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Flat Directions in Three-Generation Free-Fermionic String Models

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Abstract

In quasi-realistic string models that contain an anomalous $U(1)$ the non-zero Fayet-Iliopoulos term triggers the shifting of the original vacuum to a new one along some flat direction, so that SUSY is preserved but the gauge group is partially broken. The phenomenological study of these models thus requires as a first step the mapping of the space of flat directions. We investigate F- and D-flat directions in several three-generation $SU(3)_C \times SU(2)_L \times U(1)_Y$ free-fermionic string models and discuss the typical scenarios that generically arise. When they exist, we systematically construct the flat directions that preserve hypercharge, only break Abelian group factors, and can be proven to remain F-flat to all orders in the non-renormalizable superpotential.

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I. INTRODUCTION

While the investigation of the phenomenology of string models continues to face serious difficulties, such as the problem of the degeneracy of string vacua and the absence of a satisfactory scenario for supersymmetry breaking, classes of quasi-realistic models have been constructed that warrant further phenomenological study.

The quasi-realistic string models are those which possess at least the basic structure of the MSSM at the string scale (M_{String}). Such models have $N = 1$ supersymmetry, the standard model (SM) gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C$ as a part of the full gauge structure, and candidate fields for the three ordinary families and at least two SM Higgs doublets. Classes of such models which satisfy the above requirements have been constructed based on the weakly coupled heterotic superstring. In particular, we focus on models based on the free-fermionic construction [1–3]. In these constructions, the mass spectrum and superpotential (in principle to all orders in the nonrenormalizable terms) are calculable.

In addition to the standard model (observable) gauge group, the gauge structure of these models includes a non-Abelian (NAB) hidden sector and a number of additional $U(1)$'s, of which at least one is generally anomalous[1] (by which we mean the charge trace over all matter states is non-zero). The appearance of such an anomaly will play a crucial role in the phenomenology of the model. The SM hypercharge is a linear combination of the nonanomalous $U(1)$'s (or perhaps of the $U(1)$'s that can arise after the non-Abelian hidden sector group is broken), which is chosen taking into account some basic phenomenological requirements.

In general, the particle spectrum is such that there are many additional matter multiplets along with the MSSM particle content. Most of the states can be classified according to those which are representations of the observable sector NAB gauge group, representations of the hidden sector NAB gauge group, and NAB singlets (i.e., fields which are singlets under NAB gauge groups but can carry $U(1)$ charges). However, the division between the observable and hidden sector gauge groups is tenuous at best, because most of the fields are charged under the Abelian gauge groups. In some models, there also are "mixed" states which are non-singlets under both the observable and hidden sector NAB gauge groups. Such states, if present, may have important consequences for the phenomenology of the model.

The Green-Schwarz anomaly cancellation mechanism at genus-one in string theory generates a constant Fayet-Iliopoulos (FI) contribution to the D- term of the anomalous $U(1)$ [6–9] which is proportional to the trace of the anomalous charge over all of the fields in the model. The FI term would break supersymmetry in the original string vacuum, but it triggers string-scale vacuum expectation values for certain scalar fields. In the new shifted vacuum the D- and F- flatness constraints are satisfied, supersymmetry is restored, and the anomalous $U(1)$ is broken. There are many possibilities for this vacuum shifting, and thus an analysis of the space of D- and F- flat directions is the necessary first step before addressing the phenomenology of the model.

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[1]Conditions sufficient to keep all $U(1)$'s anomaly free [3] have recently been discussed in [4] and [5], with an anomaly-free semi-GUT presented in [5].

In a previous paper [10], we developed a method to classify in a systematic manner a subset of the flat directions of a general perturbative string model with an anomalous $U(1)$. While there is no reason a priori why a flat direction should not involve fields in non-trivial representations of NAB groups, we chose to analyze those flat directions formed only from the NAB singlets for simplicity. Our method involves determining a set of holomorphic gauge invariant monomials (HIM's) [11] that characterizes the moduli space of flat directions under the nonanomalous $U(1)$'s. A straightforward classification of the singlet fields of the model according to their charges under a conveniently defined combination of $U(1)$ charges (or of the HIM's according to their anomalous charges) can then determine by inspection if it is possible to form a flat direction that can cancel the anomalous D- term and preserve $U(1)_Y$. When such flat directions can be formed, we presented a systematic way to classify the subset of these flat directions which can be proved to be F- flat to all orders in the nonrenormalizable superpotential (and to all orders in string genus perturbation expansion). We demonstrated our method in [10] by applying it to a prototype free-fermionic string model, Model 5 of Chaudhuri, Hockney and Lykken in [12], for which the results were particularly straightforward.

In this paper, we apply this method to a number of free-fermionic three-generation string models, those presented in [13], [14], and [12]. We determine for each model whether it is possible to construct hypercharge-preserving flat directions involving the NAB singlet fields which can cancel the FI D-term. For the models in which this is the case, we examine the space of such flat directions in detail.

In Section II, we describe the construction and general properties of the models under consideration. We discuss our analysis of determining whether one can construct good flat directions and explain our procedure for determining a viable hypercharge in Section III. In Section IV, we present the results for each model. The summary and conclusions are given in Section V.

II. $SU(3)_C \times SU(2)_L \times U(1)_Y$ MODELS OF PERTURBATIVE HETEROTIC STRINGS

A. Standard Model Free-Fermionic Embeddings

Quasi-realistic four-dimensional perturbative heterotic string models generally contain gauge structures that extend beyond the rank four SM $SU(3)_C \times SU(2)_L \times U(1)_Y$. In string models, gauge groups with ranks significantly larger than four are a generic by-product of conformal anomaly cancellation. The gauge structure of a stringy SM or (semi)-GUT can be expressed in the form,

$$\{SU(3)_C \times SU(2)_L \in \mathcal{G}\}_{\rm obs} \times \mathcal{G}^{\rm NA}_{\rm hid} \times \prod_n U(1)_n \times \prod_p \Delta_p \,, \tag{1}$$

where $\mathcal{G}$ denotes a possible SM GUT or semi-GUT embedding, $\mathcal{G}^{\rm NA}_{\rm hid}$ contains the hidden sector NAB gauge factors, and Δ_p represents possible local discrete symmetries. The various Abelian $U(1)_n$ charges may be carried by non-trivial matter representations of one or both

$\mathcal{G}_{\rm obs}$ and $\mathcal{G}^{\rm NA}_{\rm hid}$ non-Abelian gauge groups. Mixed states that transform under non-trivial representations of both $\mathcal{G}_{\rm obs}$ and $\mathcal{G}^{\rm NA}_{\rm hid}$ can be present.

One method of string model construction relies on world-sheet free fermions to represent the internal degrees of freedom necessary for conformal anomaly cancellation [1,2]. In this construction, four dimensional heterotic[2] models of this type contain 18 internal left-moving real world-sheet free fermions, $\psi^{r=3,20}$ and 44 right-moving real world-sheet free fermions, $\bar{\psi}^{r=21,64}$, in addition to the two left-moving real world-sheet fermions $\psi^{r=1,2}$ whose indices denote transverse spacetime directions.

In each free-fermionic model, a collection of boundary vectors $\{\vec{\alpha}\}$ allowed by modular invariance specifies the phase changes that the 64 world-sheet fermions ψ^r can undergo after they traverse the two non-contractible loops, l_α or l_β, on the genus-one world-sheet. While there are two genus-one loops, there is only one independent basis set of boundary vectors. Modular invariance requires that the two sets of allowed boundary conditions for the two loops are identical. Transporting a fermion ψ around the loop l_α (or l_β) results in appearance of a phase

$$\psi^r \rightarrow -\mathrm{e}^{i\pi\alpha_r}\psi^r, \tag{2}$$

for rational α_r in the range $-1 < \alpha_r \leq 1$. For complex fermions $\psi^j_c \equiv \psi^{r_1} + i\psi^{r_2}$,[3] these boundary vectors are associated with a charge lattice formed by $\{\vec{Q}_{\vec{\alpha}}\}$, where each charge vector has components

$$(Q_{\vec{\alpha}})_j = \frac{\alpha_j}{2} + F_j. \tag{3}$$

F_j is a number operator for fermion oscillator excitations with eigenvalues $\{0, \pm 1\}$ for non-periodic fermions and $\{0, -1\}$ for periodic.

In heterotic strings (with the world-sheet supersymmetric sector as left-moving and the world-sheet bosonic sector as right-moving) each right-moving complex fermion corresponds to a local $U(1)$ symmetry, whose massless generator is produced by the world-sheet simple current

$$U_j =: \psi^{j*}_c \psi^j_c : \,. \tag{4}$$

Simple currents have normalizations

$$\langle U_j, U_j \rangle = 1\,. \tag{5}$$

In contrast, each left-moving complex fermion is only associated with a global $U(1)$ symmetry.

Some of the 20 left-moving and 44 right-moving real world-sheet fermions ψ^r cannot always be paired (as a result of differing boundary conditions) to form left- or right-moving

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[2]We choose the left-movers as the fermions assigned to carry world-sheet supersymmetry.

[3]The indices j and r denote complex and real fermions, respectively.

complex fermions. Instead, a left-mover and a right-mover may be paired to form a non-chiral Ising fermion, or some left-movers and/or right movers may remain unpaired, forming chiral Ising fermions. In both of these cases, the boundary conditions are limited to $\alpha_r = 0, 1$. Clearly, models must contain an even number of both left- and right-moving real Ising fermions. For every two right-moving Ising fermions the rank of the gauge group is reduced by one (independent of how these fermions divide into chiral or non-chiral Ising types).

To date, essentially only two primary embeddings of SM simple roots on charge lattices have been used in free-fermionic models (although many other embeddings are possible). The SM embedding used in [13] and [14] (and all other "NAHE" class models [15]) is also the minimal embedding. That is, the necessary root charges are obtained using the lowest possible number of complex world-sheet fermions, which is five. No fewer than five are sufficient because level-one $SU(n)$ roots can only be obtained by breaking $SO(2n) \to SU(n) \times U(1)$ (the exception to this rule is associated with $SU(2) \times SU(2) \equiv SO(4)$). Thus, the minimal free-fermionic SM embedding is,

$$\mathcal{G} = SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \, , \tag{6}$$

where the rank 3 algebra, $SU(3)_C \times U(1)_C \in SO(6)$, originates in the charges of three complex fermions, $\bar{\psi}_c^{j=1,2,3}$, and that of the rank 2 algebra, $SU(2)_L \times U(1)_L \in SO(4)$, from those of two additional complex fermions, $\bar{\psi}_c^{j=4,5}$. In terms of the five charges of the $3 + 2$ complex fermions, the simple roots for $SU(3)_C$ and $SU(2)_L$ are

$$\begin{aligned} SU(3)_C : (&\ \ 1, -1, \ \ 0; \ \ 0, \ \ 0) &(7)\\ (&\ \ 0, \ \ 1, -1; \ \ 0, \ \ 0) &(8)\\ SU(2)_L : (&\ \ 0, \ \ 0, \ \ 0; \ \ 1, -1). &(9) \end{aligned}$$

In contrast, the seven models in [12] involve a non-minimal SM charge embedding, requiring eight complex fermions,

$$\begin{aligned} SU(3)_C : (&\ \tfrac{1}{2}, \ \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -1, \ \ 0, \ \ 0, \ \ 0) &(10)\\ (&\ 0, \ \ 0, \ \ 0, \ \ 1, \ \tfrac{1}{2}, \ \tfrac{1}{2}, -\tfrac{1}{2}, \ \tfrac{1}{2}) &(11)\\ SU(3)_C : (&\ \tfrac{1}{2}, \ \tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \ \ 1, \ \ 0, \ \ 0, \ \ 0) \, . &(12) \end{aligned}$$

The general form of hypercharge candidates in minimal SM embedding models is

$$U(1)_Y = b_C U(1)_C + b_L U(1)_L + \sum_n b_n U(1)_n \, , \tag{13}$$

where the b's are rational coefficients. It has been argued by claims of phenomenological necessity [16] that viable hypercharge definitions are limited to

$$b_C = \frac{1}{3}; \ b_L = \pm\frac{1}{2}; \ b_n = 0. \tag{14}$$

Generally the "+" sign is chosen, as is true in particular for [13] and [14]. The choice of the opposite sign corresponds to an exchange of particle identities $u^c \leftrightarrow d^c$, $e^c \leftrightarrow \nu^c$, and $H_U \leftrightarrow H_D$.

In the non-minimal SM embedding of [12] there are no apparent special Abelian symmetries like $U(1)_C$ and $U(1)_L$, so the generic expression for a hypercharge candidate is simply

$$U(1)_Y = \sum_n b_n U(1)_n \,. \tag{15}$$

Phenomenologically viable possibilities for b_n in this class will be analyzed in the subsequent section, including those presented for models CHL4 through CHL6 in [12].

Generally the currents U_n of the Abelian symmetries $U(1)_n$ are not themselves simple currents U_j as defined in (4) and (5). Instead, most of the U_n are linear combinations of simple currents,

$$U_n = \sum_j b'_{n,j} U_j \,. \tag{16}$$

Often such linear combinations are not normalized to one, i.e.,

$$\langle U_n,\ U_n \rangle \neq 1 \,. \tag{17}$$

Differing hypercharge definitions can significantly alter the phenomenology of the effective field theory. In particular, the definition determines the value of k_Y for $U(1)_Y$ [17].[4] From the low-energy point of view, the value of k_Y is especially important with regard to shifts in the string unification scale M_{string}. In string models, k_Y and the corresponding levels, k_3 and k_2, of $SU(3)_C$ and $SU(2)_L$, are related at tree level by

$$g_i^2 k_i = g^2 = g^2_{\text{string}}/2, \tag{18}$$

where $g_{Y,2,3}$ are the canonical gauge couplings of $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively defined so that $Y = \frac{1}{6}$ for a quark doublet Q_L and $\operatorname{tr} t_a t_b = \frac{1}{2}\delta_{ab}$ (corresponding to the highest root ψ normalized to $\psi^2 = 1$) for the generators t_a of the fundamental representations of $SU(2)$ and $SU(3)$. At one-loop, these gauge couplings obey the renormalization group equations of the effective field theory:

$$\frac{16\pi^2}{g_i^2(\mu)} = k_i \frac{16\pi^2}{g^2} + b_i \ln \frac{M^2_{\text{string}}}{\mu^2} + \Delta_i, \tag{19}$$

for $g_i = g_Y,\, g_2,\, g_3$. b_i are the one-loop beta-function coefficients and Δ_i are "threshold" corrections from the infinite tower of massive string states. $M_{\text{string}} \sim 5 \times 10^{17}$ GeV is the one-loop corrected coupling unification scale [18].

k_Y can be determined by studying a fermion-fermion-gauge coupling. While k_i is limited to positive integer values for NAB groups, k_i is not necessarily integer in the Abelian case, but can take on positive rational values. When $SU(3)_C \times SU(2)_L$ is embedded in $SU(5)$ (or is embedded in $SU(5)$ in a string model, but $SU(5)$ is broken by string boundary conditions

[4] In string theory k_Y relates the definition of hypercharge Y to the hypercharge contribution h_Y to the total conformal dimension, $h_Y = \frac{Y^2/k_Y}{2}$, of a physical state.

– as in minimally embedded "NAHE" free-fermionic models [15]), the lowest possible value for k_Y is $\frac{5}{3}$. Note however that values of $k_Y < \frac{5}{3}$ lead to somewhat better agreement between the string scale and MSSM unification scale (assuming the MSSM particle content).

For a generic hypercharge

$$Y \equiv \sum_n b_n Q_n = \sum_j b_j Q_j, \tag{20}$$

k_Y can be expressed as [12,19],

$$k_Y = 2\sum_n b_n^2 \langle U_n, U_n\rangle = 2\sum_j b_j^2 \,, \tag{21}$$

where each U_n is the current associated with the corresponding $U(1)_n$, which are linear sums of the $U(1)_j$ with unit norm. The factor of two in eq. (21) is a result of differing string and field-theoretic conventions with regard to traces of non-Abelian gauge generators [18] (and the related highest root normalizations). While the canonical field-theoretic choice is $\mathrm{tr}\, t_a t_b = \frac{1}{2}\delta_{ab}$ (with $\psi^2 = 1$) for $SU(n)$ gauge groups, in string theory the corresponding choice is $\mathrm{tr}\, t_a t_b = \delta_{ab}$ (with $\psi^2 = 2$) instead[5]. When the gauge group includes an anomalous $U(1)$ there is a simpler way of computing k_Y, as will be explained in the next sub-section.

B. Anomalous $U(1)$ and Flat Directions

The appearance of anomalous $U(1)$'s in four-dimensional string models has been discussed extensively [4,5,10,20,21]. In the original free-fermionic charge basis of (3), models usually contain more than one $U(1)_n$ with $\mathrm{Tr}Q_n \neq 0$. However, the anomaly can be transferred into a single $U(1)_A$ through the unique rotation

$$U(1)_{\mathrm{A}} \equiv c_A \sum_n \{\mathrm{Tr}Q_n\} U(1)_n, \tag{22}$$

with c_A a normalization factor. Following this rotation, a complete orthogonal basis $\{U(1)_a\}$ may be formed from the non-anomalous components of the original set of $\{U(1)_n\}$.

The elimination of all triangle anomalies except those involving one or three $U(1)_A$ gauge bosons is guaranteed by the Green-Schwarz (GS) relations,

$$\frac{1}{k_m k_A^{1/2}} \mathop{\mathrm{Tr}}_{G_m} T(R) Q_A = \frac{1}{3k_A^{3/2}}\mathrm{Tr}Q_A^3 = \frac{1}{k_a k_A^{1/2}}\mathrm{Tr}Q_a^2 Q_A = \frac{1}{24 k_A^{1/2}}\mathrm{Tr}Q_A \equiv 8\pi^2\delta_{\mathrm{GS}}\,, \tag{23}$$

$$\begin{aligned}\frac{1}{k_m k_a^{1/2}} \mathop{\mathrm{Tr}}_{G_m} T(R) Q_a &= \frac{1}{3k_a^{3/2}}\mathrm{Tr}Q_a^3 = \frac{1}{k_A k_a^{1/2}}\mathrm{Tr}Q_A^2 Q_a = \frac{1}{(k_a k_b k_A)^{1/2}}\mathrm{Tr}Q_a Q_{b\neq a} Q_A \\ &= \frac{1}{24 k_a^{1/2}}\mathrm{Tr}Q_a = 0\,,\end{aligned} \tag{24}$$

[5]A factor of two was not included in the definition of hypercharge level used in [12]. Thus, the correct values of the k_Y's in the models presented in [12] are actually twice that given in [12], and are all greater than $\frac{5}{3}$.

where k_m is the level of the gauge group G_m and $2T(R)$ is the index of the representation R, defined by

$$\mathrm{Tr}\, T_a^{(R)} T_b^{(R)} = T(R)\delta_{ab} \,. \tag{25}$$

In a generic field-theoretic model, (22) would not necessarily place the entire anomaly into a single $U(1)_A$. The GS relations result from stringy modular invariance constraints and guarantee the consistency of the model. The physical content behind these relations is that the mixed anomalies are canceled by the pseudoscalar partner of the string dilaton, which couples universally to all gauge groups. The relations (23) can be used to compute k_Y once the states in the massless spectrum and their charges are known, without further knowledge of the string origin of each state [22].

The standard anomaly cancellation mechanism [6–9] generates a FI D-term,

$$\xi = \frac{g^2_{\rm string} M_{\rm P}^2}{192\pi^2} \mathrm{Tr} Q_A \,, \tag{26}$$

where $M_{\rm P} = M_{\rm Planck}/\sqrt{8\pi}$ with $M_{\rm Planck} \sim 1.2 \times 10^{19}$ GeV.

The FI D- term is calculable in perturbative string theory, since it is a genus-one string effect when determining masses [7] (and a genus-two effect when calculating the dilaton tadpole [9]). The FI D- term triggers a shift to a nearby deeper vacuum with non-zero VEVs for the scalar components φ_i of supermultiplets Φ_i such that the D-flatness constraints are satisfied[6]

$$D_{\rm A} = \sum_i Q_i^{(A)} |\varphi_i|^2 + \xi = 0 \ , \tag{27}$$

$$D_a = \sum_i Q_i^{(a)} |\varphi_i|^2 = 0, \tag{28}$$

along with F- flatness,

$$F_i = \frac{\partial W}{\partial \Phi_i} = 0; \;\; W = 0. \tag{29}$$

C. String Selection Rules for Superpotential Terms

The (perturbative) superpotential for the low-energy effective field theory of an underlying string model is significantly more constrained than a generic field-theoretic superpotential, resulting from additional world-sheet symmetries than simply those that translate into

[6]Our convention for defining $D_{\rm A}$ is that the corresponding D- term in the Lagrangian is $\frac{1}{2k_A} g^2 D_{\rm A}^2$, and similarly for D_a.

standard gauged spacetime symmetries. Consequently, stringy superpotentials will generically have fewer terms at a given order than one would expect simply from gauge group charge conservation.

Coupling coefficients for a superpotential term of order $(K+3)$, with $K \geq 0$, can be cast in terms of a $(K+3)$–point tree-level string amplitude A_{K+3} of the form, [23,24]

$$A_{K+3} = g\frac{C_K I_K}{(\pi M_{\rm P})^K}\,, \tag{30}$$

where g is the gauge coupling at $M_{\rm string}$ and C_K is a coefficient of $\mathcal{O}(1)$ that encompasses different renormalization factors in the OPE of string vertex operators (including the target space gauge group Clebsch-Gordan coefficients). For the renormalizable $(K=0)$ trilinear terms, the I_0 factor reduces to a correlation function f_0 of world-sheet coordinates, whereas for non-renormalizable $(K>0)$ terms, I_K is an integral of a correlation function f_K over $K-3$ world-sheet coordinates

$$I_K = \int d^2 z_3 \cdots d^2 z_{K+2} f_K(z_1=\infty, z_2=1, z_3, \cdots, z_{K+2}, z_{K+3}=0), \tag{31}$$

where z_i is the world-sheet coordinate of the conformal dimension $(1,1)$ vertex operator V_i for the $i^{\rm th}$ physical heterotic string state. By $SL(2,C)$ world-sheet invariance, three coordinates may be fixed. Generally, $z_1 \to \infty$, $z_2 = 1$, and $z_{K+3} = 0$ is chosen.

In the free-fermionic construction, the vertex operator V_i may be factored into a product of vertex operators associated with (i) the Lorentz spacetime momentum factor, V_i^{mom}; (ii) the Lorentz spacetime spin factor, V_i^{spin}; (iii) the BRST superconformal ghost charge, V_i^{BRST}; (iv) the global left-moving and local right-moving Abelian symmetry groups, $\prod_{n'} V_i^{U(1)^{global}_{n'}}$ and $\prod_n V_i^{U(1)^{local}_n}$, respectively; (v) the local right-moving NAB symmetry groups $\prod_m V_i^{\mathcal{G}_m}$; (vi) the non-chiral Ising functions, $\prod_q V_i^{NCI_q}$; and (vii) the chiral Ising functions $\prod_{q'} V_i^{CI_{q'}}$.

