

Forward-Backward Asymmetry in $B \rightarrow X_s \ell^+ \ell^-$ at the NNLL Level

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Abstract

We report the results of a new calculation of soft-gluon corrections in $B \rightarrow X_s \ell^+ \ell^-$ decays. In particular, we present the first calculation of bremsstrahlung and corresponding virtual terms to the lepton forward-backward asymmetry, which allows us to systematically include all contributions to this observable beyond the lowest non-trivial order. The new terms are important, for instance the position of the zero of the asymmetry receives corrections of $O(10\%)$. Using a different method, we also provide an independent check of recently published results on bremsstrahlung and infrared virtual corrections to the dilepton-invariant mass distribution.

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1 Introduction

Flavour-changing neutral-current (FCNC) processes are a very useful tool to understand the nature of physics beyond the Standard Model (SM). The stringent bounds obtained from $B \rightarrow X_s \gamma$ on various non-standard scenarios (see e.g. [1, 2, 3, 4]) are a clear example of the importance of theoretically clean FCNC observables in discriminating new-physics models.

Generally, inclusive rare decay modes of the B meson are theoretically clean observables. For instance the decay width $\Gamma(B \rightarrow X_s \gamma)$ is well approximated by the partonic decay rate $\Gamma(b \rightarrow s \gamma)$, which can be analysed in renormalization-group-improved perturbation theory. Non-perturbative contributions play only a subdominant role and can be calculated in a model-independent way by using the heavy-quark expansion. The inclusive $B \rightarrow X_s \ell^+ \ell^-$ transition, which is starting to be accessible at B factories [5], represents a new source of theoretically clean observables, complementary to the $B \rightarrow X_s \gamma$ rate. In particular, kinematic observables such as the invariant dilepton mass spectrum and the forward–backward (FB) asymmetry in $B \rightarrow X_s \ell^+ \ell^-$ provide additional clean information on short-distance couplings not accessible in $B \rightarrow X_s \gamma$ [6].

Non-perturbative corrections to $B \rightarrow X_s \ell^+ \ell^-$ scaling with $1/m_b^2$ and $1/m_c^2$ can be calculated quite analogously to those entering the $B \rightarrow X_s \gamma$ rate [7, 8, 9, 10]. Here the $c\bar{c}$ resonances represent a more serious problem since, for specific values of the dilepton invariant mass, $c\bar{c}$ states can be produced on shell. However, these resonances can be removed by appropriate kinematic cuts in the invariant mass spectrum. In the *perturbative window*, namely when $m_{\ell^+ \ell^-} \lesssim m_b/2$, theoretical predictions for the invariant mass spectrum are dominated by the purely perturbative contributions, and a theoretical precision comparable with the one reached in the decay $B \rightarrow X_s \gamma$ is possible. The B factories will soon provide statistics and resolution needed for the measurements of $B \rightarrow X_s \ell^+ \ell^-$ kinematic distributions. Precise theoretical estimates of the SM expectations are therefore needed in order to perform new significant tests of flavour physics.

In this paper we complete one important step of this program: we present a new calculation of QCD-bremsstrahlung and the corresponding virtual corrections in $B \rightarrow X_s \ell^+ \ell^-$. This effect has already been evaluated in Refs. [11, 12] for the dilepton invariant-mass spectrum. Here we perform the calculation using a different technique, namely using a full dimensional regularization of infrared divergences (both soft and collinear ones). We anticipate that our results agree with those of [11, 12] for the dilepton invariant-mass distribution. Moreover, our technique allows us obtain the first computation of the soft-gluon (and corresponding virtual) corrections in the FB asymmetry: the missing ingredient needed for a systematic evaluation of this observable beyond the lowest non-trivial order. Our phenomenological analysis shows that these new contributions are rather important, for instance the position of the zero of the asymmetry receives corrections of $O(10\%)$.

The paper is organized as follows: in Section 2 we set up the framework of the calculation; we briefly review the status of the various ingredients needed for a systematic analysis of the partonic process $b \rightarrow s(g)\ell^+ \ell^-$ in perturbation theory. In Section 3 we present the basic expressions needed to describe the spectrum and the FB asymmetry of $B \rightarrow X_s \ell^+ \ell^-$ in the so-called next-to-next-to-leading logarithmic (NNLL) approximation. The details of the

calculation, including a definition of the four-particle phase space and a discussion on the regularization scheme, which avoids any ambiguity due to the definition γ_5 , are presented in Section 4. Section 5 contains a brief phenomenological analysis of our results, mainly focused on the determination of the zero of the FB asymmetry, and Section 6 a short summary.

