included already in the

definition of the sigmas, it obviously can be applied by multiplying that track' s contribution to each element of **M** and $\underline{K}$.

Summary

The algorithm is then defined. First get a set of tracks crossing the crystal to be aligned, then for each track let u_m and v_m be the coordinates measured by the crystal. For this track find its equation in the form i) in the crystal frame and its projection errors, which give the sigmas once convoluted with the measurement errors. Add each **M** and $\underline{K}$ element contribution following the formulae of Appendix I, and multiply by the weight W if required.

At the end a set of alignment parameters $\underline{P}$ is obtained by solving the system v), which describes the reconstructed position of that crystal.

An application

The algorithm just described is a general method to improve the alignment of a single crystal given a set of "fitted tracks". The next logical step is to test the algorithm in a simulated framework close to one we shall meet in one of the steps of the alignment procedure previously described.

This simulation cannot be done at the moment by using the ALEPH montecarlo and reconstruction programs, because the possibility of small misalignment of the tracking detectors is not yet fully implemented. For the minivertex detector in particular, the general data structure describing misalignment of the crystals is previewed and implemented in the data base, but Galeph and Julia don' t use this information yet and everithing is described in terms of the nominal positions of the crystals. The implementation of this possibility would in fact be a waste of time while the geometrical design of the detector itself is not already fixed.

The framework

Since this is the situation, we referred to the particular case depicted as a possibility in the second step of the alignment procedure. In this case cosmic muons are used to align the crystals with respect to each other.

A montecarlo program has been written to simulate the detector with misaligned positions for the crystals, to generate cosmic muon tracks crossing it, to perform the alignment of the crystals and then to compare the reconstructed positions with the true ones.

It should be noted that, as already stated, this procedure may not be appliable, due to a delayed insertion of the detector. This exercise however has been useful to prepare, check and tune a set of subroutines that will be used for any kind of alignment of the minivertex crystals.

The program COMICS

This program starts by generating the positions of the crystals, e.g. a set of six **true** parameters for each crystal. The translation alignment parameters are defined in the ALEPH reference frame. Three possibilities are given: the crystals are left in their nominal position or the parameters are given random independent values according to gaussian distributions with given width or they are given by hand the desired displacements.

Then a loop starts over the crystals. For each crystal a statistics of N cosmic muon tracks is accumulated. Each track has suitable characteristics: first it crosses four crystals of the detector, one of which is the wanted one; then it can be generated isotropically or following a $\cos^n$(theta) law, typical of true cosmic rays [Wachsmuth]. The number N is determined accordingly: it is equal for all crystals for isotropic tracks or calculated taking into account the orientation of the crystal, its acceptance with respect to the rest of the detector, and the exponent n of the distribution law one has chosen. For each track the coordinates (after having simulated the detector response) read in the other three crystals are used to perform a least squares fit. The crystals' relative positions are determined by their reconstructed alignment parameters, which are trivially zeroes before any alignment has been performed for them. After the fit a trasformation to our crystal reference frame gives all the numbers needed to accumulate the **M** matrix and the $\underline{K}$ vector elements, as we have seen.

After N tracks the system v) is solved for our crystal and its reconstructed alignment parameters are stored (and optionally taken into account in each following use of this crystal information) and the loop continues on the other crystals.

This main loop can be repeated several times if required, each time starting from the crystal positions reconstructed in the previous iteration.

A complete printout is generated including the running conditions, the true and reconstructed parameters for each crystal, the r.m.s. values of the distribution over the crystals of the difference between the reconstructed and the true value for each parameter, a set of histograms and an optional full debug information for selected events.

After a careful study the requirement of using double precision for almost all the real variables in the program has been established.

The results

The most important output by far is constituted by the r.m.s. value defined above. We refer to this value as **the result** for each parameter.

These results are shown in figs. 4-7 as functions of the number of tracks used by the alignment program. These plots are not clearly readable at first sight and a lot of explanations are needed. First notice that the problem we have been confronted with is symmetrical for displacements along the ALEPH X axis with respect to the Y axis and for theta with respect to psi, and it' s not a surprise that the results are equal for these variables and 4 plots are sufficient for 6 parameters.

The first test has been devoted to determine the intrinsic precision of the method in this framework with respect to each parameter. The program has been run several times for different total numbers of tracks/crystal (X axis in figs. 4-7), with the following running conditions:

1) perfect starting alignment of the crystals to the nominal position;
2) isotropic tracks;
3) same number of tracks for all crystals;
4) use of nominal positions to transform coordinates;
5) alignment performed at the end of "data taking";
6) average reconstructed translation in each direction forced to 0.

In practice we tried to see how well the program would reconstruct a 0 for each parameter with uniform illumination over the crystals. The errors (r.m.s. values) obtained are displayed in figs. 4-7 as crosses. One can notice a good $1/\sqrt{N}$ behaviour as expected, since only statistical errors were present. Deviations from the correct slope for the crosses referring to low number of tracks are explained by the fact that solving the 6x6 system subtracts six degrees of freedom from the number of tracks/crystal used.