The correlation function f_K similarly factors into products of correlation functions for each of these classes of vertex operators [23],

$$\begin{aligned} f_K(z) = &\langle \prod_{i=1}^{K+3} V_i^{mom}\rangle \langle \prod_{i=1}^{K+3} V_i^{spin}\rangle \langle \prod_{i=1}^{K+3} V_i^{BRST}\rangle \prod_{n'}\langle \prod_{i=1}^{K+3} V_i^{U(1)^{global}_{n'}}\rangle \prod_{n}\langle \prod_{i=1}^{K+3} V_i^{U(1)^{local}_{n}}\rangle \\ &\prod_m\langle \prod_{i=1}^{K+3} V_i^{\mathcal{G}_m}\rangle \prod_q\langle \prod_{i=1}^{K+3} V_i^{NCI_q}\rangle \prod_{q'}\langle \prod_{i=1}^{K+3} V_i^{CI_{q'}}\rangle\,. \end{aligned} \tag{32}$$

The spacetime spin correlator for two spacetime fermions and $K+1$ scalars is trivial, contributing only a factor of

$$\langle S_\alpha(z_1) S_\beta(z_2)\rangle = (z_1 - z_2)^{-1/2} \tag{33}$$

to f_K, where S_α is a conformal field representing a Lorentz spinor. With the exception of the Ising correlation functions, the remaining correlation functions in the vertex operators have exponential form. For an Abelian symmetry or the BRST ghost charge it is

$$\langle\prod_i \mathrm{e}^{iQ_iH}\rangle = \prod_{i<j} z_{ij}^{Q_iQ_j}\,, \tag{34}$$

while for a non-Abelian symmetry the correlation function is

$$\langle\prod_i \mathrm{e}^{i\vec{Q}_i\cdot\vec{J}}\rangle = \prod_{i<j} z_{ij}^{\vec{Q}_i\cdot\vec{Q}_j}\,, \tag{35}$$

where $z_{ij} \equiv z_i - z_j$ (in this language, $Q_i = -ic$ is imaginary for ghost systems). Non-conservation of any (local or global) Abelian or non-Abelian charge, i.e., a case of $\sum_i Q_i \neq 0$ or $\sum_i \vec{Q}_i \neq \vec{0}$, yields $I_K = 0$. On the other hand, the vertex operators must contribute a total BRST superconformal ghost charge of -2 to cancel the superconformal ghost charge carried by the vacuum [25]. The spacetime momentum correlation function is

$$\langle\prod_i \mathrm{e}^{i\frac{1}{2}\vec{K}_i\cdot\vec{X}_i}\mathrm{e}^{i\frac{1}{2}\vec{K}_i\cdot\vec{\overline{X}}_i}\rangle = \prod_{i<j} |z_{ij}|^{\frac{1}{2}\vec{K}_i\cdot\vec{K}_j}\,. \tag{36}$$

Ising correlators (both non-chiral and chiral classes) are non-trivial. For example, there are six types of conformal fields (including the identity operator) associated with a non-chiral Ising fermion: a left-moving real world-sheet fermion $f(z)$, its right-moving counterpart $\overline{f}(\overline{z})$, the energy operator $\epsilon(z,\overline{z}) \equiv f\overline{f}$, and spin fields $\sigma_+(z,\bar{z})$ and $\sigma_-(z,\bar{z})$ (also known as order/disorder operators). Correlators involving the spin fields of a given Ising fermion are non-zero if and only if they can be factored into combinations of

$$\langle\sigma_+\sigma_+\rangle, \quad \langle\sigma_-\sigma_-\rangle, \quad \langle\sigma_+\sigma_- f\rangle, \quad \text{and} \quad \langle\sigma_+\sigma_-\overline{f}\rangle, \tag{37}$$

while correlators not involving the spin fields require an even number of both f and $\overline{f}$.

Chiral Ising correlation functions have additional subtleties over non-chiral Ising correlators, but under certain conditions [2,26], they may be represented in terms of vertex operators of "broken $U(1)$ charges." When these conditions are satisfied (such as in the CHL models), the chiral Ising fermions may actually be paired in a vertex operator and associated with a "broken charge" pair $\pm|Q|$. Then the vertex operator of the set of all chiral Ising fields, can be written for a physical state as

$$V_i^{CI} = \prod_{q'} V_i^{CI_{q'}}, \tag{38}$$

whose two charge vectors, $\vec{Q}^{(i)}$ and $-\vec{Q}^{(i)}$, differ only by an overall sign. The correlation function $\langle\prod_{i=1}^{K+3} V_i^{CI}\rangle$ is nonzero when there is a choice of signs such that $\sum_i \pm|\vec{Q}|^{(i)} = \vec{0}$.

Both conservation of global world-sheet charges and the Ising field correlation selection rules are truly stringy effects. In Section IV, we shall see examples in which these stringy effects reduce the number of superpotential terms at a given order otherwise allowed by gauge invariance [27].

1. Picture Changing and Charge Conservation

Satisfying both conservation of global world-sheet charges and the Ising field selection rules of (37) in the superpotential terms of the effective field theory of a string model has its subtleties. The complications stem from cancelling the vacuum BRST ghost charge [28,25] (mentioned above).

The correlation function f_K is generated by the product of the vertex operators V_i for the $K+3$ superfields Φ_i forming the candidate superpotential term,

$$f_K = \langle V_1^f(z_1 \to \infty) V_2^f(z_2 = 1) V_3^b(z_3) \cdots V_{K+3}^b(z_{K+3} = 0) \rangle \,, \tag{39}$$

where $V_i^{f\,(b)}$ is the fermionic (bosonic) part of the complete superfield vertex operator V_i. The BRST ghost charge associated with the canonical fermionic (bosonic) vertex operator is $-\frac{1}{2}$ (-1). Conformal invariance allows a physical vertex operator with a given ghost charge to be "picture-changed" into an equivalent vertex operator with a new ghost charge differing by an integer value from the first. However, as we discussed, cancellation of the vacuum ghost charge anomaly requires that the net ghost charge for f_K be -2. Since in the canonical picture the net ghost charge is $-2 + (3-K)$, the last K bosonic vertex operators should be picture-changed to carry a ghost charge of 0.[7] That is,

$$V^b_{4\,(-1)} \cdots V^b_{K+3\,(-1)} \to V^b_{4\,(0)} \cdots V^b_{K+3\,(0)} \,, \tag{40}$$

with ghost charge of a vertex operator explicitly denoted by the subscript in parenthesis.

The spin-$\frac{3}{2}$ supercurrent $T_{3/2}$ of the $N = 2$ world-sheet supersymmetry acts as the picture-changing operator for a superfield vertex operator, increasing the vertex operator ghost charge by one unit:

$$V_{(c+1)}(z) = \lim_{w \to z} \mathrm{e}^c \, T_{3/2}(w) V_{(c)}(z) \,. \tag{41}$$

$T_{3/2}$ can be separated into three components distinguished by their respective charges (appearing as superscripts) under the $U(1)_{N=2}$ current (of the $N = 2$ global world-sheet supersymmetry) also present in the $N = 2$ algebra:

$$T_{3/2} = T^0_{3/2} + T^{-1}_{3/2} + T^{+1}_{3/2} \,, \tag{42}$$

where the superscripts denote world-sheet charges. The canonical bosonic (fermionic) vertex operators with ghost charge -1 $(-\frac{1}{2})$ also carry $+1$ $(-\frac{1}{2})$ charge under $U(1)_{N=2}$. Thus, when acting on the ghost charge -1 vertex operators, only the $T^{-1}_{3/2}$ component of $T_{3/2}$ will lead to conservation of f_K's total $U(1)_{N=2}$ charge in (32) and (39).

When all 20 left-moving real world-sheet fermions $\psi^{j=1,20}$ have only periodic/antiperiodic boundary conditions,[8] the 18 internal fermions can be regrouped into six sets of three fermions (and appropriately relabelled):

$$\{(\psi^1\psi^2),\ (x, y, \omega)^{i=1 \text{ to } 6}\} \,. \tag{43}$$

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[7] Any other set of picture-changes that similarly yield a net ghost charge of -2 for f_K would also be acceptable and would generate the same superpotential terms.

[8] $N = 2$ algebras for free-fermionic models containing left-moving fermions with rational, non-integer boundary conditions have been studied in [29].

While spacetime supersymmetry requires fermions x^{2j-1} and x^{2j} to combine into complex fermions $X^{2j-1,2j} \equiv \frac{1}{\sqrt{2}}(x^{2j-1}+ix^{2j})$, for $j=1$ to 3, the various y^i and ω^i are unconstrained and may form either complex or Ising types. In terms of the 20 real fermions (and two transverse world-sheet bosons $X_{\mu=1,2}$) the world-sheet supercurrent is

$$T_{3/2}(z) = \psi^\mu \partial X_\mu + i\sum_{i=1}^{6} x^i y^i \omega^i \tag{44}$$

while the $U(1)_{N=2}$ generator (connected with spacetime supersymmetry) is

$$J(z) =: X^{12*}X^{12} : + : X^{34*}X^{34} : + : X^{56*}X^{56} : . \tag{45}$$

Eq. (45) indicates that the global $U(1)_{N=2}$ charge of a state is the sum of all three of its $X^{2j-1,2j}$ charges. Complex fermions like the X can be replaced by real bosons S, using the $U(1)$ current boson/fermion identity,

$$i\partial_z S(z) =: X^* X : . \tag{46}$$

This equivalence allows $J(z)$ to be expressed as

$$J(z) = i\partial_z (S_{12} + S_{34} + S_{56}) . \tag{47}$$

A related bosonized form of $T_{3/2}(z)$ is easily separated into the three components identified in (42) by their $U(1)_{N=2}$ charges:

$$T^{0}_{3/2}(z) = \psi^\mu \partial X_\mu \tag{48}$$
$$T^{-1}_{3/2}(z) = \mathrm{e}^{-iS_{12}}\tau_{12} + \mathrm{e}^{-iS_{34}}\tau_{34} + \mathrm{e}^{-iS_{56}}\tau_{56} \tag{49}$$
$$T^{+1}_{3/2}(z) = -(T^{-1}_{3/2}(z))^*, \tag{50}$$

where,

$$\tau_{ij} \equiv \frac{i}{\sqrt{2}}(y^i\omega^i + iy^j\omega^j). \tag{51}$$

The form of $T^{-1}_{3/2}$ in (49) implies that picture-changing a V^b_{-1} operator into a V^b_0 operator via (41), $T^{-1}_{3/2}$ simultaneously alters both the $U(1)_{N=2}$ charge and some y- and ω-related charges or Ising fields. Consider, for example, the effect of the first component, $\mathrm{e}^{-iS_{12}}(y^i\omega^i)$, of $T^{-1}_{3/2}$ on a generic V^b_{-1}. The operator $\mathrm{e}^{-iS_{12}}$ decreases the X^{12} charge (and thus also the total $U(1)_{N=2}$ charge) by 1. Further, if y^1 is an Ising fermion, then a spin field $\sigma^{y^1}_+$ ($\sigma^{y^1}_-$) in V^b_{-1} will be converted into the opposite spin field type, $\sigma^{y^1}_-$ ($\sigma^{y^1}_+$), in V^b_0 by the y^1 factor in this $T^{-1}_{3/2}$ component. If, however, there were no such Ising spinor in V^b_0, then a new Ising fermion excitation y^1 would appear in V^b_0. On the other hand, if y^1 was part of a complex fermion, e.g., $Y^{1,3} \equiv \frac{1}{\sqrt{2}}(y^1+iy^3)$, then the y^1 operator in the $T^{-1}_{3/2}$ component would create two separate terms in V^b_0: one term would have its $Y^{1,3}$ charge raised by one unit and the other term would have its $Y^{1,3}$ charge lowered by one unit. Raising and lowering of the charge would both occur because in terms of $Y^{1,3(*)}$, $y^1 = \frac{1}{\sqrt{2}}(Y^{1,3}+iY^{1,3*})$. That is, y^1

contains both charge raising and lowering operators. The ω^1 in factor $T^{-1}_{3/2}$ component would act similarly on V^b_{-1}.

Therefore, terms that seem to be allowed (disallowed) prior to picture-changing may actually be disallowed (allowed). Of particular importance is that picture-changed bosonic vertex operators V^b_0 can contain several different terms due to (i) the six separate terms in $T^{-1}_{3/2}(z)$ and (ii) operators in each term of $T^{-1}_{3/2}(z)$ that can sometimes act as both raising and lowering operators of y- and ω-associated charges. When a correlation function f_K is under examination, all possible combinations of terms in the K pictures-changed vertex operators must be considered.

We now illustrate the technique explicitly, by considering a specific example. The free-fermionic model presented in [13] contains among its $N = 1$ spacetime superfields four denoted as H_{39}, H_{37}, H_{32}, and H_{30} (these fields are identified in our Tables IIIa and IIIb as S_{43}, S_{28}, S_{24}, and S_7, respectively). Their gauge charges can be found in Table 2 of [13]. This is the same set of four non-Abelian singlets for which in [24] we computed the integral I_1 of the correlation function f_1. In Table I we list left-moving global $U(1)$ charges and non-chiral Ising fields in the fermion components of the superfields H_{39} and H_{37} and in the bosonic components of H_{32} and H_{30}.

The model in [13] contains six complex left-moving world-sheet fermions: $\psi^{12} = \psi^1 + i\psi^2$, $X^{12} = x^1 + ix^2$, $Y^{16} = y^1 + i\omega^6$, $W^{13} = y^1 + i\omega^3$, $X^{34} = x^3 + ix^4$, and $Y^{36} = y^3 + iy^6$. Table I only lists charges under the first real fermion component of a complex fermion.

Additionally, the model possesses six non-chiral Ising pairs: $(y^2, \bar{\psi}^{38})$, $(\omega^2, \bar{\psi}^{44})$, $(y^4, \bar{\psi}^{40})$, $(\omega^4, \bar{\psi}^{46})$, $(y^5, \bar{\psi}^{41})$, and $(\omega^5, \bar{\psi}^{47})$. There are no anti-periodic excitations from any of the right-moving $\bar{\psi}$ components in non-chiral Ising fermion pairs for any of the four superfields. Thus, the corresponding $\bar{\psi}$ are not relevant to the picture-changing discussion below.

We choose to picture change H_{30}. From Table I we see that the net S_{34} and S_{56} charges are both zero before picture changing, while the net S_{12} charge is $+1$. Thus, we must use the $\mathrm{e}^{-iS_{12}}\tau_{12}$ component of $T^{-1}_{3/2}$ to picture change H_{30} and cancel the S_{12} pre-picture changed charge. Before picture changing, the Ising field correlations for $(y^2, \bar{\psi}^{38})$ and $(\omega^2, \bar{\psi}^{44})$ are non-zero ($\langle\sigma^+\sigma^+\rangle$ and $\langle\sigma^-\sigma^-\rangle$, respectively), while the net Y^{16} and W^{13} global $U(1)$ charges are both -1 and -1, respectively.

The y^2 and ω^2 Ising correlations remain unaffected by the $y^1\omega^1$ component of $\tau_{12} \equiv \frac{i}{\sqrt{2}}(y^1\omega^1 + iy^2\omega^2)$. In contrast, the raising operator in y^1 cancels the pre-picture change net Y^{16} charge by altering H_{30}'s Y^{16} charge from $-\frac{1}{2}$ to $\frac{1}{2}$. The raising operator in ω^1 similarly cancels the W^{13} charge.

Determining if the string amplitude is non-zero is straightforward for low values of K. This process might seem unwieldy for increasingly higher values of K; however, simple arguments based on invariance [25] of outcome under differing choices of picture-changed fields permit a more tenable approach which does not require that picture changing be performed on $(K-3)$ individual states. Instead, essentially only the total picture-changing effect from K various components of $T^{-1}_{3/2}$ need act on an effective "composite field." The charges and Ising fields of the effective composite state are the summations of the separate respective charges of the original $K+3$ *canonical* fields. The "composite field" charges for the W_4 example above are given in the "net charges" row of Table I. A zero net "charge" appears in an Ising field column if the Ising correlation function of the four fields is already non-zero.

For a non-zero string amplitude, there must be an allowed set of $K+3=\sum_{i=1}^{6} P_i$, for $P_i \in \{\mathrm{Z}^+, 0\}$, picture changing operators

$$\sum_{P_1} \mathrm{e}^{-iS_{12}}(y_1\omega_1) + i\sum_{P_2} \mathrm{e}^{-iS_{12}}(y_2\omega_2) + \sum_{P_3} \mathrm{e}^{-iS_{34}}(y_3\omega_3) + i\sum_{P_4} \mathrm{e}^{-iS_{34}}(y_4\omega_4) + \sum_{P_5} \mathrm{e}^{-iS_{56}}(y_5\omega_5) + i\sum_{P_6} \mathrm{e}^{-iS_{56}}(y_6\omega_6), \tag{52}$$

that both contains an appropriate set of total $U(1)$ charges to cancel the corresponding charges of the "composite field" and provides for non-zero Ising y's and ω's correlations.

Cancellation of the total $U(1)_{N=1}$ charge from two canonical fermion fields and $K-2$ canonical scalar fields, $Q^{net}_{N=2} = 2\times(-1/2)+(K-2)\times(1) = K-3$, is trivially accomplished by any $\{P_i\}$ set specifying the picture-changing operators in (52), since $(\sum_{i=1}^{6} P_i)\times(-1) = -(K+3)$. However, separate cancellation of the three charge components $Q^{net}_{i,i+1}$ of $Q^{net}_{N=2}$ associated with the three $S_{i,i+1}$, imposes the first requirement for a good picture-changing charge cancellation: each $Q^{net}_{i,i+1}$ must be a positive integer. Then cancellation of each $Q^{net}_{i,i+1}$ requires choices of P_i and P_{i+1} such that

$$(P_i + P_{i+1})(-1) = -Q^{net}_{i,i+1}. \tag{53}$$

Values of the six P_i must lead to cancellation of all y_i/ω_i-related charges. Our previous discussion of $\tau_{i,i+1}$ operators implies that P_i and $P_{i'}$ lead to cancellation of a particular charge $Q_{i,i'}$ associated with a complex fermion formed from real fermions f_i and $f_{i'}$, (where f is y or ω) if and only if

$$Q_{i,i'} = P_i + P_i' \pmod 2 \tag{54}$$

$$|Q_{i,i'}| \le P_i + P_i'. \tag{55}$$

Searching for possible $\{P_i\}$ solutions to these constraints and corresponding Ising-related constraints can be easily performed by a simple computer subroutine, making an efficient determination of high order terms in the superpotential feasible.

III. FLAT DIRECTION ANALYSIS

A. Classification of Fields

Our strategy to find the set of flat directions which satisfy the flatness constraints (27), (28), and (29) is the following. First, the moduli space of flat directions under the nonanomalous $U(1)$'s only is determined. This space of flat directions can be described by a basis of independent holomorphic gauge invariant (under the non-anomalous $U(1)$'s) monomials (HIM's) [11], or equivalently by a larger superbasis[9] of all one dimensional HIM's (i.e., with

[9]The elements of this superbasis are not linearly independent. However, it has the advantage that any flat direction can be expressed simply as a product of the elements in the superbasis.

one free VEV unconstrained by the flatness conditions). The HIM's of the basis and the superbasis are then classified according to their anomalous charge. If the sign of the anomalous charge of a given HIM is opposite to that of the FI term some free VEV in the flat direction will adjust itself to cancel the anomalous D-term (D_A). The F-flatness conditions are then addressed for this D- flat direction.

However, for the purpose of determining if there exist NAB singlet flat directions which can cancel D_A, there is a useful classification of the fields in the model which can show immediately if such a flat direction is possible, as we explained in [10]. For the sake of completeness, we repeat the strategy of this classification here. One defines an auxiliary charge $\overline{Q}$ as a linear combination of non-anomalous $U(1)$ charges

$$\overline{Q}_j \equiv \sum_{a=2}^{m} \alpha_a Q_j^{(a)}, \tag{56}$$

where the α_a are chosen for convenience trying to maximize the number of fields for which $Q_A = \overline{Q}$. This relation cannot hold for all the chiral fields in the model so we define the quantities

$$\hat{Q}_j = Q_j^A - \overline{Q}_j, \tag{57}$$

and classify all the chiral fields in three different types, depending on the sign of $\hat{Q}_j$:

$$\begin{aligned}
&\Phi_j^+, \ \text{if } \ \hat{Q}_j > 0, \\
&\Phi_j^0, \ \text{if } \ \hat{Q}_j = 0, \\
&\Phi_j^-, \ \text{if } \ \hat{Q}_j < 0.
\end{aligned} \tag{58}$$

With this classification and knowledge of the sign of the FI term (26), we can determine which fields are required for a flat direction that satisfies (27). The statement is as follows: Theorem: *If* $\xi > 0$ (< 0), *any flat direction must contain at least one of the fields* Φ_j^- (Φ_j^+).

Therefore, for each model our strategy is to determine $\overline{Q}$, classify the NAB singlets according to their values of $\hat{Q}_j$ and use the theorem to determine if the model has flat directions formed out of NAB singlets. If $\xi > 0$ (< 0) and no singlets of the type Φ_j^- (Φ_j^+) exist, there is no possibility of forming D- flat directions out of singlets only which can cancel the FI term. In that case, fields that transform under non-trivial representations of NAB groups (either from the hidden sector or the observable sector) must get VEVs along the flat directions of the model.

If the $\hat{Q}_j$ classification of the fields is such that D- flat directions can be formed out of the singlet fields, we follow our method of [10]: first we determine $U(1)_Y$ as a linear combination of the non-anomalous $U(1)$'s, as described in detail below, select the singlets with zero hypercharge (such that $\mathcal{G}_{\rm obs}$ remains unbroken), and check whether fields of the appropriate type (Φ_j^- or Φ_j^+) remain with $Y = 0$. We then construct a basis of HIM's, or equivalently the superbasis of all one dimensional HIM's, that describe the moduli space of non-anomalous flat directions. The space of flat directions that also have $D_A = 0$ is a subspace of this. It can be formed by combining the directions in the basis or superbasis, ensuring that the anomalous charge[10] of the resulting direction has sign opposite to ξ.

These D- flat directions are then tested for F- flatness. As discussed in [10], a subset of the D- flat directions can be proved to be F- flat to all orders in the non-renormalizable superpotential by imposing the constraints of gauge invariance and knowledge of the superpotential to a given order. This subset, referred to as Type-B directions, are those in which one cannot form total gauge singlet holomorphic operators from the fields that form the direction[11]. Then, for these directions there are only a finite number of possible terms in the superpotential which could lift F-flatness. Whether these terms are in fact present (i.e., are allowed by string selection rules) is then checked explicitly. In contrast, Type-A directions involve fields which can form gauge group singlets, so that terms which could lift F- flatness could occur to all orders in the superpotential.

