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The Peccei-Quinn Axion in the Next-to-Minimal Supersymmetric Standard Model

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Abstract

We discuss the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with a Peccei-Quinn (PQ) $U(1)$ symmetry. When this symmetry is dynamically broken by the Higgs mechanism, the resulting pseudo-Nambu-Goldstone boson takes the role of an axion. Although much of the allowed parameter space for low values of the PQ scale has been ruled out, many scenarios with a PQ scale $\gtrsim 10^9$ GeV remain untested, allowing the NMSSM PQ axion to provide a solution to the strong CP problem and be a good dark matter candidate. Unfortunately the new particle states are so decoupled that they would not be observable at future colliders, and the NMSSM would appear indistinguishable from the minimal model. However, we show that in order to maintain vacuum stability, such a model requires that the heavy Higgs boson states have masses that lie close to approximately $\mu \tan\beta$. Therefore, a measurement of the Heavy Higgs boson masses at the LHC would allow one to either rule out the NMSSM PQ axion, or provide tantalizing circumstantial evidence for its existence.

Introduction: The Strong CP Problem and the Axion

For some some time after its formulation, one of the principle strengths of Quantum Chromodynamics (QCD) was thought to be its automatic conservation of parity (P) and charge-conjugation–parity (CP) symmetries. The only renormalizable P and CP violating term that may be added to the QCD Lagrange density is the "θ-term",

$$\mathcal{L}_\theta = \theta_{\rm eff} \frac{\alpha_s}{8\pi} F^{\mu\nu\, a} \tilde{F}^a_{\mu\nu}, \tag{1}$$

where $F^{\mu\nu\, a}$ is the gluon field strength and $\tilde{F}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma\, a}$ is it dual; $\theta_{\rm eff}$ is the effective θ-parameter after diagonlization of the quark mass matrix, i.e. $\theta_{\rm eff} = \theta + \arg\det\, M_q$. It is straightforward to show that this term is a total derivative allowing its integral over space-time to be written as a boundary term at infinity. Therefore, it was thought, its integral will vanishes in the vacuum, and the θ-term may be safely ignored.

However, it was soon realized that such a term could not be ignored if the vacuum has non-trivial topological structure [1–3]. Indeed, even if set to zero by hand in the QCD Lagrange density, it will be regenerated when contributions from instanton solutions are included in the path integral. Its space-time integral does not necessarily vanish but is proportional to the winding number (Pontryagin index) of the field configuration. The θ-term will then contribute intrinsically non-perturbative CP violation, i.e. its effects will be invisible to perturbation theory. Since no CP violation has been observed in QCD, $\theta_{\rm eff}$ must be very small.

This can be quantified by examining the electric dipole moment of the neutron, d_n: the CP violation induced by the θ-term leads to a neutron electric dipole moment of order $|d_n| \approx |\theta_{\rm eff}| 10^{-16} e$ cm [4], which must be compared to the current experimental limit $d_n < 0.63 \times 10^{-25} e$ cm [5]. Therefore $|\theta_{\rm eff}| \lesssim 10^{-9}$, naturally leading to the question: why is CP violation in QCD so small? This is known as the "strong CP problem".

The axion provides a very natural solution to the strong CP problem. It was realized that the θ-term could be absorbed by making a redefinition (an axial rotation) of the quark fields [2]. If the quarks have zero mass the Lagrange density will be unchanged except for the removal of the θ-term, and theories with differing values of $\theta_{\rm eff}$ all represent the same physics. In essence, the θ-term can be rotated away using the global U(1) axial symmetry of the model. However, if the quarks have non-zero mass then this rotation will introduce complex phases into the quark mass matrix and the theory will still be CP-violating.

Peccei and Quinn [6] pointed out that if a new global axial symmetry, a Peccei-Quinn (PQ) symmetry, is introduced then it could be used to remove the θ-term instead. When this PQ symmetry is dynamically broken by the vacuum structure it will result in a pseudo-Nambu-Goldstone boson known as the axion [7]. It is only a “pseudo”-Nambu-Goldstone boson because the PQ symmetry is not exact — it is explicitly broken by the triangle anomaly providing a non-perturbative axion-gluon coupling. This axion-gluon coupling has two effects. Firstly, it will provide a non-zero axion mass due to mixing with the pion, which is approximately given by

$$M_a = \frac{f_\pi m_\pi}{4\langle\phi_a\rangle}\sqrt{\frac{4m_u m_d}{(m_u+m_d)^2}}\,[1+\mathcal{O}(m_{u,d}/m_s)] \approx 0.6\times10^{-3}\mathrm{eV}\left[\frac{10^{10}\mathrm{GeV}}{f_a}\right], \tag{2}$$

where m_u, m_d and m_s and the up, down and strange quark masses respectively, f_π and m_π are the pion decay constant and the pion mass, and ϕ_a is the axion field. Secondly, the axion-gluon coupling introduces an effective term into the Lagrange density of the same form as the θ-term, Eq.(1), so that the CP-violating terms become

$$\mathcal{L}_{\theta\ \mathrm{eff}} = \left(\theta_{\mathrm{eff}} - \frac{\phi_a}{f_a}\right)\frac{\alpha_s}{8\pi}F^{\mu\nu\,a}\tilde{F}^a_{\mu\nu}, \tag{3}$$

where f_a is the axion decay constant. However, the potential for ϕ_a is also a function of $(\theta_{\mathrm{eff}} - \phi_a/f_a)$ and so the axion field relaxes to a vacuum-expectation-value (VEV) given by $\langle\phi_a\rangle = f_a\theta_{\mathrm{eff}}$. The θ-term is canceled and the strong CP problem is solved.

