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k-string tensions in the 4-d SU(N)-inspired dual Abelian–Higgs-type theory

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Abstract: The k-string tensions are explored in the 4-d $[\mathrm{U}(1)]^{N-1}$-invariant dual Abelian–Higgs-type theory. In the London limit of this theory, the Casimir scaling is found in the approximation when small-sized closed dual strings are disregarded. When these strings are treated in the dilute-plasma approximation, explicit corrections to the Casimir scaling are found. The leading correction due to the deviation from the London limit is also derived. Its N-ality dependence turns out to be the same as that of the first non-trivial correction produced by closed strings. It also turns out that this N-ality dependence coincides with that of the leading correction to the k-string tension, which emerges by way of the non-diluteness of the monopole plasma in the 3-d SU(N) Georgi–Glashow model. Finally, we prove that, in the latter model, Casimir scaling holds even at monopole densities close to the mean one, provided the string world sheet is flat.

Keywords: Lattice Gauge Field Theories; Phenomenological Models; Nonperturbative Effects; Confinement.

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Contents

1. Introduction

The spectrum of k-string tensions has been extensively explored in recent years, both on the lattice [1]–[7] and in the continuum limit [8]–[13]. A k-string is a string joining k quarks with k antiquarks. Alternatively, it can be defined as a string between sources that carry charge k (N-ality) with respect to the center group, Z_N, of the original SU(N) group. The knowledge of the tensions of k-strings, σ_k, is likely to shed some light on the dynamics of confinement by providing some new "phenomenological" input that models of confinement have to satisfy, see e.g. [14, 15].

Some properties of the k-string spectrum follow directly from first principles. The ratio of σ_k to the tension of the fundamental string, σ_1, in an SU(N) pure gauge theory is invariant under the interchange of quarks and antiquarks, $k \leftrightarrow (N-k)$. Moreover, in the large-N limit, the interactions between fundamental strings inside the composite k-string are suppressed, it is therefore expected that in this limit their spectrum fulfils the condition $\frac{\sigma_k}{\sigma_1} \overset{k-\text{fixed}}{\underset{N\to\infty}{\longrightarrow}} k$. Finally, the corrections to this large-N limit are expected to appear in powers of $1/N^2$ [12].

Analytical calculations in supersymmetric theories [16, 11], or in some approximations to Yang–Mills theories based on the string/gauge duality [17, 18], yield the so-called sine scaling for the k-string spectrum: $\frac{\sigma_k}{\sigma_1} = \frac{\sin(k\pi/N)}{\sin(\pi/N)}$. In two dimensions, one can show that the string tension ratio obeys an exact Casimir scaling: $\frac{\sigma_k}{\sigma_1} = \frac{k(N-k)}{N-1}$. The same result is obtained for the four-dimensional pure gauge theory, when the computations are based either on dimensional reduction [19] or on the low orders of some perturbative expansion; see e.g. Ref. [6] for the k-string spectrum in the strong coupling Hamiltonian formulation. Casimir scaling also appears in some models of the QCD vacuum, such as the stochastic vacuum model of QCD [8, 9] and the $[\mathrm{U}(1)]^{N-1}$ gauge-invariant Abelian-projected theory [10]. In 3-d, Casimir scaling with certain corrections has been found in Ref. [13].

It is important to stress that no analytical computation of the k-string spectrum has been done so far that could be directly applied to the case of pure gauge theory at weak coupling; neither the sine nor the Casimir scaling can be considered as *exact* results in non-supersymmetric Yang–Mills theories. While both formulae yield the correct limit as $N \to \infty$ at fixed k; at subleading order, the sine scaling shows the expected $1/N^2$ corrections, whereas the Casimir scaling leads to corrections in powers of $1/N$, thus contradicting the results of Ref. [12]. Lattice results show that both formulae give an approximate but satisfactory description of the numerical data. Different lattice simulations display some discrepancy in the spectrum, which are likely to arise from systematic errors in the computation of the string tension. At the current level of accuracy, lattice data do not allow a clear-cut distinction between the two behaviors; of course there is no theoretical reason to expect any of the two to be exact, and it would be more interesting if lattice results could become accurate enough for one to be able to pinpoint the parametric behavior of the subleading corrections to the large-N limit.

This paper parallels in its spirit Ref. [13], since it also deals with a confining Abelian-type theory, although a 4-d one. More specifically, we explore in this work k-string tensions in the 4-d $[\mathrm{U}(1)]^{N-1}$ gauge-invariant Abelian-projected theory, which is formulated in terms of dual magnetic Abelian gauge fields, neglecting the off-diagonal degrees of freedom. Unlike Ref. [10], where this was done in the Bogomol'nyi limit and on the basis of the analysis of the classical string solutions, here we will consider the model in the London limit (and in its vicinity), which corresponds to an extreme type-II dual superconductor; the k-string tensions will be derived from the string representation of the partition function. Such a representation means a reformulation of the theory in terms of the path integral over closed dual strings, which are always present in the theory and interact with the external k-string. As a brief historical remark, let us mention that, for the usual (dual) Abelian Higgs model, the reformulation of the partition function in terms of closed strings has been performed by many authors (e.g. in Ref. [20] in 4-d and in Ref. [21] in 3-d), in particular with the applications to the stochastic vacuum model [22, 23]; the Jacobian of the transformation from field to string variables has been discussed in Ref. [24]; the SU(N) generalization has been explored, in particular for studies of the θ-term [25] [see Ref. [26] for investigations of the θ-term in the SU(2)- and SU(3)-inspired cases] and for the purposes of further applications to the stochastic vacuum model at $N = 3$ [27, 29] [see also [28] for related studies at $N = 3$]; the corrections emerging in the vicinity of the London limit have also been explored [30].

The important fact is that closed strings are short-lived (virtual) objects [31], whose typical sizes are much smaller than both distances between them and size of the k-string world sheet. Therefore, since world-sheet tensors (also called vorticity tensor currents) of closed strings enter the final action in the linear combinations with the world-sheet tensor of the k-string, in the leading semi-classical approximation the interaction of closed strings with the k-string can be merely disregarded. This is precisely the approximation in which the SU(2)- and SU(3)-inspired models have been considered in Refs. [22, 23] and [27], respectively. This approximation can be further improved by treating closed strings in the dilute-plasma approximation [29]. In this paper, specifically in Section 5, we will also account for effects produced by the dilute plasma of closed dual strings.

The sketch of the paper is as follows. In the next section, the model under study will be described. In Section 3, σ_k will be derived in the London limit. In Section 4, we will explore corrections to σ_k, which emerge in the vicinity of the London limit, i.e. when Higgs bosons are not infinitely heavy. In Section 5, we will return to the London limit and consider another type of corrections, namely those produced by closed dual strings. In Section 6, the main results of the paper will be discussed once again. In Appendix A, some estimates related to the main part of the text will be performed. Finally, Appendix B is devoted to the 3-d SU(N) Georgi-Glashow model; it is complimentary to Ref. [13], where Casimir scaling in this model has been proved for an arbitrarily shaped surface, but at monopole densities much lower than the mean one. Here we prove that Casimir scaling holds already at the mean density, provided the surface is flat.

2. The model

The model we are going to deal with is the generalization of the SU(3)-inspired dual Abelian–Higgs-type theory [32] to the case of arbitrary N [10, 25]. The monopole condensation is modelled in it by the assumption that monopoles form condensates of the dual Higgs fields. This model can naturally be called an effective $[\mathrm{U}(1)]^{N-1}$ gauge-invariant Abelian-projected theory. Its Euclidean partition function reads:

$$\mathcal{Z}_k = \int \left(\prod_i |\Phi_i| \mathcal{D}|\Phi_i| \mathcal{D}\theta_i\right) \mathcal{D}\mathbf{B}_\mu \delta\left(\sum_i \theta_i\right) \exp\left\{-\int d^4x \left[\frac{1}{4}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^k_{\mu\nu}\right)^2 + \right.\right.$$
$$\left.\left. + \sum_i \left[\left|\left(\partial_\mu - ig_m \mathbf{q}_i \mathbf{B}_\mu\right)\Phi_i\right|^2 + \lambda\left(|\Phi_i|^2 - \eta^2\right)^2\right]\right]\right\}. \qquad (2.1)$$