We will restrict our analysis to type-B flat directions. Of course, in doing so we may leave out some flat directions which are in fact F- flat to all orders, but proving these directions are F- flat is a difficult task. We list the one-dimensional (zero-dimensional after cancelling the anomalous D- term) D- flat directions which remain F- flat to all orders, out of which higher dimensional flat directions may be formed. We also list the number of broken $U(1)$'s for each flat direction.

B. Hypercharge Determination

For each model, a viable hypercharge must be determined as a linear combination of the nonanomalous[12] $U(1)$'s which satisfies the following basic phenomenological criteria:

- Three generations of quarks and leptons, as well as a pair of electroweak Higgs doublets with conventional hypercharges.
- Grouping of all particles with nonzero charge under $SU(3)_C$ or $U(1)_{EM}$ into mirror pairs, such that mass terms can be generated and these particles made heavy. Oth-

[10]By gauge invariance the $\hat{Q}$ charge of a HIM coincides with its anomalous charge.

[11]The presence of the anomalous $U(1)$ is crucial to this point, as the HIM's associated with good flat directions are not invariant under $U(1)_A$.

[12]We do not consider the possibility of $U(1)_Y$ having some component along the generators of hidden sector NAB groups (broken at some scale). If that breaking is triggered by the FI term, we should consider flat directions that involve fields in nontrivial representations of these NAB groups, an analysis which is beyond the scope of this paper. We also ignore breaking at a lower scale (e.g., radiatively) for simplicity. The latter case would also significantly change the picture of SM gauge coupling unification. However, the additional matter content in these models can also modify the gauge unification. Such phenomenological issues are also beyond the scope of this paper.

erwise, there would necessarily be exactly massless colored or charged fermions in the theory, which are clearly excluded[13].

In general, the number of fields in each model that are candidates for the observable sector states is so large that a direct search for hypercharge candidates taking all combinations of possible observable sector fields would be very inefficient. Therefore, depending on the number of nonanomalous $U(1)$'s in the model, we seek the minimal number of constraints that can determine a hypercharge candidate and then check explicitly if the above conditions are satisfied.

In each model considered, there are only three candidates for the quark doublet states $(3,2)$ under $(SU(3)_C, SU(2)_L)$, so we require that these fields have the appropriate hypercharges $Y(Q_L) = \frac{1}{6}$. However, there are generally more than 6 candidates for the quark singlets $(\bar{3},1)$. In some models, there are $(\bar{3},1)$ states which are also multiplets under the NAB hidden sector gauge group, so that if the hidden sector gauge group is broken above the electroweak scale, these states may also be included in the list of candidates for the quark singlets. We scan all combinations of these fields such that we have three candidates for the right-handed up-type quarks, with hypercharge $Y(U_L^c) = -\frac{2}{3}$. If these conditions are not sufficient to determine Y, we also impose similar requirements for the existence of three right-handed down-type quarks, with $Y(D_L^c) = \frac{1}{3}$. We also impose the condition that the trace of the hypercharge over the remaining $(3,1)$ and $(\bar{3},1)$ states (not selected as quarks candidates) is zero. This is a necessary condition (but not sufficient) for these particles to obtain large masses and decouple from the low-energy theory.

Once a hypercharge candidate is found that satisfies these conditions we check explicitly for complete families and pairing of the remaining fields in vector-like pairs of equal and opposite electric charge. (Some hypercharge definitions involve continuous parameters, which can be varied in the search for pairs). When several Y definitions exist, one can use the values of k_Y to discriminate between them [12], choosing the one closer to $k_Y = 5/3$.

IV. RESULTS

We now apply the methods discussed above to several quasi-realistic string models taken from the literature, which provide an assorted sampling of the different situations one may encounter. Model FNY1 is the one constructed in ref. [13], model AF1 is taken from ref. [14] and the rest, CHL1 to CHL7, are the models presented in ref. [12]. The sign of the FI term, proportional to the sign of the trace of the anomalous charge over all of the fields in the model, is listed in Table II for the models considered.

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[13]This is the weakest reasonable assumption; one could make a stronger assumption of no unpaired $SU(2)_L$ chiral states. Such additional chiral fermions would be expected to acquire masses when $SU(2)_L$ is broken, and would have important phenomenological consequences, such as yielding a positive contribution to the electroweak S parameter. However, they are not absolutely excluded, so we do not require their absence as a condition on the hypercharge definition.

A. Model FNY1

The gauge group of this model is

$$\{SU(3)_C \times SU(2)_L\}_{\rm obs} \times \{SU(3) \times SU(2) \times SU(2)\}_{\rm hid} \times U(1)_A \times U(1)^{11}, \tag{59}$$

and the particle content includes, besides the MSSM multiplets, additional chiral superfields:

$$\begin{aligned} &1(\bar{3},1;1,1,1) + 1(3,1;1,1,1) + \\ &10(1,2;1,1,1) + 5(1,1;3,1,1,1) + 5(1,1;\bar{3},1,1) + \\ &12(1,1;1,2,1) + 15(1,1;1,1,2) + \\ &57(1,1;1,1,1)\ , \end{aligned} \tag{60}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(3), SU(2), SU(2))$ is indicated. We list the $U(1)$ charges of the non-Abelian singlets (including the right-handed leptons) in Table IIIa. The hypercharge definition is given by [13]

$$Y = \frac{1}{24}(2Q_1 + 3Q_2), \tag{61}$$

[normalized to give Y(quark doublet)$= 1/6$], and $\overline{Q}$ is given by

$$\overline{Q} = \frac{28}{3}(Q_6 - Q_{10}) - \frac{1}{3}Q_7 + 28(Q_9 - Q_{11}). \tag{62}$$

This model has a positive trace of the anomalous charge, so the FI term can be compensated only along flat directions with negative values of $\hat{Q}$. Anticipating the requirement that any such flat direction preserves hypercharge, we consider only $Y = 0$ fields. By inspection, one sees that S_7, S_{20}, S_{36}, S_{38}, and S_{46} have $Y = 0$ and $\hat{Q} < 0$, so this model has in principle the possibility of good flat directions.

The basis for the $Y = 0$ non-anomalous moduli space is presented in Table IV. An element like $M_{31} = \langle 32^2, 25^2, 6, 2, 1\rangle$ stands for the HIM $\Phi_{32}^2\Phi_{25}^2\Phi_6\Phi_2\Phi_1$. There are 41 fields with $Y = 0$ and their 41×11 charge matrix Q_i^n [i is a field index and n a $U(1)_n$ index] has rank 9 [$U(1)_Y$ and $U(1)_{11}$ are zero for this subset of fields]. This implies that the basis has 32 elements. In particular, the presence of primed copies of the fields S_5 and $\bar{S}_5$ introduces two basis elements (M_6 and M_7) which are trivially derived from M_5. The last direction in the table, M_{32}, has $\hat{Q} < 0$ and can be used, by combining it with other directions, to generate all $\hat{Q} < 0$ D- flat directions, i.e., those capable of giving $D_A = 0$.

In this model, the superbasis, formed by all one-dimensional flat directions, has a very large number of elements (ranging in the few thousands), and hence the complete determination of the superbasis loses its practical motivation. Nevertheless, all the information about the moduli space of non-anomalous dimensions is already contained in the basis and any flat direction can be expressed in terms of its elements. This moduli space contains many directions which are D_A flat (i.e., they have $\hat{Q} < 0$) and also remain F- flat. In Table V we list several examples of such directions that involve different numbers of fields. All of these directions are one-dimensional (before cancelling the FI term. The free VEV is then fixed to be of order ξ) and so break different numbers of $U(1)$'s, as indicated. As an example, the simplest direction R_1, involving only five different fields, breaks four non-anomalous $U(1)$'s. Before compensating the FI term, this direction is one-dimensional, and so the VEV's of the five fields are all related (according to the powers to which they are raised in the associated holomorphic invariant monomial). After compensating the FI term all the VEV's are then related to ξ according to

$$\frac{|\varphi_1|^2}{4} = |\varphi_4|^2 = |\varphi_{\bar{6}}|^2 = \frac{|\varphi_7|^2}{2} = \frac{|\varphi_9|^2}{2} = -\frac{\xi}{\hat{Q}(R_1)} = \frac{\xi}{224}. \tag{63}$$

The last directions presented in Table V are examples that break the maximal number of $U(1)$'s compatible with unbroken $U(1)_Y$. There is always another $U(1)$ besides hypercharge that remains unbroken.

Some of the listed directions do not have any possible superpotential term that could lift the F- flatness (except mass terms that are absent in superstring models). Many other directions are lifted already by terms in the trilinear superpotential, which at that order reads:

$$\begin{aligned} W_3 = {} & S_4 S_{18} S_{38} + S_4 S_{11} S_{27} + S_{14} S_{46} S_6 + S_8 S_{33} \bar{S}_6 + S_1 \bar{S}_2 \bar{S}_3 + \bar{S}_1 S_2 S_3 \\ & + S_{31} S_{35} S_2 + S_{17} S_7 \bar{S}_2 + S_{26} S_{28} S_1 + S_{25} S_{32} S_1 + S_{19} S_{20} \bar{S}_3. \end{aligned} \tag{64}$$

B. Model AF1

The gauge group is

$$\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SU(5) \times SU(3)\}_{\text{hid}} \times U(1)_A \times U(1)^9. \tag{65}$$

Besides the MSSM multiplets, the particle content includes additional chiral superfields:

$$\begin{aligned} & 2(\bar{3},1;1,1) + 2(3,1;1,1) + 8(1,2;1,1) + \\ & 4(1,1;5,1) + 4(1,1;\bar{5},1) + 8(1,1;1,3) + \\ & 8(1,1;1,\bar{3}) + 37(1,1;1,1)\ \ , \end{aligned} \tag{66}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(5), SU(3))$ is indicated. In Table VIa we list the 40 NAB singlets (including right-handed leptons) of the model with their $U(1)$ charges (rescaled by a factor 4 with respect to ref. [14] to make them integers). The hypercharge definition is [14]

$$Y = \frac{1}{24}(2Q_1 + 3Q_2), \tag{67}$$

[normalized to give Y(quark doublet)$= 1/6$], and $\overline{Q}$ is found to be:

$$\overline{Q} = \frac{1}{2}(-15Q_5 + 5Q_6 + 4Q_7 - 15Q_8 + Q_9). \tag{68}$$

In this model, negative values of $\hat{Q}$ are required to construct flat directions and Table VIa shows that two $Y = 0$ fields have the correct sign of $\hat{Q}$: S_7 and S_{10}. In the table, we also list two other $U(1)$ charges, defined as:

$$Q' = -26Q_1 + 3Q_8 + 5Q_9, \quad Q'' = 15Q_8 - Q_9. \tag{69}$$

Restricting our attention to $Y = 0$ fields, we see that S_{15}, S_{16} and S_{17} are the only fields with non-zero Q' charges, and all of them are negative and equal. This implies that no HIM built of non-Abelian $Y = 0$ singlets can contain these three fields, so they do not appear in this type of flat direction and we can ignore them in the following. We are then left with (note that all the fields have a mirror copy) $N^* = 13 \times 2$ ($Y = 0$) fields which have zero charge under three independent $U(1)$'s: Y, Q' and Q''. The rank of the non-anomalous charge matrix for this subset of fields is then equal to 6. Consequently, we expect a basis of non-anomalous flat directions composed of 20 elements. Two such bases are presented in Table VII. The first is constructed in such a way as to minimize the number (and power) of the fields entering the basis elements. The second contains a sub-basis of $\hat{Q} < 0$ elements. All type-B $\hat{Q} < 0$ directions can be obtained by combining the elements of this sub-basis.

The superbasis, containing all one-dimensional non-anomalous flat directions of the model, can be readily constructed and is presented in tables VIIIa and VIIIb. It contains a total of 157×2 elements (every flat direction is accompanied by another formed by the mirror copies of the fields in the first one). The superscript 0 labels $\hat{Q} = 0$ directions and the rest have $\hat{Q} = -60$ (+60 for the mirror direction not written). In total, 123 have the correct sign of $\hat{Q}$ to compensate the FI term, and thus are true flat directions. These elements (P_α with $\alpha = 1, ..., 123$) are the building blocks of all type-B D- flat directions.

Many of the D- flat directions built out of the P_α's will be lifted by F- terms. The superpotential of the model involving NAB singlets only is [14], up to the fourth order:

$$W_2 = 0, \tag{70}$$

$$\begin{aligned} W_3 = {} & S_3\overline{S}_4S_{12} + \overline{S}_3S_4\overline{S}_{12} \\ & + \overline{S}_{11}(S_5\overline{S}_8 + S_6\overline{S}_9 + S_7\overline{S}_{10} + S_{12}S_{13}) \\ & + S_{11}(\overline{S}_5S_8 + \overline{S}_6S_9 + \overline{S}_7S_{10} + \overline{S}_{12}\overline{S}_{13}) \\ & + \overline{S}_1S_2S_3, \end{aligned} \tag{71}$$

$$W_4 = 0. \tag{72}$$

The different Yukawa couplings, of order g, are not indicated explicitly. The mirror copy of the term $\overline{S}_1S_2S_3$ is absent, forbidden by world-sheet selection rules, as are all terms quadratic and quartic in the fields, that would otherwise be allowed by gauge symmetries.

Knowledge of W up to fourth order terms is nearly all that is needed to determine which P_α's are also F- flat. It turns out that only 10 of them remain flat to all orders in the presence of F- terms. The particular direction

$$P = R_{81} = \langle 1^4, \bar{2}, \bar{4}, 5, 6, 7, \overline{13}^2 \rangle, \tag{73}$$

however, requires knowledge of up to sixth order terms in W, as it can be lifted if the terms

$$W_B^{(5)} = S_1\overline{S}_2\overline{S}_4S_{11}\overline{S}_{13},$$
$$W_B^{(6)} = S_1\overline{S}_2\overline{S}_4(S_5\overline{S}_8 + S_6\overline{S}_9 + S_7\overline{S}_{10})\overline{S}_{13}, \tag{74}$$

do appear in the actual superpotential.

While gauge invariant terms like $W_B^{(5)}$ and $W_B^{(6)}$ are expected to appear in field-theoretic models unless additional symmetries are enforced *ad hoc*, the situation is different in string models. Beyond spacetime symmetries, stringy superpotential terms must always satisfy a set of world-sheet selection rules [30], as we discussed in section two. In this case, neither $W_B^{(5)}$ nor any of the three $W_B^{(6)}$ terms meet world-sheet selection requirements and, therefore, are eliminated from the superpotential. While the above terms are gauge group invariants, their corresponding five- and six-point string amplitudes are zero as a result of string effects, as explained below.

That $W_B^{(5)}$ does not survive in the superpotential is relatively easy to demonstrate: fields S_1, $\overline{S}_2$, and $\overline{S}_4$ are Ramond fields, that is, they originate in twisted world-sheet supersymmetric sectors of the string model. In contrast S_{11} and $\overline{S}_{13}$ come from the untwisted Neveu-Schwarz world-sheet supersymmetric sector of the model. A picture changed [31] set of $3+K$ (with $K \geq 1$) states can form an invariant under the global $U(1)_{N=2}$ symmetry of the $N=2$ world-sheet supersymmetry only if no more than $K-1$ fields are of the Neveu-Schwarz type [32]. Hence, a stringy candidate W_5 term with two Neveu-Schwarz fields has a vanishing five-point amplitude, since such a term is not invariant under $U(1)_{N=2}$.

The three $W_B^{(6)}$ terms cannot be discarded so easily, for they each contain exactly one Neveu-Schwarz field while up to two are allowed. For these terms we must also examine their additional global $U(1)$ world-sheet charges and/or Ising field correlation functions. From this we can demonstrate that none of the three terms appear in the superpotential: first, we can show that the pairs of fields that distinguish the three $W_B^{(6)}$ terms, i.e., $S_5\overline{S}_8$, $S_6\overline{S}_9$, and $S_7\overline{S}_{10}$, all originate in the same twisted subsector of the model. The three pairs all follow the same basic pattern with regard to their global world-sheet charges and Ising fields.

Next, we choose the fields S_1, $\overline{S}_2$, $\overline{S}_4$ to be the three that are not picture changed in any of the three terms. Then $\overline{S}_2$, $\overline{S}_4$ contribute to the six-point string amplitude two distinct Ising twist field correlators $\langle\sigma_i^+\sigma_i^-\rangle$ (i=1,2), associated, respectively, with two non-chiral Ising world-sheet fermions, that we will denote $(f_1(z), \overline{f}_1(\overline{z}))$ and $(f_2(z), \overline{f}_2(\overline{z}))$. Both before and after all possible picture changing options (including those that lead to a $U(1)_{N=2}$ conserving term) are performed on the respective sets of fields, $\{S_5\overline{S}_8\overline{S}_{13}\}$, $\{S_6\overline{S}_9\overline{S}_{13}\}$, and $\{\overline{S}_{13}S_7\overline{S}_{10}\}$, we find that there are no further contributions to either non-chiral Ising fermion's correlation functions. Since $\langle\sigma_i^+\sigma_i^-\rangle = 0$, the entire six-point string amplitudes are zero and the terms are removed from the superpotential. Thus, while we should generically expect flatness of the particular direction (73) to be broken by the appearance of $W_B^{(5)}$ and $W_B^{(6)}$ in a generic field-theoretic model, direction (73) remains flat in this string model due to additional world-sheet selection rules.

Table IX lists all one-dimensional (zero-dimensional after cancelling the FI term) P_α directions that remain F- flat to all orders. Other multi-dimensional flat directions can be built by multiplying these together in different combinations. Some exceptions arise if one combines P_α's in such a way that the fields S_{12} and S_{13} (or S_{13} and $\overline{S}_{13}$) take VEV's simultaneously. In the first case, F- terms are generated at the Yukawa level [see eq. (71)] and lift the direction while in the second case the flat direction would be of type A (it contains the HIM $S_{13}\overline{S}_{13}$) and knowledge of the superpotential to all orders would be required to ensure that no lifting term $(S_{13}\overline{S}_{13})^n$ appear.

C. Model CHL1

This model has the gauge group

$$\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SU(2)_2 \times SU(2)^2\}_{\text{hid}} \times U(1)_A \times U(1)^{13}, \tag{75}$$

and the particle content includes,[14] besides the MSSM multiplets, the additional chiral superfields:

$$\begin{aligned}
&5(3,1;1,1,1) + 5(\bar{3},1;1,1,1) + \\
&4(1,1;2,1,2) + 4(1,1;2,2,1) + 4(1,1;1,2,2) + \\
&2(1,2;2,1,1) + 2(1,1;3,1,1) + \\
&16(1,1;2,1,1) + 10(1,1;1,2,1) + 10(1,1;1,1,2) + \\
&14(1,2;1,1,1) + 84(1,1;1,1,1)\,,
\end{aligned} \tag{76}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(2)_2, SU(2)^2)$ is indicated. There are two bi-doublets $(2,2)$ under $SU(2)_L \times SU(2)_2$ that mix the observable and hidden sectors, so that only if these fields get string scale masses are the two sectors really separated. In Table X we list the 87 non-Abelian singlets of the model with their $U(1)$ charges (including right-handed leptons).

In the CHL models [12], the trace of the anomalous charge is negative, and thus we require positive values of $\hat{Q}$. $\overline{Q}$ is given by

$$\overline{Q} = \frac{32}{33}Q_6 - \frac{64}{473}Q_7 + \frac{48}{43}Q_8 - \frac{64}{129}Q_9 + \frac{176}{6063}Q_{10} - \frac{288}{2491}Q_{11} + \frac{40}{2067}Q_{12} + \frac{11}{39}Q_{13}. \tag{77}$$

As shown in Table X, there are four NAB-singlet fields with a positive value of $\hat{Q}$ so that, in principle, good flat directions may be formed. However, a viable definition of hypercharge also must be determined, to see whether flat directions exist that preserve $U(1)_Y$.

In Model CHL1, the search yielded no acceptable hypercharge which had three families, had exotic $SU(3)_C$ triplet pairing, and the possibility of the decoupling of mixed observable and hidden bi-doublet states. However, we can impose a weaker vector pairing requirement allowing for the possible breaking of part of the hidden sector NAB gauge groups (i.e., allowing the pairing between fields belonging to different representations of those groups,

---

[14]The particle contents of models CHL1 though CHL3 are presented and discussed in another paper in this series, [27].

with typical masses at the breaking scale[15]). In that case, several definitions of Y could be found. One particular example (which gives the lowest value of k_Y, $k_Y = 8/3$) is

$$\begin{aligned} Y = &\frac{1}{8}(Q_1 - Q_3) \\ &+\frac{1}{12}\left[-\frac{1}{11}Q_6 + \frac{45}{946}Q_7 + \frac{15}{172}Q_8 + \frac{5}{43}Q_9 + \frac{45}{8084}Q_{10} + \frac{69}{4982}Q_{11} + \frac{1}{106}Q_{12}\right]. \end{aligned} \quad (78)$$

The hypercharges of NAB-singlet fields according to this definition are also listed in Table X: a total of 33 fields are left with $Y = 0$ (and sufficient candidates for singlet leptons with $Y = 1$ appear). In addition, the charges of the fields under the linear combination

$$Q' = Q_1 + Q_2, \quad (79)$$

are also given. After examination of the Q' charges of the $Y = 0$ fields, we conclude that the field S_{33} cannot enter any flat direction (that preserves Y and is built of NAB singlets only). The only field with $\hat{Q} > 0$ is S_{14}, while S_{29}, S_{34}, S_{38}, S_{47}, S_{49}, S_{55} and S_{63} have negative $\hat{Q}$.

In this model, the number of elements of the superbasis is large, making its complete determination unwieldy. Therefore, we use a basis that describes the space of $Y = 0$ non-anomalous D- flat directions, which is presented in Table XI. It contains 21 elements, corresponding to the fact that there are 32 ($Y = 0$) NAB singlets left after removing S_{33}, and the $U(1)$ charge matrix for these 32 fields has rank 11 [13 minus 2 $U(1)$'s always unbroken: Y and Q']. All the basis elements have $\hat{Q}$ either zero or negative, while a positive value would be required to cancel the FI term. However, this does not necessarily imply that there are no flat directions with $\hat{Q} > 0$ (in contrast to the case with the elements of the superbasis, in which an element with the right sign of $\hat{Q}$ is required for D_A- flat directions). By the definition of the basis, any D- flat direction P can be written in the form

$$P^n = \Pi_i M_i^{n_i} = \Pi_i \left[\Pi_j \varphi_j^{m_{ij}}\right]^{n_i}, \quad (80)$$

where n, n_i and m_{ij} are integer numbers, with $n, m_{ij} > 0$, and the n_i not necessarily positive. The only condition for P^n to be acceptable is that all the fields appearing in it are raised to a positive power, which is not equivalent to requiring positive n_i's. From eq. (80) it follows

$$\hat{Q}(P^n) = \sum_i n_i \hat{Q}(M_i) = \sum_i n_i \left[\sum_j m_{ij} \hat{Q}(\varphi_j)\right], \quad (81)$$

opening the possibility of obtaining $\hat{Q}(P^n) > 0$ via $n_i < 0$ for some $\hat{Q}(M_i) < 0$. Whether this can be realized depends on the details of the model. In the following we illustrate how the knowledge of the basis can be used to prove general statements about flat directions.

