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Robust Estimates of Track Parameters and Spatial Resolution for Endcap Muon Chamber

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Abstract

A robust technique with sub-optimal weight function (M-estimate) was applied to investigate track fitting in cathode strip chambers (CSCs) and determine the CSC spatial resolution. The comparative analysis with the conventional least squares method was made on simulated data and experimental data from the Dubna ME1/1 prototype. The obtained results definitely prove a necessity of using robust track fitting for a reliable estimation of muon chamber spatial resolution.

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1 Introduction

The CMS muon system should provide a high spatial resolution under conditions of heavy background. Cathode strip chambers (CSCs), i.e. six-layer multiwire proportional chambers with a strip cathode readout, are used as muon detectors in a forward region of the CMS and are located behind the calorimetric system. The required azimuthal spatial resolution (σ_m) for CMS muon endcap CSCs is of an order of hundreds μm. About 10-20% of muon hits in CSC will be contaminated by different sources, but we consider here two the most essential of them: (i) secondary electromagnetic (e.m.) particles (γ and e^-/e^+) entering a muon detector from a calorimeter with a muon and (ii) δ-electrons producing by muon passed through the matter of a muon detector. As a result, the error distribution differs from the normal (Gaussian) distribution and tends to have long non-Gaussian "tails". As it is well-known [1, 2] conventional least squares (LSQ) estimates loose their optimal properties in such cases. On the contrary, robust M-estimates [2] are much less sensitive to data contamination.

Thus, the aims of our work are:

- to make a mathematical model of a muon detector with noise taking into account e.m. secondaries and δ-electrons stochastically distributed along the muon track;
- to apply a robust approach for track fitting under conditions of heavy background;
- to make comparative analysis of track parameters obtained by robust technique and by LSQ method;
- to estimate a spatial resolution of CSC prototype in the most reliable way.

2 Mathematical inference

Let us consider a linear regression dependence

$$x_i = \sum_{j=1}^{p} \phi_j(z_i) \cdot \theta_j \; + \; e_i \; , \;\; i = 1, \ldots, N, \tag{1}$$

where $\phi_j(z)$ - known p linearly independent geometric functions (e.g., $1, \; z, \; z^2 \ldots$); z_i - a coordinate of the i-th detector plane; x_i - a response of the i-th detector plane (a result of a measurement); e_i - an accidental measurement error in this detector plane; θ_j - unknown regression parameters ($j = 1, \ldots, p$) which should be estimated by use of data sample; N - a number of detector planes used for fitting.

We use so-called gross-error model [2] of a contaminated distribution of measurement errors e_i:

$$f(e) = (1-\epsilon) \cdot g(e) \; + \epsilon \cdot h(e) \;\; , \tag{2}$$

where ϵ is a parameter of contamination; $g = N(0, \sigma_m^2)$ is the Gauss distribution and h is some long-tailed noise distribution.

Using the maximum likelihood method ($L = \prod_{i=1}^{N} f(e_i) \;\longrightarrow\; max$) we obtain weighted least squares equations

$$\sum_{i}^{N} w_i \cdot [x_i - \sum_{j'}^{p} \phi_{j'}(z_i) \cdot \theta_{j'}] \cdot \phi_j(z_i) = 0 \; , \; j = 1, \ldots, p \tag{3}$$

with some optimal weights w_i dependent on relations of g and h distributions [3].

However as it was pointed out in [4] a polynomial expansion of these optimal weights up to the fourth order leads to the approximation

$$w(e_i) = \begin{cases} \left(1 - \frac{e_i^2}{\sigma^2 \cdot c_T^2}\right)^2 & , e_i^2 \le c_T^2 \cdot \sigma^2 \\ 0 & , e_i^2 > c_T^2 \cdot \sigma^2 \end{cases} , \tag{4}$$

which in fact are the famous Tukey's bi-weights and are easier to calculate than optimal ones.

The parameter σ can be obtained from the likelihood equation $\frac{\partial L}{\partial \sigma} = 0$. We choose cutting parameter $c_T = 3 \div 4$. If there is no a priori information one can take $w_i^{(0)} = 1$ but in our case we use some better initial values described below and can carry out an iterative re-weighted LSQ-procedure (3) for robust parameters estimation. Following Huber we name this procedure as descending M-estimate.

It should be remarked that we use a well-known jack-knife procedure. This algorithm is also named as splitting of a sample and consists of a check of statistical inferences by one-by-one rejecting the points with maximal deviations.