The second test has been devoted to study the alignment power of the method when an initial displacement is present, simulating the different illumination given by the angular distribution of cosmic rays on each crystal. The running conditions were the following:

1) initial errors on alignment parameters randomly extracted under gaussians of different widths; two hypotheses have been explored:

scenario n. 1	scenario n. 2
$\sigma_x = \sigma_y = \sigma_z = 20\ \mu m$	$= 200\ \mu m$
$\sigma_\theta = \sigma_\phi = \sigma_\psi = 1$ mrad	$= 10$ mrad;

2) track orientation randomly extracted following a $\cos^2(\text{theta})$ law;
3) computed number of tracks for each crystal taking into account illumination and acceptance;
4) use of reconstructed position to transform coordinates;
5) alignment performed at the end of "data taking";
6) average reconstructed translation in each direction forced to 0.

With these conditions we tried to see how well the program would reconstruct unknown alignment parameters distributed with independent errors of the same order of magnitude as previewed in the "first step" of the procedure. Notice that in this case the number of tracks/crystal used is not obviously defined. Given the kind of illumination used there is a difference of a constant factor of ~20 between the less and the more illuminated crystal. We have chosen as a convention the number of tracks crossing the less illuminated crystal to be the X axis value on the plots. This means that the horizontal axis for crosses has no direct relation with the horizontal axis for the other symbols in figs. 4-7.

After several tests, a weight of $1/\sqrt{\chi^2_{fit}}$ has been applied to each track contribution to slightly improve the results obtained. These are shown as circles in figs. 4-7, the upper line referring to the worst initial conditions, the lower one to the most favourable ones. You can immediately notice almost constant values for the errors, independent of statistics used. The fact is that the alignment of each crystal heavily uses the coordinates from very few crystals (2-4) just facing it in the other barrel. So the reconstructed parameters for each crystal are strongly influenced by the difference between true and nominal alignment parameters, averaged over very few crystals, and this becomes the most important source of error by far. This is confirmed by the fact that lower and upper series of circles show an identical behaviour. In order to reduce this source of error we tried a slightly different strategy.

The third test follows exactly the same conditions used for the second one apart from the fact that the alignment (the solution of the system) has not been performed just once at the end of the "data taking". It has been performed several times instead, using an illumination/crystal equivalent to 50 tracks/less-illuminated-crystal each time. The number 50 has been decided because the behaviour of the previous curves (circles) indicates that there is no advantage in using more than 50 tracks at a time for the precisions obtained. The results can be seen as full dots in figs. 4-7.

A dramatic improvement can be noticed, in particular for the angular parameters and the worst initial conditions. Starting from the best initial conditions one soon reaches a constant value, given by the statistical error coming from the fixed number of tracks/iteration corresponding to 50 tracks/less-illuminated-crystal. In this case a choice of 100 would have lead to even better results.

Discussion of the results

The first logical objection to the plots in figs. 4-7 is the use of an horizontal scale with different meaning for the two kind of results. This has been unavoidable since uniform illumination is required to check the constant term and slope of the first test (crosses), a relevant tool in ensuring that no systematic effect or loss of precision is hidden in the program. On the contrary more realistic oriented applications (circles and dots) require a $\cos^2$(theta) cosmic ray

angular distribution.

Even if this is true, another plot showing the precision of the method with the non-uniform illumination would have provided more comparable figures in weighting the relative relevance of the various sources of errors.

In evaluating the results one must consider that the choice of independent gaussian errors on alignment parameters as starting condition is not realistic. In the most probable hardware configuration only different thirds of the detector could be relatively misaligned of (say) 200 μm. The construction procedure put severe constraints on several relative displacements that can only help in approaching the true alignment parameters. As an example crystals will be mounted glued in pairs on alumina boards, and the relative distance between each pair is measured with excellent resolution before mounting, and cannot change anymore. Extra improvements in the results may derive from a careful study of adequate stepping for the succesive alignment iterations.

A test has confirmed that always using the same 50 tracks for iterative alignment (full dots) doesn' t make the results worse. This study has not been extended to all the configurations in order not to consume excessive CPU time, since is very reasonable that the identity of the sample used doesn' t matter when the statistical error is not the dominating one.

Given realistic values to the expected cosmic rate at ALEPH, the percentage of muons of high enough momentum to neglect multiple scattering, the percentage of non-showering muons, the cosmic trigger efficiency, the dead time of the electronics and the reconstruction efficiency, 50 tracks/less-illuminated-detector mean an acquisition time of little more than one month.

It' s not a very long (nor very short) period, but this seems enough to lower by a substantial factor the uncertainties over crystal alignment parameters, even under the very pessimistic assumptions we' ve made. This is enough for an application that, as already underlined, is more an exercise with tools we can utilize at the moment than a definitive statement on our alignment capability.

