

The Cosmological Constant and the Deconstruction of Gravity

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Abstract

Witten has presented an argument for the vanishing of the cosmological constant in $2 + 1$ dimensions. This argument is crucially tied to the specific properties of $(2 + 1)$ -dimensional gravity. We argue that this reasoning can be deconstructed to $3 + 1$ dimensions under certain conditions. Our observation is also tied to a possibility that there exists a well-defined UV completion of $(3 + 1)$ -dimensional gravity.

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1 Introduction and Summary

The cosmological constant has been an enigma in theoretical physics since it was first realized that in any simple field theoretic notion of quantum gravity, power divergences lead to large renormalization, sensitive to the largest scales available in the theory [1]. In terms of naive power-counting, the vacuum energy corresponds to a relevant operator. One might go further to say that a lack of understanding of power divergences is at the root of each of the basic theoretical problems in particle physics including the various hierarchy problems and the aforementioned cosmological constant problem. It is important to realize that in the case of power divergences, it is not enough to come up with a mechanism for canceling the parameter at a given scale; rather, it must be canceled all the way into the infrared (IR).

The cosmological constant problem has become perhaps even more acute given the recent astronomical data suggesting the existence of a positive but small vacuum energy density, being roughly in proportion to the present mass density of the universe [2]. Thus we are faced with a two-fold cosmological constant problem [1]: first, why is the vacuum energy small and second, why is the vacuum energy in proportion to the current mass density? In this article we will address the first question.

Field theories that are non-renormalizable (and hence ill-defined in the ultraviolet (UV)) may be defined through a certain process of dimensional reduction referred to as deconstruction [3]. This has been demonstrated in theories with internal gauge symmetries, for example, in the context of five-dimensional Yang-Mills theories. One dimension of the IR theory is put on a lattice and the resulting theory may be thought of as a Goldstone realization of a UV four-dimensional gauge theory. In this way, the continuum higher dimensional theory is thought of as the *infrared* limit of a lower dimensional theory. An important aspect of these constructions then is motivating why the theory has this infrared behavior.

It is enticing to think of gravity in this context: from a four-dimensional field theoretic point of view, Einstein's general relativity is famously perturbatively non-renormalizable. In order to extend the deconstruction ideas to gravity, we must confront the spacetime general coordinate and Lorentz symmetries. Thus in this paper, we explore the idea that four-dimensional quantum gravity may be *defined* through deconstruction. It is particularly convenient to work in the vierbein formalism. It was shown [4] long ago that three-dimensional gravity is a Chern-Simons (CS) gauge theory and thus is a well-defined quantum theory. In particular its UV character is sensible because it is topological. The deconstruction to $3 + 1$ dimensions would follow the path of regarding a three-dimensional theory (a close cousin of CS gravity coupled to matter) as a lattice version of a four-dimensional theory.

Of course, there exists a rather large body of evidence that gravitational theories should be thought of, in some way, as local theories in one fewer dimension. First, a purely gravitational theory has no local degrees of freedom in the usual sense of a local quantum field theory. In the work of 't Hooft [5] and Susskind [6], it was realized that an interpretation of this is that a gravitational theory is holographic

— the observables are not extensive, but related to co-dimension one structures. This is of course supported by the thermodynamics of black holes, where entropy is proportional to the area of the event horizon [7]. But most impressively, the idea has been given a concrete realization in the *AdS/CFT* construction and its relatives [8].

In the case of four-dimensional gravity, we might then try to follow this path directly and construct its physics in terms of a three-dimensional field theory. In realistic models with positive cosmological constant, this might mean some version of a de Sitter/Euclidean CFT correspondence [9]. Although the existence of such a correspondence has not been established conclusively, a variety of consequences have been considered in Refs. [10].

Note that the existing holographic duals of gravity, provided by the *AdS/CFT* correspondence, are defined in terms of non-gravitational theories. On the other hand, there are proposals for a non-perturbative definition of gravity which involve gravitational degrees of freedom. Perhaps the most notable example of this type is Matrix theory [11], which in some sense can be viewed as an example of “bulk” holography.

Finally, more than twenty years ago Weinberg suggested the idea of “asymptotic safety” which essentially advocates the existence of a UV fixed point for $(3 + 1)$ -dimensional gravity [12], in the sense of a Wilson-Fisher ϵ -expansion.¹ One of the main points of this article is precisely the claim that $(3 + 1)$ -dimensional gravity does indeed have a short distance fixed point given in terms of $(2 + 1)$ -dimensional gravity coupled to $(2 + 1)$ -dimensional matter.

