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The Light Gluino Mass Window Revisited

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Abstract

The precise measurements of the "electroweak observables" performed at LEP and SLC are well consistent with the standard model predictions. Deviations from the standard model arising from vacuum polarization diagrams (also called "weak loop corrections") have been constrained in a model-independent manner with the ε formalism. Within the same formalism, additional deviations from new physics production processes can also be constrained, still in a model-independent way. For instance, a 95% C.L. limit of

$$\Delta\Gamma_{\mathrm{had}} < 3.9\,\mathrm{MeV}$$

is set on the partial width of any purely hadronic exotic contribution to Z decays. When applied to the $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ process, it allows an absolute lower limit to be set on the gluino mass,

$$m_{\tilde{g}} > 6.3\,\mathrm{GeV}/c^2 \text{ at } 95\% \text{ C.L.},$$

which definitely closes the so-called light gluino mass window.

1 Introduction

The precise measurement of the Z total decay width at LEP and its agreement with its standard model prediction [1] (with $m_{\rm H} = 78^{+48}_{-31}\,{\rm GeV}/c^2$)

$$\Gamma_{\rm Z}^{\rm exp} = (2495.2 \pm 2.3)\,{\rm MeV} \quad \text{and} \quad \Gamma_{\rm Z}^{\rm SM} = (2495.9 \pm 2.4)\,{\rm MeV} \tag{1}$$

are often exploited to constrain the cross section of new physics processes [2]. Indeed, under the assumption that new physics contributions to the Z width are exclusively positive (as is the case for processes kinematically allowed at $\sqrt{s} = m_{\rm Z}$), this agreement allows a 95% confidence level (C.L.) limit of

$$\Delta\Gamma_{\rm Z} < 6.4\,{\rm MeV} \tag{2}$$

to be set on any exotic contribution to $\Gamma_{\rm Z}$.

However, extensions of the standard model, such as supersymmetry or technicolor, generate a whole set of new particles, which may or may not be produced in e^+e^- collisions at $\sqrt{s} = m_{\rm Z}$. The particles that are too heavy to be produced in Z decays may still contribute to the Z width through vacuum polarization diagrams with a generally undetermined sign. It may therefore well occur that negative contributions be sizeable and invalidate the widely used aforementioned limit on $\Delta\Gamma_{\rm Z}$.

It is the purpose of this note to derive model-independent limits on additional contributions to Z decays, and to use these limits to unambiguously constrain the light gluino mass window. It is indeed controversial if a light gluino $\tilde{g}$ of mass below $5\,{\rm GeV}/c^2$ is phenomenologically viable [3, 4]. A review of existing limits and of the related weak points can be found in Ref. [5]. In particular, a study of the QCD colour factors from four-jet angular correlations and the differential two-jet rate in Z decays, performed by ALEPH, allowed a 95% C.L. lower limit of $6.3\,{\rm GeV}/c^2$ to be set on $m_{\tilde{g}}$ [6]. However, it was argued by the light gluino defenders [3] that this limit was to be weakened because *(i)* the theory uncertainties were too aggressive; and *(ii)* no next-to-leading-order mass corrections were available for the four-jet angular correlations.

This note is organized as follows. In Section 2, the reader is reminded of the model-independent parametrization of the weak loop corrections to the electroweak observables according to the ε formalism [7]. This formalism is extended in Section 3 to the corrections caused by any new physics production process, in either the hadronic, the leptonic or the invisible final state. The result is applied to the $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ process in Section 4 and a model-independent lower limit on the gluino mass is obtained.

2 Parametrizing the weak loop corrections

Vacuum polarization contributions have been parametrized in a model-independent way by several authors. Here, the choice was made to parametrize the basic "electroweak observables", *i.e.*, those sensitive to the weak loop corrections, with the (linearized) ε formalism, according to [7]