In the meanwhile, what we obtained demonstrates that the statistics problems will disappear when Z^o events start to be observed, and few days of data taking will surely permit a stable alignment between crystals and with respect to the other ALEPH tracking devices.

The results from this first attempt allow us to feel confident that a software alignment of the minivertex detector within the specifications given by its own resolution is well within reach, either before (if possible), or soon after LEP operations start.

References:

1) L.Bosisio et al. - ALEPH NOTE 84/130
2) R.W.Forty - ALEPH 87-91 NOTE 87-15
3) H.Wachsmuth - ALEPH 87-64 NOTE 87-12

APPENDIX I

the matrix equation is $M \cdot \vec{P} = \vec{K}$. In the following expansion a sum over tracks is implied for M and $\vec{K}$ elements.

$$
\begin{bmatrix}
\left(\frac{A_1^2}{\sigma_v^2}+\frac{A_2^2}{\sigma_u^2}\right) & \left(-\frac{A_1}{\sigma_v^2}\right) & \left(-\frac{A_2}{\sigma_u^2}\right) & \left(-\frac{A_1^2B_1}{\sigma_v^2}-\frac{A_2^2B_1}{\sigma_u^2}\right) & \left(\frac{A_1B_2}{\sigma_v^2}-\frac{A_2B_1}{\sigma_u^2}\right) & \left(\frac{A_1^2B_2}{\sigma_v^2}+\frac{A_2^2B_2}{\sigma_u^2}\right) \\
\left(-\frac{A_1}{\sigma_v^2}\right) & \left(\frac{1}{\sigma_v^2}\right) & \emptyset & \left(\frac{A_1B_1}{\sigma_v^2}\right) & \left(-\frac{B_2}{\sigma_v^2}\right) & \left(-\frac{A_1B_2}{\sigma_v^2}\right) \\
\left(-\frac{A_2}{\sigma_u^2}\right) & \emptyset & \left(\frac{1}{\sigma_u^2}\right) & \left(\frac{A_2B_1}{\sigma_u^2}\right) & \left(\frac{B_1}{\sigma_u^2}\right) & \left(-\frac{A_2B_2}{\sigma_u^2}\right) \\
\left(-\frac{A_1^2B_1}{\sigma_v^2}-\frac{A_2^2B_1}{\sigma_u^2}\right) & \left(\frac{A_1B_1}{\sigma_v^2}\right) & \left(\frac{A_2B_1}{\sigma_u^2}\right) & \left(\frac{A_1^2B_1^2}{\sigma_v^2}+\frac{A_2^2B_1^2}{\sigma_u^2}\right) & \left(-\frac{A_1B_1B_2}{\sigma_v^2}+\frac{A_2B_1^2}{\sigma_u^2}\right) & \left(-\frac{A_1^2B_1B_2}{\sigma_v^2}-\frac{A_2^2B_1B_2}{\sigma_v^2}\right) \\
\left(\frac{A_1B_2}{\sigma_v^2}-\frac{A_2B_1}{\sigma_u^2}\right) & \left(-\frac{B_2}{\sigma_v^2}\right) & \left(\frac{B_1}{\sigma_u^2}\right) & \left(-\frac{A_1B_1B_2}{\sigma_v^2}+\frac{A_2B_1^2}{\sigma_u^2}\right) & \left(\frac{B_2^2}{\sigma_v^2}+\frac{B_1^2}{\sigma_u^2}\right) & \left(\frac{A_1B_2^2}{\sigma_v^2}-\frac{A_2^2B_1B_2}{\sigma_u^2}\right) \\
\left(\frac{A_1^2B_2}{\sigma_v^2}+\frac{A_2^2B_2}{\sigma_u^2}\right) & \left(-\frac{A_1B_2}{\sigma_v^2}\right) & \left(-\frac{A_2B_2}{\sigma_u^2}\right) & \left(-\frac{A_1^2B_1B_2}{\sigma_v^2}-\frac{A_2^2B_1B_2}{\sigma_u^2}\right) & \left(\frac{A_1B_2^2}{\sigma_v^2}-\frac{A_2^2B_1B_2}{\sigma_u^2}\right) & \left(\frac{A_1^2B_2^2}{\sigma_v^2}+\frac{A_2^2B_2^2}{\sigma_u^2}\right)
\end{bmatrix}
$$

the matrix M

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \theta \\ \varphi \\ \psi \end{bmatrix} \qquad \begin{bmatrix} \left(-\frac{A_1(V_m-B_1)}{\sigma_v^2} - \frac{A_2(U_m-B_2)}{\sigma_u^2}\right) \\ \left(\frac{V_m-B_1}{\sigma_v^2}\right) \\ \left(\frac{U_m-B_2}{\sigma_u^2}\right) \\ \left(\frac{A_1B_1(V_m-B_1)}{\sigma_v^2} + \frac{A_2B_1(U_m-B_2)}{\sigma_u^2}\right) \\ \left(\frac{B_2(V_m-B_1)}{\sigma_v^2} - \frac{B_1(U_m-B_2)}{\sigma_u^2}\right) \\ \left(-\frac{A_1B_2(V_m-B_1)}{\sigma_v^2} - \frac{A_2B_2(U_m-B_2)}{\sigma_u^2}\right) \end{bmatrix}$$

