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COMMENTS ON THE MULTIPERIPHERAL PRODUCTION OF CLUSTERS

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ABSTRACT

A scheme of multiperipheral production of clusters of mass  $\sim 1.5$  GeV and  $\langle P_T \rangle \simeq 850$  MeV is shown to be consistent with the phenomenological information available at present. Assorted comments on the implications, advantages, and shortcomings of such a scheme are also made.

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Recent comprehensive data on large-angle particle production at the ISR<sup>1)</sup> show that the central plateau-like region for the production of  $\pi$ , K, and  $\bar{p}$  has a non-negligible energy dependence also in this energy range  $\{[1/\sigma_{\text{tot}}]/[d\sigma/dy]$  ( $y = 0$ ,  $p_T \approx \langle p_T \rangle$ ) increases by about 30% for  $\pi^-$  production, when  $\sqrt{s}$  is increased from 23 GeV to 63 GeV see Fig. 1 for a typical set of data}.

If these data are plotted against  $s^{-1/2}$  as in Fig. 1 (or against  $s^{-1/4}$  as done in Ref. 1), it is apparent that a parametrization of the form  $f(p_T)[1 - \Delta(s/s_0)^{-p}]$  is quite acceptable, and therefore asymptotically scaling (and logarithmic multiplicity) can be recovered. The problem is to establish what are the implications of the observed size of  $\Delta$  ( $\Delta \sim 8$ , with  $p = 1/2$ ,  $s_0 = 1 \text{ GeV}^2$  for the straight line of Fig. 1) and, in particular, whether this effect is quantitatively acceptable in a multiperipheral (MP) picture.

We will consider here a simple MP ladder model for particle production, possibly with  $\alpha_p > 1$ . Terms that arise from Pomeron-Pomeron interactions are known to be numerically small and to cancel to a large extent in the one particle spectra<sup>2)</sup>. The main source of energy dependence for particle production in a ladder model is given by the opening up of cylindrical phase space<sup>3-5)</sup>. This effect is controlled by the parameter that limits the invariant momentum transfers  $t_i$  along the chain and, in the Muller language, it can be translated in terms of non-leading singularities<sup>3)</sup> (spaced by integers, as long as the propagators are analytic) and of complex poles<sup>5)</sup>. I believe that the former effect is quantitatively the relevant one here, and I will neglect the latter. (Undoubtedly, a more refined analysis should include both.)

A numerical evaluation has allowed me to check that this effect can indeed be represented with sufficient accuracy by the analytical form

$$\frac{1}{\sigma_T} \frac{d\sigma}{dy} (y=0, s, p_T) = f(p_T) \left( 1 - \frac{m^4}{\langle p_T \rangle^2 s_0} \left( \frac{s}{s_0} \right)^{-1/2} \right) \quad (1)$$

suggested in Ref. 3, for the range of variables and parameters in which I will be interested in this note. In Eq. (1)  $m$  is the mass of the produced particle and  $p_T$  its transverse momentum. The form (1) stems from the fact that a single parameter dampens in multiperipheralism the production of heavy masses (via the energy and longitudinal components of the momentum transfer) and the transverse momenta, and holds provided that  $m$  is larger than the masses of the projectile and beam (see Ref. 3 for a more detailed form).

If we were to use a model with direct  $\pi$  emission, the energy dependence thus obtained would be too small by one order of magnitude, as shown in Fig. 3 of Ref. 3, and the interpretation of the data of Ref. 1 would indeed constitute a problem.

But we know already that a multiperipheral model (MPM) with direct  $\pi$  emission is unattractive for a number of reasons.

i) Such models almost inevitably lead to a negative correlation between the average number of  $\pi^0$  and the number of prongs<sup>6)</sup>, in clear contradiction to experiment<sup>7)</sup>.

ii) The dependence of the size of the short-range correlations on the number of prongs in semi-inclusive experiments<sup>8)</sup> favours the interpretation of the short-range correlations in terms of weakly correlated clusters, rather than, for example, non-leading Muller terms<sup>9)</sup>. Also the shape of the inclusive short-range correlations seems to suggest the isotropic decay of massive clusters<sup>10)</sup>.