---

[15] An alternative possibility is that the shift to a SUSY preserving vacuum requires non-zero VEV's of fields that transform under a non-trivial representation of some NAB hidden sector group. In either case preservation of $U(1)_Y$ requires the breaking of the NAB group. We focus on the first possibility, as we are interested in exploring vacuum shifts involving NAB-singlet fields only. In this case, the NAB group can be broken at a lower scale, unrelated to the anomalous $U(1)$ breaking.

In this model and with the definition of Y given above, the only field with positive $\hat{Q}$ is S_{14}, so, if a P exists for which $\hat{Q}(P) > 0$, its definition (80) in terms of the basis elements must include M_{19} and/or M_{21} raised to some positive power, because these are the only basis elements that contain S_{14}. In both elements, S_{14} appears in combination with S_{49}, which neutralizes its positive $\hat{Q}$. We are then forced to include in (80) some element which contains also S_{49} but appears raised to a negative power so as to cancel the power of S_{49} in the final expression for P. The only basis element available for this purpose is M_{20}, but it cannot have a negative power in (80) because it contains the field S_{34} which appears only in this basis element and then, the final expression for P would contain a negative power of S_{34}, which cannot be accepted. This proves that no D- flat direction exists with $\hat{Q} > 0$.

If we do not fix the hypercharge definition from the beginning and include all the NAB singlets in the analysis, the moduli space of non-anomalous flat directions is larger and is described by the basis presented in Table XII. The number of basis elements is 74 [87 (fields)-13(rank)] and only one of them has $\hat{Q}$ non-zero (and negative). In this case, however, flat directions with $\hat{Q} > 0$ exist. As an example,

$$P = \langle 1^7, \overline{5}^7, 6^{15}, 7^6, 14^6, 16^4, 20^{17}, 21^{22}, 27^2, 42^2, 75^5 \rangle, \tag{82}$$

has $\hat{Q} = 64 \times 12$. Its expression in terms of the basis elements is

$$P = \frac{1}{M_{73}^3} \times \left\{ \frac{M_{42}^{17} M_{21}^3 M_{74}^6 M_{22}^3 M_{31}^5 M_1^3 M_{39}^5 M_{13}^2 M_{57}^2 M_{34} M_7^5}{M_{16}^5 M_{62}^2 M_8 M_6^8 M_{14} M_9 M_{56}^5 M_5 M_{12}^{17}} \right\}. \tag{83}$$

This gives an explicit example of a model in which the basis has no element with good $\hat{Q}$ but good $\hat{Q}$ D- flat directions exist[16].

D. Model CHL2

The gauge group of this model is

$$\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SO(7) \times SU(2)_2^2\}_{\text{hid}} \times SU(2)^4 \times U(1)_A \times U(1)^7, \tag{84}$$

and the particle content [27] includes additional chiral superfields:

$$\begin{aligned}
&3(1,1;8,1,2,1,1,1,1) + (1,1;8,1,1,1,2,1,1) + (1,1;7,3,1,1,1,1,1) +\\
&(1,1;1,2,3,1,1,1,1) + (1,1;7,2,1,1,1,1,1) + (1,1;7,1,1,1,1,1,1) +\\
&2(1,1;1,2,2,2,1,1,1) + 2(1,1;1,2,2,1,1,1,2) + (1,1;1,2,1,1,1,2,2) +\\
&(1,1;1,2,1,2,2,1,1) + 3(1,1;1,1,1,1,2,2,1) + 2(1,1;1,1,1,1,2,1,2) +\\
&3(1,1;1,1,1,2,1,1,2) + (1,1;1,1,1,1,1,2,2) + (1,1;1,1,1,1,2,1,2) +\\
&3(1,1;1,1,1,2,1,2,1) + 3(1,1;1,3,1,1,1,1,1) + (1,1;1,1,3,1,1,1,1) +
\end{aligned}$$

---

[16] This is in contrast with the situation for the fields themselves: no good flat directions can exist if the model does not contain fields with the appropriate value of $\hat{Q}$.

$$
\begin{aligned}
&10(1,1;1,2,1,1,1,1,1)+4(1,2;1,2,1,1,1,1,1)+(1,2;1,2,2,1,2,1,1)+\\
&(1,2;1,1,1,1,1,2,2)+4(1,2;1,1,1,1,1,1,1)+3(\bar{3},1;1,2,1,1,1,1,1)+\\
&(3,1;1,2,1,1,1,1,1)+(3,1;1,1,1,2,2,1,1)+2(\bar{3},1;1,1,1,1,1,1,1)+\\
&16(1,1;1,1,1,1,1,1,1)\ ,
\end{aligned} \tag{85}
$$

where the representation under $(SU(3)_C, SU(2)_L; SO(7), SU(2)^2_2, SU(2)^4)$ is indicated. In Table XIII we list the 19 non-Abelian singlets (including right-handed leptons) of the model with their $U(1)$ charges.

$\overline{Q}$ is given by

$$
\overline{Q} = \frac{1}{3}(28Q_4 + Q_5). \tag{86}
$$

Inspection of the list of singlets in this model shows that all of the fields either have zero or negative values of $\hat{Q}$, so that in this model non-Abelian fields are required for a flat direction. We conclude that the shifting to a SUSY preserving vacuum is necessarily accompanied by the spontaneous breaking of some non-Abelian group. To be more precise, we find that only a single hidden sector non-Abelian field has a positive $\hat{Q}$ value. This field is a doublet under a level-one $SU(2)$ and both level-two $SU(2)$'s, which implies that in the least these three gauge groups must be broken if the SM is to survive after anomaly cancellation.

For completeness, we construct the basis of non-anomalous flat directions built out of NAB singlets, which can be useful for more general discussions when non-singlet fields are also included. In Table XIII we also list the charges under the linear combination of $U(1)$'s defined as:

$$
Q' = 6Q_1 + 4Q_4 - Q_5 + Q_7. \tag{87}
$$

We see that S_{11}, S_{12}, S_{13} and S_{14} are the only fields with non-zero Q' charges, and all of them are positive and equal. This implies that no HIM built of non-Abelian singlets can contain these four fields, so they do not appear in this type of flat direction and we can ignore them in the following discussion. We are then left with 15 fields which have zero charge under (87). The rank of the non-anomalous charge matrix for this subset of fields is then equal to 6 and we expect a basis of non-anomalous flat directions composed of 9 elements. Such a basis is presented in Table XIV.

E. Model CHL3

The gauge group is

$$
\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SO(5)^2 \times SU(2)_2\}_{\text{hid}} \times U(1)_A \times U(1)^{11}, \tag{88}
$$

and the particle content [27] includes the additional chiral superfields:

$$
\begin{aligned}
&4(\overline{3},1;,1,1,1)+4(3,1;1,1,1)+8(1,2;1,1,1)+\\
&4(1,1;1,1,3)+16(1,1;1,4,1)+8(1,1;4,1,1)+\\
&2(1,1;1,5,1)+2(1,1;5,1,1)+(1,1;5,5,1)+\\
&4(1,1;4,1,2)+2(1,1;1,4,2)+2(1,2;4,1,1)+\\
&76(1,1;1,1,1)\ ,
\end{aligned} \tag{89}
$$

where the representation under $(SU(3)_C, SU(2)_L; SO(5), SO(5), SU(2)_2)$ is indicated. In Table XV we list the 79 non-Abelian singlets (including right-handed leptons) of the model with their $U(1)$ charges.

In this model, $\overline{Q}$ is :

$$\overline{Q} = -\frac{1}{5}(6Q_8 + 9Q_{10} - 4Q_{11}). \tag{90}$$

As in CHL2, all of the fields have either zero or negative values of $\hat{Q}$ so that non-Abelian fields are required to cancel the FI term along any flat direction. We present in Table XVI the basis of non-anomalous D- flat directions for non-Abelian singlets. Imposing 11 non-anomalous D- term conditions on 79 fields leads to a moduli space of 68 dimensions.

F. Model CHL4

The gauge group of this model is

$$\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SU(4)_2 \times SU(2)_2\}_{\text{hid}} \times U(1)_A \times U(1)^6, \tag{91}$$

and the particle content beyond the MSSM consists of the chiral superfields:

$$\begin{aligned} &12(1,2;1,1) + 2(3,1;1,1) + 2(\bar{3},1;1,1) + \\ &2(1,1;6,2) + 2(1,1;\bar{4},2) + 4(1,1;6,1) + \\ &14(1,1;4,1) + 10(1,1;\bar{4},1) + 3(1,1;1,3) + 8(1,1;1,2) + \\ &48(1,1;1,1) \ , \end{aligned} \tag{92}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(4)_2, SU(2)_2)$ is indicated. In Table XVII we list the 51 non-Abelian singlets (including right-handed leptons) of the model with their $U(1)$ charges.

Phenomenological considerations lead to the hypercharge definition [12]

$$Y = \frac{1}{24}\left(-\frac{3}{5}Q_1 + \frac{27}{80}Q_2 + \frac{3}{10}Q_3 + \frac{1}{6}Q_4 - \frac{1}{6}Q_5 + \frac{5}{48}Q_6\right), \tag{93}$$

[normalized to give Y(quark doublet)$= 1/6$]. As previously explained, k_Y for this definition of hypercharge can be readily calculated using the universal GS relation with only the knowledge of the charges of the massless spectrum of the string model. In this way, we find $k_Y = 35/12$, a factor of 2 larger than the k_Y quoted in [12], and thus greater than 5/3. This discrepancy by a factor of 2 affects all other determinations of k_Y presented in [12] and seems to eliminate the examples with $k_Y < 5/3$.

In this model, $\overline{Q}$ is particularly simple:

$$\overline{Q} = -\frac{1}{2}Q_6. \tag{94}$$

As is shown in Table XVII, $\hat{Q}$ is negative or zero for all of the non-Abelian singlet fields, while $\mathrm{Tr}Q_A < 0$, so it is not possible to form a good flat direction without utilizing the non-Abelian fields. From Table XVII one concludes also that alternative definitions of Y do not change this situation.

Even if non-Abelian fields would have to take non-zero VEV's along any true flat direction, it may be necessary to give VEV's to singlets as well. For this reason we present, as in previous cases, the basis of non-anomalous D- flat directions for non-Abelian $Y = 0$ singlets in Table XVIII. There are 17 $Y = 0$ fields plus 3 copies, and 5 non-anomalous $U(1)$'s besides hypercharge so that the number of elements of the basis is 12 (plus 3 more involving copies of fields, that are not shown).

G. Model CHL5

This model, with gauge group

$$\{SU(3)_C \times SU(2)_L\}_{\rm obs} \times \{SU(4)_2 \times SU(2)_2\}_{\rm hid} \times U(1)_A \times U(1)^6, \tag{95}$$

was already considered in ref. [10] to which we refer the reader for further details. The model contains a NAB singlet with $\hat{Q} > 0$ that can appear in several flat directions with good $\hat{Q} > 0$. A total of 5 one-dimensional type-B flat directions were found that could be used as building blocks for directions which are D- and F- flat to all orders. The phenomenological analysis of the model along some of these directions will be presented elsewhere [33].

H. Model CHL6

The gauge group of this model is

$$\{SU(3)_C \times SU(2)_L\}_{\rm obs} \times \{SU(2)_2^2\}_{\rm hid} \times U(1)_A \times U(1)^{10}, \tag{96}$$

and it includes the additional chiral superfields:

$$\begin{aligned}
&3(\overline{3},1;1,1) + 3(3,1;1,1) + 6(1,2;1,1) + \\
&4(1,2;2,1) + 26(1,1;2,1) + 40(1,1;1,2) + \\
&1(1,1;3,3) + 1(1,1;3,1) + 1(1,1;1,3) + 49(1,1;1,1)\ \ ,
\end{aligned} \tag{97}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(2)_2, SU(2)_2)$ is indicated. In Table XIX we list the 52 non-Abelian singlets (including right-handed leptons) of the model with their $U(1)$ charges.

$\overline{Q}$ is given by

$$\overline{Q} = -\frac{1}{5}(6Q_7 + 9Q_9 - 4Q_{10}). \tag{98}$$

Inspection of the list of singlets in this model shows that all of the fields have zero values of $\hat{Q}$, except for S_{24} with $\hat{Q} = -144$, which is of sign opposite to the FI term, so that hidden (or observable) non-Abelian fields must take non-zero VEV's along a flat direction.

The basis of non-anomalous flat directions is presented in Table XX. The number of elements is obtained subtracting from the total number of fields (N=52) the rank of the charge matrix (10) giving a dimension equal to 42 (in this case, we explicitly include primed fields).

I. Model CHL7

The gauge group of this model is

$$\{SU(3)_C \times SU(2)_L\}_{\text{obs}} \times \{SO(7)_2 \times SU(2)_2\}_{\text{hid}} \times U(1)_A \times U(1)^8, \tag{99}$$

and the particle content includes, besides the MSSM multiplets, additional chiral superfields:

$$\begin{aligned}&14(1,2;1,1) + 6(3,1;1,1) + 4(\bar{3},1;1,1) + \\ &1(1,2;1,2) + 1(\bar{3},1;1,2) + 4(1,1;8,1) + \\ &1(1,1;7,1) + 4(1,1;1,3) + 31(1,1;1,2) + \\ &94(1,1;1,1) \ ,\end{aligned} \tag{100}$$

where the representation under $(SU(3)_C, SU(2)_L; SU(4)_2, SU(2)_2)$ is indicated. In Table XXI we list the 81 non-Abelian singlets (including right-handed leptons. The 13 fields with zero charges under all of the Abelian groups are not listed) of the model with their $U(1)$ charges.

$\overline{Q}$ is given by

$$\overline{Q} = -4Q_3 - 4Q_4 + Q_6 - 2Q_7 - \frac{4}{3}Q_8, \tag{101}$$

and there are fields with positive $\hat{Q}$ that can in principle form a good flat direction. Once again, a viable hypercharge must be determined to ensure that such good flat directions preserve the SM gauge group.

Upon further inspection of the list of states, one can see that in this model there is an additional mixed state which is a color antitriplet and hidden sector doublet $(\bar{3},1;1,2)$. Thus, to enforce triplet pairing (i.e., to avoid an exactly massless colored fermion),the hidden sector $SU(2)$ must be broken to have the appropriate number of degrees of freedom.

Even allowing for the breaking of any group [except $SU(3)_C \times U(1)_{EM}$], no definition of Y as a linear combination of the non-anomalous $U(1)$'s exists that gives full vector pairing of the additional multiplets present in this model. As explained in section III.B we did not consider the possibility that the hypercharge definition involves the $U(1)$'s that arise from the breaking of the hidden sector NAB gauge group. In case such a possibility can indeed be realized it can be useful to know the basis of non-anomalous flat directions of all NAB singlets, which is presented in Table XXII.

V. CONCLUSIONS

We have applied the strategy developed in [10] for the classification of flat directions to several quasi-realistic models [with an anomalous $U(1)$] taken from the literature [13,14,12]. The results are summarized in Table II and offer a survey of the different possibilities that can be encountered in this type of analyses.

- In some cases (CHL2, CHL3, CHL4 and CHL6) it is possible to show that the FI term ξ cannot be compensated by giving VEV's to non-Abelian singlet fields only. The technical reason is that no such fields exist with $\hat{Q}$ of sign opposite to that of ξ (or of $\mathrm{Tr}Q_A$, as listed in Table II). This holds irrespective of the definition of hypercharge and is the reason why we did not search for a viable Y in some of these models (marked '?' in the corresponding column of Table II). For these models we thus conclude that the vacuum shifting triggered by the FI term is necessarily accompanied by the reduction of the rank of the non-Abelian group. The analysis of flat directions involving fields in non trivial representations of the non-Abelian groups is beyond the scope of this paper. Such an analysis could address the issue of whether the SM gauge group will be necessarily broken.

- In other models (e.g., CHL1), even if they contain non-Abelian singlets with good $\hat{Q}$ (and possibly flat directions) it can happen that, after determining a viable definition of hypercharge Y, no flat directions remain that preserve $U(1)_Y$. This can happen because no $Y = 0$ fields are left with good $\hat{Q}$ or, in a more subtle way, because, even if they exist the charge structure of the fields conspire to produce directions with only the wrong sign of $\hat{Q}$ (this was the case of CHL1 with the particular choice of Y). If that $U(1)_Y$ is to survive unbroken in the low-energy effective theory, the vacuum restabilization must be accompanied by the breaking of some non-Abelian gauge group.

- Models exist (CHL7) for which no definition of hypercharge is phenomenologically viable (the presence of massless charged particles in the spectrum cannot be avoided).

- Other models (FNY1, AF1, CHL5) are more successful and have both a viable hypercharge and hypercharge-preserving flat directions. In these cases, our conservative aim is to classify all such directions that can be proven to be flat to all orders. This can be done by constructing the superbasis that contains all one-dimensional directions which are D- flat for the non-anomalous $U(1)$'s. Of these directions, only those that carry an anomalous charge of sign opposite to ξ are also D_A-flat. The F- flatness to all orders of these particular directions (and some combinations of them) can be assessed by knowing the superpotential up to a finite order, and thus those directions which are both D- and F- flat can in principle be classified.

  This program can be readily completed [10] for model CHL5, which contains a small number of all-order flat directions. The number of flat directions increases for model AF1 and hence the superbasis is large, but the number of flat directions which are F-flat to all orders is still relatively small. However, for model FNY1 the superbasis is too large to be of practical use. This model simply contains too many flat directions to give a complete classification (even though in principle this can be done).

  For these models we find and list several flat directions that break different numbers of $U(1)$'s. In general, not only $U(1)_Y$ survives down to low-energies but other $U(1)$'s remain unbroken at the string scale. The fate of these additional Abelian factors depends on the details of the model and was addressed, for example, in refs. [34,24].

Having found a classification of the different vacua to which a particular model can relax, the next step is to analyze the spectrum, gauge group and superpotential of the resulting model, and investigate their phenomenological consequences. This analysis for model CHL5 is currently under investigation [33].

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Field	ψ^{12}	S_{12}	y^1	ω^1	y^2	ω^2	S_{34}	y^3	ω^3	y^4	ω^4	S_{56}	y^5	ω^5	y^6	ω^6
$H_{39,\,ferm\,(-\frac{1}{2})}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	σ^-	0	0		σ^-	0	$-\frac{1}{2}$	0	0		
$H_{37,\,ferm\,(-\frac{1}{2})}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	σ^-	0	0		σ^-	0	$-\frac{1}{2}$	0	0		
$H_{32,\,bos\,(-1)}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	σ^+	0	0	0		0	0	$\frac{1}{2}$	σ^+	0		
$H_{30,\,bos\,(-1)}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	σ^+	0	0	0		0	0	$\frac{1}{2}$	σ^+	0		
net charges	0	+1	−1	−1	0	0	0	0		0	0	0	0	0		
$T^{-1}_{3/2}$ term	0	−1	+1	+1	0	0	0	0		0	0	0	0	0		

Table I: Picture-changing example in a W_4 term $H_{39}H_{37}H_{32}H_{30}$ in [13]. These fields are identified in our Tables IIIa and IIIb as S_{43}, S_{28}, S_{24}, and S_7. The “net charges” row contains the charge vector formed from the four canonical H fields that must be cancelled by picture-changing affects.

Model	Tr Q_A	Y def.	D-flat	D-flat ($Y=0$)	F-flat
FNY1	1344	Y	Y	Y	Y
AF1	720	Y	Y	Y	Y
CHL1	−3072	Y	Y	N	–
CHL2	−2688	?	N	–	–
CHL3	−3456	?	N	–	–
CHL4	−2016	Y	N	–	–
CHL5	−1536	Y	Y	Y	Y
CHL6	−3456	Y	N	–	–
CHL7	−768	N	Y	–	–

Table II: Summary of results for the different models considered. The second column gives the total trace of the anomalous $U(1)$. The third shows whether a viable definition of hypercharge exists (a question mark indicates that no answer to that question is required to proceed with the analysis). The fourth and fifth columns relate to the existence of D-flat directions, in the latter case imposing also that $Y=0$ along them. The last column reports the existence of D- flat directions which remain F- flat.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_Y	$\hat{Q}$
$S_1\surd$	−36	0	0	4	16	0	0	108	0	0	0	0	0	0
$S_2\surd$	−28	0	0	4	−32	0	0	84	0	0	0	0	0	0
$S_3\ \surd$	−8	0	0	0	48	0	0	24	0	0	0	0	0	0
$S_4\ \surd$	−8	0	0	0	0	0	−8	−200	0	0	0	0	0	0
$S_5{}^{(\prime)}\ \surd^{(\prime)}$	0	0	0	0	0	0	0	0	4	0	0	0	0	0
$S_6\ \surd$	0	0	0	0	0	8	0	0	0	0	0	0	0	0
S_7	−24	−3	2	−2	−17	2	0	−96	0	1	−3	0	0	−112
S_8	−16	3	−2	0	−3	4	2	−64	2	−1	3	0	0	0
S_9	−12	3	−2	−6	−15	2	4	−20	0	−1	3	0	0	0
S_{10}	−12	0	4	−2	10	2	−2	−20	2	2	0	2	1	0
S_{11}	−12	3	−2	0	−3	0	6	36	−2	−1	3	0	0	0
S_{12}	−8	3	2	2	11	2	4	−32	2	−1	−3	−2	1	−112
S_{13}	−8	3	2	2	−13	2	0	−144	−2	−1	−3	2	1	0
S_{14}	−8	3	−2	0	−3	−4	−2	136	−2	−1	3	0	0	112
S_{15}	−8	0	−4	−2	10	−2	2	80	2	2	0	2	−1	0
S_{16}	−8	0	−4	−2	−14	−2	−2	−32	2	2	0	2	−1	0
S_{17}	−4	3	−2	6	−15	−2	0	180	0	−1	3	0	0	112
S_{18}	−4	3	−2	0	−3	0	2	236	2	−1	3	0	0	112
S_{19}	−4	0	0	0	24	0	0	12	2	−2	6	0	0	112
S_{20}	−4	0	0	0	24	0	0	12	−2	2	−6	0	0	−112
S_{21}	−4	0	4	−2	−14	2	2	68	2	2	0	2	1	0
S_{22}	−4	−3	−2	−2	13	2	0	−156	−2	1	3	2	−1	0
S_{23}	−4	3	2	2	11	−2	−4	68	2	−1	−3	2	1	112
S_{24}	0	−3	2	2	11	−2	0	168	0	−3	−3	0	0	112
S_{25}	20	0	0	−2	−8	−2	2	−4	0	2	6	0	0	0
S_{26}	20	−3	2	−6	−9	2	0	108	0	1	−3	0	0	0
S_{27}	20	−3	2	0	3	0	2	164	2	1	−3	0	0	0
S_{28}	16	3	−2	2	−7	−2	0	−216	0	−1	3	0	0	0
S_{29}	16	0	0	4	4	0	0	−48	2	2	6	0	0	0
S_{30}	16	0	0	4	4	0	0	−48	−2	−2	−6	0	0	0
S_{31}	16	0	0	−2	16	2	2	8	0	2	6	0	0	0
S_{32}	16	0	0	−2	−8	2	−2	−104	0	−2	−6	0	0	0
S_{33}	16	−3	2	0	3	4	−2	64	−2	1	−3	0	0	0
S_{34}	12	6	4	2	−10	2	2	20	0	0	0	0	2	0
S_{35}	12	0	0	−2	16	−2	−2	−92	0	−2	−6	0	0	0
S_{36}	12	−3	2	6	−9	−2	4	−92	0	1	−3	0	0	−112
S_{37}	12	0	−4	2	−10	−2	2	20	2	−2	0	−2	−1	0
S_{38}	12	−3	2	0	3	0	6	−36	−2	1	−3	0	0	−112
S_{39}	8	6	4	−4	2	0	0	−24	−2	0	0	0	2	0
S_{40}	8	6	−4	−4	2	0	0	−24	−2	0	0	0	0	0

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_Y	$\hat{Q}$
S_{41}	8	6	−4	2	−10	−2	−2	−80	0	0	0	0	0	0
S_{42}	8	6	4	2	14	−2	2	32	0	0	0	0	2	0
S_{43}	8	3	−2	−2	13	2	0	144	0	3	3	0	0	0
S_{44}	8	−3	−2	−2	−11	−2	−4	32	2	1	3	2	−1	112
S_{45}	8	−3	−2	−2	13	−2	0	144	−2	1	3	−2	−1	0
S_{46}	8	−3	2	0	3	−4	2	−136	2	1	−3	0	0	−112
S_{47}	8	0	4	2	−10	2	−2	−80	2	−2	0	−2	1	0
S_{48}	8	0	4	2	14	2	2	32	2	−2	0	−2	1	0
S_{49}	4	6	−4	2	14	2	−2	−68	0	0	0	0	0	0
S_{50}	4	0	−4	2	14	−2	−2	−68	2	−2	0	−2	−1	0
S_{51}	4	3	2	2	−13	−2	0	156	−2	−1	−3	−2	1	0
S_{52}	4	−3	−2	−2	−11	2	4	−68	2	1	3	−2	−1	−112

Table IIIa: List of non-Abelian singlet fields and their $U(1)$ gauge charges for model FNY1, with hypercharge and $\hat{Q} = Q_A - \overline{Q}$ as defined in eqs. (61) and (62) respectively. A $\surd$ indicates the presence of another field with equal and opposite $U(1)$ charges, while a $'$ indicates the presence of another field with identical $U(1)$ charges.