2 Theoretical framework

Within inclusive B decay modes such as $B \rightarrow X_s \gamma$ or $B \rightarrow X_s \ell^+ \ell^-$, short-distance QCD corrections lead to a sizeable modification of the pure electroweak contribution, generating large logarithms of the form $\alpha_s^n(m_b) \log^m(m_b/M_{\text{heavy}})$, where $M_{\text{heavy}} = O(M_W)$ and $m \leq n$ (with $n = 0, 1, 2, \dots$). A suitable framework to achieve the necessary resummations of these large logs is the construction of an effective low-energy theory with five quarks, obtained by integrating out the heavy degrees of freedom. With the help of renormalization-group (RG) techniques, one can then resum the series of leading logarithms (LL), next-to-leading logarithms (NLL), and so on:

$$\alpha_s^n(m_b) \log^n(m_b/M) \quad [\text{LL}], \quad \alpha_s^{n+1}(m_b) \log^n(m_b/M) \quad [\text{NLL}], \dots \quad (2.1)$$

The effective five-quark low-energy Hamiltonian relevant to the partonic process $b \rightarrow s \ell^+ \ell^-$ can be written as

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum C_i(\mu, M_{\text{heavy}}) \mathcal{O}_i(\mu), \quad (2.2)$$

where

$$\begin{aligned} \mathcal{O}_1 &= (\bar{s} \gamma_\mu T^a P_L c) (\bar{c} \gamma^\mu T_a P_L b), & \mathcal{O}_2 &= (\bar{s} \gamma_\mu P_L c) (\bar{c} \gamma^\mu P_L b), \\ \mathcal{O}_3 &= (\bar{s} \gamma_\mu P_L b) \sum_q (\bar{q} \gamma^\mu q), & \mathcal{O}_4 &= (\bar{s} \gamma_\mu T^a P_L b) \sum_q (\bar{q} \gamma^\mu T_a q), \\ \mathcal{O}_5 &= (\bar{s} \gamma_\mu \gamma_\nu \gamma_\rho P_L b) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho q), & \mathcal{O}_6 &= (\bar{s} \gamma_\mu \gamma_\nu \gamma_\rho T^a P_L b) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho T_a q), \\ \tilde{\mathcal{O}}_7 &= \frac{e}{16\pi^2} \bar{m}_b(\mu) (\bar{s} \sigma^{\mu\nu} P_R b) F_{\mu\nu}, & \tilde{\mathcal{O}}_8 &= \frac{g_s}{16\pi^2} \bar{m}_b(\mu) (\bar{s} \sigma^{\mu\nu} T^a P_R b) G_{\mu\nu}^a, \\ \tilde{\mathcal{O}}_9 &= \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell), & \tilde{\mathcal{O}}_{10} &= \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \end{aligned} \quad (2.3)$$

define the complete set of relevant dimension-6 operators and $C_i(\mu, M_{\text{heavy}})$ the corresponding Wilson coefficients.

Within this framework, QCD corrections are twofold: corrections related to the Wilson coefficients, and those related to the matrix elements of the various operators, both evaluated at the low-energy scale $\mu \approx m_b$. As the heavy fields are integrated out, the top-quark-, W -, and Z -mass dependence is contained in the initial conditions of the Wilson coefficients, determined by a matching procedure between full and effective theory at the high scale (Step 1). By means of RG equations, the $C_i(\mu, M_{\text{heavy}})$ are then evolved at the low scale (Step 2). Finally, the QCD corrections to the matrix elements of the operators are evaluated at the low scale (Step 3).

Compared with the effective Hamiltonian relevant to $b \rightarrow s \gamma$, Eq. (2.3) contains the $O(\alpha_{\text{em}})$ additional operators $\tilde{\mathcal{O}}_9$ and $\tilde{\mathcal{O}}_{10}$. Moreover, it turns out that the first large logarithm of the

form $\log(m_b/M_W)$ arises already without gluons, because the operator \mathcal{O}_2 mixes into $\tilde{\mathcal{O}}_9$ at one loop. This possibility, which has no equivalent in the $b \rightarrow s\gamma$ case, leads to the following ordering of contributions to the decay amplitude

$$\begin{aligned} & [\alpha_{\text{em}} \log(m_b/M)] \alpha_s^n(m_b) \log^n(m_b/M) \quad [\text{LL}] , \\ & [\alpha_{\text{em}} \log(m_b/M)] \alpha_s^{n+1}(m_b) \log^n(m_b/M) \quad [\text{NLL}] , \dots \end{aligned} \quad (2.4)$$

Technically, to perform the resummation, it is convenient to transform these series into the standard form (2.1). This can be achieved by redefining magnetic, chromomagnetic and lepton-pair operators as follows [13, 14]:

$$\mathcal{O}_i = \frac{16\pi^2}{g_s^2} \tilde{\mathcal{O}}_i , \quad C_i = \frac{g_s^2}{(4\pi)^2} \tilde{C}_i , \quad (i = 7, \dots, 10). \quad (2.5)$$

This redefinition enables one to proceed in the standard way, or as in $b \rightarrow s\gamma$, in the three calculational steps discussed above [13, 14]. At the high scale, the Wilson coefficients can be computed at a given order in perturbation theory and expanded in powers of α_s :

$$C_i = C_i^{(0)} + \frac{\alpha_s}{(4\pi)} C_i^{(1)} + \frac{\alpha_s^2}{(4\pi)^2} C_i^{(2)} + \dots \quad (2.6)$$

Obviously, the Wilson coefficients of the new operators \mathcal{O}_{7-10} at the high scale start at order α_s only. Then the anomalous-dimension matrix has the canonical expansion in α_s and starts with a term proportional to α_s :

$$\gamma = \frac{\alpha_s}{4\pi} \gamma^{(0)} + \frac{\alpha_s^2}{(4\pi)^2} \gamma^{(1)} + \frac{\alpha_s^3}{(4\pi)^3} \gamma^{(2)} + \dots \quad (2.7)$$

In particular, after the reshufflings in (2.5) the one-loop mixing of the operator \mathcal{O}_2 with \mathcal{O}_9 appears formally at order α_s .