One might be initially puzzled by a suggestion that $(3 + 1)$ -dimensional gravity can be defined in terms of $(2 + 1)$ -dimensional gravity coupled to $(2 + 1)$ -dimensional matter. After all, $(3 + 1)$ -dimensional gravity has local degrees of freedom. However, $(2 + 1)$ -dimensional gravity, viewed as a CS gauge theory, is purely topological. There are no propagating, local gravitational degrees of freedom. How can then a $(2 + 1)$ -dimensional theory of matter coupled to gravity account for the local, propagating, $(3 + 1)$ -dimensional degrees of freedom, such as gravitational waves? What our proposal suggests is that “most” of the degrees of freedom of $(3 + 1)$ -dimensional gravitational theory arise from the non-gravitational part of its $(2 + 1)$ -dimensional UV completion. The UV completion of $(3 + 1)$ -dimensional gravity is “holographic” in this sense.

If four-dimensional gravity may be thought of as a three-dimensional theory in a useful way, what of the cosmological constant? Several years ago, Witten [13] observed that peculiar properties of $(2 + 1)$ -dimensional gravity can lead to vanishing vacuum energy in $2 + 1$ dimensions. No precise mechanism for connecting this to four dimensions has been presented, although Witten’s context was firmly rooted in the duality between M-theory and the strong coupling limit of Type IIA or heterotic strings [14]. Can this mechanism be used instead in our context to provide insight into the vacuum energy in four dimensions? In this note we argue that Witten’s

¹In particular, by considering $(2 + \epsilon)$ -dimensional gravity.

reasoning can be deconstructed to $3 + 1$ dimensions under very specific conditions.²

The crucial observation we make in this paper is that provided one can define a UV completion of $(3 + 1)$ -dimensional gravity in terms of purely $(2 + 1)$ -dimensional gravitational and matter data, then the argument of Witten can be deconstructed to $3 + 1$ dimensions. We motivate our argument by recalling a remarkable fact from *classical* general relativity which states that in the presence of a space-like Killing field, $3 + 1$ vacuum general relativity is equivalent to $(2 + 1)$ -dimensional general relativity coupled to an $SO(2, 1)$ non-linear σ -model [16, 17]. We then proceed to provide a *quantum* analogue of this classical theorem and argue that a full quantum theory of $(3 + 1)$ -dimensional general relativity can be defined at short distance in terms of $(2 + 1)$ -dimensional gravity coupled to $(2 + 1)$ -dimensional matter. This then provides support for the claim that Witten’s observation about the vanishing $(2 + 1)$ -dimensional vacuum energy may be also valid in the world we observe.

2 The Cosmological Constant in $2 + 1$ Dimensions

It was observed by Witten [13] that supersymmetry in $2 + 1$ dimensions can lead to vanishing vacuum energy in the absence of a mass degenerate spectrum of bosonic and fermionic states. The vacuum state is supersymmetric, and therefore the cosmological constant is zero, but the excited states are not mass degenerate because unbroken global supercharges do not exist. Having unbroken global supercharges in the theory, which is what leads to the mass degeneracy of the bose-fermi spectrum in the first place, necessitates the existence of spinor fields that are covariantly constant at infinity. In $2 + 1$ dimensions any excited state gives a conical geometry whose deficit angle prohibits spinor fields with covariantly constant asymptotics. Thus, there is no mass degeneracy of bose-fermi excitations. The non-degeneracy of the spectrum of low-energy excitations is, however, suppressed by powers of the three-dimensional Newton constant under the assumption of weak gravitational coupling [14].

Although a precise realization of Witten’s argument about a supersymmetric vacuum with non-supersymmetric excitations apparently does not exist in the literature, Becker, Becker, and Strominger [18] provide an instructive construction with a solitonic ground state.³ We briefly review their considerations.

Becker, Becker and Strominger considered an $N = 2$ abelian Higgs model in $2 + 1$ dimensions [20] and studied a Nielsen-Olesen vortex [21] configuration in this theory. The solitonic configuration breaks half the supersymmetry. When this model is coupled to supergravity, the $(2 + 1)$ -dimensional gravitational background of this soliton has a particular asymptotic behavior describing a conical geometry

$$ds^2 = -dt^2 + \frac{dzd\bar{z}}{|z|^{2M/M_{Pl}}}, \quad (1)$$

²This would perhaps imply alternative interpretations of the recent astronomical data [2]. Such interpretations are explored in Ref. [15].