	$r_1 : r_2 =$ 3:6	4:20	5:11	9:12	10:19	15:18	7:38	8:44	13:40	14:46	16:41	17:47
Singlet S_i	Q						I					
S_1	.5			.5		$-.5$						
$\bar{S}_1$	.5			.5		$-.5$						
S_2	.5			$-.5$		.5						
$\bar{S}_2$	.5			$-.5$		.5						
S_3	$-.5$			.5		.5						
$\bar{S}_3$	$-.5$			.5		.5						
S_4	$-.5$			.5		.5						
$\bar{S}_4$	$-.5$			.5		.5						
S_5	.5			$-.5$		.5		$\bar{f}$				
$\bar{S}_5$	.5			$-.5$		.5		$\bar{f}$				
$\S'_5$	.5			.5		$-.5$	$\bar{f}$					
$\bar{S}'_5$	.5			.5		$-.5$	$\bar{f}$					
S_6	$-.5$			.5		.5						
$\bar{S}_6$	$-.5$			.5		.5						
S_7		.5		.5			σ^-				σ^+	
S_8	.5				$-.5$					σ^-	σ^+	
S_9			$-.5$			.5		σ^-	σ^-			
S_{10}		.5		.5				σ^+				σ^-
S_{11}	.5				$-.5$					σ^+	σ^-	
S_{12}		.5		.5				σ^-			σ^-	
S_{13}			$-.5$			.5	σ^+		σ^+			
S_{14}	.5				$-.5$				σ^-			σ^-
S_{15}		.5		.5				σ^-				σ^-
S_{16}			.5			.5	σ^-			σ^-		
S_{17}		$-.5$		.5			σ^-				σ^+	
S_{18}	.5				$-.5$				σ^+			σ^+
S_{19}	.5				.5					σ^+		σ^-
S_{20}	.5				$-.5$					σ^+		σ^-
S_{21}			.5			.5	σ^+			σ^-		
S_{22}			$-.5$			.5	σ^-		σ^+			
S_{23}		$-.5$		.5				σ^-			σ^-	
S_{24}		.5		.5			σ^-				σ^+	
S_{25}			.5			.5		σ^-		σ^+		
S_{26}			.5			.5		σ^+	σ^-			
S_{27}	.5				.5					σ^+	σ^-	
S_{28}			$-.5$			.5		σ^+	σ^-			
S_{29}	.5				.5				σ^+		σ^+	
S_{30}	.5				$-.5$				σ^-		σ^-	

	$r_1 : r_2 =$ 3:6	4:20	5:11	9:12	10:19	15:18	7:38	8:44	13:40	14:46	16:41	17:47
Singlet S_i	Q						I					
S_{31}		.5		.5			σ^+					σ^+
S_{32}			$-.5$			.5		σ^-		σ^+		
S_{33}	.5				.5					σ^-	σ^+	
S_{34}		.5		.5			σ^-					σ^+
S_{35}		$-.5$		.5			σ^+					σ^+
S_{36}		.5		.5			σ^+				σ^+	
S_{37}		$-.5$		.5				σ^-				σ^-
S_{38}	.5				.5				σ^+			σ^+
S_{39}	.5				.5				σ^-		σ^-	
S_{40}	.5				.5				σ^+		σ^+	
S_{41}		.5		.5			σ^+					σ^+
S_{42}			.5			.5		σ^+		σ^+		
S_{43}			$-.5$			.5		σ^+	σ^-			
S_{44}		$-.5$		.5				σ^+			σ^-	
S_{45}			.5			.5	σ^-		σ^+			
S_{46}	.5				.5				σ^-			σ^-
S_{47}		$-.5$		.5				σ^+				σ^-
S_{48}			$-.5$			.5	σ^+			σ^-		
S_{49}			.5			.5		σ^-		σ^+		
S_{50}			$-.5$			.5	σ^-			σ^-		
S_{51}			.5			.5	σ^+		σ^+			
S_{52}		.5		.5				σ^+			σ^-	

Table IIIb: The non-gauge worldsheet charges of the 60 NAB singlets in model FNY1. r_1 and r_2 specify the two real fermions ψ_{r_1} and ψ_{r_2} comprising either a complex left-moving fermion when $r_2 \leq 20$, or a non-chiral Ising fermion when $r_2 > 20$. A global $U(1)$ charge Q carried by a singlet S_i is listed in the column of the complex worldsheet fermion associated with the charge. Likewise, a conformal field $I \in \{f, \bar{f}, \sigma^+, \sigma^-\}$ of a non-chiral Ising fermion carried by a singlet is listed in the column of the appropriate Ising fermion.

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{17} = \langle 33, 8, \overline{6} \rangle$	0
$M_2 = \langle 2, \overline{2} \rangle$	0	$M_{18} = \langle 20, 19, \overline{3} \rangle$	0
$M_3 = \langle 3, \overline{3} \rangle$	0	$M_{19} = \langle 49, 25, 24, 7 \rangle$	0
$M_4 = \langle 4, \overline{4} \rangle$	0	$M_{20} = \langle 43, 28, 24, 7 \rangle$	0
$M_5 = \langle 5, \overline{5} \rangle$	0	$M_{21} = \langle 40, 29, 24, 7 \rangle$	0
$M_6 = \langle 5', \overline{5} \rangle$	0	$M_{22} = \langle 41, 31, 24, 7 \rangle$	0
$M_7 = \langle 5, \overline{5}' \rangle$	0	$M_{23} = \langle 46, 40, 33, 19, 2 \rangle$	0
$M_8 = \langle 6, \overline{6} \rangle$	0	$M_{24} = \langle 38, 27, 14, 8, 4 \rangle$	0
$M_9 = \langle 32, 25, 1 \rangle$	0	$M_{25} = \langle 43, 36, 24, 9, 4 \rangle$	0
$M_{10} = \langle 28, 26, 1 \rangle$	0	$M_{26} = \langle 31, 30, 14, 7, 5 \rangle$	0
$M_{11} = \langle \overline{3}, \overline{2}, 1 \rangle$	0	$M_{27} = \langle 35, 29, 18, 7, \overline{5} \rangle$	0
$M_{12} = \langle 35, 31, 2 \rangle$	0	$M_{28} = \langle 30, 27, 25, 9, 4, 1 \rangle$	0
$M_{13} = \langle 27, 11, 4 \rangle$	0	$M_{29} = \langle 33, 32, 29, 9, \overline{6}, 1 \rangle$	0
$M_{14} = \langle 38, 18, 4 \rangle$	0	$M_{30} = \langle 49, 46, 26, 19, \overline{5}, 2 \rangle$	0
$M_{15} = \langle 46, 14, 6 \rangle$	0	$M_{31} = \langle 35^2, 25^2, 6, 2, 1 \rangle$	0
$M_{16} = \langle 17, 7, \overline{2} \rangle$	0	$M_{32} = \langle 38, 14, 9, 7, 5, 4, 1^2 \rangle$	-112

Table IV: Basis of the moduli space of $Y = 0$ non-anomalous D-flat directions of model FNY1.

FLAT DIRECTION	Dim.	# $U(1)$'s	$-\hat{Q}/112$
$R_1 = \langle 1^4, 4, \overline{6}, 7^2, 9^2 \rangle$	0	4	2
$R_2 = \langle 1^3, 2, \overline{6}, 7^4, 19^2, 40^2 \rangle$	0	5	2
$R_3 = \langle 1^2, 7^2, 9, 14, 29, 35 \rangle$	0	5	1
$R_4 = \langle 1^2, \overline{4}, \overline{6}, 7^6, 14^2, 19^2, 49^2 \rangle$	0	6	2
$R_5 = \langle 1, \overline{5}, \overline{6}, 7^3, 18, 19, 49 \rangle$	0	6	1
$R_6 = \langle 1, 5, 7^3, 14^2, 19, 38, 49 \rangle$	0	6	1
$R_7 = \langle 1^3, \overline{6}, 7^4, 9, 14, 19, 49 \rangle$	0	6	2
$R_8 = \langle 1^2, 3, \overline{4}, 7^6, 14^2, 19^2, 41^2 \rangle$	0	6	2
$R_9 = \langle 1, 3^2, 5, 7^5, 8^2, 14^4, 36 \rangle$	0	6	2
$R_{10} = \langle 1^3, 3, 4, 5, 7^3, 9^2, 14^2, 36 \rangle$	0	7	2
$R_{11} = \langle 1^2, 3, 5, 7^5, 10, 14^2, 19, 41 \rangle$	0	7	2
$R_{12} = \langle 1^6, 3, \overline{6}, 7^8, 9^2, 14^2, 19^2, 41^2 \rangle$	0	7	4
$R_{13} = \langle 1^5, \overline{6}, 7^6, 9, 14, 19^2, 40, 41 \rangle$	0	7	3
$R_{14} = \langle 1, 2, \overline{6}, 7^8, 14^4, 19^4, 27^2, 38^2, 49^4 \rangle$	0	8	2
$R_{15} = \langle 1^5, 3^2, 5, 7^9, 9^2, 14^4, 19^2, 36, 41^2 \rangle$	0	8	4
$R_{16} = \langle 1^6, 3^3, \overline{6}, 7^{16}, 11^2, 14^4, 18^2, 19^4, 41^4 \rangle$	0	8	6
$R_{17} = \langle 1^6, 3, \overline{6}^3, 7^{16}, 11^2, 14^4, 18^2, 19^4, 49^4 \rangle$	0	8	6
$R_{18} = \langle 1, 3^2, 7^6, 8^2, 14^4, 18, 29, 35, 36 \rangle$	0	8	2
$R_{19} = \langle 1, 3^2, 7^6, 8^2, 14^4, 19, 27, 36, 41 \rangle$	0	8	2
$R_{20} = \langle 1, 3^2, 7^6, 8, 14^3, 18, 19, 36, 41 \rangle$	0	8	2
$R_{21} = \langle 1, 3^2, 7^6, 8^3, 14^5, 27, 29, 35, 36 \rangle$	0	8	2
$R_{22} = \langle 1^3, 3^2, 7^8, 9, 14^3, 18, 19^2, 36, 41^2 \rangle$	0	8	3
$R_{23} = \langle 1^5, 3^2, 7^{10}, 9^2, 14^4, 19^3, 27, 36, 41^3 \rangle$	0	8	4
$R_{24} = \langle 1^3, 3^3, 7^{12}, 11, 14^4, 18^2, 19^3, 36, 41^3 \rangle$	0	8	4
$R_{25} = \langle 1^2, 3, 7^6, 14^2, 18, 19^2, 36, 40, 41 \rangle$	0	8	2
$R_{26} = \langle 1^4, \overline{4}, 7^9, 14^3, 19^4, 35, 38, 41^3, 43 \rangle$	0	8	3
$R_{26} = \langle 1^8, 5, \overline{6}, 7^{15}, 11^3, 14^4, 19^5, 35^2, 41^3, 43^2 \rangle$	0	9	6
$R_{27} = \langle 1^7, 5^2, \overline{6}^2, 7^{15}, 11^3, 14^5, 19^4, 35, 43, 49^3 \rangle$	0	9	6
$R_{28} = \langle 1^6, \overline{6}, 7^9, 9, 11, 14^2, 19^3, 35, 41^2, 43 \rangle$	0	9	4
$R_{29} = \langle 1^{12}, \overline{6}^2, 7^{23}, 11^4, 14^5, 18, 19^8, 35^3, 41^5, 43^3 \rangle$	0	9	9
$R_{30} = \langle 1^9, \overline{6}^4, 7^{21}, 11^3, 14^5, 18^2, 19^6, 35, 43, 49^5 \rangle$	0	9	8

Table V: List of some type-B D-flat directions that are F-flat to all orders for the model FNY1. The dimension of the direction, after cancellation of the Fayet-Iliopoulos term, is indicated in the second column. The third column gives the number of non-anomalous $U(1)$'s broken along the flat direction and the fourth lists the corresponding values of $-\hat{Q}/112$.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_Y	$\hat{Q}$	Q'	Q''
$S_1\surd$	-16	0	0	0	4	0	0	-8	0	0	0	0	0	0
$S_2\surd$	-10	-3	2	2	-2	0	0	-5	-1	-15	0	0	0	0
$S_3\surd$	-6	3	-2	-2	6	0	0	-3	1	15	0	0	0	0
$S_4\surd$	-6	3	-2	2	-6	0	0	-3	1	15	0	0	0	0
$S_5\surd$	-4	0	0	4	0	0	-8	8	0	0	0	0	0	0
$S_6\surd$	-4	0	0	4	0	4	4	8	0	0	0	0	0	0
$S_7\surd$	-4	0	0	4	0	-4	4	8	0	0	0	-60	0	0
$S_8\surd$	-4	0	0	-4	0	0	-8	8	0	0	0	0	0	0
$S_9\surd$	-4	0	0	-4	0	4	4	8	0	0	0	0	0	0
$S_{10}\surd$	-4	0	0	-4	0	-4	4	8	0	0	0	-60	0	0
$S_{11}\surd$	0	0	0	8	0	0	0	0	0	0	0	0	0	0
$S_{12}\surd$	0	0	0	4	-12	0	0	0	0	0	0	0	0	0
$S_{13}\surd$	0	0	0	4	12	0	0	0	0	0	0	0	0	0
$S_{14}\surd$	-8	-3	-2	0	0	4	-4	1	-3	15	-1	0	144	-60
S_{15}	6	6	-4	2	2	-2	-2	-2	0	0	0	0	-156	0
S_{16}	6	6	-4	-2	2	2	-2	-2	0	0	0	30	-156	0
S_{17}	6	6	-4	0	-4	0	4	-2	0	0	0	0	-156	0
S_{18}	6	-3	-2	-2	-2	2	2	-7	3	-15	-1	60	12	60
S_{19}	6	-3	-2	2	-2	-2	2	-7	3	-15	-1	30	12	60
S_{20}	6	-3	-2	0	4	0	-4	-7	3	-15	-1	60	12	60
S_{21}	2	6	4	2	2	2	2	6	0	0	2	0	-156	0
S_{22}	2	6	4	-2	2	-2	2	6	0	0	2	-30	-156	0
S_{23}	2	6	4	0	-4	0	-4	6	0	0	2	0	-156	0
S_{24}	2	3	2	2	2	2	2	-9	-3	15	1	0	-12	-60
S_{25}	2	3	2	-2	2	-2	2	-9	-3	15	1	-30	-12	-60
S_{26}	2	3	2	0	-4	0	-4	-9	-3	15	1	0	-12	-60

Table VIa: List of non-Abelian singlets for model AF1, with hypercharge and $\hat{Q} = Q_A - \overline{Q}$ as defined in eqs. (67) and (68), respectively. Also shown are the Q' and Q'' charges defined in eq. (69).

	$r_1 : r_2 =$ 3:6	4:17	8:14	9:12	10:19	15:18	5:43	7:38	11:45	13:40	16:41	20:48
Singlet S_i	Q						I					
S_1						.5	σ^-	σ^-	σ^-	σ^-		
$\bar{S}_1$						.5	σ^-	σ^+	σ^-	σ^+		
S_2				.5			σ^-	σ^+			σ^-	σ^+
$\bar{S}_2$				.5			σ^+	σ^+			σ^+	σ^+
S_3	.5								σ^-	σ^+	σ^-	σ^+
$\bar{S}_3$	.5								σ^+	σ^+	σ^-	σ^-
S_4	.5								σ^+	σ^+	σ^-	σ^-
$\bar{S}_4$	.5								σ^-	σ^+	σ^-	σ^+
S_5						.5	σ^+	σ^-	σ^+	σ^+		
$\bar{S}_5$						.5	σ^+	σ^+	σ^+	σ^-		
S_6						.5	σ^+	σ^-	σ^-	σ^-		
$\bar{S}_6$						.5	σ^+	σ^+	σ^-	σ^+		
S_7						.5	σ^-	σ^+	σ^+	σ^+		
$\bar{S}_7$						.5	σ^-	σ^-	σ^+	σ^-		
S_8						.5	σ^+	σ^+	σ^+	σ^-		
$\bar{S}_8$						.5	σ^+	σ^-	σ^+	σ^+		
S_9						.5	σ^+	σ^+	σ^-	σ^+		
$\bar{S}_9$						.5	σ^+	σ^-	σ^-	σ^-		
S_{10}						.5	σ^-	σ^-	σ^+	σ^-		
$\bar{S}_{10}$						.5	σ^-	σ^+	σ^+	σ^+		
S_{11}	.5			.5		$-.5$						
$\bar{S}_{11}$	.5			.5		$-.5$						
S_{12}	$-.5$			.5		.5						
$\bar{S}_{12}$	$-.5$			.5		.5						
S_{13}	.5			$-.5$		.5						
$\bar{S}_{13}$	.5			$-.5$		.5						
S_{14}	.5			$-.5$		.5						
$\bar{S}_{14}$	.5			$-.5$		.5						
S_{15}	.5				.5					σ^+	σ^-	
S_{16}		.5		.5				σ^+				σ^-
S_{17}			$-.5$			.5	σ^-		σ^-			
S_{18}	.5				.5					σ^+	σ^-	
S_{19}		$-.5$		.5				σ^+				σ^+
S_{20}			.5			.5	σ^-		σ^+			
S_{21}	.5				.5					σ^+	σ^+	
S_{22}		.5		.5				σ^+				σ^+
S_{23}			.5			.5	σ^+		σ^+			
S_{24}	.5				$-.5$					σ^-	σ^+	
S_{25}		.5		.5				σ^-				σ^-
S_{26}			$-.5$			.5	σ^+		σ^-			

Table VIb: The non-gauge worldsheet charges of the 40 NAB singlets in AF1.

BASIS A	$\hat{Q}$	BASIS B	$\hat{Q}$
$M_1 = \langle 1, \bar{1} \rangle$	0	$P_1 = \langle 1^3, 5, 6, 10, \overline{13} \rangle$	-60
$M_2 = \langle 2, \bar{2} \rangle$	0	$P_2 = \langle 1^3, 5, 7, 9, \overline{13} \rangle$	-60
$M_3 = \langle 3, \bar{3} \rangle$	0	$P_3 = \langle 1^3, 5, 9, 10, 12 \rangle$	-60
$M_4 = \langle 4, \bar{4} \rangle$	0	$P_4 = \langle 1^3, 6, 7, 8, \overline{13} \rangle$	-60
$M_5 = \langle 5, \bar{5} \rangle$	0	$P_5 = \langle 1^2, 2, 4, 5, 9, 10 \rangle$	-60
$M_6 = \langle 6, \bar{6} \rangle$	0	$P_6 = \langle 1^3, \bar{3}, 4, 5, 9, 10 \rangle$	-60
$M_7 = \langle 7, \bar{7} \rangle$	0	$P_7 = \langle 1^3, \bar{5}, 6, 7, 8^2, 12 \rangle$	-60
$M_8 = \langle 8, \bar{8} \rangle$	0	$P_8 = \langle 1^3, 5, \bar{6}, 7, 9^2, 12 \rangle$	-60
$M_9 = \langle 9, \bar{9} \rangle$	0	$P_9 = \langle 1^3, 5, 6, \bar{7}, 10^2, 12 \rangle$	-60
$M_{10} = \langle 10, \overline{10} \rangle$	0	$P_{10} = \langle 1^3, 5, 6, 7, \overline{11}, \overline{13} \rangle$	-60
$M_{11} = \langle 11, \overline{11} \rangle$	0	$P_{11} = \langle 1^3, 5, 6, 7, \overline{12}, \overline{13}^2 \rangle$	-60
$M_{12} = \langle 12, \overline{12} \rangle$	0	$P_{12} = \langle 1^3, 5^2, \bar{8}, 9, 10, \overline{13} \rangle$	-60
$M_{13} = \langle 13, \overline{13} \rangle$	0	$P_{13} = \langle 1^3, 5, 9, 10, 11, \overline{13} \rangle$	-60
$M_{14} = \langle 1, 2, \bar{3} \rangle$	-60	$P_{14} = \langle 1^3, 6^2, 8, \bar{9}, 10, \overline{13} \rangle$	-60
$M_{15} = \langle 3, \bar{4}, 12 \rangle$	0	$P_{15} = \langle 1^3, 7^2, 8, 9, \overline{10}, \overline{13} \rangle$	-60
$M_{16} = \langle 5, \bar{8}, \overline{11} \rangle$	0	$P_{16} = \langle 1^3, 8, 9, 10, 12^2, 13 \rangle$	-60
$M_{17} = \langle 6, \bar{9}, \overline{11} \rangle$	0	$P_{17} = \langle 2^3, 3^3, 5, 6, 10, \overline{13} \rangle$	-60
$M_{18} = \langle 7, \overline{10}, \overline{11} \rangle$	0	$P_{18} = \langle 1^4, \bar{2}, \bar{4}, 5, 6, 7, \overline{13}^2 \rangle$	-60
$M_{19} = \langle 11, \overline{12}, \overline{13} \rangle$	0	$P_{19} = \langle 1^3, 3, \bar{4}, 5, 6, 7, \overline{13}^2 \rangle$	-60
$M_{20} = \langle 1^3, 5, 6, 10, \overline{13} \rangle$	-60	$N_1 = \langle 1, \bar{1} \rangle$	0

Table VII: Two different bases of the moduli space of non-anomalous D-flat directions of the model AF1.