The last of the three steps, however, requires some care: among the operators with a non-vanishing tree-level matrix element, only \mathcal{O}_9 has a non-vanishing coefficient at the LL level. Therefore, at this level, only the tree-level matrix element of this operator ($\langle \mathcal{O}_9 \rangle$) has to be included. At NLL accuracy the QCD one-loop contributions to $\langle \mathcal{O}_9 \rangle$, the tree-level contributions to $\langle \mathcal{O}_7 \rangle$ and $\langle \mathcal{O}_{10} \rangle$, and the electroweak one-loop matrix elements of the four-quark operators have to be calculated. Finally, at NNLL precision, one should in principle take into account the QCD two-loop corrections to $\langle \mathcal{O}_9 \rangle$, the QCD one-loop corrections to $\langle \mathcal{O}_7 \rangle$ and $\langle \mathcal{O}_{10} \rangle$, and the QCD corrections to the electroweak one-loop matrix elements of the four-quark operators.

Let us briefly review the present status of these perturbative contributions to decay rate and FB asymmetry of $B \rightarrow X_s \ell^+ \ell^-$: the complete NLL contributions to the decay amplitude can be found in [13, 14]. Since the LL contribution to the rate turns out to be numerically rather small, NLL terms represent an $O(1)$ correction to this observable. On the other hand, since a non-vanishing FB asymmetry is generated by the interference of vector ($\sim \mathcal{O}_{7,9}$) and

axial-vector ($\sim \mathcal{O}_{10}$) leptonic currents, the LL amplitude leads to a vanishing result and NLL terms represent the lowest non-trivial contribution to this observable.

For these reasons, a computation of NNLL terms in $B \rightarrow X_s \ell^+ \ell^-$ is needed if we aim at the same numerical accuracy as achieved by the NLL analysis of $B \rightarrow X_s \gamma$ [15]. Large parts of the latter can be taken over and used in the NNLL calculation of $B \rightarrow X_s \ell^+ \ell^-$. However, this is not the full story. In particular, the bremsstrahlung corrections presented in this paper for the first time are a crucial ingredient, necessary for a complete evaluation of the FB asymmetry to NNLL precision. And in the case of the differential rate, the full NNLL enterprise is still not complete:

[Step 1] The full computation of initial conditions to NNLL precision has been presented in Ref. [16]. The authors did the two-loop matching for all the operators relevant to $b \rightarrow s \ell^+ \ell^-$ (including a confirmation of the $b \rightarrow s \gamma$ NLL matching results of [17, 18]). The inclusion of this NNLL contribution removes the large matching scale uncertainty (around 16%) of the NLL calculation of the $b \rightarrow s \ell^+ \ell^-$ decay rate.

[Step 2] Thanks to the reshufflings of the LL series, most of the NNLL contributions to the anomalous-dimension matrix can be derived from the NLL analysis of $b \rightarrow s \gamma$. In particular the three-loop mixing of the four-quark operators \mathcal{O}_{1-6} into \mathcal{O}_7 and \mathcal{O}_8 can be taken over from Ref. [15]. The only missing piece for a full NNLL analysis of the $b \rightarrow s \ell^+ \ell^-$ decay rate is the three-loop mixing of the four-quark operators into \mathcal{O}_9 . In [16] an estimate was made, which suggests that the numerical influence of these missing NNLL contributions on the branching ratio of $b \rightarrow s \ell^+ \ell^-$ is small. Interestingly, since the FB asymmetry has no contributions proportional to $|\langle \mathcal{O}_9 \rangle|^2$, this missing term is not needed for a NNLL analysis of this observable.

[Step 3] Within the $B \rightarrow X_s \gamma$ calculation at NLL, the two-loop matrix elements of the four-quark operator \mathcal{O}_2 for an on-shell photon were calculated in [19] and quite recently confirmed in [20]. An independent numerical check of these results has been performed and will be presented in [21]. This calculation was extended in [11] to the case of an off-shell photon (for small squared dilepton mass), which corresponds to a NNLL contribution relevant to the decay $B \rightarrow X_s \ell^+ \ell^-$. In the dilepton spectrum, this calculation reduces the error corresponding to the uncertainty of the low-scale dependence from $\pm 13\%$ down to $\pm 6.5\%$. This step also includes the bremsstrahlung and virtual contributions that are discussed in the present paper.