iii) The wide numerical discrepancy between the value of  $\langle p_T \rangle^2 \approx 0.1 \text{ GeV}^2$  and  $\langle t \rangle \geq 0.8 \text{ GeV}^2$  [obtained assuming the strong ordering<sup>11)</sup> that presumably leads to an underestimate]. Strictly speaking, this fact is not a real difficulty for MP, but only a very unattractive feature. This means that the cut-off in  $p_T$  is not directly operated by the cut-off in the transverse part of the momentum transfer, but only indirectly via the cut-off in the longitudinal part of the same<sup>12)</sup>, and it becomes hard to tell the difference between MP, and, for example, the uncorrelated jet model<sup>13)</sup>. In particular, all our MP intuition based essentially on the features of the weak-coupling limit (random walk, etc.) would badly fail, and this has led to a series of incorrect statements in the past<sup>14)</sup>.

iv) An additional mysterious feature in models with direct pion emission is the exponential behaviour of the  $p_T$  distribution down to very small values of  $p_T$ <sup>1)</sup>.

If we want to save MP dynamics we are therefore led to consider the MP production of clusters, and it is logical to examine the extreme possibility in which clustering is dominantly the origin of the short-range correlations. In a sense this is a step backwards, because it means essentially giving up one of the successes of the MPM, i.e. the prediction of short-range correlations! But it is a price worth paying, because it immediately solves the difficulties (i) and (ii) and, as we will see later, to a large extent also (iii) and (iv).

From (ii) we know that the favourite parameters for the clusters are a mass  $M_C \approx 1.5 \text{ GeV}$  (for mesonic clusters) and an average decay multiplicity of 3-4 particles [with small dispersion around the average<sup>9,10)</sup>].

Assuming  $M_C = 1.5 \text{ GeV}$ , the only parameter that controls the size of the energy dependence is the transverse momentum of the clusters  $\langle P_T \rangle$ . The line in Fig. 1 is obtained taking  $d\sigma/dP_T^2 = c e^{-aP_T^2}$ , with  $a = 1.1 \text{ GeV}^{-2}$ , corresponding to  $\langle P_T \rangle = \frac{1}{2} \sqrt{\pi/a} \approx 850 \text{ MeV}$ . This value might seem surprisingly large, but is in fact in very good agreement with the known  $p_T$  distribution of the  $\pi$  and with what is known on azimuthal correlations. To check these points I have computed the

$p_T$  distribution of the pions decaying from a cluster of  $M_C = 1.5$  according to four-body phase space, and obtained the points shown in Fig. 2. We can see that the value of  $\langle p_T \rangle$  of the order of 850 MeV does reproduce very well the phenomenological  $e^{-6p_T}$  distribution, and that a smaller value of  $\langle p_T \rangle$ , like, for example, 700 MeV, already produces too steep a behaviour of the type  $e^{-7.5p_T}$ . The value of  $\langle p_T \rangle$  also affects the nature of the short-range azimuthal correlations (SRAC). It is obvious that in the two limits  $p_T \lesssim \langle p_T \rangle$  and  $p_T \gg \langle p_T \rangle$  one should see strong SRAC, respectively negative and positive.

The available data<sup>15)</sup> show us an important dependence of the SRAC on the charge of the two particles, that is presumably due to the Bose-Goldhaber effect. Considering here for simplicity the sum over the different charge combinations, the asymmetry parameter  $A = N(\Delta\phi > \pi/2) - N(\Delta\phi < \pi/2) / N_{\text{tot}}$  seems to be constant around a value  $\approx 0.07$  (for non-diffractive events) at  $\Delta y \gtrsim 1$ , and to decrease to about 0.03 at small  $\Delta y$ .

Let me try to interpret these data in our scheme. Since I propose a MP cluster production, the azimuthal correlations are, technically speaking, of short-range<sup>16)</sup>, and we expect that the transverse momentum of a cluster is balanced by the next, and maybe the next but one neighbour. Unfortunately, this means that typically 3 to 5 clusters (i.e. 12-20 particles) are involved in the process, faking in practice long-range negative correlations even at ISR energies. To have a rough estimate of the SRAC expected at small  $\Delta y$ , we have to assess the probability that a pair of particles found at small  $\Delta y$  actually come from the same cluster. A simple counting argument suggests that this probability should not be far from one half, and therefore that the asymmetry parameter for particles that decay from the same cluster should be around zero. In Fig. 3 I show the dependence of A on  $\langle p_T \rangle$  in the simple phase-space decay model used to obtain Fig. 2, and we see again that a value  $\langle p_T \rangle \approx 0.8-0.9$  GeV is favoured.