$R^0_1 = \langle 1, \overline{1} \rangle$	$R^0_{41} = \langle \overline{1}, 2, 4, \overline{6}, 9, 13 \rangle$	$R_{81} = \langle 1^4, \overline{2}, \overline{4}, 5, 6, 7, \overline{13}^2 \rangle$
$R^0_2 = \langle 2, \overline{2} \rangle$	$R_{42} = \langle 1^2, 2, 4, 7, 8, 9 \rangle$	$R_{82} = \langle 1^2, 2, 4, 5, 6, 10, \overline{11} \rangle$
$R^0_3 = \langle 3, \overline{3} \rangle$	$R^0_{43} = \langle \overline{1}, 2, 4, \overline{7}, 10, 13 \rangle$	$R_{83} = \langle 1^2, 2, 4, 5, 7, 9, \overline{11} \rangle$
$R^0_4 = \langle 4, \overline{4} \rangle$	$R_{44} = \langle 1^3, \overline{3}, 4, 5, 9, 10 \rangle$	$R_{84} = \langle 1^2, 2, 4, 6, 7, 8, \overline{11} \rangle$
$R^0_5 = \langle 5, \overline{5} \rangle$	$R_{45} = \langle 1^3, \overline{3}, 4, 6, 8, 10 \rangle$	$R_{85} = \langle 1^2, 2, 4, 8, 9, 10, 11 \rangle$
$R^0_6 = \langle 6, \overline{6} \rangle$	$R_{46} = \langle 1^3, \overline{3}, 4, 7, 8, 9 \rangle$	$R_{86} = \langle 1, 2^2, 4^2, 8, 9, 10, 13 \rangle$
$R^0_7 = \langle 7, \overline{7} \rangle$	$R_{47} = \langle 1^3, \overline{5}, 6, 7, 8^2, 12 \rangle$	$R_{87} = \langle 1^3, \overline{3}, 4, \overline{5}, 6, 7, 8^2 \rangle$
$R^0_8 = \langle 8, \overline{8} \rangle$	$R_{48} = \langle 1^3, 5, \overline{6}, 7, 9^2, 12 \rangle$	$R_{88} = \langle 1^3, \overline{3}, 4, 5, \overline{6}, 7, 9^2 \rangle$
$R^0_9 = \langle 9, \overline{9} \rangle$	$R_{49} = \langle 1^3, 5, 6, \overline{7}, 10^2, 12 \rangle$	$R_{89} = \langle 1^3, \overline{3}, 4, 5, 6, \overline{7}, 10^2 \rangle$
$R^0_{10} = \langle 10, \overline{10} \rangle$	$R_{50} = \langle 1^3, 5, 6, 7, \overline{11}^2, 12 \rangle$	$R_{90} = \langle 1^3, \overline{3}, 4, 5, 6, 7, \overline{11}^2 \rangle$
$R^0_{11} = \langle 11, \overline{11} \rangle$	$R_{51} = \langle 1^3, 5, 6, 7, \overline{11}, \overline{13} \rangle$	$R_{91} = \langle 1^3, 3, \overline{4}, 5, 6, 7, \overline{13}^2 \rangle$
$R^0_{12} = \langle 12, \overline{12} \rangle$	$R_{52} = \langle 1^3, 5, 6, 7, \overline{12}, \overline{13}^2 \rangle$	$R_{92} = \langle 1^3, \overline{3}, 4, 5, 6, 10, \overline{11} \rangle$
$R^0_{13} = \langle 13, \overline{13} \rangle$	$R_{53} = \langle 1^3, 5, 6, 10, \overline{11}, 12 \rangle$	$R_{93} = \langle 1^3, \overline{3}, 4, 5, 7, 9, \overline{11} \rangle$
$R^0_{14} = \langle 1, \overline{2}, \overline{3} \rangle$	$R_{54} = \langle 1^3, 5, 7, 9, \overline{11}, 12 \rangle$	$R_{94} = \langle 1^3, \overline{3}, 4, 6, 7, 8, \overline{11} \rangle$
$R^0_{15} = \langle \overline{5}, 8, 11 \rangle$	$R_{55} = \langle 1^3, 5^2, \overline{8}, 9, 10, \overline{13} \rangle$	$R_{95} = \langle 1^3, \overline{3}, 4, 8, 9, 10, 11 \rangle$
$R^0_{16} = \langle \overline{6}, 9, 11 \rangle$	$R_{56} = \langle 1^3, 5, 9, 10, 11, \overline{13} \rangle$	$R_{96} = \langle 1^3, \overline{3}^2, 4^2, 8, 9, 10, 13 \rangle$
$R^0_{17} = \langle \overline{7}, 10, 11 \rangle$	$R_{57} = \langle 1^3, 6, 7, 8, \overline{11}, 12 \rangle$	$R_{97} = \langle 2^3, 3^2, 4, \overline{5}, 6, 7, 8^2 \rangle$
$R^0_{18} = \langle \overline{11}, 12, 13 \rangle$	$R_{58} = \langle 1^3, 6^2, 8, \overline{9}, 10, \overline{13} \rangle$	$R_{98} = \langle 2^3, 3^2, 4, 5, \overline{6}, 7, 9^2 \rangle$
$R^0_{19} = \langle 3, \overline{4}, 12 \rangle$	$R_{59} = \langle 1^3, 6, 8, 10, 11, \overline{13} \rangle$	$R_{99} = \langle 2^3, 3^2, 4, 5, 6, \overline{7}, 10^2 \rangle$
$R^0_{20} = \langle 1, \overline{2}, \overline{4}, 12 \rangle$	$R_{60} = \langle 1^3, 7^2, 8, 9, \overline{10}, \overline{13} \rangle$	$R_{100} = \langle 2^3, 3^2, 4, 5, 6, 7, \overline{11}^2 \rangle$
$R^0_{21} = \langle 5, \overline{6}, \overline{8}, 9 \rangle$	$R_{61} = \langle 1^3, 7, 8, 9, 11, \overline{13} \rangle$	$R_{101} = \langle 2^3, 3^4, \overline{4}, 5, 6, 7, \overline{13}^2 \rangle$
$R^0_{22} = \langle 5, \overline{7}, \overline{8}, 10 \rangle$	$R_{62} = \langle 1^3, 8, 9, 10, 11, 12 \rangle$	$R_{102} = \langle 2^3, 3^2, 4, 5, 6, 10, \overline{11} \rangle$
$R^0_{23} = \langle 6, \overline{7}, \overline{9}, 10 \rangle$	$R_{63} = \langle 1^3, 8, 9, 10, 11^2, \overline{13} \rangle$	$R_{103} = \langle 2^3, 3^2, 4, 5, 7, 9, \overline{11} \rangle$
$R^0_{24} = \langle 5, \overline{8}, \overline{12}, \overline{13} \rangle$	$R_{64} = \langle 1^3, 8, 9, 10, 12^2, 13 \rangle$	$R_{104} = \langle 2^3, 3^2, 4, 6, 7, 8, \overline{11} \rangle$
$R^0_{25} = \langle \overline{6}, 9, 12, 13 \rangle$	$R_{65} = \langle 2^3, 3^2, 4, 5, 9, 10 \rangle$	$R_{105} = \langle 2^3, 3^2, 4, 8, 9, 10, 11 \rangle$
$R^0_{26} = \langle \overline{7}, 10, 12, 13 \rangle$	$R_{66} = \langle 2^3, 3^2, 4, 6, 8, 10 \rangle$	$R_{106} = \langle 2^3, 3, 4^2, 8, 9, 10, 13 \rangle$
$R^0_{27} = \langle 3, \overline{4}, 11, \overline{13} \rangle$	$R_{67} = \langle 2^3, 3^2, 4, 7, 8, 9 \rangle$	$R_{107} = \langle 2^3, 3^3, \overline{5}, 6, 7, 8^2, 12 \rangle$
$R^0_{28} = \langle 1, \overline{2}, \overline{4}, 11, \overline{13} \rangle$	$R_{68} = \langle 2^3, 3^3, 5, 6, 10, \overline{13} \rangle$	$R_{108} = \langle 2^3, 3^3, 5, \overline{6}, 7, 9^2, 12 \rangle$
$R_{29} = \langle 1^3, 5, 6, 10, \overline{13} \rangle$	$R_{69} = \langle 2^3, 3^3, 5, 7, 9, \overline{13} \rangle$	$R_{109} = \langle 2^3, 3^3, 5, 6, \overline{7}, 10^2, 12 \rangle$
$R_{30} = \langle 1^3, 5, 7, 9, \overline{13} \rangle$	$R_{70} = \langle 2^3, 3^3, 5, 9, 10, 12 \rangle$	$R_{110} = \langle 2^3, 3^3, 5, 6, 7, \overline{11}^2, 12 \rangle$
$R_{31} = \langle 1^3, 5, 9, 10, 12 \rangle$	$R_{71} = \langle 2^3, 3^3, 6, 7, 8, \overline{13} \rangle$	$R_{111} = \langle 2^3, 3^3, 5, 6, 7, \overline{11}, \overline{13} \rangle$
$R_{32} = \langle 1^3, 6, 7, 8, \overline{13} \rangle$	$R_{72} = \langle 2^3, 3^3, 6, 8, 10, 12 \rangle$	$R_{112} = \langle 2^3, 3^3, 5, 6, 7, \overline{12}, \overline{13}^2 \rangle$
$R_{33} = \langle 1^3, 6, 8, 10, 12 \rangle$	$R_{73} = \langle 2^3, 3^3, 7, 8, 9, 12 \rangle$	$R_{113} = \langle 2^3, 3^3, 5, 6, 10, \overline{11}, 12 \rangle$
$R_{34} = \langle 1^3, 7, 8, 9, 12 \rangle$	$R_{74} = \langle 2^3, 4^3, 5, 9, 10, \overline{12}^2 \rangle$	$R_{114} = \langle 2^3, 3^3, 5, 7, 9, \overline{11}, 12 \rangle$
$R^0_{35} = \langle 3, \overline{4}, 5, \overline{8}, \overline{13} \rangle$	$R_{75} = \langle 2^3, 4^3, 6, 8, 10, \overline{12}^2 \rangle$	$R_{115} = \langle 2^3, 3^3, 5^2, \overline{8}, 9, 10, \overline{13} \rangle$
$R^0_{36} = \langle 3, \overline{4}, 6, \overline{9}, \overline{13} \rangle$	$R_{76} = \langle 2^3, 4^3, 7, 8, 9, \overline{12}^2 \rangle$	$R_{116} = \langle 2^3, 3^3, 5, 9, 10, 11, \overline{13} \rangle$
$R^0_{37} = \langle 3, \overline{4}, 7, \overline{10}, \overline{13} \rangle$	$R_{77} = \langle 1^2, 2, 4, \overline{5}, 6, 7, 8^2 \rangle$	$R_{117} = \langle 2^3, 3^3, 6, 7, 8, \overline{11}, 12 \rangle$
$R^0_{38} = \langle \overline{1}, 2, 4, \overline{5}, 8, 13 \rangle$	$R_{78} = \langle 1^2, 2, 4, 5, \overline{6}, 7, 9^2 \rangle$	$R_{118} = \langle 2^3, 3^3, 6^2, 8, \overline{9}, 10, \overline{13} \rangle$
$R_{39} = \langle 1^2, 2, 4, 5, 9, 10 \rangle$	$R_{79} = \langle 1^2, 2, 4, 5, 6, \overline{7}, 10^2 \rangle$	$R_{119} = \langle 2^3, 3^3, 6, 8, 10, 11, \overline{13} \rangle$
$R_{40} = \langle 1^2, 2, 4, 6, 8, 10 \rangle$	$R_{80} = \langle 1^2, 2, 4, 5, 6, 7, \overline{11}^2 \rangle$	$R_{120} = \langle 2^3, 3^3, 7^2, 8, 9, \overline{10}, \overline{13} \rangle$

Table VIIIa: Superbasis of the moduli space of non-anomalous D-flat directions of model AF1.

$R_{121} = \langle 2^3, 3^3, 7, 8, 9, 11, \overline{13} \rangle$	$R_{134} = \langle 2^3, 4^3, \overline{5}^2, 6, 8^3, 10, 13^2 \rangle$	$R_{147} = \langle 2^3, 4^3, 6, \overline{7}^2, 8, 10^3, 13^2 \rangle$
$R_{122} = \langle 2^3, 3^3, 8, 9, 10, 11, 12 \rangle$	$R_{135} = \langle 2^3, 4^3, 5, \overline{6}^2, 9^3, 10, 13^2 \rangle$	$R_{148} = \langle 2^3, 4^3, 6, 7, 8, \overline{11}, \overline{12}^2 \rangle$
$R_{123} = \langle 2^3, 3^3, 8, 9, 10, 11^2, \overline{13} \rangle$	$R_{136} = \langle 2^3, 4^3, 5, 6, 10, \overline{11}, \overline{12}^2 \rangle$	$R_{149} = \langle 2^3, 4^3, 6, 7, 8, \overline{11}^3, 13^2 \rangle$
$R_{124} = \langle 2^3, 3^3, 8, 9, 10, 12^2, 13 \rangle$	$R_{137} = \langle 2^3, 4^3, 5, 6, 10, \overline{11}^3, 13^2 \rangle$	$R_{150} = \langle 2^3, 4^3, 6, 7, 8, \overline{12}^3, \overline{13} \rangle$
$R_{125} = \langle 2^3, 4^3, \overline{5}, 6, 7, 8^2, \overline{12}^2 \rangle$	$R_{138} = \langle 2^3, 4^3, 5, 6, 10, \overline{12}^3, \overline{13} \rangle$	$R_{151} = \langle 2^3, 4^3, \overline{6}, 8, 9^2, 10, 13^2 \rangle$
$R_{126} = \langle 2^3, 4^3, \overline{5}^3, 6, 7, 8^4, 13^2 \rangle$	$R_{139} = \langle 2^3, 4^3, \overline{5}^2, 7, 8^3, 9, 13^2 \rangle$	$R_{152} = \langle 2^3, 4^3, 6, 8, 10, \overline{11}^2, 13^2 \rangle$
$R_{127} = \langle 2^3, 4^3, 5, \overline{6}, 7, 9^2, \overline{12}^2 \rangle$	$R_{140} = \langle 2^3, 4^3, 5, \overline{7}^2, 9, 10^3, 13^2 \rangle$	$R_{153} = \langle 2^3, 4^3, \overline{7}, 8, 9, 10^2, 13^2 \rangle$
$R_{128} = \langle 2^3, 4^3, 5, \overline{6}^3, 7, 9^4, 13^2 \rangle$	$R_{141} = \langle 2^3, 4^3, 5, 7, 9, \overline{11}, \overline{12}^2 \rangle$	$R_{154} = \langle 2^3, 4^3, 7, 8, 9, \overline{11}^2, 13^2 \rangle$
$R_{129} = \langle 2^3, 4^3, 5, 6, \overline{7}, 10^2, \overline{12}^2 \rangle$	$R_{142} = \langle 2^3, 4^3, 5, 7, 9, \overline{11}^3, 13^2 \rangle$	$R_{155} = \langle 2^3, 4^3, 8, 9, 10, 11, \overline{12}^2 \rangle$
$R_{130} = \langle 2^3, 4^3, 5, 6, \overline{7}^3, 10^4, 13^2 \rangle$	$R_{143} = \langle 2^3, 4^3, 5, 7, 9, \overline{12}^3, \overline{13} \rangle$	$R_{156} = \langle 2^3, 4^3, 8, 9, 10, \overline{11}, 13^2 \rangle$
$R_{131} = \langle 2^3, 4^3, 5, 6, 7, \overline{11}^2, \overline{12}^2 \rangle$	$R_{144} = \langle 2^3, 4^3, \overline{5}, 8^2, 9, 10, 13^2 \rangle$	$R_{157} = \langle 2^3, 4^3, 8, 9, 10, \overline{12}, 13 \rangle$
$R_{132} = \langle 2^3, 4^3, 5, 6, 7, \overline{11}^4, 13^2 \rangle$	$R_{145} = \langle 2^3, 4^3, 5, 9, 10, \overline{11}^2, 13^2 \rangle$	– – – – – – – – – – – – – –
$R_{133} = \langle 2^3, 4^3, 5, 6, 7, \overline{12}^4, \overline{13}^2 \rangle$	$R_{146} = \langle 2^3, 4^3, \overline{6}^2, 7, 8, 9^3, 13^2 \rangle$	– – – – – – – – – – – – – –

Table VIIIb: Superbasis of the moduli space of non-anomalous D-flat directions of model AF1 (continued).

FLAT DIRECTION	Dim.	# $U(1)$'s
$R_1 = \langle 1^3, 5, 6, 10, \overline{13} \rangle$	0	4
$R_2 = \langle 1^3, 5, 7, 9, \overline{13} \rangle$	0	4
$R_3 = \langle 1^3, 5, 9, 10, 12 \rangle$	0	4
$R_4 = \langle 1^3, 6, 7, 8, \overline{13} \rangle$	0	4
$R_5 = \langle 1^3, 6, 8, 10, 12 \rangle$	0	4
$R_6 = \langle 1^3, 7, 8, 9, 12 \rangle$	0	4
$R_7 = \langle 1^2, 2, 4, 5, 9, 10 \rangle$	0	5
$R_8 = \langle 1^2, 2, 4, 6, 8, 10 \rangle$	0	5
$R_9 = \langle 1^2, 2, 4, 7, 8, 9 \rangle$	0	5
$R_{10} = \langle 1, 2^2, 4^2, 8, 9, 10, 13 \rangle$	0	6

Table IX: List of type-B D-flat directions that are F-flat to all orders for the model AF1. The dimension of the direction, after cancellation of the Fayet Iliopoulos term, is indicated in the second column. The third column gives the number of non-anomalous $U(1)$'s broken along the flat direction.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_Y	$\hat{Q}$	Q'
$S_1\surd$	40	0	0	0	0	0	−8	16	0	−4	336	−40	−248	136	0	0	0
$S_2\surd$	20	0	0	0	0	0	2	18	−10	−32	−114	−20	−124	68	−1/2	0	0
$S_3\surd$	16	0	4	0	0	0	0	−44	0	−32	−64	−64	112	−48	−1/2	0	4
$S_4\surd$	16	0	−4	0	0	0	0	−44	0	−32	−64	−64	112	−48	−1/2	0	−4
$S_5\surd$	12	0	0	0	0	0	0	−22	12	−26	8	8	−332	−36	−1/2	0	0
$S_6\surd$	12	0	0	0	0	0	−12	2	0	−22	−44	−232	−124	−36	−1/2	0	0
S_7	36	0	0	0	0	0	−10	−46	30	8	−178	−84	−12	20	0	0	0
S_8	12	0	0	0	4	0	0	−22	12	−26	8	8	−332	−36	−1/2	0	0
S_9	12	0	0	0	4	0	−12	2	0	−22	−44	−232	−124	−36	−1/2	0	0
S_{10}	12	0	0	0	0	0	8	−16	12	−6	220	408	28	92	1/2	0	0
S_{11}	12	0	0	0	0	0	−16	32	−12	2	116	−72	444	92	1/2	0	0
S_{12}	12	0	0	0	−4	0	0	−22	12	−26	8	8	−332	−36	−1/2	0	0
S_{13}	12	0	0	0	−4	0	−12	2	0	−22	−44	−232	−124	−36	−1/2	0	0
S_{14}	8	0	0	2	−2	−2	4	14	−2	34	58	−36	−96	−152	0	64	0
S_{15}	8	0	0	0	0	0	18	30	−2	30	222	−60	264	−24	1/2	0	0
S_{16}	8	0	0	−2	2	2	4	14	−2	34	58	−36	−96	−152	1/2	64	0
S_{17}	4	2	−2	−2	2	0	12	20	16	−4	−272	104	−76	−12	1/2	0	0
S_{18}	4	2	−2	−2	2	0	2	−4	−26	−6	202	108	76	116	1/2	0	0
S_{19}	4	2	−2	−2	2	0	0	44	4	0	−324	−136	132	−12	1/2	0	0
S_{20}	4	0	0	0	0	0	12	86	12	−10	40	40	−388	−12	0	0	0
S_{21}	4	0	0	0	0	0	2	−70	−22	0	−110	172	388	116	0	0	0
S_{22}	4	0	0	0	0	0	0	110	0	−6	−12	−200	−180	−12	0	0	0
S_{23}	4	−2	2	2	−2	0	12	20	16	−4	−272	104	−76	−12	−1/2	0	0
S_{24}	4	−2	2	2	−2	0	2	−4	−26	−6	202	108	76	116	−1/2	0	0
S_{25}	4	−2	2	2	−2	0	0	44	4	0	−324	−136	132	−12	−1/2	0	0
S_{26}	0	0	0	2	−2	2	−8	38	10	18	86	180	56	−128	1/2	64	0
S_{27}	0	0	0	−2	2	−2	−8	38	10	18	86	180	56	−128	1	64	0
S_{28}	−44	0	0	0	0	0	8	6	−8	−2	300	112	−196	−124	0	0	0
S_{29}	−44	0	0	0	0	0	2	−26	2	12	34	−60	52	260	0	−128	0
S_{30}	−44	0	0	0	0	0	−2	−18	−10	20	−354	116	−44	4	0	0	0
S_{31}	−44	0	0	0	0	0	−4	30	−20	2	248	−128	12	−124	0	0	0
S_{32}	−32	0	0	0	0	0	−6	−10	−10	18	−186	284	192	96	1/2	0	0
S_{33}	−28	2	2	−2	2	0	−2	4	−14	−18	−106	−12	−244	−44	0	0	4
S_{34}	−28	0	0	2	−2	−2	−6	−32	12	−2	56	56	220	84	0	−64	0
S_{35}	−28	0	0	0	0	0	4	−8	0	2	−168	208	484	−44	1/2	0	0
S_{36}	−28	0	0	−2	2	2	−6	−32	12	−2	56	56	220	84	1/2	−64	0
S_{37}	−28	−2	−2	2	−2	0	−2	4	−14	−18	−106	−12	−244	−44	−1	0	−4
S_{38}	−24	0	0	2	2	2	−4	30	2	−2	6	100	−16	200	0	−64	0
S_{39}	−24	0	0	0	0	0	−10	−46	22	−14	−90	4	152	−184	0	0	0
S_{40}	−24	0	0	−2	−2	−2	−4	30	2	−2	6	100	−16	200	1/2	−64	0
S_{41}	−20	0	0	0	4	0	−2	−18	10	32	114	20	124	−68	1/2	0	0
S_{42}	−20	0	0	0	0	4	−8	16	12	−14	−140	48	−84	−68	0	0	0
S_{43}	−20	0	0	0	0	0	−8	16	−8	−26	424	48	−84	−68	0	0	0