In principle, a complete NNLL calculation of the $B \rightarrow X_s \ell^+ \ell^-$ rate would require also the calculation of the renormalization-group-invariant two-loop matrix element of the operator \mathcal{O}_9 . Similarly to the missing piece of the anomalous-dimension matrix, also this (scale-independent) contribution does not enter the FB asymmetry at NNLL accuracy.

3 Basic expressions

We normalize all the observables by the semileptonic decay rate in order to reduce the uncertainties due to bottom quark mass and CKM angles:

$$\Gamma[b \rightarrow X_c e \bar{\nu}_e] = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 f(z) \kappa(z) . \quad (3.1)$$

Here $z = m_c^2/m_b^2$ ($m_{b,c}$ denote pole quark masses), $f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z$ is the phase-space factor and

$$\kappa(z) = 1 - \frac{2\alpha_s(m_b) h(z)}{3\pi f(z)} \quad (3.2)$$

takes into account QCD corrections (the function $h(z)$ has been given analytically in [22] and is quoted in the appendix). The normalized dilepton invariant mass spectrum is then defined as

$$R(s) = \frac{\frac{d}{ds} \Gamma(B \rightarrow X_s \ell^+ \ell^-)}{\Gamma(B \rightarrow X_c e \bar{\nu})} , \quad (3.3)$$

where $s = (p_{\ell^+} + p_{\ell^-})^2/m_b^2$. The other important observable is the forward–backward lepton asymmetry:

$$A_{\text{FB}}(s) = \frac{1}{\Gamma(B \rightarrow X_c e \bar{\nu})} \int_{-1}^1 d \cos \theta_\ell \frac{d^2 \Gamma(B \rightarrow X_s \ell^+ \ell^-)}{ds d \cos \theta_\ell} \text{sgn}(\cos \theta_\ell) , \quad (3.4)$$

where θ_ℓ is the angle between ℓ^+ and B momenta in the dilepton centre-of-mass frame. It was shown in [8] that $A_{\text{FB}}(s)$ is identical to the energy asymmetry introduced in [23].

Following closely — but not exactly — the notation of Refs. [11, 12], we present here some useful formulae that allow us to systematically take into account corrections beyond the NLL level for the partonic contributions to these two observables:

$$\begin{aligned} R(s) = \frac{\alpha_{\text{em}}^2}{4\pi^2} \left| \frac{V_{tb}^* V_{ts}}{V_{cb}} \right|^2 \frac{(1-s)^2}{f(z)\kappa(z)} & \left\{ 4 \left(1 + \frac{2}{s} \right) |C_7^{\text{new}}(s)|^2 \left(1 + \frac{\alpha_s}{\pi} \tau_{77}(s) \right) \right. \\ & + (1+2s) \left[|C_9^{\text{new}}(s)|^2 + |C_{10}^{\text{new}}(s)|^2 \right] \left(1 + \frac{\alpha_s}{\pi} \tau_{99}(s) \right) \\ & \left. + 12 \text{Re} [C_7^{\text{new}}(s) C_9^{\text{new}}(s)^*] \left(1 + \frac{\alpha_s}{\pi} \tau_{79}(s) \right) + \frac{\alpha_s}{\pi} \delta_R(s) \right\} , \quad (3.5) \end{aligned}$$

$$\begin{aligned} A_{\text{FB}}(s) = -\frac{3\alpha_{\text{em}}^2}{4\pi^2} \left| \frac{V_{tb}^* V_{ts}}{V_{cb}} \right|^2 \frac{(1-s)^2}{f(z)\kappa(z)} & \left\{ s \text{Re} [C_{10}^{\text{new}}(s)^* C_9^{\text{new}}(s)] \left(1 + \frac{\alpha_s}{\pi} \tau_{910}(s) \right) \right. \\ & \left. + 2 \text{Re} [C_{10}^{\text{new}}(s)^* C_7^{\text{new}}(s)] \left(1 + \frac{\alpha_s}{\pi} \tau_{710}(s) \right) + \frac{\alpha_s}{\pi} \delta_{\text{FB}}(s) \right\} . \quad (3.6) \end{aligned}$$

With respect to Refs. [11, 12], we have introduced a new set of effective coefficients, defined as

$$C_7^{\text{new}}(s) = \left(1 + \frac{\alpha_s}{\pi} \sigma_7(s) \right) \tilde{C}_7^{\text{eff}} - \frac{\alpha_s}{4\pi} \left[C_1^{(0)} F_1^{(7)}(s) + C_2^{(0)} F_2^{(7)}(s) + \tilde{C}_8^{\text{eff}(0)} F_8^{(7)}(s) \right]$$

$$\begin{aligned}
C_9^{\text{new}}(s) &= \left(1 + \frac{\alpha_s}{\pi} \sigma_9(s)\right) \tilde{C}_9^{\text{eff}}(s) - \frac{\alpha_s}{4\pi} \left[C_1^{(0)} F_1^{(9)}(s) + C_2^{(0)} F_2^{(9)}(s) + \tilde{C}_8^{\text{eff}(0)} F_8^{(9)}(s) \right] \\
C_{10}^{\text{new}}(s) &= \left(1 + \frac{\alpha_s}{\pi} \sigma_9(s)\right) \tilde{C}_{10}^{\text{eff}}.
\end{aligned} \tag{3.7}$$

The C_i^{new} have the advantage of encoding all dominant matrix-element corrections, which leads to an explicit s dependence in all of them.