$$\begin{aligned}
\Gamma_Z &= \Gamma_Z^0\,(1+1.35\varepsilon_1-0.46\varepsilon_3+0.35\varepsilon_b), && (3)\\
R_\ell &= R_\ell^0\,(1+0.28\varepsilon_1-0.36\varepsilon_3+0.50\varepsilon_b), && (4)\\
\sigma_{had} &= \sigma_{had}^0\,(1-0.03\varepsilon_1+0.04\varepsilon_3-0.20\varepsilon_b), && (5)\\
g_V/g_A &= (g_V/g_A)^0\,(1+17.6\varepsilon_1-22.9\varepsilon_3), && (6)\\
R_b &= R_b^0\,(1-0.06\varepsilon_1+0.07\varepsilon_3+1.79\varepsilon_b), && (7)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_Z^0 &= 2489.46\,(1+0.73\delta\alpha_S-0.35\delta\alpha)\,\mathrm{MeV}, && (8)\\
R_\ell^0 &= 20.8228\,(1+1.05\delta\alpha_S-0.28\delta\alpha), && (9)\\
\sigma_{had}^0 &= 41.420\,(1-0.41\delta\alpha_S-0.03\delta\alpha)\,\mathrm{nb}, && (10)\\
(g_V/g_A)^0 &= 0.075619-1.32\delta\alpha, && (11)\\
R_b^0 &= 0.2182355, && (12)
\end{aligned}$$

are the Born approximations of the corresponding observables, *i.e.,* without any weak loop corrections, and where the pure QCD- and QED-corrections were parametrized as

$$\delta\alpha_S = \frac{\alpha_S(m_Z)-0.119}{\pi} \quad \text{and} \quad \delta\alpha = \frac{\alpha(m_Z)-\frac{1}{128.90}}{\alpha(0)}. \qquad (13)$$

In the standard model, or in any theory that does not predict new open processes in e^+e^- collisions at $\sqrt{s}=m_Z$, the three ε's can then be fit to the precise measurements of LEP and SLC [1], summarized in Table 1. In this fit, the value of the strong and

Table 1: Precise LEP and SLC measurements of the Z lineshape parameters (Γ_Z, R_ℓ, σ_{had}), of g_V/g_A and of R_b, together with their correlation matrix. The last two measurements have been taken here as uncorrelated with the first three [8].

Observable	Measurement	Correlation matrix				
Γ_Z	2495.2 ± 2.4 MeV	1.00				
R_ℓ	20.767 ± 0.025	−0.023	1.00			
σ_{had}	41.540 ± 0.037 nb	−0.045	−0.297	1.00		
g_V/g_A	0.07408 ± 0.00068	0.00	0.00	0.00	1.00	
R_b	0.21644 ± 0.00065	0.00	0.00	0.00	0.00	1.00

electromagnetic coupling constants were taken to be

$$\alpha_S(m_Z) = 0.1183 \pm 0.0020\ [9] \quad \text{and} \quad \alpha(m_Z)^{-1} = 128.95 \pm 0.05\ [1]. \qquad (14)$$

The validity of the latter is ensured in extensions of the standard model with only heavy new particles by the decoupling properties of QED, which allow the heavy particle contributions to be safely neglected in the running of α from 0 to m_Z. The value of α_S is well constrained by measurements performed directly at the Z resonance and does not suffer from this kind of uncertainties.

The result of the fit, given in Table 2, is consistent with that presented in Ref. [7], up to small deviations (less than 1σ or thereabout) caused by recent updates of the measurements and different variables included in the fit.

Table 2: Result of the fit of the ε's to the precise measurements of the five observables of Table 1. Also indicated, for comparison, is the result presented in Ref. [7]

	$\varepsilon_1 \times 10^3$	$\varepsilon_3 \times 10^3$	$\varepsilon_{\rm b} \times 10^3$
This fit	5.4 ± 1.0	5.3 ± 0.9	-5.5 ± 1.4
Ref [7]	4.3 ± 1.2	4.5 ± 1.1	-3.8 ± 1.9

3 Model-independent limits on additional Z decays

The fitted ε values are usually interpreted in the standard model to predict the value of the Higgs boson mass, or to constrain new theories in which additional vacuum polarization diagrams would modify the ε's. Here, advantage is taken of the redundancy of the quantities in Eqs. (4) to (8) to set instead model-independent limits on additional Z decays, which would be caused by the existence of new particles light enough to be produced in $\rm e^+e^-$ collisions at $\sqrt{s} = m_{\rm Z}$.