The experimental bounds on the existence of the axion are already rather strict [8]. The non-observation of an axion in collider experiments and rare decays (e.g. quarkonium decays) rules out models where the PQ scale (f_a) is of the order of the electroweak scale. However, these bounds can always be avoided by increasing the PQ scale [9, 10], or equivalently reducing the axion mass, thereby reducing the axion's couplings to known particles.

In order to constrain this “invisible axion” one must consider astrophysical constraints [11]. Since a low mass axion is expected to be emitted during star cooling, f_a may be constrained by insisting that the axion does not significantly alter the observed stellar evolution. Stars in globular clusters are the most sensitive to these effects [12-13]. Additionally, the neutrino signal from SN 1987A indicates that it is cooled mainly by neutrino emission rather than by emission of an “invisible axion” [14]. Together these observations place a limit of roughly $f_a \gtrsim 10^9$ GeV (translating via Eq.(2) to $M_a \lesssim 0.01$ eV).

Intriguingly, at scales just above this limit the axion is seen to be a good dark matter candidate. Indeed, it was shown in Ref.[15] that in the standard thermal scenario, and many inflationary models, the dark matter axion's PQ scale is predicted to be $f_a \approx 3\times10^{10}$ GeV. If the PQ scale becomes too much larger the axion contribution to dark matter may become *too great*, thereby over-closing the universe and thus providing an upper limit on f_a. However, this upper bound is very model dependent. We will see later that the main results of this letter do not depend on the fine details of the axion mass limits, but only that the PQ scale be very large.

In this letter, we will briefly describe the PQ symmetric Next-to-Minimal Supersymmetric Standard Model (NMSSM), which is the minimal supersymmetric extension of Standard Model that can provide an axion. We will examine the Higgs boson mass spectrum of the model and see that the lightest pseudoscalar Higgs boson is the "invisible axion", and will subsequently be unobservable at colliders for the foreseeable future. However, we will show that in order to keep the mass-squared of the lightest scalar Higgs boson positive, one must constrain the heavy Higgs bosons to lie in a very specific mass window. We will provide one-loop expressions for this mass window in a very good approximation. Therefore, this model provides a prediction for the heavy Higgs boson masses which may be confirmed or ruled out at the next generation of colliders.

The PQ Symmetric NMSSM

One model that provides an axion is the PQ symmetric NMSSM [15–18]; this has the same field content as the Minimal Supersymmetric Standard Model (MSSM) except for the inclusion of an extra Higgs singlet superfield $\hat{S}$. Its superpotential is given, in an obvious notation, by

$$W = \hat{u}^c\,\mathbf{h_u}\hat{Q}\hat{H}_u - \hat{d}^c\,\mathbf{h_d}\hat{Q}\hat{H}_d - \hat{e}^c\,\mathbf{h_e}\hat{L}\hat{H}_d + \lambda\hat{S}(\hat{H}_u\hat{H}_d). \tag{4}$$

The usual Higgs–higgsino mass term $\mu\hat{H}_u\hat{H}_d$ seen in the MSSM has been replaced by the term $\lambda\hat{S}(\hat{H}_u\hat{H}_d)$ coupling the new singlet Higgs field, $\hat{S}$, to the Higgs doublets, H_d and H_u, where λ is a dimensionless parameter. The Higgs–higgsino mass term will be recovered when the scalar component, S, of the new singlet superfield gains a VEV of $\langle S\rangle = \mu/\lambda$.

In the MSSM, the dimensionful parameter, μ, is constrained to be of the order of the electroweak scale in order to give the correct pattern of electroweak symmetry breaking, even although it has no *a priori* relation to the electroweak scale. The question of why two seemingly unrelated scales should be the same is known as the "μ-problem" [21]. The original formulation of the NMSSM was intended to answer this question by dynamically linking the scale μ to a VEV of a Higgs field, S, and thereby to the electroweak scale.

The superpotential, Eq.(4), has no dimensionful couplings and exhibits a U(1) PQ symmetry, which will be carried over into the Lagrange density. In the MSSM this PQ symmetry is *explicitly* broken by the Higgs-higgsino mass term $\mu\hat{H}_u\hat{H}_d$; in the PQ symmetric NMSSM the PQ symmetry is only *dynamically* broken when S gains a non-zero VEV, giving rise to a near massless pseudo–Nambu–Goldstone boson — the axion[1]. Therefore the PQ symmetric NMSSM is the minimal

[1]The axion is only a "pseudo"–Nambu–Goldstone boson since the PQ symmetry is explicitly broken by the triangle anomaly, giving it a small mass, Eq.(2).

supersymmetric extension of the Standard Model that can provide an axion. In fact, it is a supersymmetric version of the DFSZ axion model [10].

