Deformation of the N=Z nucleus ^{76}Sr using β-decay studies

E. Nácher,* A. Algora,† B. Rubio, J.L. Taín, and D. Cano-Ott‡
Instituto de Física Corpuscular, CSIC-Univ. de Valencia, E-46071 Valencia,Spain

S. Courtin, Ph. Dessagne, F. Maréchal, Ch. Miehé, and E. Poirier
Institut de Recherches Subatomiques, IN_2P_3-CNRS, F-67037 Strasbourg Cedex 2, France

M.J.G. Borge, D. Escrig, A. Jungclaus,§ P. Sarriguren, and O. Tengblad
Instituto de Estructura de la Materia, CSIC, E-28040 Madrid, Spain

W. Gelletly
Department of Physics, University of Surrey, Guildford, GU2 5XH, United Kingdom

L.M. Fraile¶ and G. Le Scornet
ISOLDE, EP Division, CERN, CH-1211 Geneva, Switzerland
(Dated: 4th March 2004)

A novel method of deducing the deformation of the N=Z nucleus ^{76}Sr is presented. It is based on the comparison of the experimental Gamow-Teller strength distribution B(GT) from its β-decay with the results of QRPA calculations. This method confirms previous indications of the strong prolate deformation of this nucleus in a totally independent way. The measurement has been carried out with a large Total Absorption gamma Spectrometer, "Lucrecia", newly installed at CERN-ISOLDE.

PACS numbers: 21.10.Pc, 23.40.-s, 23.40.Hc, 27.50.+e, 29.30.Kv, 29.40.Mc

The shape of the atomic nucleus is conceptually one of the simplest of its macroscopic properties to visualise. However, it turns out to be one of the more difficult properties to measure. In general terms we now have a picture of how the nuclear shape varies across the Segrè Chart. Nuclei near to the closed shells are spherical. In contrast nuclei with the valence nucleons in between two shells have deformed shapes with axial symmetry and the extent of the quadrupole deformation is quite well described as being proportional to the product N_pN_n of the numbers of pairs of valence protons (N_p) and neutrons (N_n) [1]. This picture is underpinned by both the Shell and Mean Field models of nuclear structure. Experiment and theory concur that, as the N_pN_n parameterisation would suggest, nuclei rapidly deform as we add only a small number of valence nucleons to the magic numbers. Thus nuclei in the middle of the $f_{7/2}$ shell turn out to be deformed even although the numbers of valence nucleons are relatively small.

Experimentally this picture is supported by a mass of independent observations: the strongly enhanced quadrupole transition rates between low-lying states, the strongly developed rotational bands built on low-lying states, and measurements of ground state quadrupole moments. Where we have evidence of the shapes of ground and excited states in the same nucleus they are, in general but not always, the same. It turns out that in some cases nuclear states with different shapes co-exist in the same nucleus [2].

The nuclei with N≈Z and A≈70-80 are of particular interest in this context. Such nuclei enjoy a particular symmetry since the neutrons and protons are filling the same orbits. This, and the low single-particle level density, lead to rapid changes in deformation with the addition or subtraction of only a few nucleons. In terms of Mean Field models these rapid changes arise because of the proximity in energy of large energy gaps for protons and neutrons at Z,N=34 and 36 on the oblate and Z,N=38 on the prolate side of the Nilsson diagram. As a result Mean Field calculations predict the existence of several energy minima with quite different shapes in some of these nuclei [3, 4]. Evidence of this co-existence has been found for instance in Se and Kr nuclei [5, 6], and it is also predicted for the lightest Sr isotopes [7]. Thus it is of considerable interest to map out the deformation of the ground and excited states of nuclei in this region. This is easier said than done, however. There are a number of methods to measure the deformation of the ground state in unstable nuclei based on the interaction of the electric quadrupole moment of the nucleus with an external electric field gradient [8, 9]. These techniques are not applicable to nuclei with J=0 or 1/2, moreover they very seldom give the sign of the quadrupole moment and hence cannot distinguish between oblate and prolate shapes.