the vector $\vec{P}$ the vector $\vec{K}$

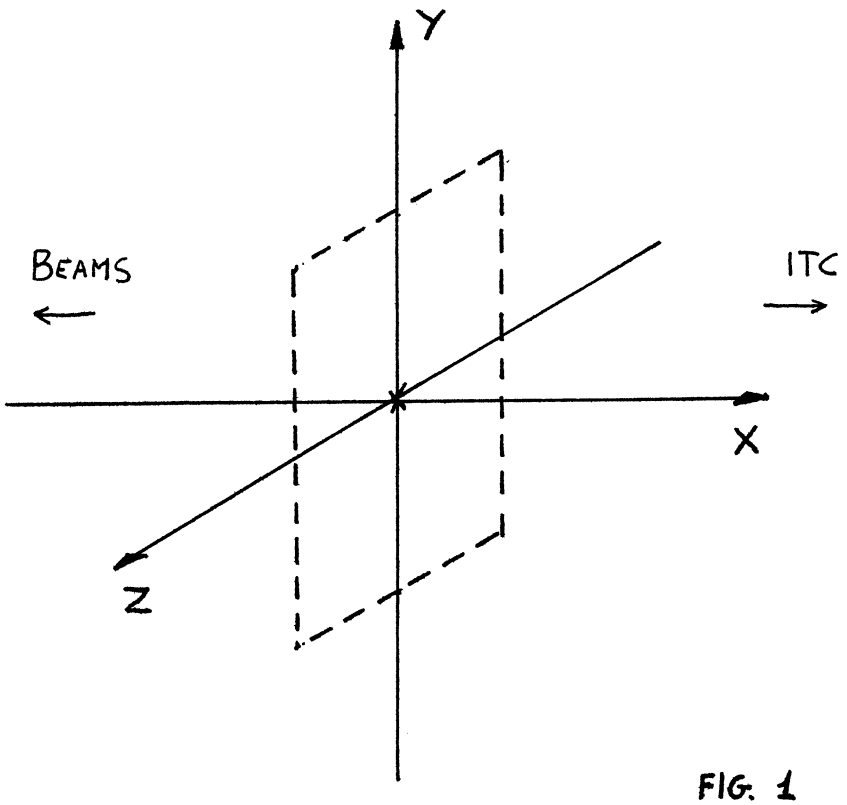
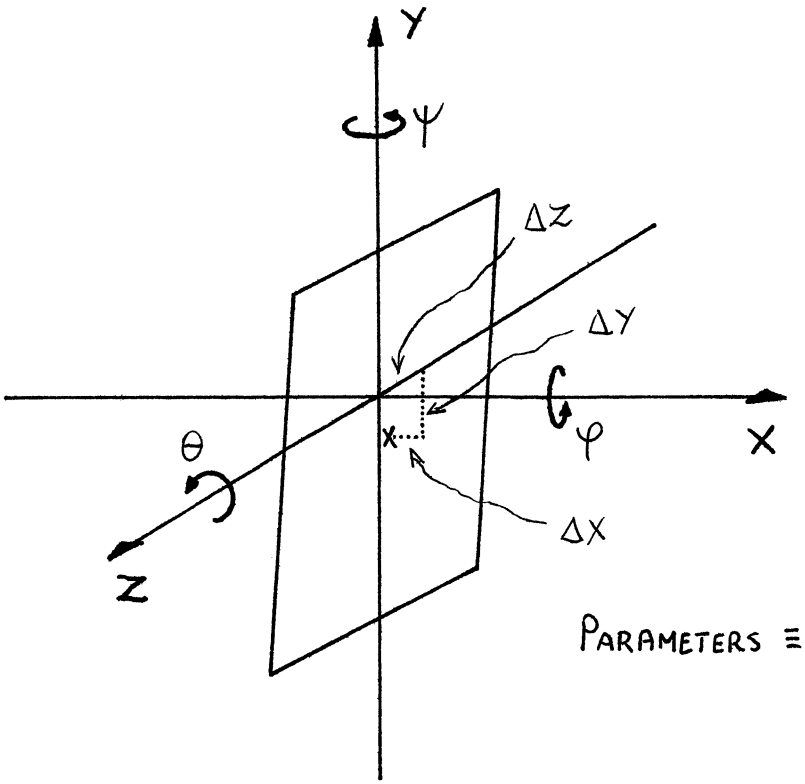