The axion constraints mentioned in the introduction must also be applied here and so models with $\langle S\rangle$ of the order of the electroweak scale are ruled out. In the more usual formulation of the NMSSM this is avoided by adding a term $\frac{1}{3}\kappa\hat{S}^3$ to the superpotential; this *explicitly* breaks the PQ symmetry, giving the 'axion' a mass and avoiding the constraints. Here, in order to preserve a near massless axion, we insist that $\langle S\rangle \gtrsim 10^9$ GeV. Therefore, the PQ symmetric NMSSM no longer links $\langle S\rangle$ to the electroweak scale and *cannot* be considered as a solution to the μ-problem. Since μ must remain of order the electroweak scale, $\lambda = \mu/\langle S\rangle$ must be very small and the μ-problem is re-expressed as: why is λ so small? We will not attempt to answer this question here.

The axion within the context of the NMSSM has also been discussed in Ref.[19]. In that study, the term $\frac{1}{3}\kappa\hat{S}^3$ was included in the superpotential, explicitly breaking the PQ symmetry, but it was pointed out that in the limit where the soft supersymmetry breaking parameters associated with λ and κ vanish, the model will contain an additional approximate $U(1)_R$ symmetry. This symmetry is dynamically broken by the vacuum, giving rise to an 'R-axion'. Unfortunately the mass of this R-axion becomes rather large, forbidding its use in solving the strong CP problem, but nevertheless the model has interesting phenomenological consequences.

The superpotential, Eqn.(4), leads to the tree–level Higgs potential [17]:

$$V = V_F + V_D + V_{\rm soft}, \tag{5}$$

with

$$\begin{aligned} V_F &= |\lambda S|^2(|H_u|^2 + |H_d|^2) + |\lambda H_u H_d|^2, & (6)\\ V_D &= \frac{1}{8}\bar{g}^2(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2}g^2|H_u^\dagger H_d|^2, & (7)\\ V_{\rm soft} &= m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + [\lambda A_\lambda S H_u H_d + {\rm h.c.}], & (8) \end{aligned}$$

where $\bar{g} = \sqrt{g^2 + g'^2}$ with g and g' being the gauge couplings of $SU(2)_L$ and $U(1)$ interactions respectively, and we have adopted the notation $H_u H_d \equiv \epsilon_{\alpha\beta}(H_u)^\alpha(H_d)^\beta = H_u^+ H_d^- - H_u^0 H_d^0$. The first two terms, V_F and V_D, are the F and D terms derived from the superpotential in the usual way, while $V_{\rm soft}$ contains the soft supersymmetry–breaking parameters A_λ, m_{H_u}, m_{H_d} and m_S.

The vacuum of the model may be rendered neutral by a suitable application of a $SU(2)_L \times U(1)_Y$ gauge transformation, and rendered real by exploiting the PQ symmetry. The vacuum is then given by

$$\langle H_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle H_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle S\rangle = \frac{1}{\sqrt{2}}v_s, \tag{9}$$

with v_s, v_u, and v_d real and positive. The requirement for this vacuum to be a local minimum provides three relations, linking the three soft mass parameters to the three VEVs of the Higgs fields:

$$
\begin{aligned}
m_{H_d}^2 &= \frac{1}{8}\bar{g}^2(v_u^2 - v_d^2) - \frac{1}{2}\lambda^2 v_u^2 + \frac{1}{\sqrt{2}}A_\lambda \lambda v_s \frac{v_u}{v_d} - \frac{1}{2}\lambda^2 v_s^2, && (10)\\
m_{H_u}^2 &= \frac{1}{8}\bar{g}^2(v_d^2 - v_u^2) - \frac{1}{2}\lambda^2 v_d^2 + \frac{1}{\sqrt{2}}A_\lambda \lambda v_s \frac{v_d}{v_u} - \frac{1}{2}\lambda^2 v_s^2, && (11)\\
m_S^2 &= -\frac{1}{2}\lambda^2 v^2 + \frac{1}{\sqrt{2}}\lambda A_\lambda \frac{v_u v_d}{v_s}; && (12)
\end{aligned}
$$

as usual, we have written $v \equiv \sqrt{v_u^2 + v_d^2}$.

The extra singlet fields mix with the Higgs doublet fields, increasing the rank of the scalar and pseudoscalar mass–squared mixing matrices by one each. After an initial rotation of the Higgs doublet fields by an angle β, defined as usual via $\tan\beta \equiv v_u/v_d$ and outlined in detail in Ref.[20], the 2×2 pseudoscalar mass matrix is given by

$$
M_A^2 \begin{pmatrix} 1 & \frac{1}{2}\sin 2\beta \cot\beta_s \\ \frac{1}{2}\sin 2\beta \cot\beta_s & \frac{1}{4}\sin^2 2\beta \cot^2\beta_s \end{pmatrix}. \tag{13}
$$