Here we present an alternative method to deduce whether the ground state shape of an unstable nucleus is oblate or prolate, and apply it to the N=Z=38 nucleus ^{76}Sr, the most deformed nucleus in the region according to Mean Field calculations [4] and previous in-beam experiments [10]. The method is based on an accurate measurement of the Gamow-Teller strength distribution, B(GT), as a function of excitation energy in the daughter nucleus, and relies on the technique of Total Absorption gamma Spectroscopy (TAgS) which will be explained later. The theoretical idea was suggested by Hamamoto *et al.* [11] and pursued by Sarriguren *et al.* [12]. According to them, one can study the deformation of the ground state of a particular nucleus by measuring the B(GT) distribution of its β-decay. In these references the authors calculate the B(GT) distributions for various nuclei in the region for the deformations minimizing the ground state energy. In some cases, the results differ markedly with the shape of the ground state of the parent, especially for the light Kr and Sr isotopes.

A precise determination of the B(GT) distribution is required for such studies and this is far from trivial. Traditional high resolution techniques, based on the use of high purity Germanium (HPGe) detectors to measure the γ-rays emitted after the β-decay, often fail to detect significant but very fragmented strength at high excitation energy in the daughter nucleus. This is mainly due to three factors: the low photo-peak efficiency of HPGe detectors for high energy γ-rays, the high fragmentation of the B(GT) at high excitation energy, and the fragmentation of the gamma de-excitation of the levels in the daughter through many different gamma cascades. Together they cause the so-called *Pandemonium effect* [13]: many weak cascades de-exciting levels at high energy can remain undetected leading to large systematic errors in the determination of the B(GT). This is the reason why, even although Refs. [14] and [15] give the first indication of the prolate character of the ^{76}Sr ground state, one must determine the B(GT) distribution more accurately over the whole Q_{EC} window to provide conclusive proof.

The alternative, the Total Absorption gamma Spectroscopy technique, avoids these systematic errors. The basis of this method is the detection of the entire energy of the gamma cascades rather than individual γ-rays. For this purpose one needs a high efficiency detector with acceptable resolution for gammas such as the inorganic scintillators NaI(Tl) or BaF_2. Furthermore, this detector must have a geometry as close as possible to 4π to absorb the complete cascade energy. If this is achieved one can measure directly the β intensity I_β(E) as a function of the excitation energy in the daughter nucleus, and from this one can extract the B(GT) distribution.

In this paper we present the results of TAgS measurements on the decay of the N=Z nucleus ^{76}Sr. A comparison of the results of this measurement with the calculations of Ref. [12] allows us to establish the prolate character of the ground state of ^{76}Sr without ambiguity.

With the aim of measuring the β-decay of nuclei far away from the stability line with the Total Absorption technique, a spectrometer called "Lucrecia" has been installed at the ISOLDE mass separator at CERN. It consists of a large NaI(Tl) crystal of cylindrical shape (L=∅=38 cm) with a cylindrical hole (∅=7.5 cm) at right angles to the symmetry axis. The purpose of the hole is twofold: on the one hand it allows the beam pipe (coming from the separator) to enter up to the centre of the crystal, thus allowing on-line activity of very short half-life (>5 ms) to be deposited here and measured. On the other hand it allows us to place ancillary detectors inside for the detection of the positrons (β^+-decay), electrons (β^--decay) or X-rays (EC process) produced in the decay. Surrounding the whole setup there is a shielding box 19 cm thick made of four layers: polyethylene-lead-copper-aluminium. In Figure 1 a schematic view of the detector setup placed inside the shielding box is shown (see Ref. [16] for further details).

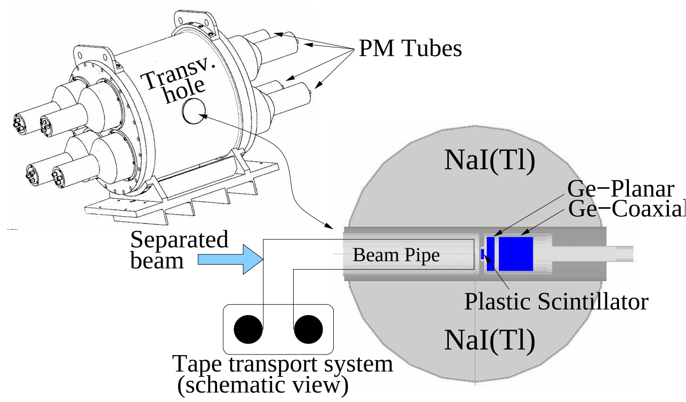

Figure 1: Detector setup for the ^{76}Sr measurement. In the upper-left part we have a 3D plot of the cylindrical NaI(Tl) detector, and in the lower part we see a transverse cut through the main crystal and the ancillary detectors, as well as the tape where the separated ^{76}Sr beam is implanted.