³For a related discussion see Ref. [19].

where $M = v^2 n$, with v the expectation value of the Higgs field and $n > 0$, is proportional to the soliton mass and M_{Pl} is the three-dimensional Planck mass. The geometry has deficit angle $\delta = 2\pi M/M_{Pl}$, and the soliton saturates the BPS bound. The gravitino gives rise to an Aharonov-Bohm phase that exactly cancels the geometric phase associated to the deficit angle of the conical singularity. However, the fermionic zero mode is not normalizable and is absent from the physical spectrum. Thus, there is no $N = 1$ supermultiplet of the unbroken supersymmetry. In this way, Witten's observation holds and the bose-fermi degeneracy of the excited states is lifted even though the solitonic ground state has zero vacuum energy.

3 Deconstructing Gauge Theories: A Summary

Before we discuss the case of $(3 + 1)$ -dimensional gravity, let us review the gauge theory case from a slightly different point of view than the original presentation [3].

Consider a gauge theory action

$$S = -\frac{1}{2g_d^2} \int d^{d-1}x dy \operatorname{tr} F_{AB}^2(x, y). \quad (2)$$

We use the notation $x^A \equiv \{x^\mu, y\}$. We wish to arrive at a theory on the space $(\mathbf{R}^{d-1} \times \Gamma)$. We then latticize y with lattice spacing a :

$$S = -\frac{a}{2g_d^2} \int d^{d-1}x \sum_j \operatorname{tr} (F_{\mu\nu,j}^2(x) + 2F_{5\nu,j}^2). \quad (3)$$

In the continuum,

$$F_{5\nu,j} = \partial_5 A_\nu - \partial_\nu A_5 + i[A_5, A_\nu]. \quad (4)$$

We define a link variable in the usual way:

$$U_{j,j+1} = \exp\left(i \int dy A_5(x, y)\right) \simeq 1 + iaA_{5,j} + \dots \quad (5)$$

Therefore

$$F_{5\nu,j} \simeq \frac{1}{a} (A_{\nu,j+1} - A_{\nu,j}) + \frac{i}{a} \partial_\nu U_{j,j+1} + \frac{1}{a} (U_{j,j+1} - 1) A_{\nu,j+1} - \frac{1}{a} A_{\nu,j} (U_{j,j+1} - 1) + \dots \quad (6)$$

(the ellipses contains less relevant terms for small a) and so

$$F_{5\nu,j} \simeq \frac{1}{a} (i\partial_\nu U_{j,j+1} + U_{j,j+1} A_{\nu,j+1} - A_{\nu,j} U_{j,j+1}) \quad (7)$$

$$= \frac{i}{a} D_\nu U_{j,j+1}. \quad (8)$$

As a result:

$$S = -\frac{a}{2g_d^2} \int d^{d-1}x \sum_j \operatorname{tr} \left(F_{\mu\nu,j}^2(x) - \frac{2}{a^2} |D_\nu U_{j,j+1}|^2 \right) \quad (9)$$

$$= -\frac{1}{2g_{d-1}^2} \int d^{d-1}x \sum_j \operatorname{tr} F_{\mu\nu,j}^2(x) + f_\pi^2 \int d^{d-1}x \sum_j \operatorname{tr} |D_\nu U_{j,j+1}|^2, \quad (10)$$

where

$$\frac{1}{g_{d-1}^2} = \frac{a}{g_d^2}, \quad f_\pi^2 = \frac{1}{ag_d^2} = \frac{1}{a^2 g_{d-1}^2}. \quad (11)$$

Let us redo this computation, as there is actually some trickery involved in the above continuum calculation.