Here, the index i runs from 1 to the number of positive roots $\mathbf{q}_i$ of the SU(N)-group, that is $N(N-1)/2$. Next, g_m is the magnetic coupling constant related to the electric one, g, by means of the Dirac quantization condition $g_m g = 4\pi n$. In what follows, we will for simplicity restrict ourselves to the monopoles possessing the minimal charge only, i.e. set $n = 1$, although the generalization to an arbitrary n is straightforward. Note that the origin of root vectors in eq. (2.1) is the fact that monopole charges are distributed along them. Further, $\Phi_i = |\Phi_i| \mathrm{e}^{i\theta_i}$ are the dual Higgs fields, which describe the condensates of monopoles, and $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$ is the field-strength tensor of the $(N-1)$-component "magnetic" potential $\mathbf{B}_\mu$. The latter is dual to the "electric" potential, whose components are diagonal gluons. Since the SU(N)-group is special, the phases θ_i of the dual Higgs fields are related to each other by the constraint $\sum\limits_i \theta_i = 0$, which is imposed by introducing the corresponding δ-function into the r.h.s. of eq. (2.1). Next, $\tilde{\mathcal{O}}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\mathcal{O}_{\lambda\rho}$, and $\mathbf{F}^k_{\mu\nu}$ is the field-strength tensor of k test quarks with colors a_i, which move along a contour C. This tensor obeys the equation $\partial_\mu \tilde{\mathbf{F}}^k_{\mu\nu} = g\mathbf{M}_k j_\nu$, where $\mathbf{M}_k \equiv \sum\limits_{i=1}^{k} \mathbf{m}_{a_i}$, $j_\mu(x) = \oint\limits_C dx_\mu(\tau)\delta(x - x(\tau))$, $\mathbf{m}_{a_i}$ is a weight vector of the group SU(N), and a_i may take values $1,\ldots,N$. Thus $\mathbf{F}^k_{\mu\nu} = -g\mathbf{M}_k\tilde{\Sigma}_{\mu\nu}$, where $\Sigma_{\mu\nu}(x) = \int\limits_\Sigma d\sigma_{\mu\nu}(x(\xi))\delta(x - x(\xi))$ is the vorticity tensor current associated with the world-sheet Σ of the open electric string, bounded by the contour C. From now on, we will omit the normalization constant in front of all the functional integrals, implying for any k the normalization condition $\mathcal{Z}_k\,[C=0] = 1$.

Next, the phases of the dual Higgs fields can be decomposed into multivalued and single-valued (also called singular and regular, respectively) parts, $\theta_i = \theta_i^{\rm sing} + \theta_i^{\rm reg}$. The fields $\theta_i^{\rm sing}$ describing closed dual strings are related to the world-sheets Σ_i of these strings by means of the equation

$$\varepsilon_{\mu\nu\lambda\rho}\partial_\lambda\partial_\rho\theta_i^{\rm sing}(x) = 2\pi\Sigma^i_{\mu\nu}(x) \equiv 2\pi \int\limits_{\Sigma_i} d\sigma_{\mu\nu}\left(x^{(i)}(\xi)\right)\delta\left(x - x^{(i)}(\xi)\right). \tag{2.2}$$

This equation is the covariant formulation of the 4-d analogue of the Stokes' theorem for $\partial_\mu\theta_i$, written in the local form. In eq. (2.2), $x^{(i)}(\xi) \equiv x^{(i)}_\mu(\xi)$ is a vector, that parameterizes the world-sheet Σ_i with $\xi = (\xi^1, \xi^2)$ standing for the 2-d coordinate. As far as the regular parts of the phases, $\theta_i^{\rm reg}$, are concerned, these describe single-valued fluctuations around closed strings, which are described by $\theta_i^{\rm sing}$. Note that, owing to the one-to-one correspondence between $\theta_i^{\rm sing}$ and Σ_i, established by eq. (2.2), the integration over $\theta_i^{\rm sing}$ is implied in the sense of a certain prescription of the summation over world-sheets of closed strings. One of such prescriptions, corresponding to the dilute plasma of closed strings, will be considered in Section 5. Further, by virtue of eq. (2.2), it is also possible to demonstrate that the integration measure $\mathcal{D}\theta_i$ becomes factorized into the product $\mathcal{D}\theta_i^{\rm sing}\mathcal{D}\theta_i^{\rm reg}$. Apparently, the constraint imposed by the δ-function $\delta\left(\sum\limits_i \theta_i\right)$ becomes also split as $\delta\left(\sum\limits_i \theta_i^{\rm sing}\right)\delta\left(\sum\limits_i \theta_i^{\rm reg}\right)$.

Let us further expand $|\Phi_i|$ in eq. (2.1) as $|\Phi_i| = \eta + \frac{\varphi_i}{\sqrt{2}}$ and perform the gauge transformation $g_m\mathbf{q}_i\mathbf{B}^{\rm new}_\mu = g_m\mathbf{q}_i\mathbf{B}_\mu - \partial_\mu\theta_i$, noticing that, according to eq. (2.2), $(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\theta_i^{\rm sing} = 2\pi\tilde{\Sigma}^i_{\mu\nu}$. The constraint $\sum\limits_i \theta_i^{\rm sing} = 0$ leads to the constraint for world-sheets of closed strings, which can be imposed by the δ-function $\delta\left(\sum\limits_i \Sigma^i_{\mu\nu}\right)$. Instead, the constraint imposed by $\delta\left(\sum\limits_i \theta_i^{\rm reg}\right)$ can be lifted to the exponent upon the introduction of some Lagrange multiplier. By virtue of the fact that $\sum\limits_i \mathbf{q}_i = 0$, the integration over this multiplier can be shown [27] to eventually result in an inessential overall constant factor. If we further use the orthonormality of roots,

$$\sum_i q_i^\alpha q_i^\beta = \frac{N}{2}\delta^{\alpha\beta}, \tag{2.3}$$

where $\alpha, \beta = 1, \ldots, N-1$, we obtain the following Lagrangian

$$\mathcal{L}_k = \frac{1}{2Ng_m^2}\sum_i \left[\mathbf{q}_i\left(g_m\mathbf{F}_{\mu\nu} - 4\pi\mathbf{M}_k\tilde{\Sigma}_{\mu\nu}\right) + 2\pi\tilde{\Sigma}^i_{\mu\nu}\right]^2 + \frac{m^2}{2}\mathbf{B}_\mu^2 +$$

$$+\sum_i \left[\frac{1}{2}(\partial_\mu\varphi_i)^2 + \frac{m_H^2}{2}\varphi_i^2 + \sqrt{2}g_m^2\eta\varphi_i(\mathbf{q}_i\mathbf{B}_\mu)^2\right]. \tag{2.4}$$

The masses of the dual vector boson and the dual Higgs field here read $m = g_m\eta\sqrt{N}$ and $m_H = 2\eta\sqrt{\lambda}$.

3. London limit

In terms of the Landau–Ginzburg parameter $\kappa \equiv \frac{m_H}{m}$, the London limit (LL) is defined as $\ln\kappa \gg 1$. Let us now proceed with the string representation of the model (2.4) in this limit. Notice that, since we would like our model to be consistent with QCD, we must have $g = \sqrt{\bar{\lambda}/N}$, where $\bar{\lambda}$ remains finite in the large-N limit. The definition of the London limit then yields

$$\lambda \gg \frac{(2\pi \mathrm{e} N)^2}{\bar{\lambda}}. \tag{3.1}$$

In general, in order to have confinement (i.e. the type-II dual superconductivity) in the large-N limit, one should demand that λ grows with N at least as $\mathcal{O}(N^2)$. If λ grows with N faster than N^2, $\lambda = \mathcal{O}(N^{2+\epsilon})$ where $\epsilon > 0$, κ grows with N too, $\kappa = \mathcal{O}(N^{\epsilon/2})$, making the London limit deeper. In what follows, we will adopt the minimal requirement, $\lambda = \mathcal{O}(N^2)$, necessary for confinement in the large-N limit of the model (2.1). As we have seen, only in this case, κ is N-independent.

In the London limit, the partition function of our model has the form

$$\mathcal{Z}_k^{\mathrm{LL}} = \int\left(\prod_i \left[d\Sigma^i_{\mu\nu}\right]\right)\delta\left(\sum_i \Sigma^i_{\mu\nu}\right)\int \mathcal{D}\mathbf{B}_\mu \exp\left(-\int d^4x\mathcal{L}_k^{\mathrm{LL}}\right), \tag{3.2}$$

where the Lagrangian reads

$$\mathcal{L}_k^{\mathrm{LL}} = \frac{1}{2Ng_m^2}\sum_i \left[\mathbf{q}_i\left(g_m\mathbf{F}_{\mu\nu} - 4\pi\mathbf{M}_k\tilde{\Sigma}_{\mu\nu}\right) + 2\pi\tilde{\Sigma}^i_{\mu\nu}\right]^2 + \frac{m^2}{2}\mathbf{B}_\mu^2.$$

The symbol $\left[d\Sigma^i_{\mu\nu}\right]$ in eq. (3.2) is a formal expression for the sum over string world-sheets, whose concrete form will be specified below.

