

# Proposal of a second generation of quantum-gravity-motivated Lorentz-symmetry tests: sensitivity to effects suppressed quadratically by the Planck scale<sup>1</sup>

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## ABSTRACT

Over the last few years the study of possible Planck-scale departures from classical Lorentz symmetry has been one of the most active areas of quantum-gravity research. We now have a satisfactory description of the fate of Lorentz symmetry in the most popular noncommutative spacetimes and several studies have been devoted to the fate of Lorentz symmetry in loop quantum gravity. Remarkably there are planned experiments with enough sensitivity to reveal these quantum-spacetime effects, if their magnitude is only linearly suppressed by the Planck length. Unfortunately, in some quantum-gravity scenarios even the strongest quantum-spacetime effects are suppressed by at least two powers of the Planck length, and many authors have argued that it would be impossible to test these quadratically-suppressed effects. I here observe that advanced cosmic-ray observatories and neutrino observatories can provide the first elements of an experimental programme testing the possibility of departures from Lorentz symmetry that are quadratically Planck-length suppressed.

The recent interest in the possibility that (classical) Lorentz symmetry might be only an approximate symmetry of quantum spacetime was originally ignited by phenomenological analyses based on mostly-heuristic arguments [1, 2, 3, 4]. It has now matured into a detailed technical understanding of the fate of Lorentz symmetry in quantum-gravity approaches based on noncommutative geometry [5, 6, 7] and we also have several insightful results concerning the loop-quantum-gravity approach [8, 9, 10, 11].

In approaches involving spacetime discretization, such as loop quantum gravity, we are essentially finding [8, 9, 10, 11] that the discretized spacetime observables are incompatible with continuous Lorentz-symmetry transformations. Lorentz symmetry remains a good approximate symmetry, since it is violated only by correction terms whose magnitude is governed by the

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ratio  $L_p/\lambda$ , where  $L_p \simeq 10^{-33}cm$  is the Planck length and  $\lambda$  is a much larger characteristic length scale present in the physical context (*e.g.*, the wavelength of a particle).

When the quantum-gravity approach is based on noncommutative geometry Lorentz symmetry is not necessarily broken, but it must at least be “deformed” in an appropriate sense [6, 7]. Classical Lorentz symmetry is described through a standard Lie algebra, with an associated ordinary description of the action of the symmetry generators on products of fields. A deformed action of the Lorentz generators is required by consistency with fact that fields in a noncommutative geometry are themselves noncommutative. The action of symmetry generators on products of fields requires the introduction of additional structures (the so-called co-algebra sector [12]), effectively replacing a Lie algebra of symmetries by a Hopf algebra of symmetries.

There is a common prediction of these otherwise different descriptions of the fate of Lorentz symmetry: Planck-scale-modified dispersion relations. These are primarily characterized, from a phenomenological perspective, through the presence of a lowest- $L_p$ -order correction

$$E^2 \simeq p^2 + m^2 - \eta(L_p E)^n p^2, \quad (1)$$

where  $\eta$  is a coefficient of order 1, whose precise value may depend on the specific model, and  $n$ , the lowest power of  $L_p$  that leads to a nonvanishing contribution, is also model-dependent. In any given noncommutative geometry one finds a definite value of  $n$ , and it appears to be equally easy [6, 7] to construct noncommutative geometries with  $n = 1$  or with  $n = 2$ . In loop quantum gravity one might typically expect to find  $n = 2$ , but certain scenarios [8] have been shown to lead to  $n = 1$ .

The difference between  $n = 1$  and  $n = 2$  is very significant from a phenomenological perspective. Already with  $n = 1$ , which corresponds to effects that are linearly suppressed by the Planck length, the correction term in Eq. (1) is very small: assuming  $\eta \simeq 1$ , for particles with energy  $E \sim 10^{12}eV$ , the highest-energy particles we produce in laboratory, it represents a correction of one part in  $10^{16}$ . Of course, the case  $n = 2$  pays the even higher price of quadratic suppression by the Planck length and for  $E \sim 10^{12}eV$  its effects are at the  $10^{-32}$  level.

The realization that quantum-gravity effects (whether or not they involve departures from classical Lorentz symmetry) are inevitably suppressed by the smallness of the Planck length generated for many decades the conviction [13] that no guidance from experiments could be obtained in the study of the quantum-gravity problem. Recently this pessimistic view was revised [11, 14] thanks to the results obtained in the mentioned Lorentz-symmetry studies [1, 2, 3, 4, 6, 7, 8, 9, 10] and in other quantum-gravity studies not directly connected with the fate of Lorentz symmetry [15, 16, 17]. These studies showed that in the case of effects with linear dependence on the Planck length there is a handful of experiments that allow testing.