To achieve this, we put the entire theory on a lattice and take the continuum limit in all but the y direction (which retains lattice spacing a):

$$S_{latt} = \sum_P \sigma_P \left(-1 + \frac{1}{2N} \text{tr}(U_P + U_P^\dagger) \right), \quad (12)$$

where $\text{tr} 1 = N$ and σ is an appropriate numerical scaling factor. P denotes a plaquette, which we can think of as a sum over lattice points, and a sum over pairs of directions \hat{A}, \hat{B} and

$$U_{AB}(n) = U_A(n)U_B(n + \hat{A})U_A^\dagger(n + \hat{B})U_B^\dagger(n), \quad (13)$$

where $U_A(n)$ is a link field, which in the continuum limit goes to the Wilson line. In the present case, we split the index A into $\mu, 5$ with lattice spacings ϵ, a . There are two types of terms in eq. (12):

$$S_{latt} = \sum_{\mu\nu} \sigma_{\mu\nu} \left(-1 + \frac{1}{2N} \text{tr}(U_{\mu\nu} + U_{\mu\nu}^\dagger) \right) + \sum_{\mu} \sigma_{\mu 5} \left(-1 + \frac{1}{2N} \text{tr}(U_{\mu 5} + U_{\mu 5}^\dagger) \right). \quad (14)$$

The first term will go in the continuum limit to $-\frac{\sigma_{\mu\nu}}{2}\epsilon^4 \text{tr} F_{\mu\nu}^2$, while the second term yields $-\frac{\sigma_{\mu 5}}{2}\epsilon^2 \text{tr} D_\mu U_5 (D_\mu U_5)^\dagger$, where

$$D_\mu U_5 = \partial_\mu U_5 + i(A_{\mu,j} U_5 - U_5 A_{\mu,j+1}). \quad (15)$$

Thus, we arrive at

$$S_{latt} = - \sum_{\mu\nu} \frac{\sigma_{\mu\nu}}{2} \epsilon^{5-d} \int d^{d-1}x \sum_j \left\{ \text{tr} F_{\mu\nu,j}^2(x) + \frac{\sigma_{\mu 5}}{\sigma_{\mu\nu} \epsilon^2} \text{tr} |D_\mu U_5|^2 \right\}. \quad (16)$$

By appropriate scalings of the parameters, we may obtain:

$$S = - \frac{1}{2g_{d-1}^2} \int d^{d-1}x \sum_j \text{tr} F_{\mu\nu,j}^2(x) + f_\pi^2 \int d^{d-1}x \sum_j \text{tr} |D_\nu U_5|^2 \quad (17)$$

with $f_\pi = 1/(g_{d-1}a)$.

We note that the link field U_5 is a bifundamental, transforming as $U_5 \rightarrow V_j U_5 V_{j+1}^{-1}$. The essential non-perturbative information used at this point is that fermion condensation can induce the effective σ -model action in the IR. This then points to the degrees of freedom of an $SU(N)$ quiver theory [3]. Thus the UV completion of a non-renormalizable five-dimensional gauge theory is a very specific quiver theory [3].

Unfortunately, if we wish to obtain the continuum limit in the quantum theory, we must take $a \rightarrow 0$ holding g_d fixed. This scales $g_{d-1} \rightarrow \infty$, and thus the infrared dynamics is in fact significantly different than the classical theory would indicate.

4 Towards a UV Completion of $(3 + 1)$ -dimensional Einstein-Hilbert Gravity

Now we are ready to address the question of deconstruction of $(3 + 1)$ -dimensional gravity from the point of view of Ref. [3]. As we have reviewed above, the scenario has been initially applied to certain non-renormalizable gauge theories. Given a certain set of similarities between pure gravity and non-abelian gauge theories it is natural to wonder whether the deconstruction techniques can be successfully applied to gravity [22].

There exist many similarities between gravity and gauge theory. These are evident, for example, in the MacDowell-Mansouri approach [23]; the approach to $3 + 1$ gravity based on Ashtekar variables [24]; the approach to $(2 + 1)$ -dimensional general relativity based on CS theory [4]; the close relation between the topological BF theory and gravity in any dimension [25]; the appearance of an induced Chern Simons theory in the context of $(3 + 1)$ -dimensional gravity on manifolds with a boundary [26], etc.

Given the fact that deconstruction provides a procedure for defining UV completions of certain, in principle, non-renormalizable field theories, it is only natural to ask whether a similar reasoning can be applied to $(3 + 1)$ -dimensional gravity, while remembering that the $(2 + 1)$ -dimensional (pure gravity) theory is well defined. In other words, is it possible to deconstruct $(2 + 1)$ -dimensional CS coupled to certain matter fields into a pure $(3 + 1)$ -dimensional gravity?