For instance, such new particles could be produced and decay in such a way that they contribute only to hadronic Z decays (all quark flavours). Let $\varepsilon_{\rm NP}^{\rm had}$ be the ratio of this new partial width $\Gamma_{\rm NP}$ to the total decay width of the Z without this new contribution. The first three observables are changed as follows,

$$\Gamma_{\rm Z} \longrightarrow \Gamma_{\rm Z}\left(1 + 1.00\varepsilon_{\rm NP}^{\rm had}\right), \qquad [\Gamma_{\rm Z} + \Gamma_{\rm NP}] \tag{15}$$

$$R_\ell \longrightarrow R_\ell\left(1 + 1.43\varepsilon_{\rm NP}^{\rm had}\right), \qquad [(\Gamma_{\rm had} + \Gamma_{\rm NP})/\Gamma_\ell] \tag{16}$$

$$\sigma_{\rm had} \longrightarrow \sigma_{\rm had}\left(1 - 0.57\varepsilon_{\rm NP}^{\rm had}\right), \qquad \left[\frac{12\pi}{m_{\rm Z}^2}\frac{\Gamma_{\rm ee}(\Gamma_{\rm had} + \Gamma_{\rm NP})}{(\Gamma_{\rm Z} + \Gamma_{\rm NP})^2}\right] \tag{17}$$

while (g_V/g_A) and $R_{\rm b}$ remain untouched. If the technical definition of the original ε's is modified with respect to [7] in such a way that they still only account for the weak loop corrections, these changes modify in turn Eqs. (4) to (6) according to

$$\Gamma_{\rm Z} = \Gamma_{\rm Z}^0\,(1 + 1.35\varepsilon_1 - 0.46\varepsilon_3 + 0.35\varepsilon_{\rm b} + 1.00\varepsilon_{\rm NP}^{\rm had}), \tag{18}$$

$$R_\ell = R_\ell^0\,(1 + 0.28\varepsilon_1 - 0.36\varepsilon_3 + 0.50\varepsilon_{\rm b} + 1.43\varepsilon_{\rm NP}^{\rm had}), \tag{19}$$

$$\sigma_{\rm had} = \sigma_{\rm had}^0\,(1 - 0.03\varepsilon_1 + 0.04\varepsilon_3 - 0.20\varepsilon_{\rm b} - 0.57\varepsilon_{\rm NP}^{\rm had}), \tag{20}$$

and Eqs. (7) and (8) still apply. Similarly, a new physics contribution to the sole invisible decay width would modify the equations according to

$$\Gamma_{\rm Z} = \Gamma_{\rm Z}^0\,(1 + 1.35\varepsilon_1 - 0.46\varepsilon_3 + 0.35\varepsilon_{\rm b} + 1.00\varepsilon_{\rm NP}^{\rm inv}), \tag{21}$$

$$\sigma_{\rm had} = \sigma_{\rm had}^0\,(1 - 0.03\varepsilon_1 + 0.04\varepsilon_3 - 0.20\varepsilon_{\rm b} - 2.00\varepsilon_{\rm NP}^{\rm inv}), \tag{22}$$

and a new physics contribution to the leptonic decay width only (democratically in the three lepton flavours) to

$$\Gamma_Z = \Gamma_Z^0\,(1 + 1.35\varepsilon_1 - 0.46\varepsilon_3 + 0.35\varepsilon_b + 1.00\varepsilon_{NP}^{\ell}), \quad (23)$$

$$R_\ell = R_\ell^0\,(1 + 0.28\varepsilon_1 - 0.36\varepsilon_3 + 0.50\varepsilon_b - 9.89\varepsilon_{NP}^{\ell}), \quad (24)$$

$$\sigma_{had} = \sigma_{had}^0\,(1 - 0.03\varepsilon_1 + 0.04\varepsilon_3 - 0.20\varepsilon_b + 7.89\varepsilon_{NP}^{\ell}). \quad (25)$$

In each of the three cases, the new physics contribution ε_{NP} can be fitted together with the other three ε's to the five measured quantities. The results of the three fits, all compatible with $\varepsilon_{NP} = 0$, are listed in Table 3. Conservative upper limits on the ε_{NP}'s (*i.e*, on the new physics branching fractions) were derived at the 95% confidence level by integrating their probability density functions in the physical region ($\varepsilon_{NP} > 0$) only. These limits are reported in Table 4, together with the corresponding limits on the new physics partial width and on the new physics cross section at the Z peak. For completeness, 3σ- and 5σ-limits are also indicated.

Table 3: Results of the fits of the ε's to the precise measurements of the five observables of Table 1 when new physics Z decays are added, either in the hadronic, leptonic or invisible final state.