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_Y	$\hat{Q}$	Q'
S_{44}	-20	0	0	0	0	0	-2	70	22	0	110	204	332	-68	1	0	0
S_{45}	-20	0	0	0	0	-4	-8	16	12	-14	-140	48	-84	-68	0	0	0
S_{46}	-20	0	0	0	-4	0	-2	-18	10	32	114	20	124	-68	$1/2$	0	0
S_{47}	-20	0	0	2	2	2	6	32	12	-18	24	24	276	60	0	-64	0
S_{48}	-20	0	0	2	-2	2	6	-56	0	14	28	-160	68	60	$-1/2$	-64	0
S_{49}	-20	0	0	-2	2	-2	6	-56	0	14	28	-160	68	60	0	-64	0
S_{50}	-20	0	0	-2	-2	-2	6	32	12	-18	24	24	276	60	$1/2$	-64	0
S_{51}	-16	2	2	-2	2	0	-6	12	-14	-20	62	156	-8	48	$1/2$	0	4
S_{52}	-16	2	-2	2	2	0	-6	12	-14	-20	62	156	-8	48	0	0	0
S_{53}	-16	0	0	2	2	-2	8	6	-10	14	-22	-116	-168	176	$-1/2$	-64	0
S_{54}	-16	0	0	0	0	0	18	30	-22	18	-246	36	96	48	0	0	0
S_{55}	-16	0	0	-2	-2	2	8	6	-10	14	-22	-116	-168	176	0	-64	0
S_{56}	-16	-2	2	-2	-2	0	-6	12	-14	-20	62	156	-8	48	0	0	0
S_{57}	-16	-2	-2	2	-2	0	-6	12	-14	-20	62	156	-8	48	$-1/2$	0	-4
S_{58}	-12	0	4	0	0	0	10	46	10	16	82	-12	180	-92	$1/2$	0	4
S_{59}	-12	0	0	2	2	-2	18	8	0	-2	-4	-192	124	36	$-1/2$	-64	0
S_{60}	-12	0	0	0	0	0	4	-8	0	2	-168	-168	-236	-92	$-1/2$	0	0
S_{61}	-12	0	0	0	0	0	-12	-42	12	22	104	104	-500	36	0	0	0
S_{62}	-12	0	0	0	0	0	-24	-18	0	26	52	-136	-292	36	0	0	0
S_{63}	-12	0	0	-2	-2	2	18	8	0	-2	-4	-192	124	36	0	-64	0
S_{64}	-12	0	-4	0	0	0	10	46	10	16	82	-12	180	-92	$1/2$	0	-4
S_{65}	-8	2	2	2	-2	0	6	-12	-26	-4	34	-60	-160	24	$-1/2$	0	4
S_{66}	-8	2	-2	-2	-2	0	6	-12	-26	-4	34	-60	-160	24	0	0	0
S_{67}	-8	0	0	0	0	0	-6	-10	10	30	282	188	360	24	1	0	0
S_{68}	-8	0	0	0	0	0	14	-6	10	-14	-150	-244	56	-232	$-1/2$	0	0
S_{69}	-8	-2	2	2	2	0	6	-12	-26	-4	34	-60	-160	24	-1	0	0
S_{70}	-8	-2	-2	-2	2	0	6	-12	-26	-4	34	-60	-160	24	$-1/2$	0	-4
S_{71}	-4	2	2	-2	-2	0	0	-44	16	12	-240	136	-132	12	$1/2$	0	4
S_{72}	-4	2	2	-2	-2	0	-2	4	6	-6	362	-108	-76	-116	$1/2$	0	4
S_{73}	-4	2	2	-2	-2	0	-12	-20	4	16	-292	-104	76	12	$1/2$	0	4
S_{74}	-4	0	0	0	0	0	22	22	-2	32	54	-228	28	-116	0	0	0
S_{75}	-4	0	0	0	0	0	8	-38	32	-10	140	-48	84	-244	0	0	0
S_{76}	-4	0	0	0	0	0	2	-26	2	12	34	316	-500	140	0	0	0
S_{77}	-4	0	0	0	0	0	-4	-14	20	-6	88	-288	292	-244	0	0	0
S_{78}	-4	0	0	0	0	0	-22	22	-22	20	-70	-164	-84	140	0	0	0
S_{79}	-4	-2	-2	2	2	0	0	-44	16	12	-240	136	-132	12	$-1/2$	0	-4
S_{80}	-4	-2	-2	2	2	0	-2	4	6	-6	362	-108	-76	-116	$-1/2$	0	-4
S_{81}	-4	-2	-2	2	2	0	-12	-20	4	16	-292	-104	76	12	$-1/2$	0	-4

Table X: List of non-Abelian singlets for model CHL1, with $\hat{Q} = Q_A - \overline{Q}$, Y and Q' as defined in eqs. (77), (78) and (79) respectively.

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{12} = \langle 77, 62, 21^2, 20^2 \rangle$	0
$M_2 = \langle 54, 39, 1 \rangle$	0	$M_{13} = \langle 77, 61, 22, 21^2, 20 \rangle$	0
$M_3 = \langle 54, 43, 7 \rangle$	0	$M_{14} = \langle 45, 42, 31, 29^2, 28, \overline{1} \rangle$	-256
$M_4 = \langle 31, 21, 20, 7 \rangle$	0	$M_{15} = \langle 43, 39, 31, 30, 29^2, 20, \overline{1} \rangle$	-256
$M_5 = \langle 28, 22, 21, 7 \rangle$	0	$M_{16} = \langle 63, 45, 39, 38, 31, 30, 29, 28 \rangle$	-256
$M_6 = \langle 74, 56, 52, 7 \rangle$	0	$M_{17} = \langle 55, 47, 45, 39, 31, 30, 29, 28 \rangle$	-256
$M_7 = \langle 78, 75, 21, 20 \rangle$	0	$M_{18} = \langle 55, 45, 39, 38, 31, 29, 28, \overline{1} \rangle$	-256
$M_8 = \langle 77, 76, 22, 21 \rangle$	0	$M_{19} = \langle 49, 42, 39, 22, 21^2, 20, 14, 1 \rangle$	0
$M_9 = \langle 77, 30, 21, 20, 1 \rangle$	0	$M_{20} = \langle 49, 43, 42, 34, 31, 30, 29, 20, \overline{1} \rangle$	-256
$M_{10} = \langle 75, 30, 22, 21, 1 \rangle$	0	$M_{21} = \langle 74^2, 66^2, 56^2, 49, 45, 39^3, 38^4, 14 \rangle$	-256
$M_{11} = \langle 74, 45, 42, 21, 1 \rangle$	0	– –	– – –

Table XI: Basis of the moduli space of non-anomalous D-flat directions of model CHL1, with $Y = 0$.

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{38} = \langle 22, 21, 5, \overline{2} \rangle$	0
$M_2 = \langle 2, \overline{2} \rangle$	0	$M_{39} = \langle 60, 10, 6, \overline{5} \rangle$	0
$M_3 = \langle 3, \overline{3} \rangle$	0	$M_{40} = \langle 24, 17, 6, \overline{2} \rangle$	0
$M_4 = \langle 4, \overline{4} \rangle$	0	$M_{41} = \langle 23, 18, 6, \overline{2} \rangle$	0
$M_5 = \langle 5, \overline{5} \rangle$	0	$M_{42} = \langle 21, 20, 6, \overline{2} \rangle$	0
$M_6 = \langle 6, \overline{6} \rangle$	0	$M_{43} = \langle 57, 33, 15, 7 \rangle$	0
$M_7 = \langle 68, 32, 1 \rangle$	0	$M_{44} = \langle 51, 37, 15, 7 \rangle$	0
$M_8 = \langle 60, 35, 1 \rangle$	0	$M_{45} = \langle 58, 37, 18, 7 \rangle$	0
$M_9 = \langle 54, 39, 1 \rangle$	0	$M_{46} = \langle 64, 33, 24, 7 \rangle$	0
$M_{10} = \langle 36, 14, 2 \rangle$	0	$M_{47} = \langle 70, 65, 44, 7 \rangle$	0
$M_{11} = \langle 34, 16, 2 \rangle$	0	$M_{48} = \langle 69, 66, 44, 7 \rangle$	0
$M_{12} = \langle 49, 26, 2 \rangle$	0	$M_{49} = \langle 74, 57, 51, 7 \rangle$	0
$M_{13} = \langle 48, 27, 2 \rangle$	0	$M_{50} = \langle 74, 56, 52, 7 \rangle$	0
$M_{14} = \langle 67, 60, 2 \rangle$	0	$M_{51} = \langle 73, 57, 15, 8 \rangle$	0
$M_{15} = \langle 58, 43, 7 \rangle$	0	$M_{52} = \langle 67, 66, 25, 8 \rangle$	0
$M_{16} = \langle 78, 68, 10 \rangle$	0	$M_{53} = \langle 81, 51, 15, 12 \rangle$	0
$M_{17} = \langle 76, 68, 11 \rangle$	0	$M_{54} = \langle 69, 67, 19, 12 \rangle$	0
$M_{18} = \langle 64, 30, 3, 1 \rangle$	0	$M_{55} = \langle 68, 35, 30, 1^2 \rangle$	0
$M_{19} = \langle 58, 30, 4, 1 \rangle$	0	$M_{56} = \langle 74, 39, 32, 2, 1 \rangle$	0
$M_{20} = \langle 46, 41, 2^2 \rangle$	0	$M_{57} = \langle 74, 45, 42, 21, 1 \rangle$	0
$M_{21} = \langle 28, 7, \overline{5}, 2 \rangle$	0	$M_{58} = \langle 60^2, 44, 21, 1 \rangle$	0
$M_{22} = \langle 31, 7, \overline{6}, 2 \rangle$	0	$M_{59} = \langle 64, 61, \overline{5}, 3, 2 \rangle$	0
$M_{23} = \langle 46, 8, \overline{5}, 2 \rangle$	0	$M_{60} = \langle 64, 62, \overline{6}, 3, 2 \rangle$	0
$M_{24} = \langle 46, 9, \overline{6}, 2 \rangle$	0	$M_{61} = \langle 49, 42, 35, 14, 2, 1 \rangle$	0
$M_{25} = \langle 41, 12, \overline{5}, 2 \rangle$	0	$M_{62} = \langle 48, 45, 35, 16, 2, 1 \rangle$	0
$M_{26} = \langle 41, 13, \overline{6}, 2 \rangle$	0	$M_{63} = \langle 65, 64, 56, 41, 7, 2 \rangle$	0
$M_{27} = \langle 80, 71, \overline{5}, 2 \rangle$	0	$M_{64} = \langle 40, 38, 16, 14, 4, 3 \rangle$	0
$M_{28} = \langle 79, 72, \overline{5}, 2 \rangle$	0	$M_{65} = \langle 55, 38, 27, 14, 4, 3 \rangle$	0
$M_{29} = \langle 81, 72, \overline{6}, 2 \rangle$	0	$M_{66} = \langle 53, 40, 26, 16, 4, 3 \rangle$	0
$M_{30} = \langle 80, 73, \overline{6}, 2 \rangle$	0	$M_{67} = \langle 63, 32, 14, 11, 8, 5 \rangle$	0
$M_{31} = \langle 78, 75, \overline{6}, 2 \rangle$	0	$M_{68} = \langle 55, 35, 14, 11, 8, 5 \rangle$	0
$M_{32} = \langle 77, 76, \overline{5}, 2 \rangle$	0	$M_{69} = \langle 59, 32, 16, 12, 11, 5 \rangle$	0
$M_{33} = \langle 32, 15, 6, 5 \rangle$	0	$M_{70} = \langle 71, 70, 47, 14, 11, 5 \rangle$	0
$M_{34} = \langle 67, 54, 6, 5 \rangle$	0	$M_{71} = \langle 79, 65, 50, 16, 11, 5 \rangle$	0
$M_{35} = \langle 60, 11, \overline{6}, 5 \rangle$	0	$M_{72} = \langle 71, 70, 59, 26, 11, 5 \rangle$	0
$M_{36} = \langle 25, 18, 5, \overline{2} \rangle$	0	$M_{73} = \langle 82, 45, 42, 31, 29^2, 28 \rangle$	-256
$M_{37} = \langle 24, 19, 5, \overline{2} \rangle$	0	$M_{74} = \langle 39, 35, 29, 16, 14, 2^2, 1 \rangle$	0

Table XII: Basis of the moduli space of non-anomalous D-flat directions of model CHL1.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	$\hat{Q}$	Q'
S_1 $\surd$	40	0	0	−4	4	8	8	−8	0	0
S_2	32	−4	0	4	4	−16	0	−8	0	0
S_3	8	6	4	0	0	24	0	−12	0	0
S_4	8	6	−4	0	0	24	0	−12	0	0
S_5	0	0	8	0	0	0	16	0	0	0
S_6	0	0	−8	0	0	0	16	0	0	0
S_7	0	−4	0	0	0	0	0	24	0	0
S_8	−40	2	4	−4	−4	−8	0	−4	0	0
S_9	−40	2	−4	−4	−4	−8	0	−4	0	0
S_{10}	−40	0	0	4	−4	−8	8	8	0	0
S_{11}	−40	0	0	0	8	−8	8	8	−112	48
S_{12}	−16	0	0	0	0	−48	0	0	0	48
S_{13}	−8	2	4	0	0	−24	0	12	0	48
S_{14}	−8	2	−4	0	0	−24	0	12	0	48
S_{15}	−8	−2	4	0	0	−24	16	−12	0	0
S_{16}	−8	−2	4	0	0	−24	−16	−12	0	0
S_{17}	−8	−2	−4	0	0	−24	16	−12	0	0
S_{18}	−8	−2	−4	0	0	−24	−16	−12	0	0

Table XIII: List of non-Abelian singlet fields for model CHL2, with $\hat{Q} = Q_A - \overline{Q}$ as defined in eq. (86). Also shown is the Q' charge defined in eq. (87).

BASIS	$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0
$M_2 = \langle 2, 3, 7, 9 \rangle$	0
$M_3 = \langle 2, 4, 7, 8 \rangle$	0
$M_4 = \langle 3, 6, 7, 16 \rangle$	0
$M_5 = \langle 4, 5, 7, 18 \rangle$	0
$M_6 = \langle 1, 3, 7, 10, 18 \rangle$	0
$M_7 = \langle 1, 4, 7, 10, 16 \rangle$	0
$M_8 = \langle 3^2, 7^2, 17, 18 \rangle$	0
$M_9 = \langle 3, 4, 7^2, 15, 18 \rangle$	0

Table XIV: Basis of the moduli space of non-anomalous D-flat directions of model CHL2.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	$\hat{Q}$
$S_1\surd$	40	0	0	0	0	-4	-4	-4	-8	4	8	56	0
$S_2\surd$	24	0	0	0	4	0	0	8	0	0	-24	-24	0
$S_3\surd$	24	4	0	0	0	0	0	8	0	0	-24	-24	0
$S_4\surd$	24	-4	0	0	0	0	0	8	0	0	-24	-24	0
$S_5\surd$	24	0	0	0	-4	0	0	8	0	0	-24	-24	0
$S_6\surd$	0	4	0	0	4	0	0	0	0	0	0	0	0
$S_7\surd$	0	4	0	0	-4	0	0	0	0	0	0	0	0
$S_8\surd$	0	0	4	-4	0	0	0	0	0	0	0	0	0
$S_9\surd$	0	0	4	4	0	0	0	0	0	0	0	0	0
S_{10}	40	0	0	0	0	-4	4	-4	8	4	-24	8	0
S_{11}	28	2	-2	2	2	2	0	-8	8	-2	-12	20	0
S_{12}	28	-2	2	-2	-2	2	0	-8	8	-2	-12	20	0
S_{13}	24	0	0	0	0	0	0	8	-16	0	8	24	0
S_{14}	24	0	0	-4	0	0	0	8	-16	0	8	24	0
S_{15}	24	0	0	4	0	0	0	8	-16	0	8	24	0
S_{16}	16	0	0	0	0	-2	0	4	-12	10	-8	-16	0
S_{17}	16	0	0	0	0	-6	0	4	-12	-2	-8	-16	0
S_{18}	16	0	4	0	0	-2	0	4	-12	10	-8	-16	0
S_{19}	16	0	4	0	0	-6	0	4	-12	-2	-8	-16	0
S_{20}	16	0	0	0	4	-4	0	8	8	4	0	32	0
S_{21}	16	0	0	0	-4	-4	0	8	8	4	0	32	0
S_{22}	16	0	-4	0	0	-2	0	4	-12	10	-8	-16	0
S_{23}	16	0	-4	0	0	-6	0	4	-12	-2	-8	-16	0
S_{24}	8	0	0	0	0	4	0	8	32	-4	-8	40	0
S_{25}	4	2	2	2	2	4	-4	0	-12	4	4	-4	0
S_{26}	4	2	2	2	2	2	4	4	8	-2	12	44	0
S_{27}	4	2	2	2	2	0	-4	0	-12	-8	4	-4	0
S_{28}	4	2	-2	2	-2	4	4	0	-12	4	4	-4	0
S_{29}	4	2	-2	2	-2	2	-4	4	8	-2	12	44	0
S_{30}	4	2	-2	2	-2	0	4	0	-12	-8	4	-4	0
S_{31}	4	-2	2	-2	2	4	4	0	-12	4	4	-4	0
S_{32}	4	-2	2	-2	2	2	-4	4	8	-2	12	44	0
S_{33}	4	-2	2	-2	2	0	4	0	-12	-8	4	-4	0
S_{34}	4	-2	-2	-2	-2	4	-4	0	-12	4	4	-4	0
S_{35}	4	-2	-2	-2	-2	2	4	4	8	-2	12	44	0
S_{36}	4	-2	-2	-2	-2	0	-4	0	-12	-8	4	-4	0
S_{37}	0	0	0	0	0	2	0	-4	-4	6	-40	-96	0
S_{38}	0	0	0	0	0	2	0	-4	-36	6	24	0	0
S_{39}	0	0	0	0	0	-2	0	-4	-4	-6	-40	-96	0
S_{40}	0	0	0	0	0	-2	0	-4	-36	-6	24	0	0

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	$\hat{Q}$
S_{41}	−48	0	0	0	0	2	−4	0	−4	6	8	−48	0
S_{42}	−48	0	0	0	0	0	0	0	8	0	−16	96	−144
S_{43}	−48	0	0	0	0	0	4	4	16	0	16	0	0
S_{44}	−48	0	0	0	0	−2	−4	0	−4	−6	8	−48	0
S_{45}	−28	2	2	−2	2	−2	0	8	−8	2	12	−20	0
S_{46}	−28	−2	−2	2	−2	−2	0	8	−8	2	12	−20	0
S_{47}	−24	0	−4	0	0	0	0	−8	16	0	−8	−24	0
S_{48}	−24	0	4	0	0	0	0	−8	16	0	−8	−24	0
S_{49}	−16	4	0	0	0	4	0	−8	−8	−4	0	−32	0
S_{50}	−16	0	0	4	0	6	0	−4	12	2	8	16	0
S_{51}	−16	0	0	4	0	2	0	−4	12	−10	8	16	0
S_{52}	−16	0	0	0	0	−6	0	−4	12	14	8	16	0
S_{53}	−16	0	0	0	0	−10	0	−4	12	2	8	16	0
S_{54}	−16	0	0	−4	0	6	0	−4	12	2	8	16	0
S_{55}	−16	0	0	−4	0	2	0	−4	12	−10	8	16	0
S_{56}	−16	−4	0	0	0	4	0	−8	−8	−4	0	−32	0
S_{57}	−8	0	0	0	0	0	0	0	8	16	24	56	0
S_{58}	−8	0	0	0	0	−8	0	0	8	−8	24	56	0
S_{59}	−4	2	2	−2	−2	0	−4	0	12	8	−4	4	0
S_{60}	−4	2	2	−2	−2	−2	4	−4	−8	2	−12	−44	0
S_{61}	−4	2	2	−2	−2	−4	−4	0	12	−4	−4	4	0
S_{62}	−4	2	−2	−2	2	0	4	0	12	8	−4	4	0
S_{63}	−4	2	−2	−2	2	−2	−4	−4	−8	2	−12	−44	0
S_{64}	−4	2	−2	−2	2	−4	4	0	12	−4	−4	4	0
S_{65}	−4	−2	2	2	−2	0	4	0	12	8	−4	4	0
S_{66}	−4	−2	2	2	−2	−2	−4	−4	−8	2	−12	−44	0
S_{67}	−4	−2	2	2	−2	−4	4	0	12	−4	−4	4	0
S_{68}	−4	−2	−2	2	2	0	−4	0	12	8	−4	4	0
S_{69}	−4	−2	−2	2	2	−2	4	−4	−8	2	−12	−44	0
S_{70}	−4	−2	−2	2	2	−4	−4	0	12	−4	−4	4	0

Table XV: List of non-Abelian singlet fields for model CHL3, with $\hat{Q} = Q_A - \overline{Q}$ as defined in eq. (90).