Many things point out to the possibility that four-dimensional gravity can be defined in terms of purely three-dimensional data. For example, three-dimensional CS actions appear as natural boundary terms in the connection formulation of four-dimensional theory [26], as well as in the relation between the BF topological theory and $(3 + 1)$ -dimensional general relativity.⁴

There even exists a theorem in classical general relativity which states that for the case of space-like Killing fields $3 + 1$ gravity can be rewritten as $2 + 1$ gravity coupled to a non-linear $SO(2, 1)$ σ -model [16, 17]. More precisely, in a classical background with a space-like isometry, the metric can be put in the form

$$ds^2 = N^2(x)dr^2 + \hat{g}_{ab}(x)(dx^a + N^a(x)dr)(dx^b + N^b(x)dr). \quad (18)$$

For the case of $3 + 1$ dimensions, the vacuum classical equations of motion are particularly simple [16, 17], and reduce to $2 + 1$ -dimensional gravity coupled to scalar fields. Let us disregard the shift fields for the sake of simplicity. The equations of motion can be written in the following form

$$\hat{R}_{ab}^{(3)}(x) = 2\hat{\nabla}_a\phi(x) \cdot \hat{\nabla}_b\phi(x), \quad \hat{g}^{ab}\hat{\nabla}_a\hat{\nabla}_b\phi = 0. \quad (19)$$

Here $\hat{R}_{ab}^{(3)}$ and $\hat{\nabla}_a$ are the Ricci tensor and the covariant derivative associated with \hat{g}_{ab} , and ϕ is a scalar field arising after a field redefinition of N and \hat{g} [17]. By suitable

⁴Note also, that in the framework of the *AdS/CFT* correspondence four-dimensional Poincaré supergravity data can be reconstructed from three-dimensional conformal supergravity data [27].

rescalings, we can bring this to the form

$$\hat{R}_{ab}^{(3)} - \frac{1}{2}\hat{g}_{ab}\hat{R}^{(3)} = 8\pi G_3 T_{ab}, \quad (20)$$

where the covariantly conserved energy momentum tensor reads

$$T_{ab} = \hat{\nabla}_a\Phi\hat{\nabla}_b\Phi - \frac{1}{2}\hat{g}_{ab}\hat{\nabla}_c\Phi\hat{\nabla}^c\Phi. \quad (21)$$

Therefore, the $(3+1)$ -dimensional vacuum equations in the presence of a space-like Killing field are equivalent to the $(2+1)$ -dimensional gravity coupled to a scalar field, illustrating the more general theorem stated in Ref. [16].

This is of course only an on-shell observation. We claim that in the quantum theory, a similar condition holds *locally* and applies at the level of the action and the path integral.

To argue this, we start from the classical formulation of gravity using the d -bien and spin connection as variables, with action

$$S_{EH} = \frac{1}{G_d} \int d^d x \epsilon_{a_1 \dots a_d} \epsilon^{A_1 \dots A_d} e_{A_1}^{a_1} \dots e_{A_{d-2}}^{a_{d-2}} R_{A_{d-1} A_d}{}^{a_{d-1} a_d}, \quad (22)$$

where (note G_d has units of m^{2-d} , as $[\omega^a_b] = [R^a_b] = 1, [e^a] = L$ as forms)

$$R = d\omega + \omega \wedge \omega. \quad (23)$$

We are careful to distinguish various indices: we are on a manifold M , with tangent bundle TM . The indices A, B, \dots label vectors in TM . We also have a vector bundle V with structure group $SO(d-1, 1)$, which we will assume is (more or less) isomorphic to TM . Indices for vectors in V will be given by a, b, \dots . These latter indices eventually will be thought of as ‘‘gauge’’ indices.

Let us focus on $d = 4$. We then have

$$S_{EH} = \frac{1}{G_4} \int \epsilon_{abcd} e^a \wedge e^b \wedge R^{cd} = \frac{2}{G_4} \int d^4 x \epsilon^{\mu\nu\lambda} \epsilon_{abcd} \left(e_3^a e_\mu^b R_{\nu\lambda}{}^{cd} - e_\mu^a e_\nu^b R_{\lambda 3}{}^{cd} \right). \quad (24)$$

To go to the lattice, we have many options. In the gauge theory case, gauge covariance was maintained throughout, and the lower dimensional theory had gauge group G^N . Analogously, the simplest lattice action to take in the case of gravity would be to keep $SO(3, 1)$ invariance. We can regard ω as an $SO(3, 1)$ connection and replace it by plaquettes in the lattice version. To begin,⁵ we will suppose that the vierbein remains as a site field. The appropriate thing to do then is replace S_{EH} by a lattice version in which the curvature R_{AB} is replaced by $Im U_{AB}$. Along the lines of the calculations in the Yang-Mills theory, we find:

$$Im U_{\mu 3} = \epsilon J_\mu + \dots, \quad (25)$$

$$Im U_{\mu\nu} = -2\epsilon^2 R_{\mu\nu} + \dots \quad (26)$$

⁵It seems also that there could be a formalism where we treat e, ω as connections for $ISO(3, 1)$, and thus introduce link fields corresponding to e_3 as well. We will not follow that approach here.