Decay	$\varepsilon_1 \times 10^3$	$\varepsilon_3 \times 10^3$	$\varepsilon_b \times 10^3$	$\varepsilon_{NP} \times 10^3$
Hadronic	5.7 ± 1.0	5.5 ± 1.0	-4.6 ± 1.7	-0.70 ± 1.00
Leptonic	5.0 ± 1.3	5.1 ± 1.1	-5.6 ± 1.6	0.042 ± 0.031
Invisible	5.4 ± 1.0	5.4 ± 0.9	-4.4 ± 1.4	-0.91 ± 0.48

Table 4: Limits at 95% C.L. on the new physics branching ratio, partial width and cross section at the Z peak, in the hadronic, the leptonic and the invisible final states. The limits at 3σ (99.63% C.L.) and 5σ (99.99994% C.L.) are also indicated.

Final state	Hadronic	Leptonic	Invisible
ΔBR_{95} (10^{-3})	1.56	0.31	0.54
$\Delta\Gamma_{95}$ (MeV)	3.9	0.77	1.33
$\Delta\sigma_{95}$ (pb)	66.9	13.2	22.9
$\Delta BR_{3\sigma}$ (10^{-3})	2.52	0.43	0.93
$\Delta\Gamma_{3\sigma}$ (MeV)	6.3	1.07	2.32
$\Delta\sigma_{3\sigma}$ (pb)	108.	18.3	39.8
$\Delta BR_{5\sigma}$ (10^{-3})	4.45	0.65	1.78
$\Delta\Gamma_{5\sigma}$ (MeV)	11.1	1.61	4.45
$\Delta\sigma_{5\sigma}$ (pb)	191.	27.5	76.5

Because the existence of new particles could also modify the evolution of the electromagnetic coupling constant from 0 to m_Z, it was checked whether these limits could be affected by a variation of $\alpha(m_Z)$. This check is illustrated in Fig. 1a, where it appears that the standard model value of α yields the largest upper limits on the three partial widths. The limits of Table 4 are therefore conservative in this respect.

New light hadronic flavours would also modify the evolution of the strong coupling constant. Although, as already mentioned, α_S is constrained by measurements performed directly at the Z mass scale, it is also determined with at least as accurate low energy measurements extrapolated at m_Z. A different scaling law for the latter would modify the world average of α_S. However, a contribution from new coloured scalars or fermions would always slow down the running from low energy to m_Z so as to increase the value of $\alpha_S(m_Z)$ [10]. As shown in Fig. 1b, such an increase of α_S would render more constraining the limits on the hadronic and invisible partial widths. The limit on the leptonic width would be slightly weakened, but would anyway remain the strongest of the three constraints. The conservative choice of ignoring this effect was made throughout.

Finally, it may be argued that a new hadronic contribution to Z decays would decrease the value of α_S fitted from the Z lineshape. However, the α_S world average and its uncertainty (saturated by common theory errors), and therefore the result of the present fit, do not change noticeably when this measurement in taken out.