A natural way to satisfy the constraint $\sum\limits_i \Sigma^i_{\mu\nu} = 0$ is to set $\Sigma^i_{\mu\nu} = \mathbf{q}_i\mathbf{S}_{\mu\nu}$, where $\mathbf{S}_{\mu\nu}$ are then no longer subject to any constraint. The Lagrangian then takes the form

$$\mathcal{L}_k^{\rm LL} = \frac{1}{4g_m^2}\left(g_m \mathbf{F}_{\mu\nu} - 4\pi \mathbf{M}_k \tilde{\Sigma}_{\mu\nu} + 2\pi \tilde{\mathbf{S}}_{\mu\nu}\right)^2 + \frac{m^2}{2}\mathbf{B}_\mu^2.$$

To perform the integration over $\mathbf{B}_\mu$, let us linearize the squares in this Lagrangian upon the introduction of two auxiliary fields, $\mathbf{h}_{\mu\nu}$ and $\mathbf{k}_\mu$, as follows:

$$\mathcal{L}_k^{\rm LL} = \frac{1}{4}\mathbf{h}_{\mu\nu}^2 + \frac{i}{2}\tilde{\mathbf{h}}_{\mu\nu}\left(\mathbf{F}_{\mu\nu} - g\mathbf{M}_k \tilde{\Sigma}_{\mu\nu} + \frac{g}{2}\tilde{\mathbf{S}}_{\mu\nu}\right) + \mathbf{k}_\mu^2 + i\sqrt{2}m\mathbf{k}_\mu \mathbf{B}_\mu. \tag{3.3}$$

The integration over $\mathbf{B}_\mu$ then yields

$$\mathbf{k}_\mu = \frac{1}{m\sqrt{2}}\partial_\nu \tilde{\mathbf{h}}_{\nu\mu}, \tag{3.4}$$

and we obtain

$$\mathcal{L}_k^{\rm LL} = \frac{1}{2m^2}(\partial_\mu \tilde{\mathbf{h}}_{\mu\nu})^2 + \frac{1}{4}\mathbf{h}_{\mu\nu}^2 + \frac{ig}{2}\mathbf{h}_{\mu\nu}\left(\frac{\mathbf{S}_{\mu\nu}}{2} - \mathbf{M}_k \Sigma_{\mu\nu}\right). \tag{3.5}$$

To perform the integration over the so-called Kalb-Ramond field $\mathbf{h}_{\mu\nu}$, notice that the general solution to eq. (3.4) with respect to $\mathbf{h}_{\mu\nu}(x)$ reads

$$\mathbf{h}_{\mu\nu}(x) = -\sqrt{2}m\varepsilon_{\mu\nu\lambda\rho}\partial_\lambda^x \int d^4x' D_0(x-x')\mathbf{k}_\rho(x') + \partial_\mu \mathbf{C}_\nu(x) - \partial_\nu \mathbf{C}_\mu(x),$$

where $\mathbf{C}_\mu$ is an arbitrary vector field, and $D_0(x) = 1/(4\pi^2 x^2)$ is the Coulomb propagator. To fix $\mathbf{C}_\mu = 0$ is equivalent to impose the constraint $\partial_\mu \mathbf{h}_{\mu\nu} = 0$. On the other hand, because of the coupling of $\mathbf{h}_{\mu\nu}$ to the open-string world-sheet, $-\frac{ig}{2}\mathbf{M}_k \mathbf{h}_{\mu\nu}\Sigma_{\mu\nu}$, the saddle-point value of $\mathbf{h}_{\mu\nu}$, which saturates the respective Gaussian integral, does not obey the constraint $\partial_\mu \mathbf{h}_{\mu\nu} = 0$. To make the integration consistent, we should therefore promote $\mathbf{h}_{\mu\nu}$ by making $\mathbf{C}_\mu$ non-vanishing. For such $\mathbf{h}_{\mu\nu}$, $(\partial_\mu \tilde{\mathbf{h}}_{\mu\nu})^2$ in eq. (3.5) can be replaced by $\frac{1}{6}\mathbf{H}_{\mu\nu\lambda}^2$, where $\mathbf{H}_{\mu\nu\lambda} = \partial_\mu \mathbf{h}_{\nu\lambda} + \partial_\lambda \mathbf{h}_{\mu\nu} + \partial_\nu \mathbf{h}_{\lambda\mu}$ is the field-strength tensor of $\mathbf{h}_{\mu\nu}$. The integration over $\mathbf{h}_{\mu\nu}$ in the resulting theory with the Lagrangian

$$\mathcal{L}_k^{\rm LL} = \frac{1}{12m^2}\mathbf{H}_{\mu\nu\lambda}^2 + \frac{1}{4}\mathbf{h}_{\mu\nu}^2 + \frac{ig}{2}\mathbf{h}_{\mu\nu}\left(\frac{\mathbf{S}_{\mu\nu}}{2} - \mathbf{M}_k \Sigma_{\mu\nu}\right) \tag{3.6}$$

is straightforward (see e.g. [23] for details) and yields

$$\mathcal{Z}_k^{\rm LL} = \exp\left\{-\frac{(g\mathbf{M}_k)^2}{2}\left[\oint\limits_C dx_\mu \oint\limits_C dx'_\mu D_m(x-x') + \right.\right.$$
$$\left.\left. + \frac{m^2}{2}\int d^4x d^4x' \Sigma_{\mu\nu}(x) D_m(x-x')\Sigma_{\mu\nu}(x')\right]\right\} \times$$
$$\times \int [d\mathbf{S}_{\mu\nu}] \exp\left[-\left(\frac{gm}{4}\right)^2 \int d^4x d^4x' \mathbf{S}_{\mu\nu}(x) D_m(x-x')\mathbf{S}_{\mu\nu}(x') + \right.$$
$$\left. + \left(\frac{gm}{2}\right)^2 \mathbf{M}_k \int d^4x d^4x' \mathbf{S}_{\mu\nu}(x) D_m(x-x')\Sigma_{\mu\nu}(x')\right], \tag{3.7}$$

where $D_m = mK_1(m|x|)/(4\pi^2|x|)$ is the Yukawa propagator. The formal measure $[d\mathbf{S}_{\mu\nu}]$ here, which replaces the measure $\left[d\Sigma^i_{\mu\nu}\right]$ in eq. (3.2), implies a certain prescription of the summation over world sheets of closed strings. For the dilute-plasma model of closed strings, this measure will be found in Section 5.

According to the first term on the r.h.s. of eq. (3.7), the Yukawa part of the potential satisfies the Casimir-scaling law, since $\mathbf{M}_k^2 \equiv C_k = \frac{k(N-k)}{2N}$ [13]. Further, in the leading semiclassical approximation we are considering in this and next sections, the integral over small-sized closed strings can be disregarded, and the remaining $\Sigma_{\mu\nu} \times \Sigma_{\mu\nu}$-interaction produces the Casimir scaling also for the confining part of the potential. In fact, extracting from this interaction the string tension according to the respective general formula [33] (cf. also [25]), we obtain $\sigma_k = C_k\bar{\sigma}$, where $\bar{\sigma} = 4\pi N\eta^2 \ln\kappa$. Note that, for σ_1 to be N-independent as the quark–antiquark string tension in QCD, we should have

$$\eta \sim \frac{1}{\sqrt{N\ln\kappa}} = \mathcal{O}(N^{-1/2}). \tag{3.8}$$

In what follows, we will address corrections to the Casimir scaling, that appear from the deviation from the London limit, as well as corrections produced by closed dual strings.