For the case of Planck-scale departures from Lorentz symmetry, there are two well-established physical contexts in which the predicted effects, for  $n = 1$ , are observably large. The first context is the one of observations of gamma-ray bursts. These are far-away bursts of photons which

we plan to observe soon up to  $\sim TeV$  energies, with observatories such as GLAST [18]. According to Eq. (1) these (nearly-)simultaneously [1] emitted photons should reach our observatories with small energy-dependent time-of-arrival differences

$$\Delta t \sim \eta(L_p E)^n T , \quad (2)$$

where  $T$  is the overall time of travel and  $E$  is the highest energy among the two photons whose times of arrival are being compared. On the basis of  $v = dE/dp$ , Eq. (1) predicts a small energy dependence [1, 2, 3, 4, 6, 7, 8, 9, 10] of the speed of massless particles, and, although the energy-dependent term is very small, for gamma-ray bursters at distances as large as  $10^{10} \text{light years}$  the time-of-arrival differences, resulting from the whole long journey, could be at the observable level of  $0.01s$ , in the case of observations of  $GeV$  particles and  $n = 1$ . The analogous prediction for the case of quadratic suppression by the Planck length ( $n = 2$ ) leads to time-of-arrival differences of order  $10^{-18}s$ , which is instead much beyond the achievable sensitivities.

The differences between the  $n = 1$  and  $n = 2$  scenarios are also important in the other physical context which is being considered as a possible way to test the idea of Planck-scale departures from Lorentz symmetry. This is the context of observations of the highest-energy cosmic rays. A characteristic feature of the expected cosmic-ray spectrum, the so-called ‘‘GZK limit’’, depends on the evaluation of the minimum energy required of a cosmic ray in order to produce pions in collisions with CMBR (cosmic microwave background radiation) photons. According to ordinary Lorentz symmetry this threshold energy is  $E_{th} \simeq 5 \cdot 10^{19} eV$  and cosmic rays with energy in excess of this value should lose the excess energy through pion production. The value of a threshold energy is obtained combining energy-momentum conservation and dispersion relations. The modification (1) of the dispersion relation could of course affect the evaluation of the cosmic-ray threshold energy, depending on the type of laws of energy-momentum conservation which are implemented. In a large number of quantum-gravity studies that adopt (1) it is assumed that energy-momentum conservation is not modified, which of course comes at the cost of obtaining results for the threshold energies that reflect the existence of a preferred class of inertial observers. Several recent quantum-gravity studies have also explored the idea [6, 7, 19] of implementing a dispersion relation of type (1) without giving rise to a preferred class of inertial observers, which then necessarily requires [6] modified laws of boost transformations between inertial observers and a deformed law of energy-momentum conservation. For simplicity I focus here on the much-studied case with unmodified energy-momentum conservation, in which it has been shown [2, 3, 4, 10] that Eq. (1) induces a Planck-length-dependent contribution to the threshold energy of order

$$\Delta E_{th} \sim L_p^n E_{th}^{n+2} / \epsilon , \quad (3)$$

where  $\epsilon$  is a representative (very low) energy scale of the CMBR. Strong interest was generated by the observation [2, 3, 4, 10] that for  $n = 1$  one finds  $\Delta E_{th} \gg E_{th}$ , meaning that the value of the threshold energy is very significantly affected by this class of quantum-spacetime effects. This possibility actually receives encouragement by the (preliminary) AGASA [20] observations of the high-energy cosmic-ray spectrum, which appear to reflect [2, 3, 4, 10] a sizeable shift of the standard GZK limit.

Modifications of threshold energies that are formally similar to the one here considered occur also in the analysis of processes we study in the laboratory, but in those contexts the correction is completely negligible (essentially the relevant analog of the ratio  $E_{th}/\epsilon$  is not large enough). Instead in collisions involving a ultra-high-energy cosmic-ray and a CMBR photon the ratio  $E_{th}/\epsilon$  is large enough to compensate for the smallness of the Planck length, at least if  $n = 1$ . The case  $n = 2$  is usually not considered in the cosmic-ray literature because of the large additional suppression introduced by the extra power of  $L_p$ .

The fact that at least for  $n = 1$  these two experimental strategies (time-of-arrival analyses of gamma-ray bursts and cosmic-ray spectrum analyses) allow testing effects that are genuinely at the Planck scale has generated very strong interest, as one can infer from recent quantum-gravity reviews [11, 14]. But, as discussed above, a more satisfactory exploration of this idea of quantum-gravity-induced departures from Lorentz symmetry should also consider the case of quadratic Planck-length suppression ( $n = 2$ ), and it is generally believed [11, 14] that it will never be possible to find contexts with sensitivity to quadratically- $L_p$ -suppressed effects.

The key point that I intend to convey here is that, contrary to these common beliefs, we might soon be ready for experimental studies with good sensitivity even to effects that are quadratically suppressed by the Planck length. This can be achieved even just reconsidering the mentioned time-of-arrival analyses and cosmic-ray spectrum analyses.