FIG. 1



PARAMETERS ≡ $\{\Delta X, \Delta Y, \Delta Z, \theta, \varphi, \psi\}$

FIG. 2

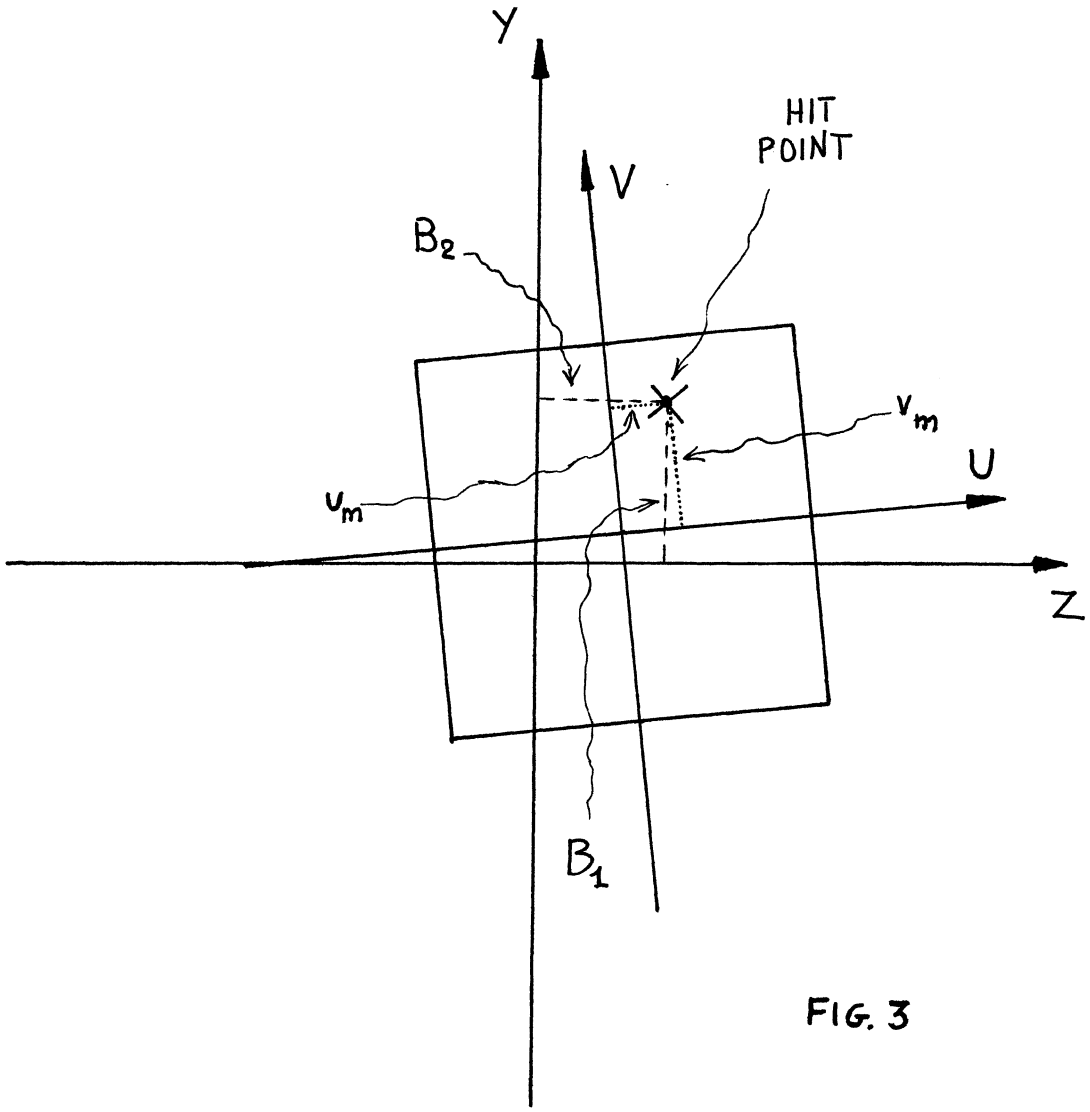
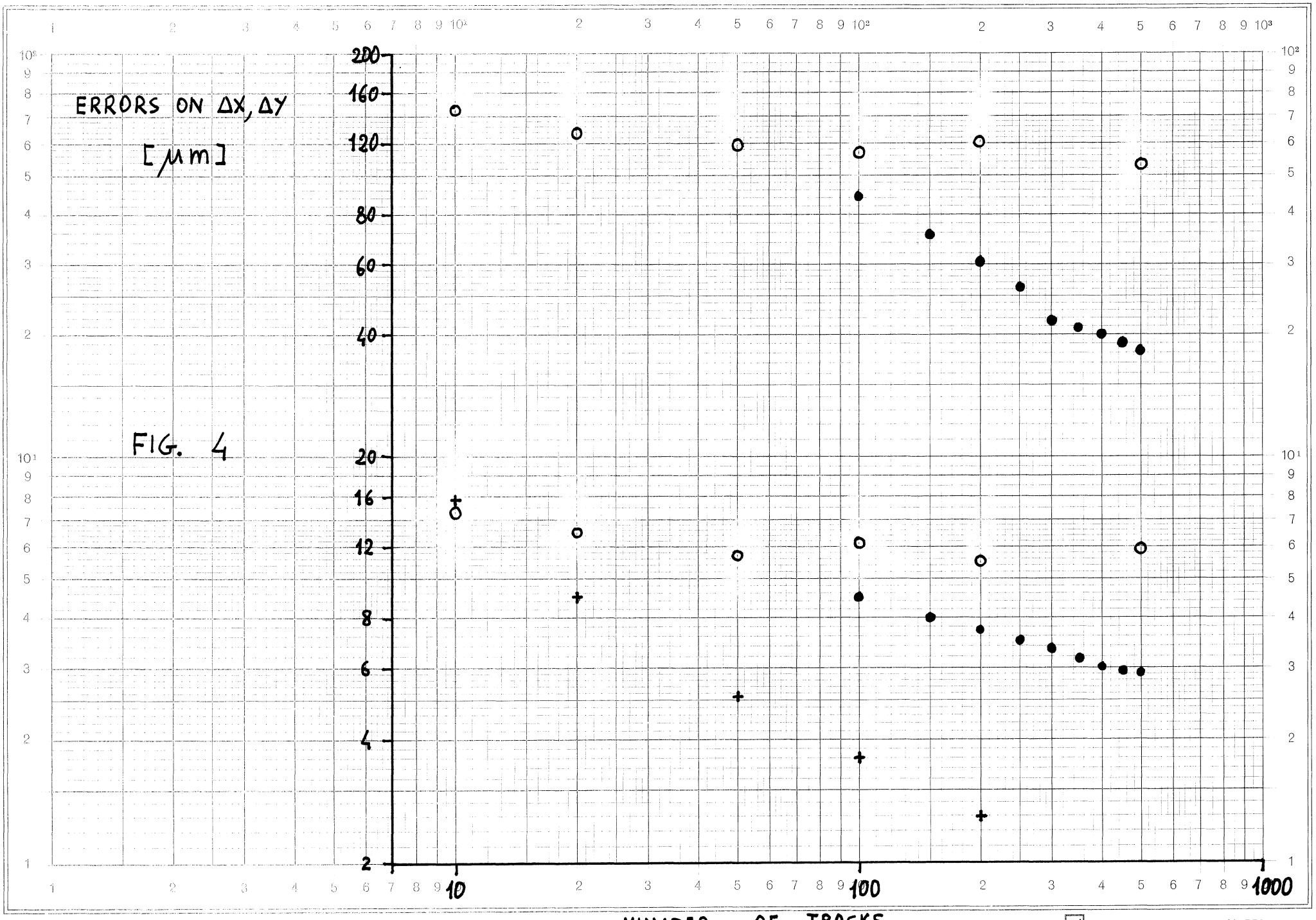