4. Corrections due to the deviation from the London limit

Let us consider the Lagrangian (2.4) with $\Sigma^i_{\mu\nu} = 0$. Introducing the $\mathbf{h}_{\mu\nu}$-field in the same way as in the London limit, we obtain the following expression:

$$\mathcal{L}_k = \frac{1}{4}\mathbf{h}^2_{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi_i)^2 + \frac{m_H^2}{2}\varphi_i^2 - \frac{ig}{2}\mathbf{M}_k\mathbf{h}_{\mu\nu}\Sigma_{\mu\nu} + i\mathbf{B}_\mu\partial_\nu\tilde{\mathbf{h}}_{\mu\nu} + \frac{m^2}{2}B^\alpha_\mu\left(\delta^{\alpha\beta} + \xi^{\alpha\beta}\right)B^\beta_\mu,$$

where the tensor $\xi^{\alpha\beta} \equiv \frac{2\sqrt{2}}{\eta N}\sum\limits_i \varphi_i q_i^\alpha q_i^\beta$ is apparently symmetric. Performing the Gaussian integration over $\mathbf{B}_\mu$, we arrive at the following substitution:

$$i\mathbf{B}_\mu\partial_\nu\tilde{\mathbf{h}}_{\mu\nu} + \frac{m^2}{2}B^\alpha_\mu\left(\delta^{\alpha\beta} + \xi^{\alpha\beta}\right)B^\beta_\mu \longrightarrow \frac{1}{2m^2}(\partial_\nu\tilde{h}^\alpha_{\mu\nu})\left(\delta^{\alpha\beta} - \xi^{\alpha\beta}\right)(\partial_\lambda\tilde{h}^\beta_{\mu\lambda}). \tag{4.1}$$

It can be shown (see Appendix A for details) that $\det^{-1/2}\left[\hat{1} + \Xi\right]$, where $\hat{1}$ and Ξ are the unit and the $\xi^{\alpha\beta}$-matrices, produces a renormalization of m and m_H, which does not violate the London limit condition, $\ln\kappa \gg 1$. Notice also that we have retained only the term linear in Ξ on the r.h.s. of eq. (4.1). As we will see below, this linear term eventually produces a correction to σ_k, which we are looking for. Instead, the omitted Ξ^2-term is shown in Appendix A to produce merely an inessential correction to m_H, which is smaller than m_H in the factor $\mathcal{O}\left(\frac{1}{\kappa\sqrt{N}}\right)$.

Next, the $\delta^{\alpha\beta}$-term on the r.h.s. of eq. (4.1) is clearly the kinetic term of the Kalb–Ramond field, that is the first term on the r.h.s. of eq. (3.5). Instead, the $\xi^{\alpha\beta}$-term on the r.h.s. of eq. (4.1) is the Higgs-inspired correction, which we will denote as $C^{\alpha\beta}\xi^{\alpha\beta}$, where $C^{\alpha\beta} \equiv -\frac{1}{2m^2}(\partial_\nu\tilde{h}^\alpha_{\mu\nu})(\partial_\lambda\tilde{h}^\beta_{\mu\lambda})$. The Gaussian integration over φ_i then leads to the following substitution:

$$\frac{1}{2}(\partial_\mu\varphi_i)^2+\frac{m_H^2}{2}\varphi_i^2+\xi^{\alpha\beta}C^{\alpha\beta}\longrightarrow -\frac{4}{(\eta N)^2}\sum\limits_i q_i^\alpha q_i^\beta q_i^\gamma q_i^\delta C^{\alpha\beta}(x)\int d^4x' D_{m_H}(x-x')C^{\gamma\delta}(x'). \tag{4.2}$$

Analogously to eq. (2.3), the orthonormality of roots yields the following formula (cf. Ref. [13]):

$$\sum\limits_i q_i^\alpha q_i^\beta q_i^\gamma q_i^\delta=\frac{N}{2(N+1)}\left(\delta^{\alpha\beta}\delta^{\gamma\delta}+\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma}\right).$$

Using this, we finally obtain the following Lagrangian:

$$\mathcal{L}_k=-C^{\alpha\alpha}+\frac{1}{4}\mathbf{h}_{\mu\nu}^2-\frac{ig}{2}\mathbf{M}_k\mathbf{h}_{\mu\nu}\Sigma_{\mu\nu}-$$

$$-\frac{2}{\eta^2N(N+1)}\int d^4x' D_{m_H}(x-x')\left[C^{\alpha\alpha}(x)C^{\beta\beta}(x')+2C^{\alpha\beta}(x)C^{\alpha\beta}(x')\right]. \tag{4.3}$$

To proceed, notice that, according to eq. (3.3), the physical meaning of the field $\mathbf{k}_\mu$ is the monopole current, which couples to the dual gauge field $\mathbf{B}_\mu$ and also possesses its own self-interaction terms. (Since for the massive vector field one always has $\partial_\mu\mathbf{B}_\mu=0$, this current is automatically conserved.) The monopole current is defined in terms of the Kalb–Ramond field by means of eq. (3.4). Using this correspondence, it is straightforward to reformulate the action, corresponding to the obtained Lagrangian (4.3), in terms of monopole currents:

$$\int d^4x\mathcal{L}_k=\int d^4x\mathbf{k}_\mu^2+m^2\int d^4xd^4x'\mathbf{k}_\mu(x)D_0(x-x')\mathbf{k}_\mu(x')+$$

$$+igm\sqrt{2}\mathbf{M}_k\int d^4xd^4x'\tilde{\Sigma}_{\mu\nu}(x)\mathbf{k}_\nu(x')\partial_\mu^x D_0(x-x')-\frac{2}{\eta^2N(N+1)}\int d^4xd^4x'D_{m_H}(x-x')\times$$

$$\times\left[k_\mu^\alpha(x)k_\mu^\alpha(x)k_\nu^\beta(x')k_\nu^\beta(x')+2k_\mu^\alpha(x)k_\mu^\beta(x)k_\nu^\alpha(x')k_\nu^\beta(x')\right], \tag{4.4}$$

where the terms are presented in the same order as they stand in eq. (4.3). The first and second terms on the r.h.s. of this equation are clearly the mass term of the current and the Coulomb interaction between currents, respectively. The third term has a form similar to the Gauss linking number of a closed surface Σ and a closed contour Γ, $L(\Sigma,\Gamma)=\int d^4xd^4x'\tilde{\Sigma}_{\mu\nu}(x)j_\nu(x')\partial_\mu^x D_0(x-x')$, where $j_\mu(x)\equiv\oint\limits_\Gamma dx_\mu(\tau)\delta(x-x(\tau))$. Our formula differs from the Gauss one in two respects: first, the surface is open, and second, instead of the classical current j_μ of a point-like particle, localized along a closed trajectory, we have the quantum field, $gm\sqrt{2}\mathbf{M}_k\mathbf{k}_\mu$, distributed over the whole space-time.

Let us now estimate the Higgs-inspired correction, given by the last term on the r.h.s. of eq. (4.4). Notice that, according to eq. (3.8), m depends on N as $m=g_m\eta\sqrt{N}\sim\sqrt{N}\cdot\frac{1}{\sqrt{N}}\cdot\sqrt{N}=\sqrt{N}$, and $m_H=\kappa m$, where κ has been naturally chosen N-independent (cf. the first paragraph of Section 3). Therefore, m_H grows with N as

$$m_H = \mathcal{O}(N^{1/2}). \tag{4.5}$$

The heaviness of the Higgs bosons, implied in both London and large-N limits, enables us to write, in the leading $1/m_H$-approximation, the following expression for the correction under study:

$$-\frac{2}{(\eta m_H)^2 N(N+1)} \int d^4x \left[(\mathbf{k}_\mu^2)^2 + 2(\mathbf{k}_\mu \mathbf{k}_\nu)^2\right]. \tag{4.6}$$

After the variation of the total action with respect to k_μ^α, the color structure of the saddle-point equation thus obtained prescribes to seek $\mathbf{k}_\mu$ in the form $\mathbf{M}k_\mu$. The equation for k_μ then reads

$$\left(1 - \frac{12C_k}{(\eta m_H)^2 N(N+1)} k_\rho^2\right) k_\mu + m^2 \int d^4x' D_0(x-x') k_\mu(x') =$$

$$= -\frac{igm}{\sqrt{2}} \int d^4x' \tilde{\Sigma}_{\mu\nu}(x') \partial_\nu^x D_0(x-x').$$

The leading-order part of this equation, without the k_ρ^2-term, can easily be converted into the differential form by acting with ∂^2 onto both its sides. The resulting equation $(\partial^2 - m^2)k_\mu = \frac{igm}{\sqrt{2}} \partial_\nu \tilde{\Sigma}_{\mu\nu}$ leads to the following leading-order saddle-point expression for k_μ: $k_\mu(x) = -\frac{igm}{\sqrt{2}} \int d^4x' D_m(x-x') \partial_\nu \tilde{\Sigma}_{\mu\nu}(x')$, which should further be substituted into eq. (4.6),

$$-\frac{6(C_k)^2}{(\eta m_H)^2 N(N+1)} \int d^4x (k_\mu^2)^2. \tag{4.7}$$

Among all the terms contained here in k_μ^2, only the surface$\times$surface one,

$$-\frac{(gm)^2}{4} \int d^4x_1 d^4x_2 \Sigma_{\mu\nu}(x_1) \Sigma_{\mu\nu}(x_2) \partial_\alpha^{x_1} D_m(x-x_1) \partial_\alpha^{x_2} D_m(x-x_2),$$

produces the desired correction to the string tension. Indeed, the integral structure of the respective part of the correction (4.7),

$$-\frac{3(C_k)^2 (gm)^4}{8N(N+1)(\eta m_H)^2} \int d\sigma_{\mu\nu}(x_1) d\sigma_{\mu\nu}(x_2) d\sigma_{\lambda\rho}(x_3) d\sigma_{\lambda\rho}(x_4) \cdot J, \tag{4.8}$$

where $J \equiv \partial_\alpha^{x_1} \partial_\alpha^{x_2} \partial_\beta^{x_3} \partial_\beta^{x_4} \int d^4x \prod_{l=1}^{4} D_m(x-x_l)$, is the same as appears in the 3-d SU(N) Georgi–Glashow model due to the non-diluteness of the monopole plasma [13]. In that paper, it was shown that this structure does produce a correction to the string tension. It can be shown that, similarly to the 3-d SU(N) Georgi–Glashow model, this correction behaves with N as $(C_k)^2/N$.