Let me see now how this picture accommodates the well-known difficulties with the value of the slope and shrinkage of the diffraction peak. Since we are assuming that clusters decay isotropically, we can consider directly the overlap function for cluster production.

What is the relation between  $\langle p_T \rangle^2$  and  $\langle t \rangle$  in this model? Consider Fig. 4 where a portion of a ladder cluster production amplitude is represented. The minimum value of  $t_{\text{exch}}$  is given by

$$|t_{\text{exch}}| = (|t_{\text{inc}}| + M_C^2) \quad M^2 / s$$

and  $M^2/s$  can be determined in terms of the cluster decay multiplicity  $n_C$  and of the coefficient of  $\ln s$  in  $\langle n_\pi \rangle$ . Assuming  $\langle n_\pi \rangle \approx 3 \ln s$ ,  $n_C = 4$ , I obtain  $s/M^2 \approx e^{4/3} \approx 4$ ,

and assuming  $t_{\text{exch}} \approx t_{\text{inc}}$ ,  $t_{\text{exch}} \approx M_C^2/3 \approx 0.75 \text{ GeV}^2$ . Hence for cluster production with the afore-mentioned parameters we are in a situation in which  $\langle P_T \rangle^2 \approx \langle t_{\text{min}} \rangle$  and hopefully the weak-coupling limit ideas like random walk are not too wrong in this regime. Assuming that indeed one can use this approach,  $\alpha'_P$  can be calculated according to Henyey's approach<sup>14)</sup>, with two important differences:

- i) The number of steps in the random walk is given by  $\bar{n}_C \approx \bar{n}_\pi/4$ .
- ii) The average length of one step is less than one half of the one used by Henyey.

In conclusion,  $\alpha'_P$  is reduced by a factor of about 20 from the estimate of Ref. 14, and yields the very small value  $\alpha'_P \approx 0.15$ , quite compatible with the data [remember that some shrinkage can be provided by s-channel saturation at small  $b$ , if  $\alpha_P(0) > 1$ <sup>17)</sup>].

Two problems remain to be solved: to explain the size of the slope of the diffraction peak and to understand why  $\alpha'_M \gg \alpha'_P$ . With reference to the former, let me remark that with MPM the proportionality between  $\bar{n}$  and  $A = -(d/dt) \ln d\sigma/dt|_{t=0}$  can be broken if the off-shell internal couplings have a very different  $t$ -dependence from the external on-shell ones. A large value of  $A$  and a small shrinkage can be obtained if the couplings of the exchanged objects to the external particles are very steep in  $t$ . This prescription might sound arbitrary, but it becomes more natural in impact parameter space. In fact, the incoming particles have a large geometrical size of about 1 fm, that has to reflect itself in a sharp diffraction peak. To this geometrical size the (quasi)-random walk of multiperipheralism adds a comparatively small contribution. The real question is in fact why is the random walk step so short, or alternatively what sets the large scale of  $\langle P_T \rangle$ , the answer being presumable the cluster mass itself.

The question why is  $\alpha'_M$  much larger than  $\alpha'_P$  I cannot answer this framework. Taking for granted that the average multiplicity in states that build up the overlap function for quantum number exchange is smaller than the corresponding one for  $P$  exchange (an assumption that is open to challenge, but has not been challenge yet), such an effect could be obtained as a feature of the strong coupling limit in MP, provided that the assumed  $t$ -dependence of the propagators is not, for example, Gaussian<sup>18)\*</sup>. However on the one hand I feel uncomfortable about the size of the effect and, on the other hand, we have introduced clusters just to recover the basic features of the weak-coupling limit.

Alternatively one might argue that a different dynamical mechanism like for instance  $\pi$  exchange with large impact parameter steps is dominant in quantum number

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\*) In the strong-coupling limit  $\langle t \rangle \gg \langle p_T \rangle^2$ , and increasing  $\bar{n}$  increases  $\langle t \rangle$ . If the propagator is, for example, a function of  $\sqrt{t_{\text{min}} + q_T^2}$  not of Gaussian type (an exponential, or an inverse power would do) increasing  $t_{\text{min}}$  allows the overlap function to absorb larger values of  $q_T^2$  without losing too much.