Let me start by reconsidering the analysis of the cosmic-ray spectrum. A large majority of related quantum-gravity studies focus on the fact that for  $n = 1$  one finds  $\Delta E_{th} \gg E_{th}$ , which would render possible [2, 3, 4, 10] the observation of cosmic rays much above the GZK limit of  $\sim 5 \cdot 10^{19} eV$  that one obtains on the basis of a standard estimate of  $E_{th}$ . The possibility to constrain the case  $n = 2$  was already considered in Ref. [3] and a possible role of the case  $n = 2$  in the cosmic-ray paradox was investigated in detail in Ref. [4], but these studies went largely unnoticed in this respect. Actually, even for  $n = 2$  one finds that  $\Delta E_{th} \sim E_{th}$ , *i.e.* the quantum-gravity correction is nonnegligible. This is easily verified by substituting the relevant energy scales in relation (3) for  $n = 2$ . While in the  $n = 1$  case the effect is very large, the case  $n = 2$  invites us to consider the case of a  $\Delta E_{th}$  which is comparable to (but not necessarily much larger than)  $E_{th}$ , and this could in turn require cosmic-ray observations with high-statistics information on the spectrum at energies close to the GZK limit. Since the Planck-scale  $n = 2$  correction  $\Delta E_{th}$  is not negligible, such detailed studies of the spectrum in the neighbourhood

of  $E_{th}$  would inevitably provide an opportunity for a significant test. Cosmic-ray observations such as the ones planned for the Auger observatory [21] do have the required capability for high-statistics studies of cosmic rays with energies close to  $E_{th}$ .

The corresponding “ $n = 2$  upgrade” of the time-of-arrival gamma-ray-burst studies actually requires an even more profound modification of the experimental strategy. As one can easily deduce from (2), in order to compensate for the extra power of  $L_p$  which is present in the case  $n = 2$  it would be necessary to compare the times of arrival of ultra-energetic particles emitted by a gamma-ray burster. Unfortunately, we only expect to observe gamma-ray-burst photons with energies up to the  $TeV$  scale, since at higher energies photons are efficiently absorbed before they reach the Earth. However current well-established models [22] predict that gamma-ray bursters should also emit a substantial amount of high-energy neutrinos. With advanced planned neutrino observatories, such as ANTARES [23], NEMO [24] and EUSO [25], it should be possible to correlate detections of high-energy neutrinos with corresponding detections of gamma-ray-burst photons. Neutrinos are of course immune from electromagnetic absorption, and therefore it is possible to observe neutrinos with energies between  $10^{14}$  and  $10^{19}$   $eV$ .

Models of gamma-ray bursters predict in particular a substantial flux of neutrinos with energies of about  $10^{14}$  or  $10^{15}$   $eV$ . Comparing the times of arrival of these neutrinos emitted by gamma-ray bursters to the corresponding times of arrival of low-energy photons the case  $n = 1$  would predict a huge<sup>2</sup> time-of-arrival difference ( $\Delta t \sim 1year$ ) and even for the case  $n = 2$  the expected time-of-arrival difference,  $\Delta t \sim 10^{-6}s$ , is within the realm of possibilities of future observatories.

Current models of gamma-ray bursters also predict some production of neutrinos with energies extending to the  $10^{19}eV$  level. For such ultra-energetic neutrinos a comparison of time-of-arrival differences with respect to soft photons also emitted by the burster should provide, assuming  $n=2$ , a signal at the level  $\Delta t \sim 1s$ , comfortably within the realm of timing accuracy of the relevant observatories. For this strategy relying on ultra-high-energy neutrinos the delicate point is clearly not timing, but rather the statistics (sufficient number of observed neutrinos) needed to establish a robust experimental result. Moreover, it appears necessary to understand gamma-ray bursters well enough to establish if there is a typical time delay after a gamma-ray-burst low-energy-photon trigger at which we should expect the arrival of neutrinos. The fact that this “time history” of the gamma-ray burst must be obtained only with precision of, say,  $1s$  (which is a comfortably large time scale with respect to the short time scales present in most gamma-ray bursts) suggests that this understanding should be achievable in the short-term

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<sup>2</sup>The strong implications of the case  $n = 1$  for neutrino astrophysics were already emphasized in Refs. [26, 27]. Ref. [27] did not at all consider the case  $n = 2$ , on which I focus here, while Ref. [26] did make a brief remark (see footnote 6) on the case  $n = 2$ , stressing that, according to the scheme there advocated, one could find no motivation for the case  $n = 2$  but in principle good sensitivity might be achieved through observations of high-energy neutrinos.

future. Hopefully the observations I am reporting here will provide additional motivation for these studies.

In summary, the next few years, following the strategies proposed here, should mark the beginning of a second generation of quantum-gravity-motivated Lorentz-symmetry studies, in which we explore even the possibility of effects suppressed quadratically by the Planck length. High-energy neutrino observatories can lead to very significant insight, which is particularly valuable since the time-of-arrival studies are not affected by the law of energy-momentum conservation, and are therefore applicable to studies of Planck-scale effects of type (1) with and without the emergence of a preferred class of inertial observers. As mentioned, a role for high-energy neutrino observations in quantum-gravity research requires some progress particularly in our understanding of gamma-ray bursts, and it is therefore an objective which we can realistically set presently as a goal and expect to achieve in a few years. Forthcoming improvements [21] in our knowledge of ultrahigh-energy cosmic rays are likely to be our first step into the realm of quadratically Planck-length-suppressed effects, but they will provide us indications that are applicable to a more limited class of quantum-gravity models, since threshold analyses are affected by the delicate issue of a possible role for the Planck length in the laws of energy-momentum conservation.

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