As we shall illustrate in detail in the next section, the terms $\sigma_i(s)$ take into account *universal* $O(\alpha_s)$ bremsstrahlung and the corresponding infrared (IR) virtual corrections proportional to the tree-level matrix elements of \mathcal{O}_{7-10} . The remaining (finite) non-universal bremsstrahlung and IR virtual corrections are encoded in rate and FB asymmetry through $\tau_i(s)$ and $\delta_{R,\text{FB}}(s)$. Analytic and numerical results for the $\tau_i(s)$ and the $\sigma_i(s)$ will be presented later on. Here we simply note that, with our definition of the C_i^{new} , the additional finite terms $\tau_i(s)$ are rather small, especially for large values of s ($|\tau_i(s)| < 0.5$ for $s > 0.3$). Substantially smaller are the bremsstrahlung corrections not related to $\mathcal{O}_{7-10} \otimes \mathcal{O}_{7-10}$, which are encoded in $\delta_{R,\text{FB}}(s)$. A complete evaluation of $\delta_R(s)$ can be found in [12], where this effect is shown to be at the $O(1\%)$ level.

The other explicit $O(\alpha_s)$ terms in (3.7) are due to virtual corrections that are infrared-safe. In particular, the two-loop functions $F_{1,2}^{(7,9)}$ and the one-loop functions $F_8^{(7),(9)}$ correspond to virtual corrections to $\mathcal{O}_{1,2}$ and \mathcal{O}_8 , respectively. These functions have been computed in Ref. [11] for small s . The coefficients $\tilde{C}_{7-10}^{\text{eff}}$, including the one-loop matrix-element contributions of \mathcal{O}_{1-6} are defined in close analogy with Ref. [14] and are reported in the appendix as a function of the true Wilson coefficients C_i . Finally, explicit expressions for the latter, evolved down at the low-energy scale, can be found in [16].

By means of expressions (3.5) and (3.6), we can more easily discuss the organization of the perturbative expansion for the two observables. According to the arguments in Section 2, the formally leading terms are obtained by setting $C_7^{\text{new}} = C_{10}^{\text{new}} = 0$, neglecting τ_i and δ_i , and identifying C_9^{new} with the leading term of \tilde{C}_9^{eff} [formally $O(1/\alpha_s)$]. At this level $A_{\text{FB}}(s)$ is clearly vanishing. At the NNL level, when $A_{\text{FB}}(s)$ receives its first non-vanishing contribution, we should retain the interference of the $O(1/\alpha_s)$ term in C_9^{new} with the leading $O(1)$ terms in $C_{7,10}^{\text{new}}$, as well as the subleading corrections in $|C_9^{\text{new}}|^2$. Within this approach, the missing ingredients for a NNLL analysis of $A_{\text{FB}}(s)$ are only σ_9 and τ_{910} .

As anticipated, the standard LL expansion is numerically not well justified, since the formally-leading $O(1/\alpha_s)$ term in C_9^{new} is much closer to an $O(1)$ term. For this reason, it has been proposed in Ref. [11] to use a different counting rule, where the $O(1/\alpha_s)$ term of C_9^{new} is treated as $O(1)$. We also believe that this approach, although it cannot be consistently extended at higher orders, is well justified at the present status of the calculation. Within this approach, the three C_i^{new} and the two observables [$R(s)$ and $A_{\text{FB}}(s)$] are all homogeneous quantities, starting with an $O(1)$ term. Then all σ_i , τ_i and δ_i functions are required for a next-to-leading order analysis of both $R(s)$ and $A_{\text{FB}}(s)$.