With the account for the obtained correction, the N-ality dependence of the ratio of string tensions is given by the formula

$$\frac{\sigma_k}{\sigma_1} = \frac{k(N-k)}{N-1}\left[1+\alpha(N)\frac{(k-1)(N-k-1)}{N}\right],$$

where the coefficient $\alpha(N) \sim N^{-1}$ and therefore, at fixed k, the whole correction vanishes in the large-N limit. As well as the Casimir-scaling term, the obtained leading Higgs-inspired correction is apparently invariant under the interchange of quarks and antiquarks, $k \leftrightarrow (N-k)$.

5. Corrections due to closed strings

To study the grand canonical ensemble of closed strings, it is necessary to replace $\mathbf{S}_{\mu\nu}$ in eq. (3.6) (with $\Sigma_{\mu\nu}$ set for a while equal to zero) by $\sum\limits_a n_a \mathbf{S}^a_{\mu\nu}$, where the n_a stand for winding numbers. It is known [31, 29] that one may restrict oneself to closed strings possessing the minimal winding numbers, $n_a = \pm 1$. That is merely because the energy of a single closed string is a quadratic function of its flux, owing to which it is energetically favorable for the vacuum to maintain two closed strings of a unit flux, rather than one string of the double flux.

Then, taking into account that the plasma of closed strings is dilute, one can perform the summation over the grand canonical ensemble of these objects, which modifies the Lagrangian (3.6), with $\Sigma_{\mu\nu} = 0$, as follows:

$$\mathcal{L}_{\text{gr. can.}} = \frac{1}{12m^2}\mathbf{H}^2_{\mu\nu\lambda} + \frac{1}{4}\mathbf{h}^2_{\mu\nu} - 2\zeta\cos\left(\frac{g}{4}\frac{|\mathbf{h}_{\mu\nu}|}{\Lambda^2}\right). \tag{5.1}$$

Here $\Lambda \equiv \sqrt{L/a^3}$ is a UV momentum cut-off with L and a denoting the characteristic distances between closed strings and their typical sizes, respectively. In the dilute-plasma approximation under study, $a \ll L$ and $\Lambda \gg a^{-1}$. Next, $\zeta \propto \mathrm{e}^{-S_0}$ stands for the fugacity (Boltzmann factor) of a single string, which has the dimension $(\text{mass})^4$, with S_0 denoting the action of a single string, $S_0 \sim \sigma_1 a^2$. Finally, it is assumed that closed strings are not too small, namely $a \geq \mathcal{O}\left(\frac{1}{gm}\right)$, so that $S_0 \gg 1$, and the mean density of the plasma, 2ζ, is exponentially small, i.e. the plasma is dilute.

Note also that, because of the Debye screening of the dual vector boson in the plasma of closed strings, its mass increases. This is clearly seen from eq. (5.1), by the increase of the mass of the Kalb–Ramond field, which represents this boson:

$$m^2 \longrightarrow M^2 = m^2\left(1+\frac{g^2\zeta}{4\Lambda^4}\right). \tag{5.2}$$

To study corrections to the k-string tension produced by closed strings, we will need to know correlation functions of these strings in the plasma. To obtain an expression for the generating functional of such correlation functions, one needs the theory to be formulated in terms of dynamical vorticity tensor currents. This can be done by recalling that, for closed strings:

$$\exp\left\{-\int d^4x\left[\frac{1}{12m^2}\mathbf{H}^2_{\mu\nu\lambda} + \frac{1}{4}\mathbf{h}^2_{\mu\nu}\right]\right\} =$$

$$= \int \mathcal{D}\mathbf{S}_{\mu\nu} \exp\left[-\left(\frac{gm}{4}\right)^2 \int d^4x d^4x' \mathbf{S}_{\mu\nu}(x) D_m(x-x') \mathbf{S}_{\mu\nu}(x') - \frac{ig}{4}\int d^4x \mathbf{h}_{\mu\nu}\mathbf{S}_{\mu\nu}\right].$$

The Kalb–Ramond field can then be integrated out by solving the saddle-point equation stemming from the respective part of the Lagrangian,

$$-2\zeta\cos\left(\frac{g}{4}\frac{|\mathbf{h}_{\mu\nu}|}{\Lambda^2}\right) + \frac{ig}{4}\mathbf{h}_{\mu\nu}\mathbf{S}_{\mu\nu}.$$

This yields the following expression for the partition function of the grand canonical ensemble of closed strings in terms of their vorticity tensor currents:

$$\mathcal{Z}_{\text{gr. can.}} = \int \mathcal{D}\mathbf{S}_{\mu\nu} \exp\left\{-\left[\left(\frac{gm}{4}\right)^2 \int d^4x d^4x' \mathbf{S}_{\mu\nu}(x) D_m(x-x') \mathbf{S}_{\mu\nu}(x') + V[\mathbf{S}_{\mu\nu}]\right]\right\}, \tag{5.3}$$

where the potential $V[\mathbf{S}_{\mu\nu}]$ reads

$$V[\mathbf{S}_{\mu\nu}] = \int d^4x \left\{\Lambda^2|\mathbf{S}_{\mu\nu}| \ln\left[\frac{\Lambda^2}{2\zeta}|\mathbf{S}_{\mu\nu}| + \sqrt{1+\left(\frac{\Lambda^2}{2\zeta}|\mathbf{S}_{\mu\nu}|\right)^2}\right] - 2\zeta\sqrt{1+\left(\frac{\Lambda^2}{2\zeta}|\mathbf{S}_{\mu\nu}|\right)^2}\right\}. \tag{5.4}$$

(Note that the operator, which describes the density of plasma at the point x, is $\Lambda^2|\mathbf{S}_{\mu\nu}(x)|$.) Comparing now eqs. (5.3), (5.4) with eq. (3.7), we see that, in the dilute-plasma model of closed strings, the formal measure $[d\mathbf{S}_{\mu\nu}]$ concretizes as $\mathcal{D}\mathbf{S}_{\mu\nu}\mathrm{e}^{-V[\mathbf{S}_{\mu\nu}]}$.

Corrections to the string tension σ_k of the open world-sheet Σ stem from the last term in the following expression for the partition function (3.7):

$$-\ln \mathcal{Z}_k^{\mathrm{LL}} =$$

$$= \frac{(g\mathbf{M}_k)^2}{2}\left[\oint_C dx_\mu \oint_C dx'_\mu D_m(x-x') + \frac{m^2}{2}\int d^4x d^4x' \Sigma_{\mu\nu}(x) D_m(x-x') \Sigma_{\mu\nu}(x')\right] -$$

$$-\ln\left\langle \exp\left[\left(\frac{gm}{2}\right)^2 \mathbf{M}_k \int d^4x d^4x' \mathbf{S}_{\mu\nu}(x) D_m(x-x') \Sigma_{\mu\nu}(x')\right]\right\rangle,$$

where the average $\langle\ldots\rangle$ over closed strings is now defined according to eqs. (5.3), (5.4). By virtue of the cumulant expansion, this term can be written as

$$-\sum_{n=1}^{\infty}\left(\frac{gm}{2}\right)^{2n} M_k^{\alpha_1}\cdots M_k^{\alpha_n}\times$$

$$\times \int d^4x_1 \cdots d^4x_n \sigma_{\mu_1\nu_1}(x_1)\cdots\sigma_{\mu_n\nu_n}(x_n)\left\langle\left\langle S^{\alpha_1}_{\mu_1\nu_1}(x_1)\cdots S^{\alpha_n}_{\mu_n\nu_n}(x_n)\right\rangle\right\rangle.$$