In analogy to $\tan\beta$, we have also defined $\tan\beta_s \equiv v_s/v$; due to the requirement that $\langle S\rangle \gtrsim 10^9$ GeV, $\tan\beta_s$ will be very large, and therefore $\cot\beta_s$ very small. In the above, we have *defined* the upper–left entry of the pseudoscalar mass–squared mixing matrix to be M_A^2. This new mass parameter replaces the soft supersymmetry–breaking parameter A_λ and becomes the mass of the MSSM pseudoscalar Higgs boson as the MSSM limit is approached, i.e. $\cot\beta_s \to 0$ with μ fixed. This treatment allows higher order loop corrections to be absorbed directly into the definition of M_A. Including one-loop top/stop corrections, it is related to A_λ by

$$
M_A^2 = \frac{2\mu}{\sin 2\beta}A_\lambda - \frac{3h_t^2}{16\pi^2}A_t F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \tag{14}
$$

where $h_t = \sqrt{2}m_t/(v\sin\beta)$ is the top-quark Yukawa coupling and A_t is its associated soft supersymmetry-breaking mass parameter. The function F is given by

$$
F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) = \frac{1}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\left[m_{\tilde{t}_1}^2 \log\left(m_{\tilde{t}_1}^2/Q^2\right) - m_{\tilde{t}_2}^2 \log\left(m_{\tilde{t}_2}^2/Q^2\right)\right] - 1 \tag{15}
$$

and m_t, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ are the top and stop masses, with Q the renormalization scale.

This pseudoscalar mass-squared matrix is easily diagonalized, revealing two mass eigenstates, which will be denoted A_1 and A_2 with the label assigned in order of increasing mass. The first of these, A_1, is the massless Nambu–Goldstone boson associated with the dynamical breaking of the PQ symmetry — the axion. The PQ symmetry ensures that it will be massless even after the inclusion of loop corrections; it only gains a very small mass via non-perturbative mixing with the pion, as described earlier. The heavier mass eigenstate, A_2, has mass

$$M_{A_2}^2 = M_A^2(1 + \frac{1}{4}\sin^2 2\beta \cot^2 \beta_s). \tag{16}$$

Since $\cot \beta_s \lesssim 10^{-7}$ GeV the heavy pseudoscalar Higgs boson reproduces the mass of the MSSM pseudoscalar with a deviation less than one part in 10^{14}.

Similarly, the symmetric 3×3 scalar Higgs mass-squared matrix is

$$M^2 = M_0^2 + \Delta, \tag{17}$$

where the entries of the tree-level contribution, M_0^2, can be written as

$$[M_0^2]_{11} = M_A^2 + (M_Z^2 - \mu^2 \cot^2 \beta_s) \sin^2 2\beta \tag{18}$$
$$[M_0^2]_{12} = -\frac{1}{2}(M_Z^2 - \mu^2 \cot^2 \beta_s) \sin 4\beta \tag{19}$$
$$[M_0^2]_{13} = -\frac{1}{4} M_A^2 \sin 4\beta \cot \beta_s \tag{20}$$
$$[M_0^2]_{22} = M_Z^2 \cos^2 2\beta + \mu^2 \cot^2 \beta_s \sin^2 2\beta \tag{21}$$
$$[M_0^2]_{23} = \frac{1}{2}(4\mu^2 - M_A^2 \sin^2 2\beta) \cot \beta_s \tag{22}$$
$$[M_0^2]_{33} = \frac{1}{4} M_A^2 \sin^2 2\beta \cot^2 \beta_s \tag{23}$$

Δ denotes higher order corrections to the scalar Higgs mass matrix [22, 23]. Including one-loop top/stop corrections these are given by [23]

$$\Delta_{11} = \frac{3h_t^2}{8\pi^2} m_t^2 \left[s_\beta^2 \log\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right] - 8\frac{a^4}{s_\beta^2} K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 8a^2 K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right], \tag{24}$$

$$\Delta_{12} = \frac{3h_t^2}{8\pi^2} m_t^2 \left[s_\beta c_\beta \log\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right] - 8\frac{a^3 b}{s_\beta^2} K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 4\frac{a}{s_\beta}(a c_\beta + b s_\beta) K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right], \tag{25}$$

$$\Delta_{13} = \frac{3h_t^2}{16\pi^2} \left[-\sqrt{2}\mu s_\beta c_\beta F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 8 m_t^2 a^2 b \frac{c_\beta}{s_\beta^2} K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right.$$
$$\left. -4 m_t^2 a \frac{c_\beta^2}{s_\beta} K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] \sqrt{2}\mu \cot \beta_s, \tag{26}$$

$$\Delta_{22} = \frac{3h_t^2}{8\pi^2} m_t^2 \left[c_\beta^2 \log\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right] - 8a^2 b^2 \frac{1}{s_\beta^2} K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 8ab\frac{c_\beta}{s_\beta} K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right], \tag{27}$$

$$\Delta_{23} = \frac{3h_t^2}{16\pi^2}\Bigg[\sqrt{2}\mu c_\beta^2 F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 8m_t^2 a^3 \frac{c_\beta}{s_\beta^2} K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) - 4m_t^2 a c_\beta K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\Bigg] \sqrt{2}\mu \cot\beta_s, \tag{28}$$

$$\Delta_{33} = -\frac{3h_t^2}{2\pi^2} m_t^2 \mu^2 a^2 c_\beta^2 \cot^2\beta_s K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{29}$$

where $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, $a = (-\mu c_\beta + A_t s_\beta)/\sqrt{2}$, and $b = (\mu s_\beta + A_t c_\beta)/\sqrt{2}$, and the functions K_1 and K_2 are

$$K_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \equiv K(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)/(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2, \tag{30}$$

$$K_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \equiv (K(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + 1)/(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2), \tag{31}$$

with

$$K(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \equiv F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) - \frac{1}{2}\log\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right]. \tag{32}$$

Closed form expressions for the scalar Higgs boson mass eigenvalues can be obtained by diagonalizing M^2. However, these results are rather lengthy and unilluminating, and will not be reproduced here.