where

$$J_\mu(x, j) = i \left(D_\mu U_3 \cdot U_3^\dagger - U_3 (D_\mu U_3)^\dagger \right). \quad (27)$$

Taking $\epsilon \rightarrow 0$, we will obtain

$$S_{EH} = \frac{2a}{G_4} \int d^3x \sum_j \epsilon_{abcd} \epsilon^{\mu\nu\lambda} \left(e_3^a e_\mu^b R_{\nu\lambda}{}^{cd} + \sigma e_\mu^a e_\nu^b J_\lambda{}^{cd} \right). \quad (28)$$

Note that in writing this action, we have essentially forced $SO(3, 1)_{(j)}$ invariance at each site j . Although U_3 is a link field, and thus transforms as $U_3 \rightarrow \Lambda_j U_3 \Lambda_{j+1}^{-1}$, the current is a tensor only under the local slice, $SO(3, 1)_{(j)}$.

Thus, we have an action of the form

$$S = \frac{1}{G_3} \sum_j \int_{\mathcal{M}_j} \epsilon_{abcd} \left[\varphi^a e^b \wedge R^{cd} + f e^a \wedge e^b \wedge J^{cd} \right] \quad (29)$$

where we have dropped the index j on fields and written $\varphi \equiv e_3$ and $f \equiv 1/a$. This action manifestly possesses $\text{Diff}_3 \times SO(3, 1)$ invariance. We can introduce a four-dimensional cosmological constant as well:

$$S = \frac{1}{G_3} \sum_j \int_{\mathcal{M}_j} \epsilon_{abcd} \left[\varphi^a e^b \wedge R^{cd} + \lambda \varphi^a e^b \wedge e^c \wedge e^d + f e^a \wedge e^b \wedge J^{cd} \right]. \quad (30)$$

This looks like a $(2 + 1)$ -dimensional ‘‘gauge theory’’ coupled to a current J . Note, however, the Latin indices are $(3 + 1)$ -dimensional, and thus this is not in any sense ‘‘ $2 + 1$ gravity’’. Furthermore, there are N copies of the symmetry group.

The UV theory could also possess σ -model terms such as

$$S_\sigma = \sum_j \int_{\mathcal{M}_j} \epsilon_{abcd} \left[J^{ac} \wedge * J^{bd} \right] \quad (31)$$

as well as other higher order terms. Our point of view here is that in the UV, we can treat the theory as containing just a set of currents with kinetic terms if necessary. As we go to the IR (the continuum limit), the current kinetic terms become *irrelevant* (e.g., eq. (31) becomes a curvature-squared term), leaving only the Einstein-Hilbert action.

Of course, an important aspect of this is that the continuum limit must exist in some sense. In fact, the original four-dimensional action is an effective theory, which is certainly only valid for probes at length scales⁶ $L \gg L_4$. Thus, if $a < L_4$, the available probes cannot tell the difference between the lattice theory and the continuum. Consequently, the region of strong three-dimensional coupling can be avoided, while staying within the region of validity of the four-dimensional theory. Essentially, classically the three and four-dimensional theories are equivalent, as constructed. We conjecture that this remains true in the quantum theory. Furthermore, we will have to suppose that the value of the four-dimensional cosmological constant is given by

⁶We use the notation L_d for the d -dimensional Planck length.

its limiting three-dimensional value. This seems obvious if we don't have to strictly take the continuum limit.