Here, $\sigma_{\mu\nu}(x) \equiv \int d^4x' D_m(x-x')\Sigma_{\mu\nu}(x')$, and $\langle\langle\cdots\rangle\rangle$ denotes a one-particle irreducible average (cumulant) of closed strings. Since the action corresponding to the partition function (5.3) contains only powers of $\mathbf{S}_{\mu\nu}(x)\mathbf{S}_{\mu\nu}(x')$, cumulants of odd orders vanish, whereas a cumulant of an even order n has the form

$$\left\langle\left\langle S^{\alpha_1}_{\mu_1\nu_1}(x_1)\cdots S^{\alpha_n}_{\mu_n\nu_n}(x_n)\right\rangle\right\rangle =$$

$$= \delta^{\alpha_1\alpha_2}\cdots\delta^{\alpha_{n-1}\alpha_n} f_{\mu_1\nu_1,\mu_2\nu_2}(x_1-x_2)\cdots f_{\mu_{n-1}\nu_{n-1},\mu_n\nu_n}(x_{n-1}-x_n) + \text{permutations.}$$

Here, $f_{\mu_1\nu_1,\mu_2\nu_2}(x_1-x_2) = \varepsilon_{\mu_1\nu_1\lambda_1\rho}\varepsilon_{\mu_2\nu_2\lambda_2\rho}\partial^{x_1}_{\lambda_1}\partial^{x_2}_{\lambda_2}\mathcal{D}(x_1-x_2)$ with $\mathcal{D}$ standing for a function whose concrete form for the case of a very dilute plasma will be made clear in a moment. As for the Lorentz structure of the f-tensors, it stems from the condition of closeness of strings, $\partial_\mu \mathbf{S}_{\mu\nu} = 0$. The color structure of the cumulant produces, for $n \equiv 2l$, the factor $(\mathbf{M}_k^2)^l = (C_k)^l$. The N-ality dependence of σ_k is therefore defined by the following formula:

$$\sigma_k = C_k\bar{\sigma} + \sum_{l=1}^{\infty}\sigma^{(l)}(C_k)^l, \tag{5.5}$$

where $\sigma^{(l)}$ are (N-dependent) coefficients of dimension [mass]^2. Note that the term with $l = 2$ in this equation produces a correction to the string tension, which has the same $(C_k)^2$-dependence as the leading Higgs-inspired correction found in the previous section.

As an example, let us finally present the lowest non-trivial two-point correlation function of closed strings, which can be derived in the approximation when the plasma is very dilute, i.e. its density is even lower than the (already exponentially small) mean one, 2ζ. In that case, the potential (5.4) becomes a quadratic functional. Including the source term, $\int d^4x \mathbf{J}_{\mu\nu}\mathbf{S}_{\mu\nu}$, into the square brackets on the r.h.s. of eq. (5.3), we obtain for this Gaussian integral:

$$\mathcal{Z}_{\text{gr. can.}}[\mathbf{J}_{\mu\nu}] \simeq \frac{1}{\mathcal{Z}_{\text{gr. can.}}[0]}\int \mathcal{D}\mathbf{S}_{\mu\nu}\exp\left\{-\left[\left(\frac{gm}{4}\right)^2\int d^4x d^4x' \mathbf{S}_{\mu\nu}(x)D_m(x-x')\mathbf{S}_{\mu\nu}(x') + \right.\right.$$

$$\left.\left. + \int d^4x\left(-2\zeta + \frac{\Lambda^4}{4\zeta}\mathbf{S}^2_{\mu\nu} + \mathbf{J}_{\mu\nu}\mathbf{S}_{\mu\nu}\right)\right]\right\} = \exp\left[-\int d^4x d^4x' \mathbf{J}_{\mu\nu}(x)\mathcal{G}(x-y)\mathbf{J}_{\mu\nu}(x')\right],$$

where $\mathcal{G}(x) \equiv \frac{\zeta}{\Lambda^4}(\partial^2 - m^2)D_M(x)$, and the mass M is defined by eq. (5.2). Imposing the condition $\partial_\mu\mathbf{S}_{\mu\nu} = 0$, one can further, similarly to Ref. [29], derive the desired two-point correlation function (string propagator):

$$\left\langle S^{\alpha}_{\mu\nu}(x)S^{\beta}_{\lambda\rho}(0)\right\rangle = \delta^{\alpha\beta}\varepsilon_{\mu\nu\gamma\sigma}\varepsilon_{\lambda\rho\xi\sigma}\frac{\zeta}{(M\Lambda^2)^2}\partial_\gamma\partial_\xi(\partial^2 - m^2)[D_0(x) - D_M(x)]. \tag{5.6}$$

It is known that, in this Gaussian approximation, all higher cumulants vanish, i.e. the terms with $l \geq 2$ in eq. (5.5) are absent. Therefore, in this very dilute plasma approximation, the Casimir-scaling law is preserved, since the terms that violate it vanish.

6. Summary

In this paper, we have explored the k-string tension spectrum in the SU(N)-inspired 4-d dual Abelian–Higgs-type theory. We have first considered the London limit of this theory and demonstrated that, in the leading semi-classical approximation when the small-sized closed dual strings are completely disregarded, the k-string tension obeys the Casimir-scaling law. In the same approximation, when closed strings are disregarded, we have further explored the leading correction to the Casimir scaling emerging due to the deviation from the London limit, i.e. due to the finiteness of the masses of the Higgs bosons. This correction turns out to have the same N-ality dependence as the correction, which one finds in the 3-d SU(N) Georgi–Glashow model when the non-diluteness of the monopole plasma is taken into account. We have then addressed another type of corrections to the Casimir scaling, which emerge in the London limit when one accounts for the dilute plasma of closed dual strings. In the leading low-density approximation, i.e. when the plasma is very dilute, the respective correction is shown not to violate the Casimir scaling. Instead, the correction of the next order in the non-diluteness has the same N-ality dependence as the above-mentioned correction emerging without closed strings in the vicinity of the London limit. Finally, we have analyzed the corrections that appear in higher orders in the non-diluteness of the plasma of closed strings. Interestingly, the $1/N$ dependence of the ratio σ_k/σ_1 does not satisfy the counting rules that were spelled out for SU(N) Yang–Mills theories in Ref. [12]. However, one has to be careful in comparing our results to the full non-Abelian gauge theory. In our model, off-diagonal degrees of freedom are disregarded, the only remnant of the non-Abelian structure being the quantization condition for the magnetic charge. The discrepancy between the results presented here (and in Ref. [13]) and those obtained in Ref. [12] suggests that a purely Abelian description of the SU(N) vacuum is not adequate to catch the full dynamics of the non-Abelian gauge theory, in agreement with the conclusions in Refs. [35, 36]. Further work along these lines is needed to clarify this issue.

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A. A few technical points

Let us first evaluate the effect produced by the determinant when integrating over the $\mathbf{B}_\mu$-field in eq. (4.1). One has

$$\det{}^{-1/2}\left[\hat{1}+\Xi\right]\simeq\exp\left[-\frac{1}{2}\mathrm{Tr}\,\Xi+\frac{1}{4}\mathrm{Tr}\,\Xi^2\right], \tag{A.1}$$

where "Tr" includes both the trace over the indices α, β and the trace in the coordinate space. Next terms have been omitted since they describe self-interactions of the φ_i-field, rather than the renormalization of m and m_H, which will be shown to be produced by the two retained terms. The first term on the r.h.s. of eq. (A.1) modifies eq. (4.2) as follows:

$$\frac{1}{2}(\partial_\mu \varphi_i)^2 + \frac{m_H^2}{2}\varphi_i^2 + \xi^{\alpha\beta}\left(C^{\alpha\beta} + \frac{zm_H^4}{2}\delta^{\alpha\beta}\right) \longrightarrow -\frac{4}{(\eta N)^2}\sum_i q_i^\alpha q_i^\beta q_i^\gamma q_i^\delta \times$$

$$\times \left[C^{\alpha\beta}(x) + \frac{zm_H^4}{2}\delta^{\alpha\beta}\right]\int d^4x' D_{m_H}(x-x')\left[C^{\gamma\delta}(x') + \frac{zm_H^4}{2}\delta^{\gamma\delta}\right],$$

where the regularization parameter z is defined by the relation $\delta^{(4)}(x)\big|_{x=0} = zm_H^4$. Note that, due to eq. (4.5) and the fact that $\delta^{(4)}(x)\big|_{x=0}$ is N-independent, z should scale with N as $\mathcal{O}(N^{-2})$. Apart from this requirement, z can be chosen at will. Up to an inessential constant addendum, the correction to the action thus reads:

$$-\frac{2zm_H^4}{(\eta N)^2}\sum_i q_i^\alpha q_i^\beta q_i^\gamma q_i^\delta \int d^4x d^4x' D_{m_H}(x-x')\left[C^{\alpha\beta}(x)\delta^{\gamma\delta} + C^{\gamma\delta}(x')\delta^{\alpha\beta}\right] =$$

$$= -\frac{2zm_H^2}{N\eta^2}\int d^4x C^{\alpha\alpha}.$$

Comparing this result with the first term on the r.h.s. of eq. (4.3), we arrive at the following renormalization of m^{-2}:

$$\frac{1}{m^2} \to \frac{1}{m^2}\left(1 + \frac{2zm_H^2}{N\eta^2}\right), \tag{A.2}$$

where the correction vanishes at large N as $\frac{2zm_H^2}{N\eta^2} = \mathcal{O}(N^{-1})$. Let us now consider the second term on the r.h.s. of eq. (A.1). It modifies the mass term on the l.h.s. of eq. (4.2) as

$$\frac{m_H^2}{2}\varphi_i^2 \to \frac{m_H^2}{2}\sum_{i,j}\varphi_i\varphi_j\left[\delta_{ij} - \frac{4zm_H^2}{(N\eta)^2}q_i^\alpha q_i^\beta q_j^\alpha q_j^\beta\right].$$