Fortunately, these exact expressions are not needed due to the very small size of $\cot\beta_s \sim \mathcal{O}(10^{-7})$. Notice that the mass-squared matrix takes the form

$$M^2 = \begin{pmatrix} A_{11} & A_{12} & C_1\cot\beta_s \\ A_{12} & A_{22} & C_2\cot\beta_s \\ C_1\cot\beta_s & C_2\cot\beta_s & B\cot^2\beta_s \end{pmatrix}. \tag{33}$$

This is true not only at tree-level but also when higher orders are included. We may reduce this matrix to block diagonal form by applying a unitary transformation defined by the 3×3 matrix

$$V^\dagger = \begin{pmatrix} \mathbf{1} - \frac{1}{2}\cot^2\beta_s\Gamma\Gamma^\dagger & -\cot\beta_s\Gamma \\ \cot\beta_s\Gamma^\dagger & 1 - \frac{1}{2}\cot^2\beta_s\Gamma^\dagger\Gamma \end{pmatrix} + \mathcal{O}(\cot^4\beta_s), \tag{34}$$

with

$$\Gamma = \left(C_1A_{22} - C_2A_{12},\ -C_1A_{12} + C_2A_{11}\right)/\det A. \tag{35}$$

Applying this transformation gives the simple form

$$VM^2V^\dagger = \begin{pmatrix} A_{11} + C_1\Gamma_1\cot^2\beta_s & A_{12} + \frac{1}{2}(C_1\Gamma_2 + C_2\Gamma_1)\cot^2\beta_s & 0 \\ A_{12} + \frac{1}{2}(C_1\Gamma_2 + C_2\Gamma_1)\cot^2\beta_s & A_{22} + C_2\Gamma_2\cot^2\beta_s & 0 \\ 0 & 0 & (B - C_2\Gamma_2 - C_1\Gamma_1)\cot^2\beta_s \end{pmatrix} + \mathcal{O}(\cot^3\beta_s). \tag{36}$$

The upper-left block consists of the usual MSSM scalar Higgs boson mass-squared matrix (to any desired number of loops) plus corrections of order $\cot^2\beta_s$. Consequently, the two heavier states, H_2 and H_3, are rather uninteresting; the MSSM scalar Higgs masses, like a heavy pseudoscalar, will be recovered with corrections of only one part in 10^{14}, which is neither experimentally observable, nor theoretically reliable since unincluded higher order corrections will present much larger deviations. This was to be expected since our NMSSM parameter choice is approaching the MSSM limit.

A prediction for M_A

The lightest Higgs boson, whose mass-squared is given by the lower-right entry, is rather more interesting. Its mass is suppressed by $\cot\beta_s$, making it effectively massless at current collider energies, but its couplings to known particles, which mainly arise from the mixing with the other scalar Higgs bosons[2], will also be tiny. Subsequently, this state would be unobservable at high energy colliders for the foreseeable future, and the low energy phenomenology would appear indistinguishable from the MSSM.

However, the expression for the lightest scalar mass shows interesting structure. Inserting the tree-level values into the lower-right entry of Eq.(36) gives the tree level mass-squared

$$M_{H_1}^2 = \mu^2 \tan^2 2\beta \cot^2\beta_s(-(x^2+y^2)(x^2-1)^2 + y^2\cos^2 2\beta)/(xy)^2 \tag{37}$$

where $x \equiv M_A \sin 2\beta/(2\mu)$ and $y \equiv M_Z \sin 2\beta/(2\mu)$.

This mass-squared must be positive in order to have a physically acceptable theory. If it is negative, the Higgs potential will be unbounded from below and the vacuum unstable. However, only the last term in the brackets of Eq.(37) is positive; $M_{H_1}^2$ will become negative for both high and low values of M_A, and a stable vacuum will be achieved only for a small range around $x \approx 1$. This is also true when loop corrections are included, as shown in Fig.(1, left).

To demonstrate this we examined 10^6 different scenarios, with M_A and $\tan\beta$ chosen randomly between 0 to 6 TeV and 3 to 30 respectively. we calculated the one-loop mass spectrum and, for every scenario with a stable vacuum, plotted a single point on the M_A–$\tan\beta$ plane of Fig.(1, right). We discarded scenarios with unstable vacua. It is immediately evident that the physically acceptable scenarios all lie within a small band around $M_A \approx 2\mu/\sin 2\beta \approx \mu\tan\beta$.

Therefore the PQ symmetric NMSSM with a large expectation value of the new singlet field makes a *prediction* for the masses of the heavy Higgs bosons. This prediction is potentially falsifiable, or verifiable, at the next generation of colliders. Furthermore, as long as $\cot\beta_s$ is small, the

[2]The Lagrangian of the model also contains new direct couplings of the new singlet state to known particles but these are also suppressed by at least one order of $\cot\beta_s$.