In order to produce the nucleus of interest (^{76}Sr), a 52 g/cm^2 Nb target was bombarded with a 1.4 GeV proton beam. The intensity was chosen to produce a counting rate of ≈3.5 kHz in the NaI crystal. In order to separate Sr selectively, a fluorination technique was used [17]. The radioactive beam was steered to the detector setup and implanted in an aluminised mylar tape which was moved every 15 seconds to transport the source to the middle of the crystal and to avoid the buildup of the daughter activity ($T_{1/2}$(^{76}Sr)=8.9 s, $T_{1/2}$(^{76}Rb)=36.8 s). During this 15 s cycle the decay of the implanted radioactive source was measured. The γ-rays following the decay (either by β^+ or by EC) were measured by the NaI(Tl) crystal and analysed without any condition on the ancillary detectors. However these detectors were very useful for the on-line control of the measurement.

In ideal conditions, if the TAgS had 100% peak efficiency over the whole energy range, the experimental spectrum measured in the NaI(Tl) cylinder would be the β intensity distribution I_β(E) convoluted with the energy resolution of the crystal and the response of the detector to the positron when applicable. In reality the detector does not have 100% peak efficiency because of the dead material inside the spectrometer (the ancillary detectors) and the transverse hole. Consequently the spectrum is modified by the response function of the detector. In other words, the relationship between the quantity of interest, $I_\beta(E)$, and the experimental data d(i) is:

$$d(i) = \sum_{j=1}^{j_{max}} R(i,j)\, I_\beta(j) \qquad \begin{pmatrix} i \equiv channel \\ j \equiv energy\ bin \end{pmatrix} \qquad (1)$$

In order to obtain $I_\beta(E)$ from our data we should solve Eq. (1). This is not a trivial task because the response matrix $R(i,j)$ can not be inverted due to the fact that it is quasi-singular in the sense that two neighbouring columns are very similar. However, there is a set of algorithms that has been developed to solve this kind of "Ill Posed" problem. In Ref. [18] there is a systematic study of three of these methods applied to the specific problem of the TAgS data. Here we have used the Expectation Maximization algorithm [19] to obtain the $I_\beta(E)$ by unfolding the experimental data. To calculate the response matrix $R(i,j)$, which is needed by the algorithm, we have used the levels and branching ratios given in Ref. [15], and the *GEANT4* simulation code. The analysis has been performed taking into account both the EC and β^+ components of the decay. A more detailed explanation on the procedure to calculate $R(i,j)$ and to analyse the data will be given in a forthcoming article.

The best check one can perform to validate the result of the analysis is to recalculate the experimental spectrum by multiplying the response function of the detector ($R(i,j)$ in Eq. (1)) by the resulting beta intensity I_β. If the analysis is properly done, this recalculated spectrum should be very similar to the real experimental spectrum. The upper part of Fig. 2 shows the experimental spectrum (shade without line) overlaid with the recalculated one (dashed line). The agreement between the two spectra is very good.

The $I_\beta(E)$ is the experimental result of this work, however, the physical information is carried by the reduced transition probability B(GT), which can be extracted from the $I_\beta(E)$ using the expresion:

$$B(GT) = \frac{I_\beta(E)}{f(Q_{EC}-E)T_{1/2}} \times 6147 \left(\frac{g_V}{g_A}\right)^2 \qquad (2)$$

where the B(GT) is averaged inside the 40 keV energy bin, and $f(Q_{EC}-E)$ is the Fermi integral which carries the information on both the phase space available in the final state and the Coulomb interaction. For the calculation of the B(GT) we have used the Q_{EC} value from Ref. [20, 21], the $T_{1/2}$ from [15] and the tabulated Fermi integral from [22].

In the lower panel of Fig. 2 the resulting B(GT) distribution is presented. The analysis gives a total B(GT) of $3.8(6) g_A^2/4\pi$ up to 5.6 MeV of which 57% is located in the resonance between 4 and 5 MeV. It should be noted that the β-delayed proton emission (S_p=3.5 MeV) in this decay has been observed at excitation energies from 4.8 to 5.8 MeV [14, 15]. However, the contribution of this component is very small, of the order of 2% in B(GT) compared to the decay through the β-delayed γ-rays studied here.