The tensor $q_i^\alpha q_i^\beta q_j^\alpha q_j^\beta$ should be proportional to δ_{ij}, which is the only tensor symmetric in indices (i,j), and the proportionality coefficient is 1. The squared Higgs mass therefore renormalizes as

$$m_H^2 \to m_H^2\left[1 - \frac{4zm_H^2}{(N\eta)^2}\right]. \tag{A.3}$$

The obtained correction vanishes at large N: $\frac{4zm_H^2}{(N\eta)^2} = \mathcal{O}(N^{-2})$, so that $\frac{4zm_H^2}{(N\eta)^2} < 1$ in the large-N limit. A more accurate condition on z, which should hold at any N, can be imposed by using the inequality of the London limit, eq. (3.1). It yields:

$$1 > \frac{4zm_H^2}{(N\eta)^2} = \frac{16\lambda z}{N^2} \gg \frac{16z}{N^2}\frac{(2\pi \mathrm{e}N)^2}{\bar\lambda} = \frac{(8\pi\mathrm{e})^2 z}{\bar\lambda},$$

i.e. at a given $\bar\lambda$, z should be chosen such that $z \ll \frac{\bar\lambda}{(8\pi\mathrm{e})^2}$. The squared Landau–Ginzburg parameter then renormalizes according to eqs. (A.2), (A.3) as

$$\kappa^2 \to \kappa^2\left[1 + \frac{2zm_H^2}{N\eta^2}\left(1 - \frac{2}{N}\right)\right] = \kappa^2\left\{1 + \mathcal{O}(N^{-1})\cdot\left[1 + \mathcal{O}(N^{-1})\right]\right\}.$$

Therefore, upon renormalization, κ remains large, and the respective correction vanishes in the large-N limit.

Let us now evaluate the omitted Ξ^2-term on the r.h.s. of eq. (4.1). This term reads

$$k_\mu^\alpha \xi^{\alpha\beta}\xi^{\beta\gamma}k_\mu^\gamma = \frac{8}{(\eta N)^2}k_\mu^\alpha k_\mu^\gamma \varphi_i\varphi_j q_i^\alpha q_i^\beta q_j^\beta q_j^\gamma, \tag{A.4}$$

with the monopole current $\mathbf{k}_\mu$ defined by eq. (3.4). Since the tensors $k_\mu^\alpha k_\mu^\gamma$, $\varphi_i\varphi_j$ are symmetric in indices (α,γ) and (i,j), respectively, it is natural to impose the following Ansatz: $q_i^\alpha q_i^\beta q_j^\beta q_j^\gamma = \mathcal{N}\delta^{\alpha\gamma}\delta_{ij}$. The proportionality coefficient $\mathcal{N}$ can readily be found: $\mathcal{N} = \frac{1}{N-1}$, and the term (A.4) therefore reads $\frac{8\mathbf{k}_\mu^2}{(\eta N)^2(N-1)}\varphi_i^2$. Furthermore, the characteristic amplitude of the $\mathbf{k}_\mu$-field can be estimated by noticing that the configuration of this field, which dominates in the partition function, is the one at which each of the first two terms on the r.h.s. of eq. (4.4) is of the order of unity. When applied to the mass term, this requirement yields $L^2 \sim |\mathbf{k}_\mu|^{-1}$, where L and $|\mathbf{k}_\mu|$ are the characteristic wavelength and the amplitude of the $\mathbf{k}_\mu$-field, respectively. Applying further the same requirement to the Coulomb interaction of monopole currents, the second term on the r.h.s. of eq. (4.4), we have $m^2|\mathbf{k}_\mu|^2L^6 \sim 1$. Substituting here the above estimate for L^2, we obtain the desired estimate for the characteristic value of the amplitude: $|\mathbf{k}_\mu| \sim m^2$. This leads to the following estimate for the magnitude of the term (A.4): $\frac{m^4}{(\eta N)^2(N-1)}\varphi_i^2$. This term therefore produces a small positive correction to m_H, whose magnitude with respect to m_H can be estimated as:

$$\frac{m^2}{\eta N\sqrt{N-1}}\frac{1}{m_H} = \mathcal{O}\left(\frac{1}{\kappa\sqrt{N}}\right).$$

B. More on Casimir scaling in the 3-d SU(N) Georgi-Glashow model

It has been proved in [13] that, in the 3-d SU(N) Georgi-Glashow model, Casimir scaling holds for an *arbitrarily shaped* surface (i.e. the world sheet of a k-string), provided that the density of monopole plasma is much lower than the mean one. The purpose of this Appendix is to show that, in case of a flat surface, Casimir scaling in this model is an *exact* result, not requiring the condition that the density of the monopole plasma is much lower than the mean one. To this end, let us consider the confining part of the k-th power of the fundamental Wilson loop, i.e. the part produced by monopoles. It has the form $\langle W_k(C)\rangle_{\rm mon} = \sum\limits_{\substack{a_1,\ldots,a_k=1 \\ (\text{with possible coincidences})}}^{N} W_{a_1,\ldots,a_k}(C)$, where

$$W_{a_1,\ldots,a_k}(C)=\frac{1}{\mathcal{Z}_{\rm mon}}\int\mathcal{D}\mathbf{B}_\mu\delta\left(\varepsilon_{\mu\nu\lambda}\partial_\nu\mathbf{B}_\lambda\right)\int\mathcal{D}\mathbf{l}\exp\left\{\int d^3x\left[-\frac{g_m^2}{2}\mathbf{B}_\mu^2+\right.\right.$$

$$\left.\left.+ig_m\mathbf{l}\partial_\mu\mathbf{B}_\mu+2\zeta\sum_i\cos\left(g_m\mathbf{q}_i\mathbf{l}\right)\right]+4\pi i\mathbf{M}_k^{(n)}\int\limits_{\Sigma(C)}d\sigma_\mu\mathbf{B}_\mu\right\},\tag{B.1}$$

where $\mathcal{Z}_{\rm mon}$ is the same functional integral, but with the last term [which describes the flux of the magnetic field through an arbitrary surface $\Sigma(C)$] set equal to zero. The constraint $\varepsilon_{\mu\nu\lambda}\partial_\nu\mathbf{B}_\lambda=0$ imposes the fact that free photons, inessential for confinement, are not taken into account. The dimensionalities of the magnetic coupling constant, g_m, dual photon field, $\mathbf{l}$, and the (exponentially small) monopole fugacity ζ are $[\text{mass}]^{-1/2}$, $[\text{mass}]^{1/2}$, and $[\text{mass}]^3$, respectively; for more details on the model and eq. (B.1) at $k=1$ see [13, 34]. In eq. (B.1), $\mathbf{M}_k^{(n)}$ again denotes the sum $\sum\limits_{i=1}^{k}\mathbf{m}_{a_i}$, where some n indices out of k can now coincide.