exchange processes, and that cluster formation is suppressed or absent in the states that build up non-vacuum exchange. Since in this framework clusters are the origin of positive short-range correlations, one could find a confirmation of this hypothesis in the observation that in  $p\bar{p}$  annihilation the correlation parameter  $f_2$  is negative also for rather large values of  $s$ <sup>19)</sup>. In spite of the fact that alternative explanations can be suggested for this effect<sup>\*)</sup>. I would be happy to know whether there are positive short-range correlations, for example, in  $\pi^-p \rightarrow$  all neutrals. Notice that this line of thought essentially gives up hope of the MP bootstrap programme, and also the relation between the asymptotic value of  $\bar{n}/\ln s$  and the value of the meson trajectory intercept<sup>22)</sup>. Personally I believe that the decrease with energy of a partial cross-section  $\sigma_n$  for  $s$  such that  $\bar{n}(s) > n$  is dictated rather by the saturation of s-channel unitarity than by t-channel dynamics<sup>\*\*)</sup>.

So far I have considered only the dominant  $\pi$  production. For K and  $\bar{p}$  production the increase in the yield over the ISR energy range observed in Ref. 1 is, respectively a factor of about 1.8 and a factor of about 2.5. Furthermore, the  $\bar{p}/\pi^-$  ratio at  $\sqrt{s} = 63$  is of the order of 8%. The suppression and the steep energy dependence of  $\bar{p}$  production are easily reproduced assuming that the  $\bar{p}$  come from the decay of somewhat heavier clusters. I find for these baryonic clusters the values  $M_B \approx 2.7$  GeV and  $\langle P_T^B \rangle \approx 1 - 1.2$  GeV. (For K production the relevant values are intermediate between the two sets.) On the basis of these estimates one can predict two characteristic features.

i) There should be a strong positive short-range azimuthal correlation between  $p$  and  $\bar{p}$ , since the rather large value of  $P_T^B$  is mainly inherited by the heavy particles, and it should, on the average, overcome the small Q value of the decay.

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\*) The fact that  $f_2$  is positive, for example, in pp interactions at sufficiently large  $s$ , means that the distribution of  $\sigma_n$  is broader than a Poissonian. In  $p\bar{p}$  this might not happen because of one of the following two reasons:

i) It could be that there is a strong correlation between the value of the mass of the mesonic system and its multiplicity. In normal pp collision the average value of this mass is about  $\sqrt{s}/2$ , but with very large dispersion, whereas in  $p\bar{p}$  annihilation it is exactly  $\sqrt{s}$ . It would be very interesting to measure the final two nucleons in pp collision and to determine the multiplicity distribution (and the relative  $f_2$ ) for fixed values of  $M^2 \approx (1 - |x_1|)(1 - |x_2|)s$ ,  $x_1$  and  $x_2$  being the Feynman parameters of the two nucleons.

ii) Alternatively it has often been argued that there could be a strong correlation between the multiplicity and the impact parameter at which a certain reaction happens<sup>20)</sup>. Taking the attitude that annihilation processes are mainly responsible in building the difference of the overlap functions of  $p\bar{p}$  and pp scattering, one would reach the conclusion that  $p\bar{p}$  annihilation is very peripheral and takes place at a well-defined value of  $b \approx 0.7$  fm<sup>21)</sup>, and this could explain the narrowness of the multiplicity distribution.

\*\*\*) I am inclined to think this way especially in view of the results of the analysis of inelastic diffraction<sup>23)</sup>. There it was found that s-channel unitarity is indeed almost saturated for a large range  $b \lesssim 0.7$  fm. How this observation is compatible with the t-channel dynamics that is basic to multiperipheralism, I cannot say.

ii) One expects, on the basis of its mass, that a baryonic cluster decays dominantly into  $N\bar{N}$  plus one or two  $\pi$ . Consequently, the presence of a  $\bar{p}$  in an event should decrease the average multiplicity of the produced  $\pi$ , in contrast with the naïve expectation based on a model of completely uncorrelated clusters<sup>24)</sup>.

The feature of the present scheme that I would like to stress as a concluding remark is that little fundamental importance should be attributed to the value  $\langle p_T \rangle \approx 300$  MeV, and that if a basic unit of length is to be chosen, it is more likely to be related to the scale of  $\sim 0.8 - 1$  GeV. I also find appealing the fact that exactly this value of  $\langle p_T \rangle$  is found in  $p\bar{p}$  annihilation events, in which cluster formation is presumably negligible, because of the negative value of  $f_2$ . In this view, the gap between small and large  $p_T$  physics, might actually be shorter to bridge than is usually assumed.