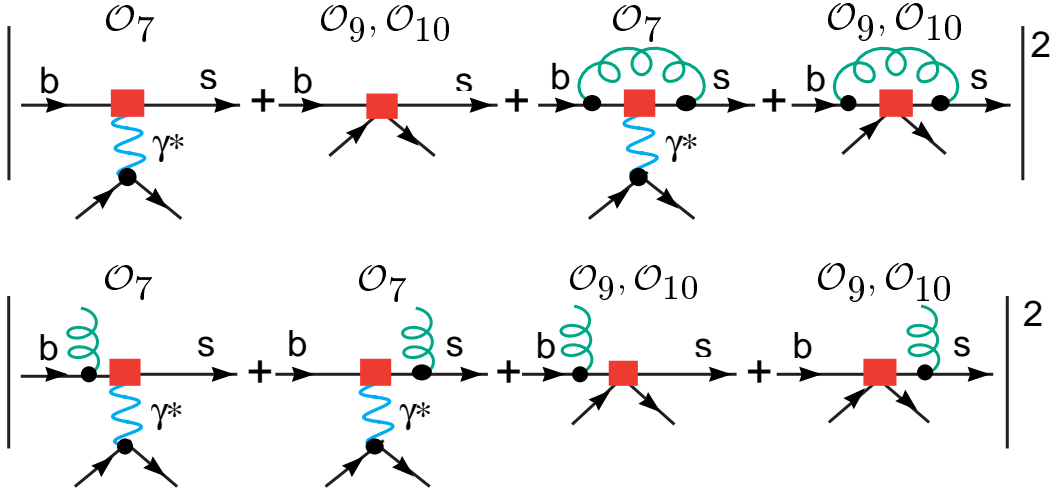


Figure 1: Virtual (up) and real (down) QCD corrections generating the terms τ_i and σ_i in Eqs. (3.5)–(3.7). The boxes denote the insertion of \mathcal{L}_{eff} in (4.1).

4 Details of the calculation

In contrast to previous calculations of $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ matrix elements, we set $m_s = 0$ and, following the approach of Ref. [24], we employ dimensional regularization to regulate all infrared singularities, including collinear divergences arising from the vanishing strange-quark mass.

The diagrams we need to compute are shown in Fig. 1, where the boxes denote the insertion of the following effective non-local Lagrangian

$$\mathcal{L}_{\text{eff}} = \kappa_F \left[\tilde{C}_9^{\text{eff}} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma^\mu \ell + \tilde{C}_{10}^{\text{eff}} \bar{s} \gamma_\mu P_L b \bar{\ell} \gamma^\mu \gamma_5 \ell + \frac{m_b}{e} \tilde{C}_7^{\text{eff}} \bar{s} \sigma_{\mu\nu} P_R b F^{\mu\nu} \right], \quad (4.1)$$

where

$$\kappa_F = \frac{\alpha_{\text{em}} G_F V_{ts}^* V_{tb}}{\pi \sqrt{2}} \quad (4.2)$$

and, for simplicity, we have omitted to explicitly show the s dependence of \tilde{C}_9^{eff} . Using (4.1) instead of the local Hamiltonian in (2.2), we can easily take into account the (two-loop) IR-divergent contributions of four-quark operators, whose one-loop $b \rightarrow s \ell^+ \ell^-$ matrix element is encoded in the non-local coefficients \tilde{C}_i^{eff} [11] (see appendix).

4.1 Virtual corrections

The amplitude for the three-body process $b(p) \rightarrow s(p_s) \ell^+(p_1) \ell^-(p_2)$ can be written as

$$\begin{aligned} \mathcal{A}_{[3]} = & \frac{2\kappa_F}{m_b} \left\{ \bar{s} P_R [m_b \gamma^\mu A(s) + (p^\mu + p_s^\mu) B(s) + O(q^\mu)] b \bar{\ell} \gamma_\mu \ell \right. \\ & \left. + \bar{s} P_R [m_b \gamma^\mu A'(s) + (p^\mu + p_s^\mu) B'(s) + O(q^\mu)] b \bar{\ell} \gamma_\mu \gamma_5 \ell \right\}, \quad (4.3) \end{aligned}$$

where $q = p_1 + p_2$ and, at the tree level,

$$A^{(0)} = \frac{1}{2}\tilde{C}_9^{\text{eff}} + \frac{1}{s}\tilde{C}_7^{\text{eff}} , \quad B^{(0)} = -\frac{1}{s}\tilde{C}_7^{\text{eff}} , \quad A'^{(0)} = \frac{1}{2}\tilde{C}_{10}^{\text{eff}} , \quad B'^{(0)} = 0 . \quad (4.4)$$

Employing the same notation, and working in $d = 4 - 2\epsilon$ dimensions, the renormalized virtual one-loop corrections in Fig. 1 are given by

$$\begin{aligned} A^{(1)} &= \frac{\alpha_s}{3\pi}\Gamma(1+\epsilon)(1-s)^{-2\epsilon}\left(\frac{m_b^2}{4\pi}\right)^{-\epsilon}\left\{\tilde{C}_9^{\text{eff}}\left[-\frac{1}{2\epsilon^2}-\frac{5}{4\epsilon}-\text{Li}_2(s)-\frac{1+2s}{2s}\ln(1-s)-3\right]\right. \\ &\quad \left.+\frac{1}{s}\tilde{C}_7^{\text{eff}}\left[-\frac{1}{\epsilon^2}-\frac{5}{2\epsilon}-2\text{Li}_2(s)-3\ln(1-s)-10-8\ln\left(\frac{\mu}{m_b}\right)\right]+O(\epsilon)\right\} , \\ B^{(1)} &= \frac{\alpha_s}{3\pi}\Gamma(1+\epsilon)(1-s)^{-2\epsilon}\left(\frac{m_b^2}{4\pi}\right)^{-\epsilon}\left\{\tilde{C}_9^{\text{eff}}\frac{1}{2s}\ln(1-s)\right. \\ &\quad \left.+\frac{1}{s}\tilde{C}_7^{\text{eff}}\left[\frac{1}{\epsilon^2}+\frac{5}{2\epsilon}+2\text{Li}_2(s)+\ln(1-s)+10+8\ln\left(\frac{\mu}{m_b}\right)\right]+O(\epsilon)\right\} , \\ A'^{(1)} &= \frac{\alpha_s}{3\pi}\Gamma(1+\epsilon)(1-s)^{-2\epsilon}\left(\frac{m_b^2}{4\pi}\right)^{-\epsilon}\tilde{C}_{10}^{\text{eff}}\left[-\frac{1}{2\epsilon^2}-\frac{5}{4\epsilon}-\text{Li}_2(s)-\ln(1-s)\right. \\ &\quad \left.-3-\frac{1}{2s}\ln(1-s)+O(\epsilon)\right] , \\ B'^{(1)} &= \frac{\alpha_s}{3\pi}\Gamma(1+\epsilon)(1-s)^{-2\epsilon}\left(\frac{m_b^2}{4\pi}\right)^{-\epsilon}\tilde{C}_{10}^{\text{eff}}\left[\frac{1}{2s}\ln(1-s)\right] . \end{aligned} \quad (4.5)$$