For the contour C located in the (x,y)-plane, the saddle-point equations stemming from eq. (B.1) read

$$ig_m\mathbf{l}'+g_m^2\mathbf{B}-4\pi i\mathbf{M}_k^{(n)}\delta(z)=0,\tag{B.2}$$

$$i\mathbf{B}'-2\zeta\sum_i\mathbf{q}_i\sin(g_m\mathbf{q}_i\mathbf{l})=0,\tag{B.3}$$

where $'\equiv d/dz$, and the natural Ansatz $\mathbf{B}_\mu=\delta_{\mu3}\mathbf{B}(z)$, $\mathbf{l}=\mathbf{l}(z)$ has been adopted. Next, as it follows from eq. (B.2), $\mathbf{B}\propto i\mathbf{M}_k^{(n)}$, therefore $W_{a_1,\ldots,a_k}(C)\to{\rm e}^{-\left(\mathbf{M}_k^{(n)}\right)^2\sigma|\Sigma(C)|}$ at $|\Sigma(C)|\to\infty$, where the string tension σ is k-independent. Since $\left(\mathbf{M}_k^{(n)}\right)^2=C_k+\frac{n^2-n}{2}$, in the limit of asymptotically large areas $|\Sigma(C)|$ of interest, we arrive at a Feynman-Kac–type formula,

$$\left\langle W_k(C)\right\rangle_{\rm mon}={\rm e}^{-C_k\sigma|\Sigma(C)|}\sum_{n=1}^{k}c_n{\rm e}^{-\frac{n^2-n}{2}\sigma|\Sigma(C)|},\tag{B.4}$$

with some positive coefficients c_n; therefore, only the case $n=0$ is relevant in eq. (B.1) (cf. ref. [13]). We should, thus, solve the system of eqs. (B.2), (B.3) with $\mathbf{M}_k^{(0)}$, which coincide with $\mathbf{M}_k$ from the main text. Setting $\mathbf{B}(z)=\mathbf{M}_kB(z)$, $\mathbf{l}(z)=\mathbf{M}_kl(z)$, we see that eq. (B.2) takes the same form as in the fundamental case, namely

$$ig_ml'+g_m^2B=4\pi i\delta(z),\tag{B.5}$$

whereas to handle eq. (B.3), one should notice that any root vector is a difference of two weight vectors, $\mathbf{q}_i\equiv\mathbf{q}_{ab}=\mathbf{m}_a-\mathbf{m}_b$. In particular, positive roots, we are dealing with, are those with $b<a$, therefore eq. (B.3) takes the form

$$\sum_{b<a}\mathbf{M}_k(\mathbf{m}_a-\mathbf{m}_b)\sin\left[g_m\mathbf{M}_k(\mathbf{m}_a-\mathbf{m}_b)l\right]=\frac{i}{2\zeta}C_kB'.$$

Using the symmetry of the expression under the sum on the l.h.s. with respect to $a \leftrightarrow b$, we can rewrite this equation as

$$\sum_{a,b=1}^{N} \mathbf{M}_k\mathbf{m}_a \left[\sin(g_m\mathbf{M}_k\mathbf{m}_a l)\cos(g_m\mathbf{M}_k\mathbf{m}_b l) - \cos(g_m\mathbf{M}_k\mathbf{m}_a l)\sin(g_m\mathbf{M}_k\mathbf{m}_b l)\right] = \frac{i}{2\zeta}C_k B'. \tag{B.6}$$

We should further perform the four sums on the l.h.s.; let us begin with the first one, $\sum_{a=1}^{N}(\mathbf{M}_k\mathbf{m}_a)\sin(g_m\mathbf{M}_k\mathbf{m}_a l)$. Apparently, there are k terms in this sum, for which $\mathbf{m}_a$ coincides with some of the k weight vectors, which enter $\mathbf{M}_k$. Using the relation $\mathbf{m}_a\mathbf{m}_b = (\delta_{ab} - N^{-1})/2$, we have for such terms $\mathbf{M}_k\mathbf{m}_a = \frac{N-1}{2N} - (k-1)\frac{1}{2N} = \frac{N-k}{2N}$. For the other $(N-k)$ terms in the sum, $\mathbf{m}_a$ does not coincide with any weight vector in $\mathbf{M}_k$, hence $\mathbf{M}_k\mathbf{m}_a = -\frac{k}{2N}$ for such terms. We, therefore, obtain

$$\sum_{a=1}^{N}(\mathbf{M}_k\mathbf{m}_a)\sin(g_m\mathbf{M}_k\mathbf{m}_a l) = k\cdot\frac{N-k}{2N}\sin\left(g_m\frac{N-k}{2N}l\right) + (N-k)\cdot\frac{k}{2N}\sin\left(g_m\frac{k}{2N}l\right) =$$

$$= C_k\left[\sin\left(g_m\frac{k}{2N}l\right) + \sin\left(g_m\frac{N-k}{2N}l\right)\right].$$

Remarkably, already this expression alone is manifestly invariant under $k \leftrightarrow (N-k)$. In the same way, we obtain for the three other sums the following expressions:

$$\sum_{b=1}^{N}\cos(g_m\mathbf{M}_k\mathbf{m}_b l) = k\cos\left(g_m\frac{N-k}{2N}l\right) + (N-k)\cos\left(g_m\frac{k}{2N}l\right),$$

$$\sum_{a=1}^{N}(\mathbf{M}_k\mathbf{m}_a)\cos(g_m\mathbf{M}_k\mathbf{m}_a l) = C_k\left[\cos\left(g_m\frac{N-k}{2N}l\right) - \cos\left(g_m\frac{k}{2N}l\right)\right],$$

$$\sum_{b=1}^{N}\sin(g_m\mathbf{M}_k\mathbf{m}_b l) = k\sin\left(g_m\frac{N-k}{2N}l\right) - (N-k)\sin\left(g_m\frac{k}{2N}l\right).$$

The l.h.s. of eq. (B.6) then takes the form $C_k N\sin\frac{g_m l}{2}$, and the whole equation becomes

$$B' + 2i\zeta N\sin\frac{g_m l}{2} = 0, \tag{B.7}$$

that also coincides with that of the fundamental case [34]. The solution to the system of equations (B.5), (B.7) reads

$$B(z) = i\frac{8m_D}{g_m^2}\frac{\mathrm{e}^{-m_D|z|}}{1+\mathrm{e}^{-2m_D|z|}}, \quad l(z) = \frac{8}{g_m}\,\mathrm{sgn}\, z\cdot\arctan\left(\mathrm{e}^{-m_D|z|}\right), \tag{B.8}$$

where $m_D = g_m\sqrt{N\zeta}$ is the Debye mass of the dual photon $\mathbf{l}$. [This mass is visible in eq. (B.1) with $C=0$, where the $\mathbf{B}_\mu$-field is integrated out to produce the kinetic term of the $\mathbf{l}$-field, and cosine is expanded up to the quadratic term.] Inserting the so-obtained field $B(z)$ into eq. (B.1), we obtain $\sigma = cg\sqrt{N\zeta}$, where $g = 4\pi/g_m$. The proportionality coefficient c here depends on the range of average of $B(z)$, since, according to eqs. (B.8), the string is exponentially thick, $|z| \lesssim m_D^{-1}$. For instance, if we simply choose the value $B(0)$, then $c = 4$. [For comparison, the value of c one obtains in the case when the density of the monopole plasma is much lower than the mean one, $\zeta N(N-1)$, is [34] π, that can be shown to approximately correspond to the following range of average: $|z| < \frac{\sqrt{6-\frac{3\pi}{2}}}{m_D}$.] The obtained string tension is manifestly k-independent.

Equation (B.4) then leads to the conclusion that, for a flat surface, Casimir scaling holds in the full sine-Gordon theory describing the monopole plasma in the 3-d Georgi-Glashow model. In another words, for a flat surface, Casimir scaling holds also at monopole densities close to the mean one, $\zeta N(N-1)$, rather than only at the densities much smaller than $\zeta N(N-1)$, as it takes place for a non-flat surface [13] [1]. It is finally reasonable to have some feeling on how much a surface should deviate from a flat one in order that the Casimir scaling starts violating, if the density of monopoles is the mean one, $\zeta N(N-1)$. To this end, let us consider a straight string (apparently corresponding to a flat surface) of a minimal possible length, m_D^{-1}. According to the first of eqs. (B.8), the field $B(z)$ at the end points of such a string decreases above and below the surface also at the distance m_D^{-1}. Therefore, if we start bending the string such that it forms a piece of a circle, the two solutions overlap with each other if the radius of this circle is $\leq m_D^{-1}$. The critical situation when this radius is equal to m_D^{-1} corresponds to the distance between the end points of the string equal to $2m_D^{-1}\sin\frac{1}{2}$. This is the minimal distance which should hold between two points, separated by the distance m_D^{-1} along the string, at which the surface can still be considered as flat from the point of view of the Casimir scaling.

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[1] It has been argued in ref. [13] that non-diluteness corrections are suppressed if $N \lesssim \mathcal{O}\left(e^{S_0/2}\right)$. Here, $S_0 = \frac{4\pi\epsilon m_W}{g^2}$ is a single-monopole action, $S_0 \gg 1$, where m_W is the W-boson mass, and the function ϵ describes quantum corrections to the classical expression, $1 \leq \epsilon < 1.8$. Although this boundary on N (above which non-diluteness effects might significantly distort Casimir scaling) is exponentially large, it nevertheless does exist for a non-flat surface. Instead, as we have just seen, for a flat surface, Casimir scaling holds at any, whatever large, N.

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