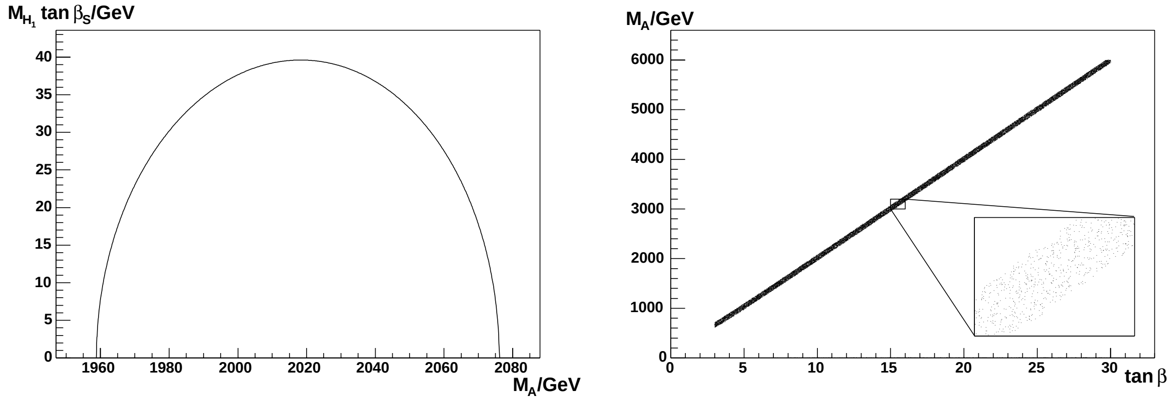

Figure 1: *Left: The dependence of the lightest scalar Higgs mass (normalized by* $\tan\beta_s$*) on* M_A*, for* $\tan\beta = 10$ *and* $\mu = 200$ *GeV. Beyond the points where the curve meets the axis the mass-squared becomes negative and the vacuum unstable. Right: The distribution of scenarios with physically acceptable vacua, with* M_A *chosen randomly between* 0 *and* 6 *TeV and* $\tan\beta$ *chosen randomly between* 3 *and* 30*. The vacuum structure constrains the value of* M_A *to lie close to approximately* $\mu\tan\beta$*. The blow-up allows individual scenario points to be seen.*

positivity or negativity of $M_{H_1}^2$ is independent of $\cot\beta_s$, and consequently the prediction of the heavy Higgs boson masses is also independent of the value of $\cot\beta_s$. Therefore, if after measuring μ and $\tan\beta$ at a future collider, the heavy pseudoscalar mass is not found to lie close to $\mu\tan\beta$ then this model is ruled out for *all large values of the singlet expectation value.* Alternatively, if the mass prediction were found to hold, it would provide very tantalizing, albeit indirect, evidence for the PQ symmetric NMSSM as a solution to the strong CP problem and for the PQ axion as a source of dark matter.

In order to compare the values of μ and $\tan\beta$ with M_A at the next generation of colliders, the vacuum stability bounds must be made more precise. Since $M_{H_1}^2 = 0$ with $M_{H_1}^2$ given by Eq.(37) is only a cubic in x^2, it can be solved to give closed form analytical expressions for the tree-level boundary. Throwing the third non-physical solution away, we find

$$x^2_{\mathrm{max/min}} = 1 - \frac{1}{3}(1+y^2)(1-\cos\gamma \pm \sqrt{3}\sin\gamma) + \Delta_\pm, \qquad (38)$$

where

$$\gamma = \frac{1}{3}\tan^{-1}\left(-\frac{\sqrt{4(1+y^2)^6 - (2(1+y^2)^3 - 27y^2\cos^2 2\beta)^2}}{2(1+y^2)^3 - 27y^2\cos^2 2\beta}\right), \qquad (39)$$

and $\Delta_\pm$ represents the higher order corrections.

Since the one-loop top/stop contributions to $\Delta_\pm$, Eqs.(24-29), are independent of M_A, $M_{H_1}^2 = 0$ remains cubic in M_A^2 when these corrections are included and we can still find a closed form solution for the limits. However, these expressions are long and complicated, and once again such complexity is not needed here. Instead we expand the one-loop corrections as a series in the small parameter y and discard terms of $\mathcal{O}(y^3)$. This gives

$$\begin{aligned}
\Delta_\pm &= \frac{1}{64\mu^2}\Bigg(\mp\frac{8}{y}\frac{s_{2\beta}^2}{c_{2\beta}}\Delta_{22} + [32\Delta_{23} + 8s_{2\beta}^2\Delta_{22} - 16 s_{2\beta}c_{2\beta}\Delta_{12}] \\
&\qquad \mp y\frac{1}{s_{2\beta}^2 c_{2\beta}}\left[8s_{2\beta}^2 c_{2\beta}^4\Delta_{11} - 8s_{2\beta}^3(1+s_{2\beta}^2)c_{2\beta}\Delta_{12} + s_{2\beta}^4(3+s_{2\beta}^2)\Delta_{22}\right. \\
&\qquad\qquad \left. -32 s_{2\beta}c_{2\beta}^3\Delta_{13} + 16 s_{2\beta}^2 c_{2\beta}^2\Delta_{23} + 32 c_{2\beta}^2\Delta_{33}\right] \\
&\qquad +16y^2\left[s_{2\beta}^2 c_{2\beta}^2\Delta_{11} + s_{2\beta}^3 c_{2\beta}\Delta_{12} - 2s_{2\beta}c_{2\beta}\Delta_{13}\right]\Bigg) + \mathcal{O}(y^3), \qquad (40)
\end{aligned}$$

where Δ_{ij} are given by Eqs.(24–29).