Our procedure realization includes some additional ideas:
a) We vary polynomial power in (4), since we observed that Tukey's weights may decrease very sharply with deviation. So we use polynomial of a second order on the 1st iteration and every time after rejecting of outliers.
b) Before zero-iteration we apply a special "base-line" procedure for selecting of initial weights. Combining measurements in groups we choose p-point base curve with a minimal sum of deviations for all other points from this curve and assign the initial weights for outliers lower than 1.

3 Monte-Carlo model and results

We build a Monte-Carlo (M.C.) mathematical model of linear regression $x = az + b$ for a straight line muon track passing through 6 equidistant CSC layers. Values of charge induced on each of cathode planes are simulated in accordance with the Landau distribution. Space distribution of this charge on the cathode plane is described by the Gatti formula [5]. The summarised charge on every strip $q_j^{(0)}$ is smeared (Δq_j) by adding a normal noise (a readout electronics noise) with the given σ_{noise} (i.e. $q_j^{(0)} \to q_j = q_j^{(0)} + \Delta q_j$) . Therefore the restored centroid of the charge distribution x_i is calculated with some error σ_m.

Simulating contamination we take into account δ-electrons and e.m. accompaniment stochastically distributed along the muon track. The contamination parameter ϵ, the number of δ-electrons in each layer and the distance between muon (x_i) and δ-electron (x_e) (the variable of exponential distribution $|x_i - x_e|$) are parametrised on the basis of the previous GEANT simulation [6] of muon passing through CSC and calorimeter matter.

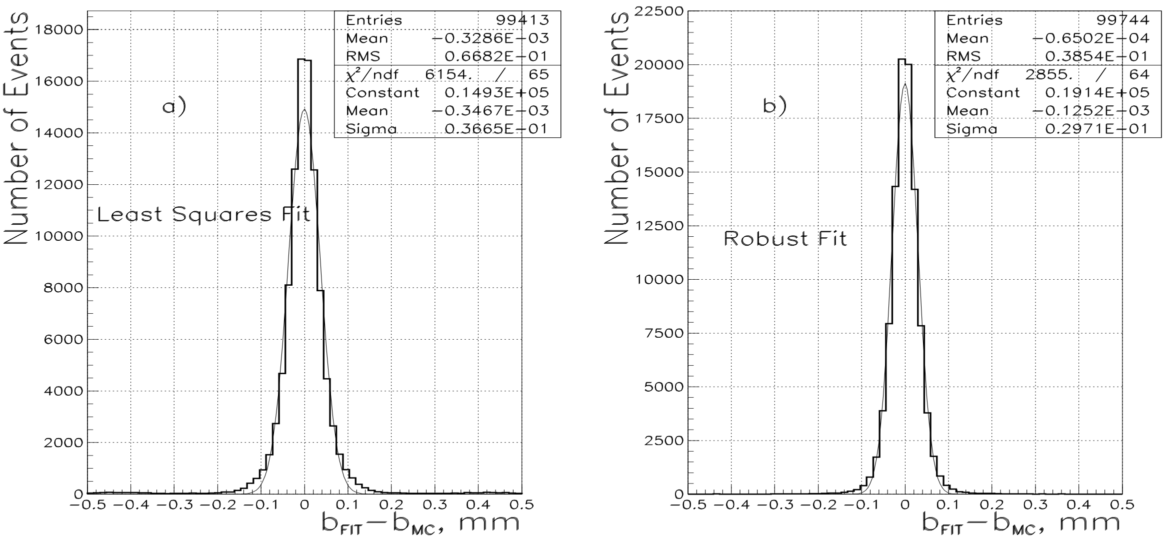

Figure 1: The distribution of deviations of intercept fitted parameters b_{FIT} from original M.C. parameters b_{MC} (fitted by a Gaussian).

As we can see from Fig.1, the distribution of intercept parameter b deviations for LSQ fitting (Fig.1a) has longer tails than the distribution for a robust approach. Track parameters obtained by the robust method (Fig.1b) have a value of root mean squares (RMS) in 1.7 times better than parameters obtained by the LSQ method. For a slope parameter a we obtained a similar result.

It is well-known from a mathematical statistics (see, e.g. [7, 8]) that any parameter estimation can be qualified by a confidence level. A percentage of events, in which at least one of parameters (a, b) lies outside of 95% confidence interval, amounts to 4.9% for robust track fitting and 22.7 % for LSQ fitting (the latter ones are quite suspicious to be used).