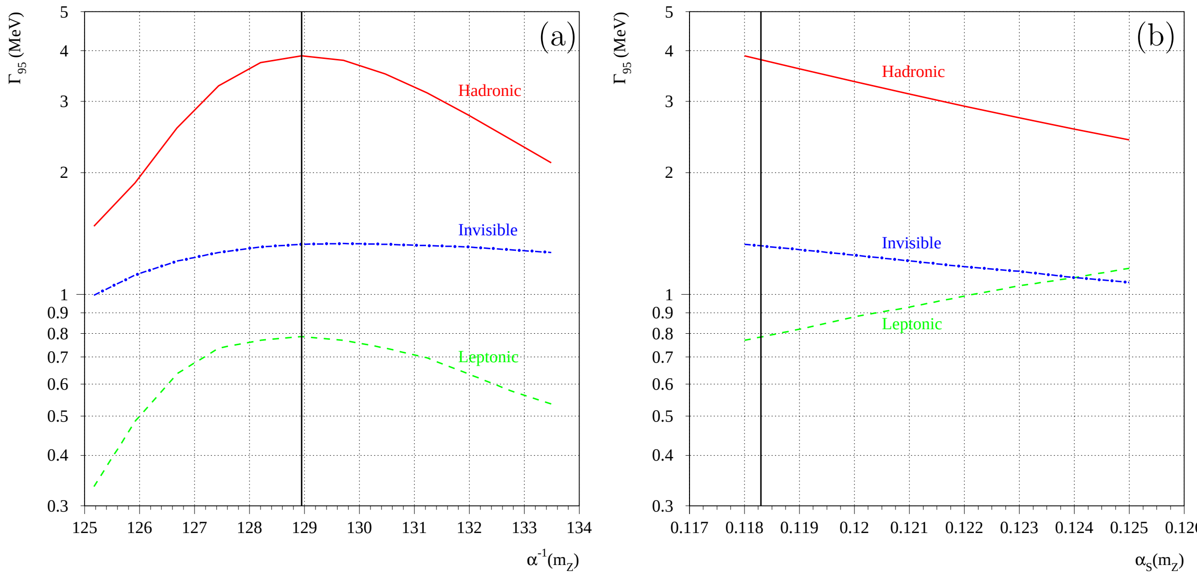

Figure 1: The 95% C.L. limits on the new physics partial width, in the hadronic (full curve), the leptonic (dashed curve) and the invisible (dot-dashed curve) as a function of (a) $1/\alpha(m_Z)$ and (b) $\alpha_S(m_Z)$. The vertical lines indicate the standard model values of the coupling constants, chosen to perform the fits.

The fit of $\varepsilon_{\rm NP}$ can be repeated in any other configuration of hadronic, leptonic and invisible contributions to the Z decays from the new physics process. Let $x_{\rm had}$, x_ℓ and $x_{\rm inv}$ be the fractions of hadronic, leptonic and invisible final states produced by the new physics process under consideration. By definition, a final state which is neither hadronic nor leptonic is called invisible, therefore $x_{\rm inv} + x_{\rm had} + x_\ell = 1$. The Z lineshape parameters of Eqs. (4) to (6) are modified according to

$$\Gamma_{\rm Z} \longrightarrow \Gamma_{\rm Z}\,[1+\varepsilon_{\rm NP}]\,, \tag{26}$$
$$R_\ell \longrightarrow R_\ell\,[1+\varepsilon_{\rm NP}\,(1.43x_{\rm had} - 9.89x_\ell)]\,, \tag{27}$$
$$\sigma_{\rm had} \longrightarrow \sigma_{\rm had}\,[1+\varepsilon_{\rm NP}\,(-2.00 + 1.43x_{\rm had} + 9.89x_\ell))]\,. \tag{28}$$

The (x_{had}, x_{inv}) plane was scanned and the fit performed at each point, yielding a limit on ε_{NP} everywhere in this plane. The corresponding 95% C.L. limit on $\Delta\Gamma_{\text{Z}}$, the new physics contribution to the Z total width, is displayed in Fig. 2a. Similarly, the limit on $\Delta\Gamma_{\text{Z}}$ under the hypothesis that the new particle production contributes to hadronic and invisible final states, and to only one lepton flavour (μ or τ), is shown in Fig. 2b.