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{35} = \langle \overline{6}, 11, 45 \rangle$	0
$M_2 = \langle 2, \overline{2} \rangle$	0	$M_{36} = \langle \overline{8}, 14, 48 \rangle$	0
$M_3 = \langle 3, \overline{3} \rangle$	0	$M_{37} = \langle \overline{9}, 18, 51 \rangle$	0
$M_4 = \langle 4, \overline{4} \rangle$	0	$M_{38} = \langle \overline{9}, 19, 50 \rangle$	0
$M_5 = \langle 5, \overline{5} \rangle$	0	$M_{39} = \langle \overline{6}, 20, 49 \rangle$	0
$M_6 = \langle 6, \overline{6} \rangle$	0	$M_{40} = \langle \overline{6}, 25, 64 \rangle$	0
$M_7 = \langle 7, \overline{7} \rangle$	0	$M_{41} = \langle \overline{6}, 26, 63 \rangle$	0
$M_8 = \langle 8, \overline{8} \rangle$	0	$M_{42} = \langle \overline{6}, 27, 62 \rangle$	0
$M_9 = \langle 9, \overline{9} \rangle$	0	$M_{43} = \langle \overline{7}, 28, 61 \rangle$	0
$M_{10} = \langle 2, \overline{3}, 7 \rangle$	0	$M_{44} = \langle \overline{7}, 29, 60 \rangle$	0
$M_{11} = \langle 2, \overline{4}, \overline{6} \rangle$	0	$M_{45} = \langle \overline{7}, 30, 59 \rangle$	0
$M_{12} = \langle 3, \overline{5}, \overline{6} \rangle$	0	$M_{46} = \langle 1, 3, 43, 56 \rangle$	0
$M_{13} = \langle 6, 12, 46 \rangle$	0	$M_{47} = \langle 10, 14, 41, 51 \rangle$	0
$M_{14} = \langle 6, 21, 56 \rangle$	0	$M_{48} = \langle 10, 14, 44, 50 \rangle$	0
$M_{15} = \langle 6, 34, 67 \rangle$	0	$M_{49} = \langle 10, 24, 38, 44 \rangle$	0
$M_{16} = \langle 6, 35, 66 \rangle$	0	$M_{50} = \langle 10, 24, 40, 41 \rangle$	0
$M_{17} = \langle 6, 36, 65 \rangle$	0	$M_{51} = \langle 10, 25, 35, 44 \rangle$	0
$M_{18} = \langle 7, 20, 56 \rangle$	0	$M_{52} = \langle 10, 25, 36, 43 \rangle$	0
$M_{19} = \langle 7, 31, 70 \rangle$	0	$M_{53} = \langle 10, 25, 46, 55 \rangle$	0
$M_{20} = \langle 7, 32, 69 \rangle$	0	$M_{54} = \langle 10, 26, 36, 41 \rangle$	0
$M_{21} = \langle 7, 33, 68 \rangle$	0	$M_{55} = \langle 10, 28, 32, 44 \rangle$	0
$M_{22} = \langle 8, 11, 46 \rangle$	0	$M_{56} = \langle 10, 29, 33, 41 \rangle$	0
$M_{23} = \langle 8, 15, 47 \rangle$	0	$M_{57} = \langle 10, 29, 45, 56 \rangle$	0
$M_{24} = \langle 8, 22, 51 \rangle$	0	$M_{58} = \langle 13^2, 47, 48 \rangle$	0
$M_{25} = \langle 8, 23, 50 \rangle$	0	$M_{59} = \langle 14, 37, 50, 58 \rangle$	0
$M_{26} = \langle 8, 28, 70 \rangle$	0	$M_{60} = \langle 14, 39, 51, 57 \rangle$	0
$M_{27} = \langle 8, 29, 69 \rangle$	0	$M_{61} = \langle 16^2, 51, 55 \rangle$	0
$M_{28} = \langle 8, 30, 68 \rangle$	0	$M_{62} = \langle 16, 17, 50, 55 \rangle$	0
$M_{29} = \langle 9, 14, 47 \rangle$	0	$M_{63} = \langle 1, 14, 37, 43, 51 \rangle$	0
$M_{30} = \langle 9, 22, 55 \rangle$	0	$M_{64} = \langle 1, 14, 39, 43, 50 \rangle$	0
$M_{31} = \langle 9, 23, 54 \rangle$	0	$M_{65} = \langle 3, 12, 38, 43, 70 \rangle$	0
$M_{32} = \langle 9, 34, 64 \rangle$	0	$M_{66} = \langle 3, 14, 50, 53, 56 \rangle$	0
$M_{33} = \langle 9, 35, 63 \rangle$	0	$M_{67} = \langle 3, 14, 51, 52, 56 \rangle$	0
$M_{34} = \langle 9, 36, 62 \rangle$	0	$M_{68} = \langle 37, 40, 41, 42^2, 43^2, 44 \rangle$	-288

Table XVI: Basis of the moduli space of non-anomalous D-flat directions of model CHL3.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_Y	$\hat{Q}$
S_1 $\surd$	16	0	0	−20	4	20	−32	−1/2	0
S_2 $\surd$	16	4	0	−12	−20	−4	−32	−1/2	0
S_3 $\surd$	16	2	−32	0	−8	−28	−32	−1/2	0
S_4 $\surd$	16	−2	0	−4	−20	32	−32	−1/2	0
S_5 $\surd$	16	−2	−32	−8	16	−4	−32	−1/2	0
S_6 $\surd$	16	−4	−32	8	−8	8	−32	−1/2	0
S_7 $\surd$	8	8	−16	4	−4	4	−16	−1/2	0
S_8 $\surd$	8	4	−16	−4	20	28	−16	−1/2	0
S_9 $\surd$	8	2	−16	12	−4	40	−16	−1/2	0
S_{10} $\surd$	8	−4	16	−16	−16	−8	−16	0	0
S_{11} $\surd$	0	6	0	−8	0	−36	0	0	0
S_{12} $\surd$	0	2	32	−12	−12	24	0	0	0
S_{13}	40	2	16	16	−8	−4	−80	0	0
S_{14}	8	2	48	20	20	−8	−16	1	0
S_{15}	8	−2	−16	4	−28	−56	−16	0	0
S_{16}	8	−6	−16	−4	−4	−32	−16	0	0
S_{17}	8	−8	−16	12	−28	−20	−16	0	0
S_{18}	0	4	0	8	−24	−24	0	0	0
S_{19}	0	−4	32	−4	−12	60	0	0	0
S_{20}	0	−2	32	−20	12	48	0	0	0
S_{21}	0	−2	0	16	−24	12	0	0	0
S_{22}	−32	−2	32	20	4	8	64	1	0
S_{23}	−24	0	48	−4	−12	−36	48	1	0
S_{24}	−24	−6	16	0	24	−36	48	1	0
S_{25}	−16	4	−32	−16	−16	40	32	−1	0
${S_{26}}^{(\prime\prime\prime)}$	−12	−4	−8	6	18	−30	24	1/2	0
${S_{27}}^{(\prime)}$	−8	0	8	6	10	14	−152	−1/2	−84
S_{28}	−8	10	−16	8	16	−28	16	0	0
S_{29}	−8	6	−16	0	40	−4	16	0	0
${S_{30}}^{(\prime)}$	−4	0	0	0	8	−20	−160	−1/2	−84
${S_{31}}^{(\prime\prime\prime)}$	−4	4	8	14	−22	10	8	0	0

Table XVII: Same as Table I but for model CHL4, with hypercharge and $\hat{Q} = Q_A - \overline{Q}$ as defined in eqs. (93) and (94) respectively.

BASIS	$\hat{Q}$
$M_1 = \langle 10, \overline{10} \rangle$	0
$M_2 = \langle 11, \overline{11} \rangle$	0
$M_3 = \langle 12, \overline{12} \rangle$	0
$M_4 = \langle \overline{10}, 12, 16 \rangle$	0
$M_5 = \langle \overline{10}, 15, 20 \rangle$	0
$M_6 = \langle 10, \overline{11}, 28 \rangle$	0
$M_7 = \langle 11, \overline{12}, 19 \rangle$	0
$M_8 = \langle \overline{12}, 18, 20 \rangle$	0
$M_9 = \langle 10, 21, 29 \rangle$	0
$M_{10} = \langle 12, 17, 29 \rangle$	0
$M_{11} = \langle 15, 19, 29 \rangle$	0
$M_{12} = \langle 16, 20, 29, 31^2 \rangle$	0

Table XVIII: Basis of the moduli space of non-anomalous D-flat directions of model CHL4, involving $Y = 0$ fields only.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	$\hat{Q}_A$
S_1 $\surd$	40	0	0	0	−4	−4	−4	−8	4	8	56	0
S_2 $\surd$	28	0	0	0	2	0	−8	8	−2	−12	20	0
S_3 $\surd$	24	0	0	0	0	0	8	−16	0	8	24	0
$S_4{}^{(\prime)}$ $\surd^{(\prime)}$	24	0	0	0	0	0	8	0	0	−24	−24	0
S_5 $\surd$	16	0	0	0	−2	0	4	−12	10	−8	−16	0
S_6 $\surd$	16	0	0	0	−6	0	4	−12	−2	−8	−16	0
S_7 $\surd$	16	0	0	0	−4	0	8	8	4	0	32	0
S_8 $\surd$	4	0	0	0	4	4	0	−12	4	4	−4	0
S_9 $\surd$	4	0	0	0	4	−4	0	−12	4	4	−4	0
S_{10} $\surd$	4	0	0	0	2	4	4	8	−2	12	44	0
S_{11} $\surd$	4	0	0	0	2	−4	4	8	−2	12	44	0
S_{12} $\surd$	4	0	0	0	0	4	0	−12	−8	4	−4	0
S_{13} $\surd$	4	0	0	0	0	−4	0	−12	−8	4	−4	0
S_{14} $\surd$	0	4	0	4	0	0	0	0	0	0	0	0
S_{15} $\surd$	0	4	0	−4	0	0	0	0	0	0	0	0
S_{16}	40	0	0	0	−4	4	−4	8	4	−24	8	0
S_{17}	8	0	0	0	4	0	8	32	−4	−8	40	0
S_{18}	0	0	0	0	2	0	−4	−36	6	24	0	0
S_{19}	0	0	0	0	−2	0	−4	−36	−6	24	0	0
S_{20}	0	0	0	0	2	0	−4	−4	6	−40	−96	0
S_{21}	0	0	0	0	−2	0	−4	−4	−6	−40	−96	0
S_{22}	−48	0	0	0	2	−4	0	−4	6	8	−48	0
S_{23}	−48	0	0	0	0	4	4	16	0	16	0	0
S_{24}	−48	0	0	0	0	0	0	8	0	−16	96	−144
S_{25}	−48	0	0	0	−2	−4	0	−4	−6	8	−48	0
S_{26}	−24	0	4	0	0	0	−8	16	0	−8	−24	0
S_{27}	−24	0	−4	0	0	0	−8	16	0	−8	−24	0
S_{28}	−16	0	4	0	6	0	−4	12	2	8	16	0
S_{29}	−16	0	4	0	2	0	−4	12	−10	8	16	0
S_{30}	−16	0	0	0	−6	0	−4	12	14	8	16	0
S_{31}	−16	0	0	0	−10	0	−4	12	2	8	16	0
S_{32}	−16	0	−4	0	6	0	−4	12	2	8	16	0
S_{33}	−16	0	−4	0	2	0	−4	12	−10	8	16	0
S_{34}	−8	0	0	0	0	0	0	8	16	24	56	0
S_{35}	−8	0	0	0	−8	0	0	8	−8	24	56	0

Table XIX: Same as Table I but for model CHL6, with $\hat{Q} = Q_A - \overline{Q}$ as defined in eq. (98).

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{22} = \langle 2, 7, 8, 25 \rangle$	0
$M_2 = \langle 2, \overline{2} \rangle$	0	$M_{23} = \langle 2, 7, 12, 22 \rangle$	0
$M_3 = \langle 3, \overline{3} \rangle$	0	$M_{24} = \langle 3^2, 26, 27 \rangle$	0
$M_4 = \langle 4, \overline{4} \rangle$	0	$M_{25} = \langle 3, 5, 26, 33 \rangle$	0
$M_5 = \langle 5, \overline{5} \rangle$	0	$M_{26} = \langle 3, 5, 27, 29 \rangle$	0
$M_6 = \langle 6, \overline{6} \rangle$	0	$M_{27} = \langle 3, 6, 26, 32 \rangle$	0
$M_7 = \langle 7, \overline{7} \rangle$	0	$M_{28} = \langle 3, 6, 27, 28 \rangle$	0
$M_8 = \langle 8, \overline{8} \rangle$	0	$M_{29} = \langle 3, \overline{5}, 16, 22 \rangle$	0
$M_9 = \langle 9, \overline{9} \rangle$	0	$M_{30} = \langle 3, \overline{6}, 20, 35 \rangle$	0
$M_{10} = \langle 10, \overline{10} \rangle$	0	$M_{31} = \langle 3, \overline{5}, 21, 34 \rangle$	0
$M_{11} = \langle 11, \overline{11} \rangle$	0	$M_{32} = \langle 3, \overline{5}, \overline{11}, \overline{12} \rangle$	0
$M_{12} = \langle 12, \overline{12} \rangle$	0	$M_{33} = \langle \overline{3}, 5, 17, 19 \rangle$	0
$M_{13} = \langle 13, \overline{13} \rangle$	0	$M_{34} = \langle \overline{3}, 6, 17, 18 \rangle$	0
$M_{14} = \langle 14, \overline{14} \rangle$	0	$M_{35} = \langle 1, 3, \overline{5}, 20, 23 \rangle$	0
$M_{15} = \langle 15, \overline{15} \rangle$	0	$M_{36} = \langle 1, 3, \overline{6}, 21, 23 \rangle$	0
$M_{16} = \langle 1, 4, \overline{7}, 23 \rangle$	0	$M_{37} = \langle 1, 8, \overline{11}, 17, 25 \rangle$	0
$M_{17} = \langle 2, 3, \overline{9}, 22 \rangle$	0	$M_{38} = \langle \overline{1}, 2, 3, 9, 31 \rangle$	0
$M_{18} = \langle 2, 3, \overline{10}, 23 \rangle$	0	$M_{39} = \langle \overline{1}, 2, 3, 13, 30 \rangle$	0
$M_{19} = \langle 2, 3, \overline{13}, 25 \rangle$	0	$M_{40} = \langle 5, \overline{7}, 23, 24, 25 \rangle$	-144
$M_{20} = \langle 2, 5, 10, 25 \rangle$	0	$M_{41} = \langle 4', \overline{4} \rangle$	0
$M_{21} = \langle 2, 6, 9, 23 \rangle$	0	$M_{42} = \langle 4, \overline{4}' \rangle$	0

Table XX: Basis of the moduli space of non-anomalous D-flat directions of model CHL6.

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	$\hat{Q}$
S_1 $\surd$	16	0	0	−2	0	0	8	−4	6	0
$S_2{}^{(\prime)}$ $\surd$	12	2	0	−2	0	4	−4	0	−6	0
$S_3{}^{(\prime)}$ $\surd$	4	2	2	−1	0	4	−8	−6	3	0
$S_4{}^{(\prime)}$ $\surd$	4	2	−2	−1	0	4	−8	−6	3	0
S_5 $\surd$	4	−2	0	4	0	−4	−4	4	0	32
$S_6{}^{(\prime)}$ $\surd^{(\prime)}$	0	4	0	0	0	0	0	0	0	0
S_7 $\surd$	0	0	0	2	4	0	−8	−4	6	32
S_8 $\surd$	0	0	0	2	−4	0	−8	−4	6	0
S_9 $\surd$	8	0	2	1	4	0	−4	2	−3	32
S_{10} $\surd$	8	0	2	1	−4	0	−4	2	−3	0
S_{11} $\surd$	8	0	−2	1	4	0	−4	2	−3	32
S_{12} $\surd$	8	0	−2	1	−4	0	−4	2	−3	0
S_{13}	12	0	0	0	−2	4	12	8	−6	0
S_{14}	12	−2	2	1	−4	−4	8	−2	9	0
S_{15}	12	−2	−2	1	−4	−4	8	−2	9	0
S_{16}	8	2	0	2	0	8	0	−4	−6	0
S_{17}	8	2	0	0	0	8	−8	8	0	32
S_{18}	8	2	0	0	−2	8	0	−4	6	0
S_{19}	8	2	0	−2	2	8	8	0	0	0
S_{20}	8	−2	0	2	0	−8	0	−4	−6	0
S_{21}	8	−2	0	0	0	−8	−8	8	0	32
S_{22}	8	−2	0	0	−2	−8	0	−4	6	0
S_{23}	8	−2	0	−2	2	−8	8	0	0	0
S_{24}	4	0	2	3	2	4	0	−2	9	32
S_{25}	4	0	0	−2	2	−4	−4	4	−12	0
S_{26}	4	0	−2	3	2	4	0	−2	9	32
S_{27}	4	−2	0	6	0	−4	4	−8	−6	0
S_{28}	4	−2	0	2	0	−4	−12	16	6	64
S_{29}	4	−2	0	−2	4	−4	20	0	6	0
S_{30}	0	2	0	4	2	0	8	4	6	32
S_{31}	0	2	0	2	−2	0	16	8	0	0
S_{32}	0	2	0	−2	2	0	−16	−8	0	0
S_{33}	0	2	0	−4	−2	0	−8	−4	−6	−32
S_{34}	−20	2	0	0	4	4	−12	−4	0	0
S_{35}	−20	0	0	2	2	4	4	4	0	0
S_{36}	−12	0	2	3	−2	4	0	6	−3	0
S_{37}	−12	0	2	1	0	4	16	−2	−3	−32
S_{38}	−12	0	2	−1	0	4	8	10	3	0
S_{39}	−12	0	0	0	0	−4	−4	−4	−12	−32
S_{40}	−12	0	0	−2	0	−4	−12	8	−6	0

NA Singlet	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	$\hat{Q}$
S_{41}	-12	0	0	-4	2	-4	4	0	-6	-32
S_{42}	-12	0	-2	3	-2	4	0	6	-3	0
S_{43}	-12	0	-2	1	0	4	16	-2	-3	-32
S_{44}	-12	0	-2	-1	0	4	8	10	3	0
S_{45}	-8	2	0	2	-2	-8	-8	0	0	0
S_{46}	-8	2	0	0	2	-8	0	4	-6	0
S_{47}	-8	2	0	0	0	-8	8	-8	0	-32
S_{48}	-8	2	0	-2	0	-8	0	4	6	0
S_{49}	-8	-2	0	2	-2	8	-8	0	0	0
S_{50}	-8	-2	0	0	2	8	0	4	-6	0
S_{51}	-8	-2	0	0	0	8	8	-8	0	-32
S_{52}	-8	-2	0	-2	0	8	0	4	6	-32
S_{53}	-4	0	2	1	-2	-4	-16	-6	3	0
S_{54}	-4	0	2	-1	0	-4	0	-14	3	-32
S_{55}	-4	0	2	-3	0	-4	-8	-2	9	0
S_{56}	-4	0	0	4	2	4	-4	8	-6	32
S_{57}	-4	0	0	2	4	4	12	0	-6	0
S_{58}	-4	0	0	2	-4	4	12	0	-6	-32
S_{59}	-4	0	0	0	4	4	4	12	0	32
S_{60}	-4	0	0	0	-4	4	4	12	0	0
S_{61}	-4	0	0	-2	-2	4	20	4	0	-32
S_{62}	-4	0	-2	1	-2	-4	-16	-6	3	0
S_{63}	-4	0	-2	-1	0	-4	0	-14	3	-32
S_{64}	-4	0	-2	-3	0	-4	-8	-2	9	0

Table XXI: List of non-Abelian singlets for model CHL7, with $\hat{Q} = Q_A - \overline{Q}$ as defined in eq. (101).

BASIS	$\hat{Q}$		$\hat{Q}$
$M_1 = \langle 1, \overline{1} \rangle$	0	$M_{35} = \langle 8, 35, 41 \rangle$	-32
$M_2 = \langle 2, \overline{2} \rangle$	0	$M_{36} = \langle 9, 58, 64 \rangle$	0
$M_3 = \langle 3, \overline{3} \rangle$	0	$M_{37} = \langle 9, 60, 63 \rangle$	0
$M_4 = \langle 4, \overline{4} \rangle$	0	$M_{38} = \langle 9, 61, 62 \rangle$	0
$M_5 = \langle 5, \overline{5} \rangle$	0	$M_{39} = \langle 10, 26, 41 \rangle$	0
$M_6 = \langle 6, \overline{6} \rangle$	0	$M_{40} = \langle 10, 57, 64 \rangle$	0
$M_7 = \langle 7, \overline{7} \rangle$	0	$M_{41} = \langle 10, 59, 63 \rangle$	0
$M_8 = \langle 8, \overline{8} \rangle$	0	$M_{42} = \langle 11, 53, 61 \rangle$	0
$M_9 = \langle 9, \overline{9} \rangle$	0	$M_{43} = \langle 11, 54, 60 \rangle$	0
$M_{10} = \langle 10, \overline{10} \rangle$	0	$M_{44} = \langle 11, 55, 58 \rangle$	0
$M_{11} = \langle 11, \overline{11} \rangle$	0	$M_{45} = \langle 12, 24, 41 \rangle$	0
$M_{12} = \langle 12, \overline{12} \rangle$	0	$M_{46} = \langle 13, 22, 34 \rangle$	0
$M_{13} = \langle 1, 2, 5 \rangle$	32	$M_{47} = \langle \overline{2}, 13, 32 \rangle$	0
$M_{14} = \langle 1, 9, 12 \rangle$	32	$M_{48} = \langle \overline{6}, 16, 48 \rangle$	0
$M_{15} = \langle 1, 10, 11 \rangle$	32	$M_{49} = \langle \overline{3}, 16, 55 \rangle$	0
$M_{16} = \langle 1, 16, 21 \rangle$	32	$M_{50} = \langle \overline{4}, 16, 64 \rangle$	0
$M_{17} = \langle 1, 17, 20 \rangle$	32	$M_{51} = \langle \overline{6}, 17, 47 \rangle$	0
$M_{18} = \langle 1, 45, 50 \rangle$	0	$M_{52} = \langle \overline{3}, 17, 54 \rangle$	0
$M_{19} = \langle 1, 46, 49 \rangle$	0	$M_{53} = \langle \overline{1}, 18, 23 \rangle$	0
$M_{20} = \langle 2, 9, 15 \rangle$	32	$M_{54} = \langle \overline{2}, 18, 25 \rangle$	0
$M_{21} = \langle 2, 11, 14 \rangle$	32	$M_{55} = \langle \overline{1}, 19, 22 \rangle$	0
$M_{22} = \langle 2, 22, 35 \rangle$	0	$M_{56} = \langle \overline{3}, 19, 53 \rangle$	0
$M_{23} = \langle 3, 20, 44 \rangle$	0	$M_{57} = \langle \overline{5}, 22, 56 \rangle$	0
$M_{24} = \langle 3, 21, 43 \rangle$	0	$M_{58} = \langle \overline{3}, 24, 33 \rangle$	0
$M_{25} = \langle 3, 23, 42 \rangle$	0	$M_{59} = \langle \overline{6}, 30, 33 \rangle$	0
$M_{26} = \langle 4, 20, 38 \rangle$	0	$M_{60} = \langle \overline{6}, 31, 32 \rangle$	0
$M_{27} = \langle 4, 21, 37 \rangle$	0	$M_{61} = \langle \overline{2}, 33, 35 \rangle$	-32
$M_{28} = \langle 4, 23, 36 \rangle$	0	$M_{62} = \langle \overline{7}, 34, 66 \rangle$	-32
$M_{29} = \langle 5, 18, 41 \rangle$	0	$M_{63} = \langle 1, 7, 39, 60 \rangle$	0
$M_{30} = \langle 5, 32, 61 \rangle$	0	$M_{64} = \langle 13, 24, 39, 64 \rangle$	0
$M_{31} = \langle 6, 20, 52 \rangle$	0	$M_{65} = \langle 2, 8, 29, 65 \rangle$	0
$M_{32} = \langle 6, 21, 51 \rangle$	0	$M_{66} = \langle 2, 27, 38, 64 \rangle$	0
$M_{33} = \langle 6, 22, 50 \rangle$	0	$M_{67} = \langle 2, 28, 37 \rangle$	0
$M_{34} = \langle 6, 23, 49 \rangle$	0	$M_{68} = \langle 64, 39, 24, 9 \rangle$	0

Table XXII: Basis of the moduli space of non-anomalous D-flat directions of model CHL7.