The above results include also self-energy corrections of the external legs, not explicitly shown in Fig. 1. Effectively, these contributions are included using on-shell renormalization conditions. For later comparison, we state here the corresponding on-shell wave-function renormalization constants for the external quark fields, where we explicitly separated ultraviolet (UV) and IR poles:

$$\begin{aligned} Z_\psi(m_b) &= 1 - \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{m_b^2}{\mu^2}\right)^{-\epsilon}\left(\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} + 4\right) , \\ Z_\psi(m_s = 0) &= 1 - \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right) . \end{aligned} \quad (4.6)$$

Because of the conservation of vector and axial currents, UV divergences cancel out completely in the terms proportional to \tilde{C}_9^{eff} and $\tilde{C}_{10}^{\text{eff}}$. On the other hand, UV divergences proportional to \tilde{C}_7^{eff} have been eliminated by the on-shell mass counterterm Z_{m_b} and by the $\overline{\text{MS}}$ renormalization of the corresponding operator using the counterterm Z_{77} :

$$\begin{aligned} Z_{77} &= 1 + \frac{\alpha_s}{4\pi}\frac{16}{3}\frac{1}{\epsilon_{\text{UV}}} , \\ Z_{m_b} &= 1 - \frac{\alpha_s}{4\pi}\frac{4}{3}\left(\frac{m_b^2}{\mu^2}\right)^{-\epsilon}\left(\frac{3}{\epsilon_{\text{UV}}} + 4\right) . \end{aligned} \quad (4.7)$$

After the UV renormalization has been performed, we suppress the μ dependence corresponding to the IR divergences in order to simplify the notation, as can be seen in Eq. (4.5).

Taking into account the full d -dimensional three-body phase space, virtual IR corrections to the rate and forward-backward asymmetry are obtained by means of Eqs. (4.3)–(4.5), isolating the $O(\alpha_s)$ terms in

$$d\Gamma_{[3]} = \frac{m_b}{2} \frac{d^{d-1}p_s}{(2\pi)^{d-1}2E_s} \frac{d^{d-1}q}{(2\pi)^{d-1}2q_0} (2\pi)^{d-1} \delta^d(p - p_s - q) \times \\ \times \frac{d^{d-1}p_1}{(2\pi)^{d-1}2E_1} \frac{d^{d-1}p_2}{(2\pi)^{d-1}2E_2} (2\pi)^d \delta^d(q - p_1 - p_2) ds \frac{1}{2} \sum_{\text{spins}} \frac{1}{3} \sum_{\text{colours}} |\mathcal{A}_{[3]}|^2 \quad (4.8)$$

4.2 Bremsstrahlung corrections to $R(s)$

The matrix element of the four-body process $b(p) \rightarrow s(p_s)g(k)\ell^+(p_1)\ell^-(p_2)$, squared, can in general be decomposed as

$$\frac{1}{2} \sum_{\text{spins}} \frac{1}{3} \sum_{\text{colours}} |\mathcal{A}_{[4]}|^2 (s, y, x_p, x_s, x_v) = H_{\mu\nu}^S L_S^{\mu\nu} + H_{\mu\nu}^A L_A^{\mu\nu}, \quad (4.9)$$

where $L_{S,A}^{\mu\nu}$ denote the leptonic tensors

$$L_S^{\mu\nu} = \text{tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) \quad \text{and} \quad L_A^{\mu\nu} = \text{tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu \gamma_5), \quad (4.10)$$

and in addition to s we have introduced the following kinematical variables

$$x_p = \frac{k \cdot p}{m_b^2}, \quad x_s = \frac{k \cdot p_s}{m_b^2}, \quad y = \frac{p \cdot (p_1 - p_2)}{m_b^2}, \quad x_v = \frac{k \cdot (p_1 - p_2)}{m_b^2}. \quad (4.11)$$