FIG. 3



Logar. Teilung / Division 1-100 und 1-1000 Einheit / Unité 82,33 mm

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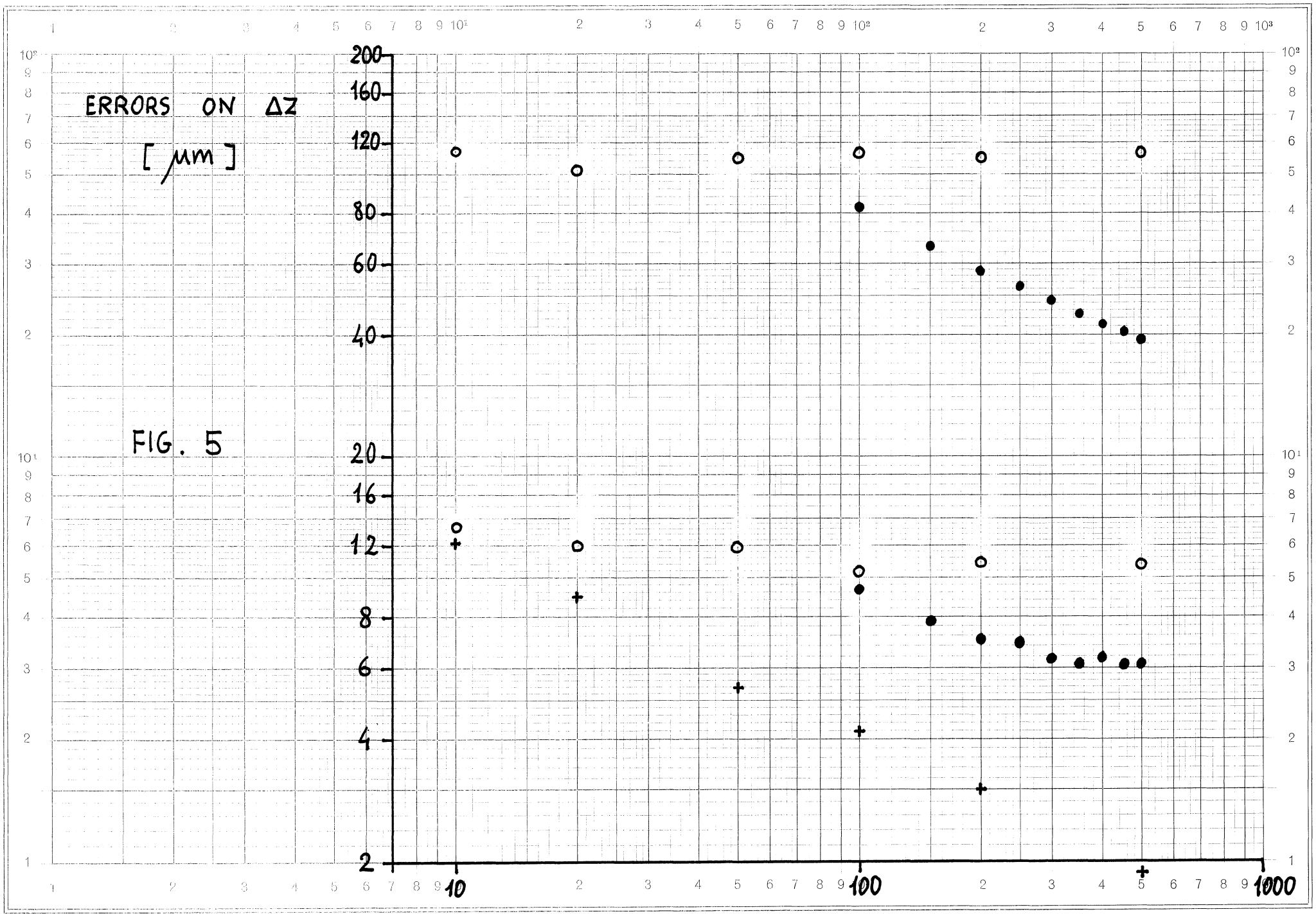

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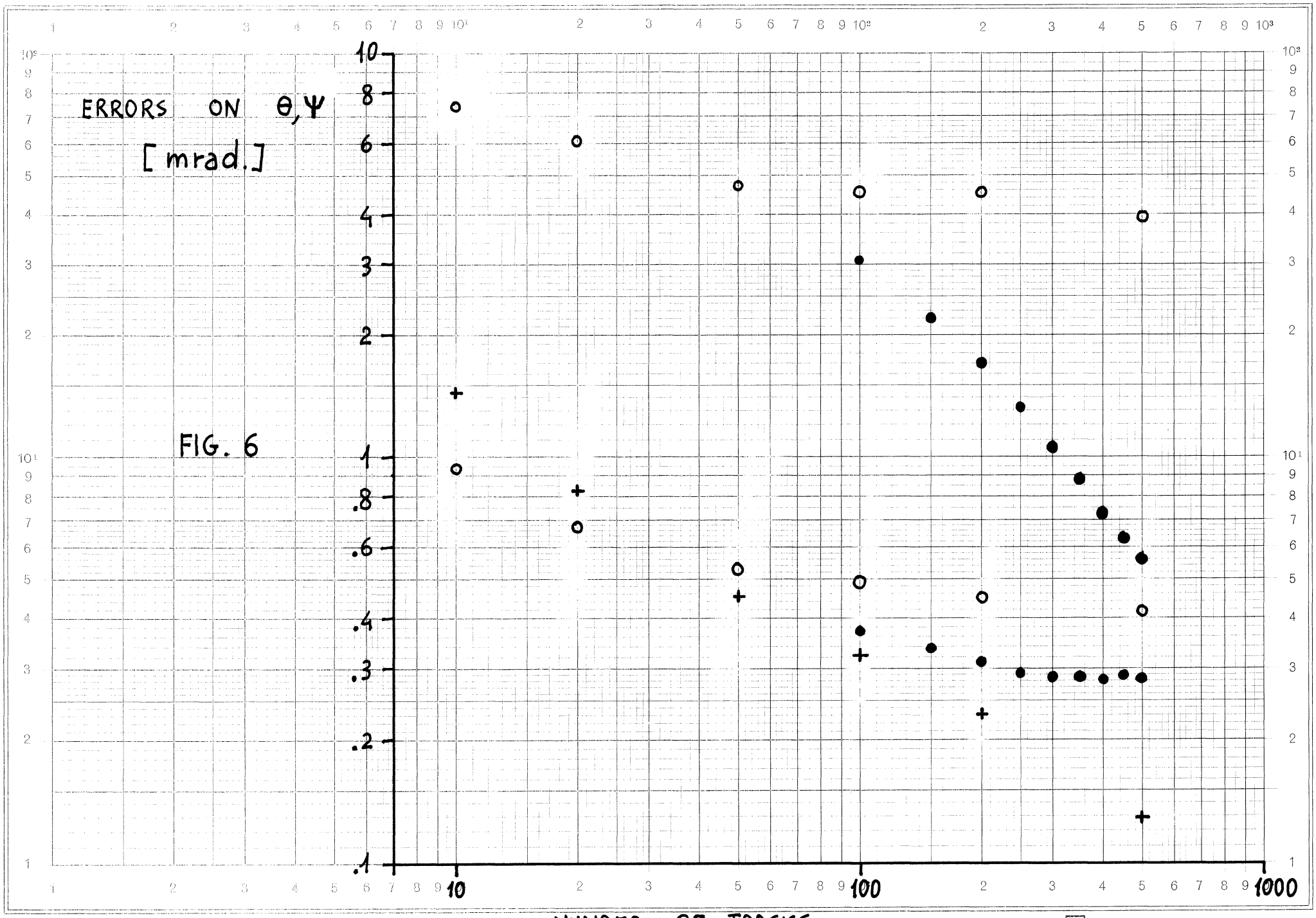