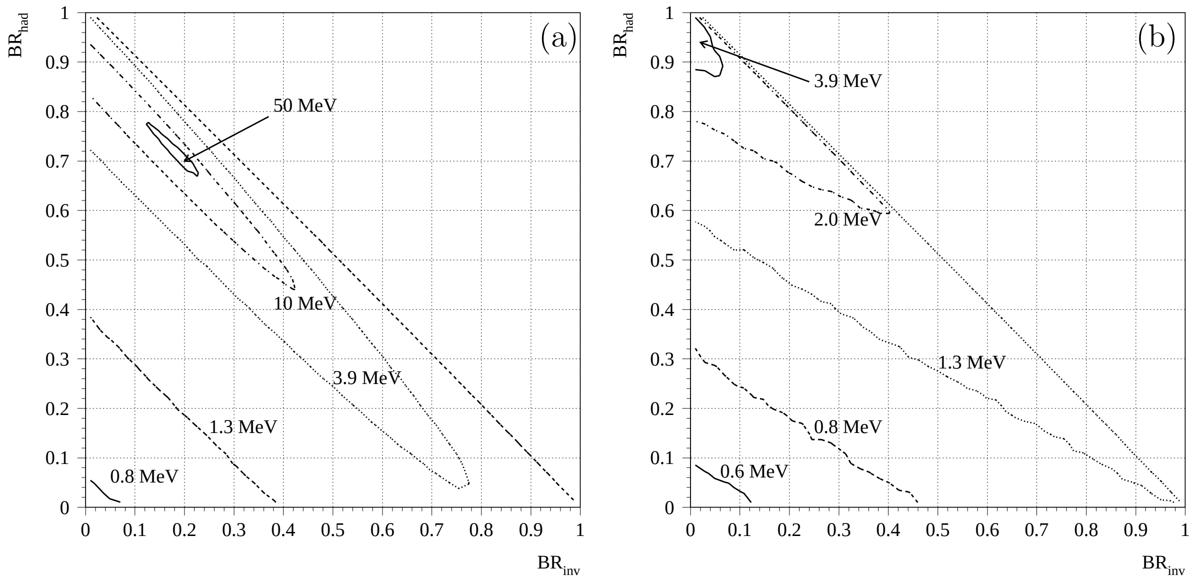

Figure 2: The 95% C.L. limit on the new physics contribution to the total decay width, as a function of the fraction of hadronic and invisible final states arising from the new particle production, with (a) leptonic decays democratic in the three flavours and (b) leptonic decays in only one flavour (μ or τ).

Very constraining limits on $\Delta\Gamma_{\text{Z}}$ are set all over the plane, but no absolute limit can be obtained when the new particle production leads to fractions in the hadronic, invisible and leptonic (three flavours) final states identical to the Z branching fractions. In this case, only Γ_{Z} depends on ε_{NP}. Equations (5) to (8) no longer yield an independent determination of ε_1 and ε_3 with meaningful accuracy, because they all depend on the same linear combination of the two quantities. As a result, the new particle contribution to the Z width can always be cancelled by the $(1.35\varepsilon_1 - 0.46\varepsilon_3)$ virtual contribution, if a sufficient amount of fine tuning takes place. Whether or not this amount of fine tuning is acceptable would need different measurements and/or more theory to decide.

4 A model-independent limit on the gluino mass

The results obtained in Section 3 can be applied to a variety of new processes. In this note, they are used to constrain the cross section of the gluino production at $\sqrt{s} = m_{\text{Z}}$ in the process

$$\text{e}^+\text{e}^- \rightarrow \text{q}\bar{\text{q}}\tilde{\text{g}}\tilde{\text{g}}, \tag{29}$$

displayed in Fig. 3 as a function of the gluino mass (from Ref. [11]).

When the gluino is light, the final state arising from this process is purely hadronic irrespective of the gluino decay and hadronization, and therefore contributes solely to the

Z hadronic decay width in all quark flavours. A 95% C.L. upper limit on the production cross section at $\sqrt{s} = m_Z$ can then be set at 67 pb (Table 4). The corresponding lower limit on the gluino mass can be read off from the curve in Fig. 3, and is

$$m_{\tilde{g}} > 6.3\,\mathrm{GeV}/c^2 \text{ at } 95\% \text{ C.L.,} \quad (30)$$
$$m_{\tilde{g}} > 4.9\,\mathrm{GeV}/c^2 \ (3\sigma \text{ limit}), \quad (31)$$
$$m_{\tilde{g}} > 3.6\,\mathrm{GeV}/c^2 \ (5\sigma \text{ limit}). \quad (32)$$