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Figure captions

- Fig. 1 : Energy dependence of  $\pi^-$  production ( $y = 0$ ,  $p_T = 500$  MeV) from Ref. 1, in the present model.
- Fig. 2 : The transverse momentum spectrum of  $\pi$  obtained in this model with  $A = 1.1$  and  $A = 1.5$  (Monte Carlo calculation). The solid line is the traditional  $e^{-6p_T}$ .
- Fig. 3 : Dependence of the azimuthal asymmetry correlation parameter on  $\langle p_T \rangle$ .
- Fig. 4 : Determination of  $\langle t \rangle$  in cluster multiperipheralism.

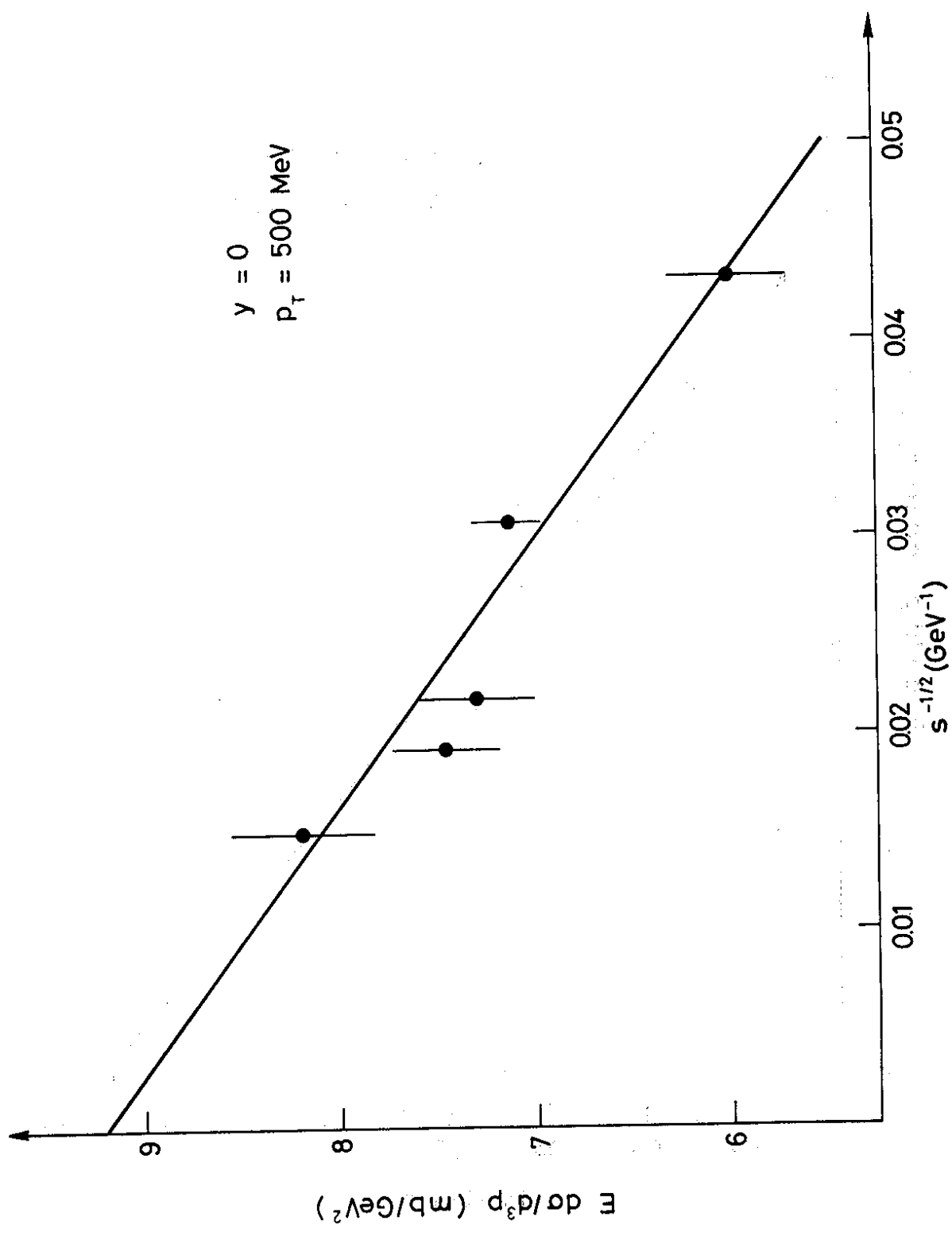


Fig. 1

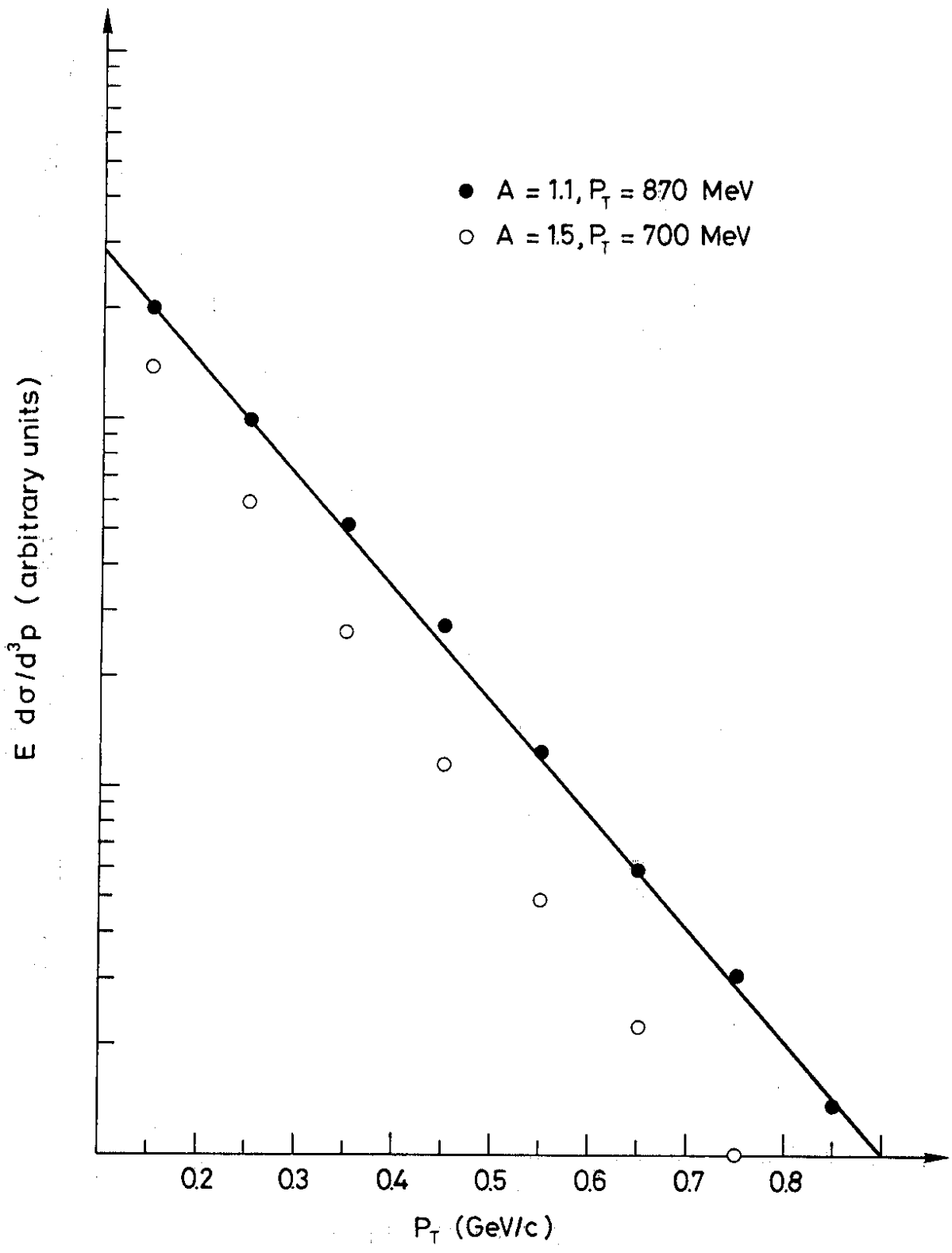


Fig. 2

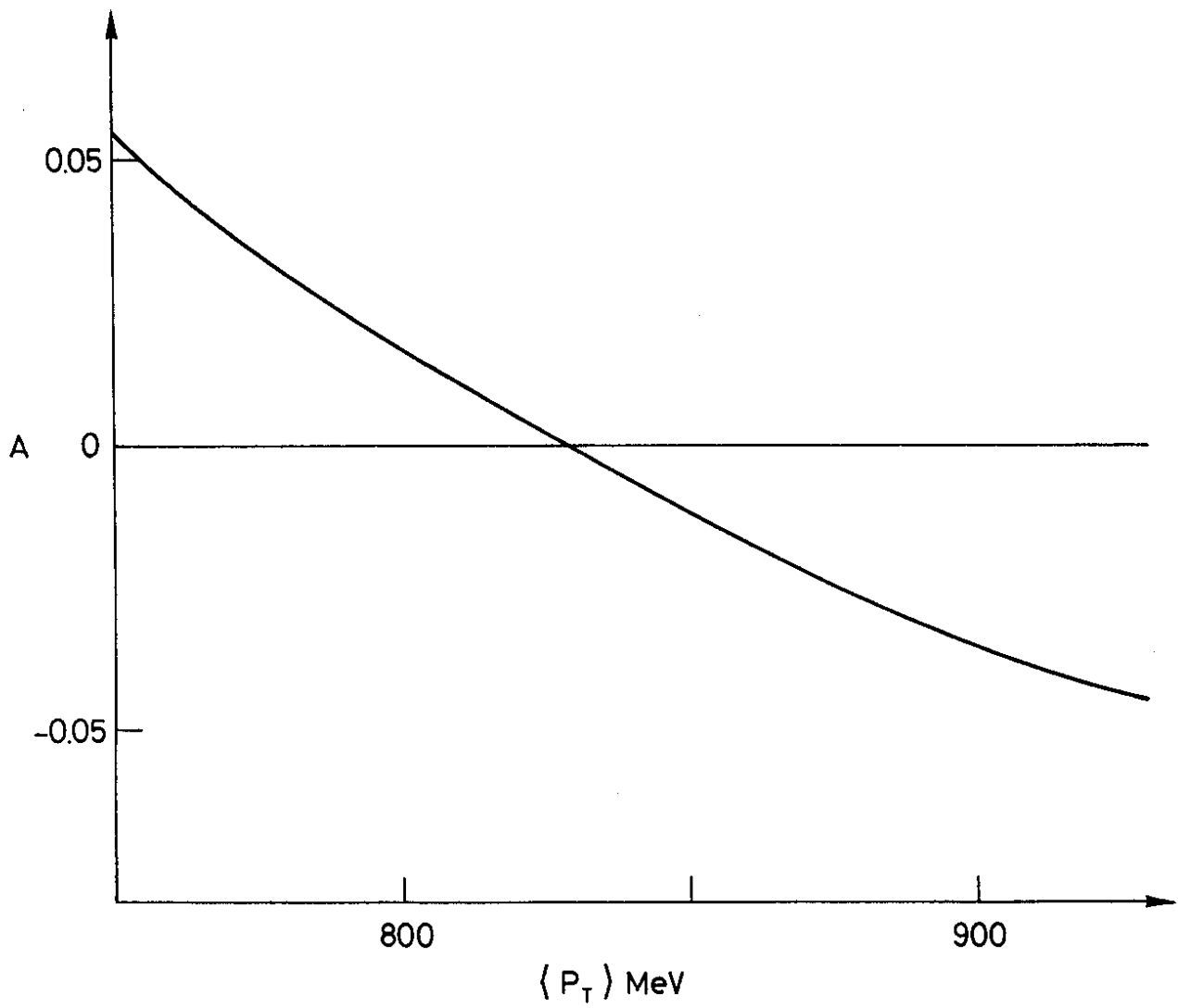


Fig. 3

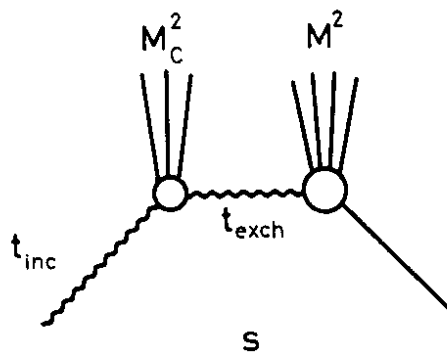


Fig. 4

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