This approximation is rather good. The non-observation of supersymmetry to date requires that $\tan\beta \gtrsim 3$ and $\mu \gtrsim 80$ GeV, giving $y \lesssim 0.34$. The discarded terms will therefore alter the one-loop corrections by at most a few percent. For more typical MSSM parameter choices, y will be even smaller; e.g. for the Snowmass reference point SPS 1a [24], $\tan\beta = 10$ and $\mu \approx 350$ GeV, giving $y \approx 0.026$

A large $\tan\beta$ expansion of the tree-level result gives a very approximate, but rather useful, "rule of thumb":

$$M_A \approx \mu\tan\beta \pm M_Z. \qquad (41)$$

The coupling of the lightest scalar Higgs boson to electrons may also be restricted by astrophysical data, allowing more stringent limits to be placed on the PQ scale. Just as for the axion, H_1 will be produced during the cooling of globular–cluster stars if its mass is below about 10 keV. The maximum value of the H_1 mass seen in Fig.(1, left) is realized[3] at $x \approx 1$; inserting this into Eq.(37) gives

$$M_{H_1}^{\max} \approx \mu \sin 2\beta \cot\beta_s, \qquad (42)$$

so the limits from star cooling cannot be avoided if $\langle S\rangle \gtrsim 2\mu\sin 2\beta \times 10^7 \gtrsim 10^{10}$ GeV, where for the last inequality we have made the reasonable assumption that $\mu \lesssim 1$ TeV and $\tan\beta > 3$.

[3] More accurately, making a series expansion in the small parameter y, the maximum (tree-level) value of M_{H_1} is found at $x = 1 + \frac{1}{2}y^2\cos^2 2\beta + \mathcal{O}(y^4)$.

Above this scale one must respect the limits on the coupling of the lightest scalar Higgs boson to electrons [13],

$$g_{H_1 e} \lesssim 1.3 \times 10^{-14}. \tag{43}$$

In the NMSSM it is easy to see that $g_{H_1 e} \approx m_e/\langle S\rangle$ and so this translates into a lower bound on the PQ scale. Combining this with the requirement that the H_1 mass be less than 10 keV for this lower bound to apply, *excludes* the values

$$2\mu \sin 2\beta \times 10^7 \lesssim \langle S\rangle \lesssim 4 \times 10^{10}\mathrm{GeV}. \tag{44}$$

Allowing the maximum and minimum values of μ and $\tan\beta$ respectively, only a rather small range of $\langle S\rangle$ values is unequivocally ruled out. However, as μ and $\tan\beta$ are allowed to move toward less extreme values, the excluded range becomes larger and soon overlaps with that disallowed by emission of the axion from globular–cluster stars, i.e. $\langle S\rangle \gtrsim 10^9$ GeV.

Finally, since the model is supersymmetric, the extra neutral singlet superfield also contains a higgsino, which will be manifest as an extra neutralino — the lightest supersymmetric particle (LSP) of the model. Once again, the large value of the PQ scale leads to it having a very small mass and being almost totally decoupled from the other particles. To a good approximation, its mass is given by $M_{\mathrm{LSP}} \approx \mu \cot^2\beta_s$, which, for $\mu \approx 10^3$ GeV and $\langle S\rangle \approx 10^{11}$ GeV translates to[4] $M_{\mathrm{LSP}} \approx 3\times10^{-6}$ eV. In contrast to the scalar and pseudoscalar Higgs bosons, R-parity conservation prevents the LSP being emitted during star cooling, so it provides no further astrophysical limits.

Summary & Conclusions

In this letter, we have discussed the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with an explicit Peccei-Quinn (PQ) symmetry. This model is the minimal supersymmetric extension of the Standard Model that can provide an axion. This axion is a pseudo–Nambu–Goldstone boson associated with the dynamical breaking of the PQ symmetry, and is manifest in this model as the lightest pseudoscalar Higgs boson; it can be used to solve the strong CP problem of QCD and is a dark matter candidate. The stellar evolution of globular cluster stars and the neutrino signal from SN 1987A provide a lower bound on the PQ breaking scale $\gtrsim 10^9$ GeV.

We have shown that in this limit simple expressions for the NMSSM Higgs boson masses can be obtained. The heavy and intermediate mass Higgs bosons have masses and couplings

[4] It is intriguing to note that this mass lies not too far from the expected neutrino mass scale.

indistinguishable from those of the corresponding MSSM. The lightest scalar and pseudoscalar (the axion) decouple from the other particles and will be invisible to future collider searches.

However, we have demonstrated that in order that the theory have a stable vacuum, i.e. in this case that the lightest scalar mass-squared be positive, the heavy mass scale M_A must lie within approximately $\mu \tan\beta \pm M_Z$. We have presented analytic expressions for these limits on M_A to one-loop top/stop accuracy.