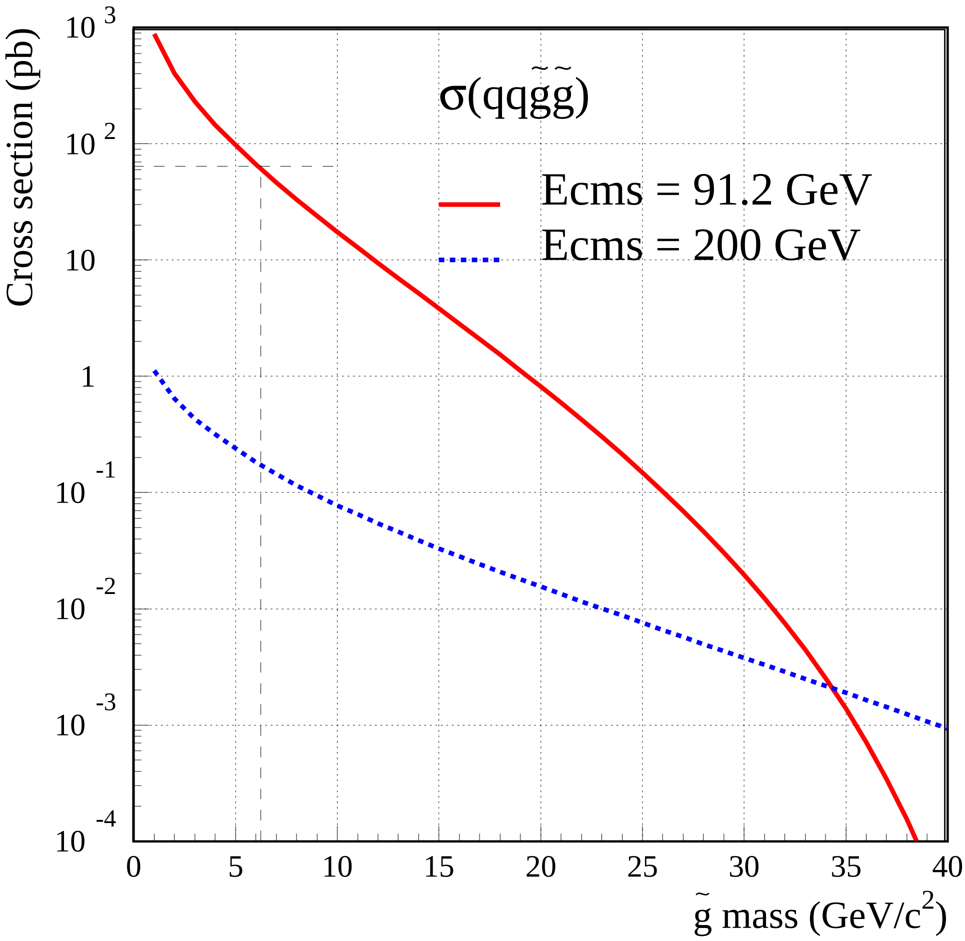

Figure 3: The production cross section of the process $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$, at $\sqrt{s} = m_Z$ (full curve) and 200 GeV (dashed curve). Also indicated is the 95% C.L. upper limit on this cross section from precise LEP and SLC measurements and the corresponding upper limit on the gluino mass (long-dashed line).

This limit confirms the one obtained by ALEPH [6] with an independent study of the QCD colour factors, which makes no use of the absolute Z decay rates, and that derived from the running of the strong coupling constant [12], which checks in addition the compatibility of the α_S measurements at all energy scales. Because it would add fully independent information and because it would avoid the conservative choices of Section 3 to be made, a combination of these results would further consolidate the light gluino exclusion. For instance, a combination of the present limit and that of Ref. [6] yields lower limits on $m_{\tilde{g}}$ of 6.8 GeV/c^2 and 5.5 GeV/c^2 at 95% and 99.63% C.L., respectively.

It is only if at least three different fine tuning processes took place, *i.e.,* if

1. other new particles were produced in association with the gluino with a cross section of the order of or larger than that of the gluino production, but still were not directly detected at LEP;

2. these processes led to final states such that the overall fractions of hadronic, invisible and leptonic new decays be similar to those of the Z decays, for all lepton and quark flavours;

3. additional new physics yielded large negative virtual contributions to the Z total decay width (from the $1.35\varepsilon_1 - 0.46\varepsilon_3$ combination) to exactly compensate this multiple new particle production;

that the limit derived with the method presented in this note would not hold. I leave it to the champions of the light gluino scenario to find a theory in which this devilish conspiracy could take place.

5 Conclusion

A method to derive model-independent limits on new physics contributions to Z decays has been presented. No general upper limit on the total Z decay width could be obtained, but very stringent constraints apply when the final states produced by the new physics process of interest are known. In particular, conservative upper limits have been put on Γ_Z of 0.55, 1.3 and 3.9 MeV in the case of purely leptonic (μ or τ), invisible and hadronic final states.

When applied to the $e^+e^- \to Z \to q\bar{q}\tilde{g}\tilde{g}$ process, it allows a model-independent lower limit to be set on the gluino mass:

$$m_{\tilde{g}} > 6.3\,\mathrm{GeV}/c^2 \text{ at } 95\%\,\mathrm{C.L.}$$

The light gluino mass window is closed.

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