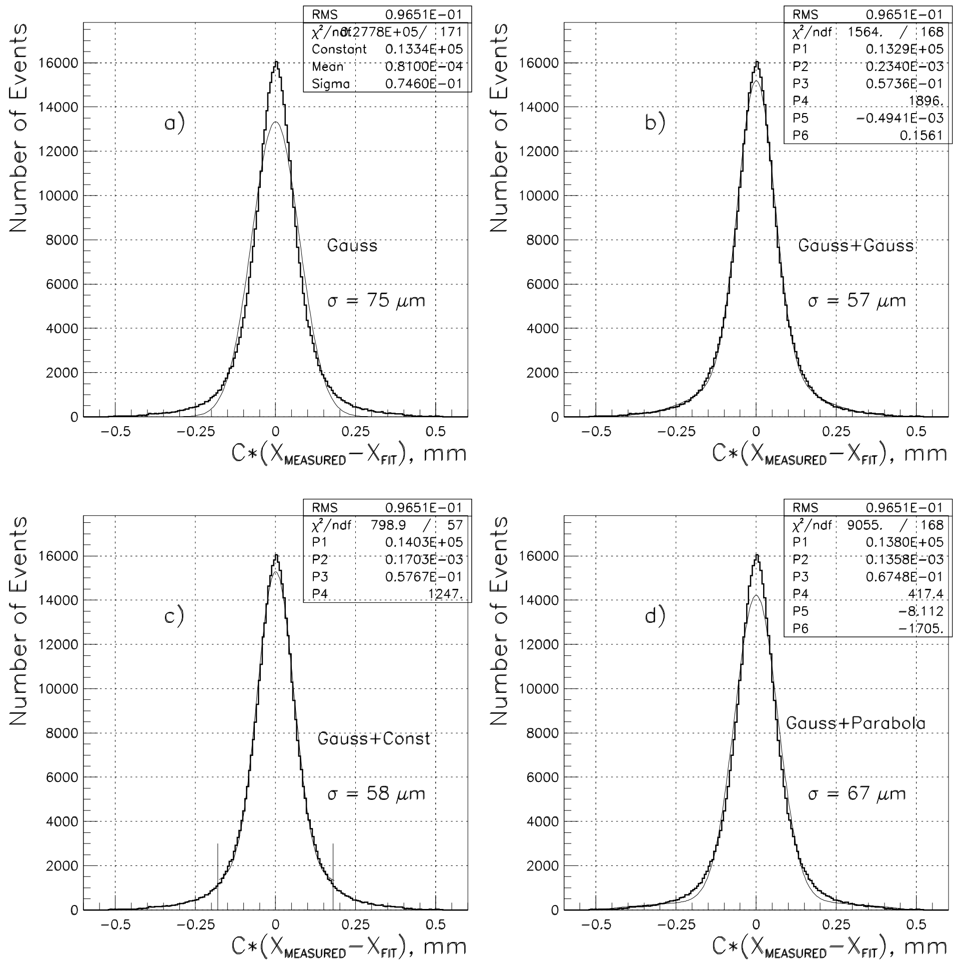

Figure 2: Distribution of normalised residuals between measured and fitted values of x_i for LSQ method on simulated data; the same distribution is fitted in different ways: a) by Gaussian, b) by two Gaussians, c) by Gaussian and constant in a region of $\pm 3\sigma_m$, d) by Gaussian and parabola.

In order to estimate a spatial resolution one should calculate normalised (due to track extrapolation) residuals for simulated events. Applying the LSQ method we obtain distribution of residuals which differs from Gaussian and has long tails (see Fig. 2). First we can conclude that RMS of distribution is essentially greater (approximately in 1.6 times) than the modelling device resolution $\sigma_m \approx 60\mu m$. Then we tried to estimate Gaussian component (which corresponds to a spatial resolution of chamber) by different fitting of this distribution. Various ways of component estimation lead to different results. On the contrary, the distribution of residuals obtained by robust track fitting (Fig.3) gives the distribution very close to a Gaussian ($RMS \approx \sigma$) and therefore we can make the only inference for the chamber resolution.