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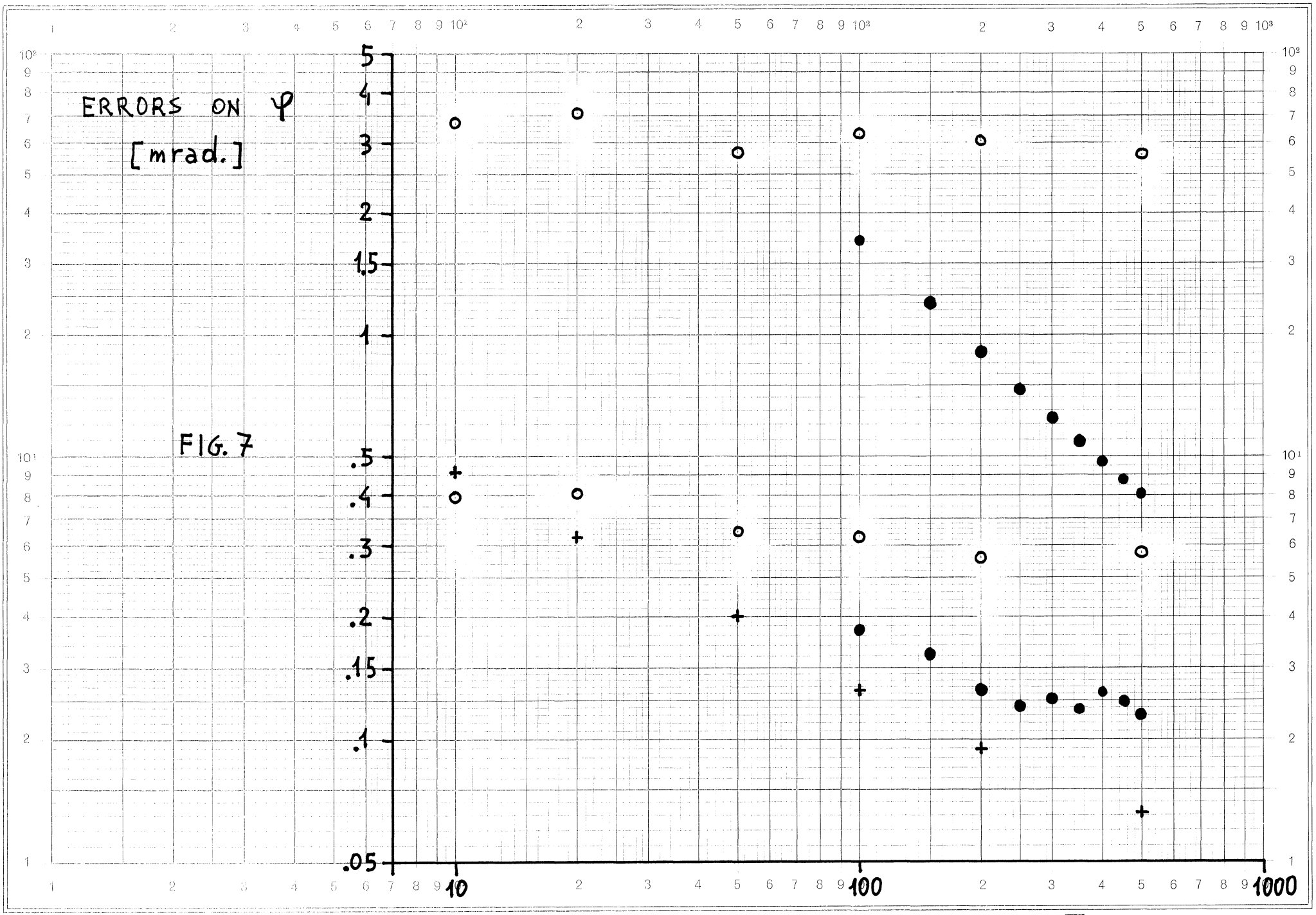

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