If, at a future collider, M_A were found to be outside this range, then the PQ symmetric NMSSM would be ruled out *for all values of the PQ scale.* This is not an unreasonable event; the restriction on M_A is unlikely to occur by chance without some other organizing principle. For example, *all* of the Snowmass MSSM reference points [24], which are considered a representative sample of MSSM scenarios, fail this criterion. It is important to stress that only the axion associated with this particular model would be ruled out; an axion could still be present via some other mechanism, and axion search experiments, such as CAST [25], the U.S. Axion Search (Livermore) [26] and the Kyoto search experiment CARRACK [27] would still be very important.

On the other hand, if the heavy Higgs boson mass scale were seen to obey the bound given by Eq.(38) we would have very exciting circumstantial evidence for the existence of an NMSSM axion. Then the role of the axion search experiments would become even more crucial.

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References

[1] A. A. Belavin, A. M. Polyakov, A. S. Shvarts and Y. S. Tyupkin, Phys. Lett. B **59** (1975) 85.

[2] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. D **14** (1976) 3432 [Erratum-ibid. D **18** (1978) 2199].

[3] R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37** (1976) 172; C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Lett. B **63** (1976) 334.

[4] V. Baluni, Phys. Rev. D **19** (1979) 2227; R. J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. B **88** (1979) 123 [Erratum-ibid. B **91** (1980) 487].

[5] P. G. Harris *et al.*, Phys. Rev. Lett. **82** (1999) 904.

[6] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38** (1977) 1440; Phys. Rev. D **16** (1977) 1791.

[7] S. Weinberg, Phys. Rev. Lett. **40** (1978) 223; F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[8] For a review, see e.g. K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66** (2002) 010001.

[9] J. E. Kim, Phys. Rev. Lett. **43** (1979) 103; M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **166** (1980) 493.

[10] A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31** (1980) 260 [Yad. Fiz. **31** (1980) 497]; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B **104** (1981) 199.

[11] For a review, see e.g. G. G. Raffelt, Phys. Rept. **198** (1990) 1.

[12] D. A. Dicus, E. W. Kolb, V. L. Teplitz and R. V. Wagoner, Phys. Rev. D **18** (1978) 1829; J. A. Grifols and E. Masso, Phys. Lett. B **173** (1986) 237.

[13] J. A. Grifols, E. Masso and S. Peris, Mod. Phys. Lett. A **4** (1989) 311; G. Raffelt and A. Weiss, Phys. Rev. D **51** (1995) 1495 [arXiv:hep-ph/9410205].

[14] J. R. Ellis and K. A. Olive, Phys. Lett. B **193** (1987) 525; G. Raffelt and D. Seckel, Phys. Rev. Lett. **60** (1988) 1793.

[15] E. P. Shellard and R. A. Battye, arXiv:astro-ph/9802216.

[16] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B **104** (1981) 199. H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120** (1983) 346; J. M. Frere, D. R. Jones and S. Raby, Nucl. Phys. B **222** (1983) 11; J. P. Derendinger and C. A. Savoy, Nucl. Phys. B **237** (1984) 307; A. I. Veselov, M. I. Vysotsky and K. A. Ter-Martirosian, Sov. Phys. JETP **63** (1986) 489 [Zh. Eksp. Teor. Fiz. **90** (1986) 838]; U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B **492** (1997) 21.

[17] J. Ellis, J. F. Gunion, H. Haber, L. Roszkowski, F. Zwirner, Phys. Rev. D **39** (1989) 844.

[18] S. F. King and P. L. White, Phys. Rev. D **52** (1995) 4183; F. Franke and H. Fraas, Int. J. Mod. Phys. A **12** (1997) 479; B. Ananthanarayan and P. N. Pandita, Int. J. Mod. Phys. A **12** (1997) 2321; U. Ellwanger and C. Hugonie, Eur. Phys. J. C **25** (2002) 297.

[19] B. A. Dobrescu and K. T. Matchev, JHEP **0009** (2000) 031 [arXiv:hep-ph/0008192].

[20] D. J. Miller, R. Nevzorov and P. M. Zerwas, arXiv:hep-ph/0304049.

[21] J. E. Kim and H. P. Nilles, Phys. Lett. B **138** (1984) 150.

[22] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257 (1991) 83 and B262 (1991) 477; M. Carena, J. R. Espinosa, M. Quiros, C. Wagner, Phys. Lett. B**355** (1995) 209; M. Carena, M. Quiros, C. Wagner, Nucl. Phys. B**461** (1996) 407; J. A. Casas, J. R. Espinosa, M. Quiros, A. Riotto, Nucl. Phys. B**436** (1995) 3.

[23] P. A. Kovalenko, R. B. Nevzorov and K. A. Ter-Martirosian, Phys. Atom. Nucl. **61** (1998) 812 [Yad. Fiz. **61** (1998) 898].

[24] B. C. Allanach *et al.*, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, Eur. Phys. J. C **25** (2002) 113 [eConf **C010630** (2001) P125] [arXiv:hep-ph/0202233].

[25] K. Zioutas *et al.*, Nucl. Instrum. Meth. A **425** (1999) 482 [arXiv:astro-ph/9801176].

[26] C. Hagmann *et al.*, Phys. Rev. Lett. **80** (1998) 2043 [arXiv:astro-ph/9801286].

[27] I. Ogawa, S. Matsuki and K. Yamamoto, Phys. Rev. D **53** (1996) 1740.