We could try to proceed further and reduce $SO(3, 1)$ to $SO(2, 1)$, to make the theory look gravitational in $2 + 1$ as well. We start by just segregating indices:

$$S = \sum_j \int_{\mathcal{M}_j} \epsilon_{\alpha\beta\gamma} \left[(\varphi^3 e^\alpha - \varphi^\alpha A) \wedge (R^{\beta\gamma} - \omega^\beta \wedge \omega^\gamma) - 2\varphi^\alpha e^\beta \wedge (D\omega)^\gamma \right. \\ \left. + \lambda\varphi^3 e^\alpha \wedge e^\beta \wedge e^\gamma - 3\lambda\varphi^\alpha A \wedge e^\beta \wedge e^\gamma - 2f_\pi e^\alpha \wedge e^\beta \wedge J^\gamma + 2f_\pi A \wedge e^\alpha \wedge J^{\beta\gamma} \right], \quad (32)$$

where $A \equiv e^3$, $(D\omega)^\gamma \equiv d\omega^\gamma + \omega^\gamma_\delta \wedge \omega^\delta$, $\omega^\alpha \equiv \omega^{\alpha,3}$, and $J^\gamma \equiv J^{\gamma,3}$. Note that with the assumed lattice, there are natural vevs:

$$\langle \varphi^3 \rangle = 1, \quad \langle \varphi^\alpha \rangle = 0, \quad \langle A_\mu \rangle = 0, \quad (33)$$

which put the background metric in the form appropriate to the chosen lattice

$$ds^2 = a^2(\Delta j)^2 + ds_{2,1}^2(j). \quad (34)$$

This metric is just a discretized form of the canonical ‘‘ADM’’ metric [28]

$$ds^2 = N^2(x, r)dr^2 + g_{\mu\nu}(x, r)[dx^\mu + N^\mu(x, r)dr][dx^\nu + N^\nu(x, r)dr]. \quad (35)$$

Here r denotes the continuum limit of the discretized lattice direction. In this discretized form the shift vector has been expanded around zero.

Thus it is natural to expand in fluctuations around this vev, (fluctuations in φ^α , A_μ correspond to modifications in the shape of the lattice) and we obtain:

$$S = S_{\oplus EH} + S_{int} \quad (36)$$

where

$$S_{\oplus EH} = \frac{1}{G_3} \sum_j \int_{\mathcal{M}_j} \epsilon_{\alpha\beta\gamma} \left[e^\alpha \wedge R^{\beta\gamma} + \lambda e^\alpha \wedge e^\beta \wedge e^\gamma \right] \quad (37)$$

and

$$S_{int} = \sum_j \int_{\mathcal{M}_j} \epsilon_{\alpha\beta\gamma} \left[-e^\alpha \wedge \omega^\beta \wedge \omega^\gamma + (\varphi^3 e^\alpha - \varphi^\alpha A) \wedge (R^{\beta\gamma} - \omega^\beta \wedge \omega^\gamma) - 2\varphi^\alpha e^\beta \wedge (D\omega)^\gamma \right. \\ \left. + \lambda\varphi^3 e^\alpha \wedge e^\beta \wedge e^\gamma - 3\lambda\varphi^\alpha A \wedge e^\beta \wedge e^\gamma - 2f e^\alpha \wedge e^\beta \wedge J^\gamma + 2f A \wedge e^\alpha \wedge J^{\beta\gamma} \right]. \quad (38)$$

In addition, we would add matter fields to S_{int} .

Provided the $(2 + 1)$ -dimensional currents $J^{\mu\nu}$ can be dynamically induced, via some unknown non-perturbative mechanism, from some other well-defined degrees of freedom, in the deep UV one would be left only with (N copies) of the $(2 + 1)$ -dimensional CS term coupled to these $(2 + 1)$ -dimensional degrees of freedom. In the intermediate range of scale we get N copies of linked $(2 + 1)$ -dimensional CS theories coupled to $(2 + 1)$ -dimensional currents.⁷ In the deep IR we recover the full $(3 + 1)$ -dimensional general relativity.⁸

⁷Our concluding picture resembles somewhat that of Ref. [29].

⁸One could also entertain the possibility of simply starting with $2 + 1$ gravity coupled to appropriate fields. In this case, $SO(3, 1)$ would have to be an accidental symmetry of the IR.

Notice that we have the right number of degrees of freedom needed to reproduce the $(3+1)$ -dimensional theory. These degrees of freedom come from the matter fields coupled to the CS theory. Thus our formulation does provide a quantum mechanical version of the classical theorems discussed above [16, 17].

As we will argue in the next concluding section, the above observations are enough to argue that Witten's mechanism for vanishing of the $(2+1)$ -dimensional cosmological constant can be lifted to $3+1$ dimensions.