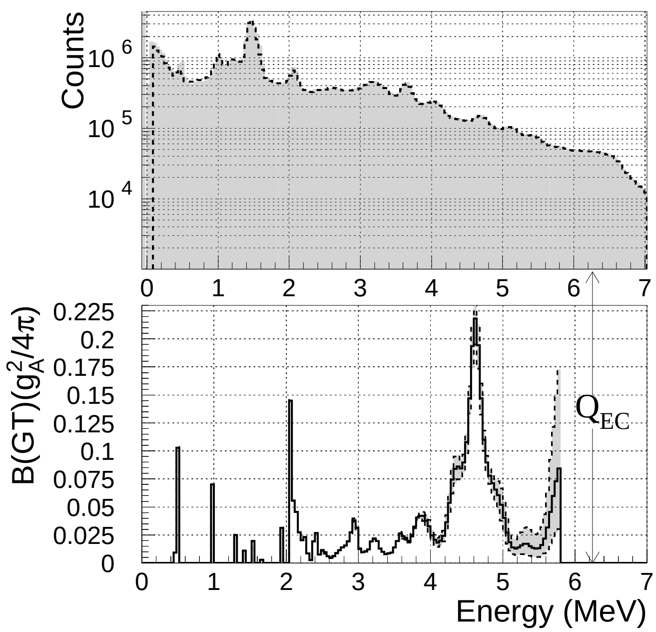

Figure 2: Upper panel: Experimental total absorption spectrum of the β-decay of ^{76}Sr overlaid with the recalculated spectrum after the analysis (see text). Lower panel: B(GT) distribution derived from the experimental data shown above. The shade represents the experimental uncertainty.

One way to compare the results with the theory is to accumulate in each energy bin the sum of the B(GT) measured up to that particular energy. Fig. 3 shows this plot in which the experimental result is compared with the theoretical calculations of Ref. [12] for both prolate and oblate shapes for the ground state of ^{76}Sr. The generally accepted quenching factor of 0.6 has been applied to the calculations.

In Ref. [12] the authors first construct the quasiparticle basis self-consistently from a deformed Hartree-Fock (HF) calculation with density-dependent Skyrme forces and pairing correlations in the BCS framework. The minima in the total HF energy vs deformation parameter plot give the possible deformations of the ground state. For the case of ^{76}Sr two minima are found, one prolate with β_2=0.41 and the other oblate with β_2=-0.13. Finally, the QRPA equations are solved with a separable residual interaction derived from the same Skyrme force used in the HF calculation. Fig. 3 shows the results using the residual interaction SK3. The agreement of the experimental results of this work (squares in Fig. 3) with the prolate shape calculation

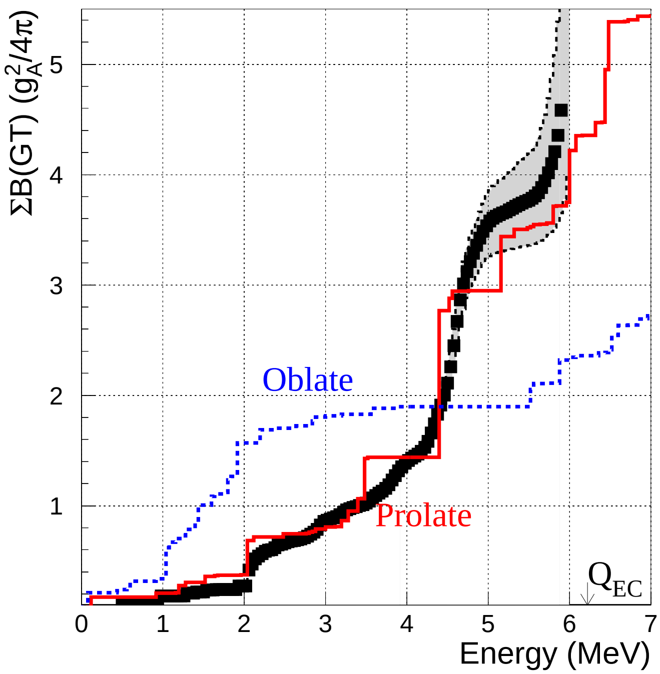

Figure 3: Accumulated B(GT) as a function of the excitation energy in the daughter nucleus. The experimental results from this work (squares) are compared with the theoretical calculations [12] assuming prolate (solid line) and oblate (dashed line) shapes for the ^{76}Sr ground state. The shade indicates the experimental uncertainty.

of [12] is very good over the energy range 0-5.6 MeV. In contrast, there is no similarity between the results of the oblate calculation and the experimental points.