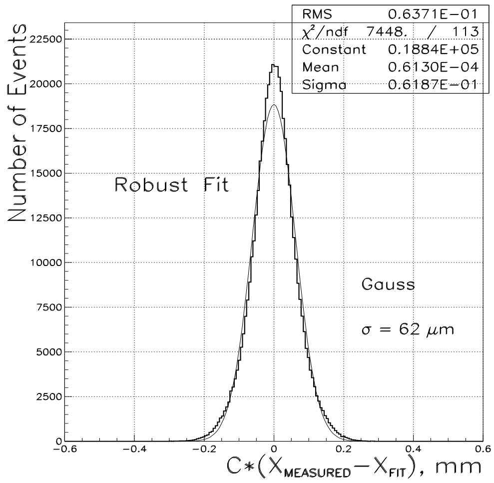

Figure 3: Distribution of normalised residuals for robust track fitting on simulated data (fitted by a Gaussian).

After testing our elaborated approach to estimate a realistic device resolution on simulated data, we apply this technique to experimental data from the detector prototype [9]. These data were obtained from the full-scale Dubna CSC prototype at the Integrated Test setup exposed on the H2 beam line of the CERN SPS accelerator. Data taking was performed with a muon beam having momenta from 100 to 300 GeV/c. As the CSC was located behind the hadron calorimeter, muon hits were hardly contaminated by δ-rays and e.m.secondaries. Residuals obtained by both robust and LSQ track fitting on experimental data are shown in Fig.4.

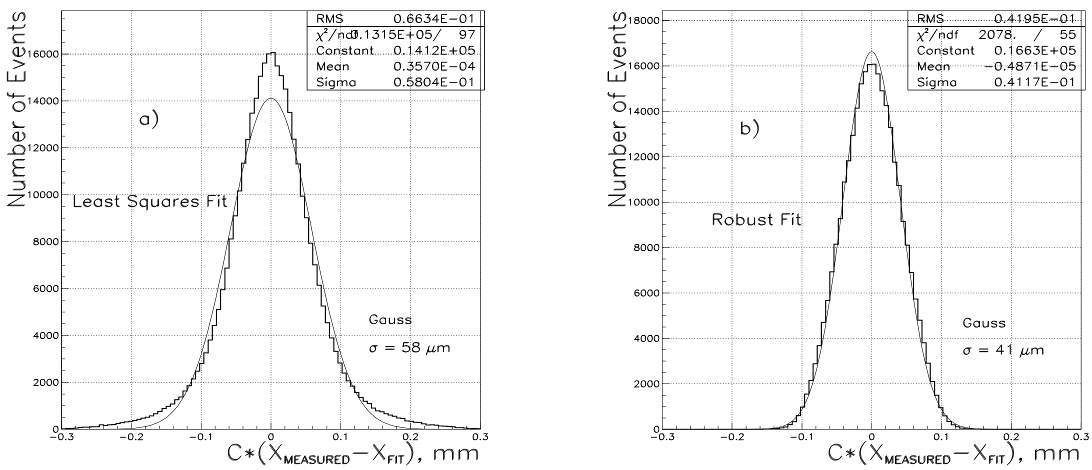

Figure 4: Distribution of normalised residuals between measured and fitted values of x_i for LSQ and robust methods on Dubna CSC prototype experimental data (fitted by a Gaussian).

We can see non-Gaussian tails in the distribution of residuals for LSQ fitting and the distribution of residuals for robust fitting which is very close to Gaussian. So we can make a definite conclusion about our proposed robust technique that this approach can be used for a reliable estimation of CSC spatial resolution.

4 Conclusions

The robust track fitting approach with the use of sub-optimal weights is proposed for muon chamber data processing under conditions of hard background instead of the conventional LSQ method.

Mathematical model of CSC is elaborated to test track fitting procedure taking into account hard contamination.

Calculations on simulated data show that the track parameters obtained by the robust procedure have up to 1.7 times better RMS than the parameters obtained by the LSQ method.

Moreover, we should point out that the part of events with robust estimated parameters, which lie out of the 95% confidential interval, are corresponding to the resting 5%. However, for LSQ fitting this part is exceeded 20%. Therefore one can conclude that the latter part of events with unsatisfactory parameter values is not possible to apply for calculation of residuals .

Long non-Gaussian tails in distributions of LSQ residuals on experimental data lead to a wide ambiguity in estimation of CSC spatial resolution. On the contrary, distributions of robust residuals are very close to Gaussian.

The obtained results definitely prove a necessity of using the robust track fitting for a reliable estimation of muon chamber spatial resolution.

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