5 The Vanishing Cosmological Constant Deconstructed

Now we argue that Witten's argument for the vanishing of $(2+1)$ -dimensional vacuum energy can be deconstructed as follows:

- 1) Assume a local spatial foliation of $(3+1)$ -dimensional spacetime.
- 2) Deconstruct the vacuum part of pure $(3+1)$ -dimensional gravity from (N copies of) $(2+1)$ -dimensional general relativity coupled to certain $(2+1)$ -dimensional matter fields represented in terms of currents as in the preceding section. Assume that $3+1$ -dimensional sources can be defined in terms of a deconstructed $2+1$ -dimensional theory. For sources represented by gauge fields this should be possible given the discussion⁹ of section 3.
- 3) In the deep UV we have (N copies of) $2+1$ gravity coupled to some $(2+1)$ -dimensional sources. Whatever the matter content of this $(2+1)$ -dimensional theory is, we know that the resulting geometry has to be conical. Thus Witten's argument applies: the vacuum is supersymmetric, yet the excited states are not.
- 4) In the range of intermediate scales, we have N linked copies of $2+1$ gravity coupled to $(2+1)$ -dimensional currents. Once again, the resulting $(2+1)$ -dimensional geometry is conical. Thus Witten's argument holds in the region between UV and IR.

Finally notice that based on dimensional grounds the mass splitting is directly proportional to the deficit angle and inversely proportional to the three-dimensional Newton constant. Thus as long as the three-dimensional Newton constant is of order one as the continuum limit is taken, and the deficit angle (on each local three-dimensional slice) is taken to scale as the inverse of the lattice spacing, the fermi-bose splitting will be finite in the infrared. These remarks may be tested by examination of the example of Ref. [18].¹⁰

⁹One might ask why a $(3+1)$ -dimensional theory with a well defined $(3+1)$ -dimensional UV behavior, such as the Standard Model, should be defined in terms of $(2+1)$ -dimensional data. The point here is that both the deconstructed $(2+1)$ -dimensional and the intrinsic $(3+1)$ -dimensional UV definitions lead to the same IR physics, and as such are indistinguishable at long distances.

¹⁰For example [18], the deficit angle produced by a mass M is $\delta = 2\pi ML_3$. Thus, the mass difference (at one-loop) between fermions and bosons should be proportional to $g^2\delta/G_N = 2\pi g^2 M$, where g is the interaction strength. In a realistic model the mass M should be deconstructed to be of the order of a TeV.

According to the outlined argument the vacuum energy is zero in the UV, and also some place in between UV and IR. But does it remain zero in the IR? That is difficult to say, given the fact that the three-dimensional coupling has to be of order one, but the physical picture would be that as one takes the lattice spacing to zero, one still has in principle an infinite number of $(2 + 1)$ -dimensional matter theories strongly coupled to $2 + 1$ gravity.

Essentially we have a deconstruction of $(2 + 1)$ -dimensional conical singularities to one-dimensional, string-like singularities in every local patch of $3 + 1$ dimensions. Thus we again end up with a claim that the vacuum state can be made supersymmetric and yet the excited states do not fall into supermultiplets because of the non-existence of the global supercharge due to the presence of the string-like defects which create a deconstructed version of the asymptotically conical three-dimensional geometry.¹¹

Within this framework the four-dimensional cosmological constant is essentially determined by the value of the three-dimensional cosmological constant. In the supersymmetric scenario the latter is zero, and so is the four-dimensional one. Yet the excited states are non-supersymmetric due to the non-existence of a global supercharge.

In order to have vanishing vacuum energy, we really need to have supersymmetry in four dimensions. The MacDowell-Mansouri approach [23] provides a unified geometric formulation of supersymmetry and gravity with the curvature constructed from the spin connection, the vierbein, and the gravitino. The analysis presented in section 4 applies also in this situation. It would be very interesting to study the deconstruction of this theory explicitly.

We conclude this article with an obvious question: assuming that the ultraviolet completion of $(3 + 1)$ -dimensional gravity is indeed given in terms of $(2 + 1)$ -dimensional gravity coupled to $(2 + 1)$ -dimensional matter as we have argued above, what are the most immediate observational consequences, in the sense of $(3 + 1)$ -dimensional gravity being modified at very short distances?

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¹¹As reviewed in Ref. [17], the information about the mass resides in $(2 + 1)$ -dimensions in the “zeroth order” behavior of the metric at infinity. This should be contrasted to the situation in $(3 + 1)$ -dimensions where the analogous information about the mass resides in the leading $1/r$ deviations from the Minkowski metric near infinity [17].

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