This agrees with the strong deformation ($\beta_2 \approx 0.4$) of ^{76}Sr already extracted from the dynamical properties observed in in-beam studies [10]. It also gives the first definitive experimental evidence of the prolate character of the ground state deformation, confirming the result indicated in [14] and [15].

We should point out here that these results prove the validity of the method of deducing the sign of the electric quadrupole moment of ground states or β-decay isomers from the study of their decay. This opens new opportunities in the study of nuclei far from the stability line where very often the first information comes from β-decay (half-life, J^π...). On the other hand the theoretical approach used in the present study has been so far restricted to nuclei in this region. The present work should encourage further theoretical studies in other regions of well deformed nuclei.

Finally, as a part of this series of experiments, we have recently published the results on ^{74}Kr decay [16]. In that paper a clear indication of shape mixing was deduced, a conclusion which is further corroborated by the present study which is free of shape admixtures.

Summarising, in this work we present the results of an experiment devoted to measuring the B(GT) distribution in the decay of the N=Z isotope ^{76}Sr. When we compare our experimental results with the theoretical calculations of Ref. [12] we conclude that the ground state of ^{76}Sr is strongly prolate ($\beta_2 \approx 0.4$), in agreement with theoretical predictions [4, 7] and with previous experimental indications [10, 14, 15]. An important consequence of the present work is the validation of the method of deducing the deformation, including the sign of the quadrupole moment, from the comparison of the β-decay TAgS results and the calculated B(GT) since the ^{76}Sr ground state is a very clean case, free of shape admixtures.

The authors feel especially grateful to J.C. Caspar, J. Devin, G. Heitz and C. Weber for all their invaluable work on the mechanical mounting of the spectrometer and the set-up of the data acquisition system. This work has been partially suported by the IN_2P_3/CNRS (France), CICYT-MCyT (Spain), EPSRC (UK), the European Comission and the DFG (Germany).

* Electronic address: Enrique.Nacher@ific.uv.es
† MTA ATOMKI, H-4026 Debrecen, Hungary
‡ CIEMAT, E-28040 Madrid, Spain
§ Univ. Autonoma de Madrid, E-28049 Madrid, Spain
¶ Univ. Complutense de Madrid, E-28040 Madrid, Spain

[1] R. Casten, Nucl. Phys. **A443**, 1 (1985).
[2] e.g. A.N. Andreyev *et al*, Nature **405**, 430 (2000).
[3] W. Nazarewicz *et al*, Nucl. Phys. **A435**, 397 (1985).
[4] P. Bonche *et al*, Nucl. Phys. **A443**, 39 (1985).
[5] J.H. Hamilton *et al*, Phys. Rev. Lett **32**, 239 (1974).
[6] C. Chandler *et al*, Phys. Rev. **C56**, R2924 (1997).
[7] A. Petrovici *et al*, Nucl. Phys. **A605**, 290 (1996).
[8] E. Davni *et al*, Phys. Rev. Lett. **50**, 1652 (1983).
[9] F. Hardeman *et al*, Phys. Rev. **C43**, 130 (1991).
[10] C.J. Lister *et al*, Phys. Rev. **C42**, R1191 (1990).
[11] I. Hamamoto *et al*, Z. Phys. **A353**, 145 (1995).
[12] P. Sarriguren *et al*, Nucl. Phys. **A691**, 631 (2001).
[13] J.C. Hardy *et al*, Phys. Lett. **71B**, 307 (1977).
[14] C. Miehé *et al*, in New Facet of Spin Giant Resonances in Nuclei, ed. by H. Sakai *et al*, 140 (1997).
[15] Ph. Dessagne *et al*, accepted in Eur. Phys. J. **A**, (2004).
[16] E. Poirier *et al*, accepted in Phys. Rev. **C**, (2004).
[17] H.L. Ravn *et al*, Nucl. Instr. and Meth. **123**, 131 (1975).
[18] D. Cano Ott, Ph.D. thesis, Univ. de Valencia (2000).
[19] A.P. Dempster *et al*, J. R. Statist. Soc. **B39**, 1 (1977).
[20] G. Audi *et al*, Nucl. Phys **A729**, 358 (2003).
[21] F. Herfurth *et al*, Proc. of the "6th Int. Conf. on Radioactive Nuclear Beams", Argonne, Sep. 2003, to be published in Nucl. Phys. **A** (2004).
[22] N.B. Gove and M.J. Martin, Nuclear Data Tables **A10**, 246 (1971).