The asymmetric leptonic tensor $L_A^{\mu\nu}$ vanishes if averaged over the full leptonic phase space (symmetric in $p_1 \leftrightarrow p_2$). On the other hand, in the case of $L_S^{\mu\nu}$ we can write

$$I_S^{\mu\nu} = (2\pi)^d \int \frac{d^{d-1}p_1}{(2\pi)^{d-1}2E_1} \int \frac{d^{d-1}p_2}{(2\pi)^{d-1}2E_2} \delta^d(q - p_1 - p_2) L_S^{\mu\nu} \\ = \frac{(1 - \epsilon)(q^2)^{-\epsilon}}{\pi^{\frac{1-2\epsilon}{2}} 2^{3-4\epsilon} \Gamma\left(\frac{5}{2} - \epsilon\right)} (q^\mu q^\nu - q^2 g^{\mu\nu}). \quad (4.12)$$

Since $I_S^{\mu\nu}$ depends only on q , the variables y and x_v do not appear in the calculation of the (symmetric) dilepton spectrum.

The explicit calculation of the real-emission diagrams in Fig. 1 leads to

$$\frac{1}{m_b^2} (q^\mu q^\nu - q^2 g^{\mu\nu}) H_{\mu\nu}^S = \frac{16\pi}{3} \kappa_F^2 \alpha_s \left\{ (1 - s) \left[-\frac{1}{x_p^2} + \frac{1-s}{x_p x_s} - \frac{2}{x_s} + (1 - \epsilon) \frac{2x_p}{x_s(1-s)} + \frac{2}{x_p} \right] \right. \\ \times \left[\frac{1 + 2s - 2\epsilon s}{2} (|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2) + \frac{4 + 2s - 4\epsilon}{s} |\tilde{C}_7^{\text{eff}}|^2 + (6 - 4\epsilon) \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}) \right] \\ + \frac{x_s(1 + 2s) - 2x_p}{x_p} (|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2) + \frac{4x_s(s - 2 + 8x_p) - 8x_p(s + 4)}{sx_p} |\tilde{C}_7^{\text{eff}}|^2 \\ \left. - \frac{4x_s + 8x_p}{x_p} \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}) + O(\epsilon) \right\}. \quad (4.13)$$

Infrared singularities arise only by terms proportional to $1/x_p^2$ or $1/x_s$: for this reason only the $O(\epsilon)$ pieces proportional to these terms (between square brackets) have been explicitly shown. Taking into account the full d -dimensional four-body phase space, the radiative differential rate can be written as

$$\begin{aligned}
\frac{d\Gamma_{[4]}}{ds} &= \frac{m_b}{2} (2\pi)^{d-1} \int \frac{d^{d-1}p_s}{(2\pi)^{d-1}2E_s} \int \frac{d^{d-1}k}{(2\pi)^{d-1}2E_g} \int \frac{d^{d-1}q}{(2\pi)^{d-1}2q_0} \delta^d(p - p_s - k - q) I_S^{\mu\nu} H_{\mu\nu}^S \\
&= \frac{C_S}{16\pi^2} s^{-\epsilon} (1-s)^{3-4\epsilon} \int_0^1 dx (1-x)^{1-2\epsilon} x^{1-2\epsilon} \int_0^1 d\omega (1-\omega)^{-\epsilon} \omega^{-\epsilon} \\
&\quad \times [1 - x\omega(1-s)]^{-2+2\epsilon} \left[\frac{1}{m_b^2} (q^\mu q^\nu - q^2 g^{\mu\nu}) H_{\mu\nu}^S \right], \tag{4.14}
\end{aligned}$$

where

$$C_S = \frac{(1-\epsilon)m_b^{5-6\epsilon}}{\pi^{2-3\epsilon}2^{9-10\epsilon}\Gamma(1-\epsilon)\Gamma\left(\frac{5}{2}-\epsilon\right)\Gamma\left(\frac{3}{2}-\epsilon\right)}. \tag{4.15}$$

The integration variables in Eq. (4.14) are defined by

$$\omega = \frac{1}{2}(1 - \cos\theta_{sk}), \quad x = \frac{2p \cdot k}{m_b^2 - q^2}, \tag{4.16}$$

where θ_{sk} is the angle between gluon and strange-quark momenta in the B rest frame, so that

$$x_p(s, x) = \frac{x(1-s)}{2}, \quad x_s(s, x, \omega) = \frac{x\omega(1-x)(1-s)^2}{2[1-x\omega(1-s)]}. \tag{4.17}$$

Performing explicitly the integrals in x and ω we obtain

$$\begin{aligned}
\frac{d\Gamma_{[4]}}{ds} &= \frac{\alpha_s}{3\pi} \kappa_F^2 C_S s^{-\epsilon} (1-s)^{2-4\epsilon} \\
&\times \left\{ \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} - 4\text{Li}_2(s) - 4\ln(s)\ln(1-s) - \pi^2 + \frac{35}{2} + \frac{1}{1-s} - \frac{s(2-3s)}{(1-s)^2} \ln(s) \right] \right. \\
&\times \left[\frac{1+2s-2\epsilon s}{2} \left(|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2 \right) + \frac{4+2s-4\epsilon}{s} |\tilde{C}_7^{\text{eff}}|^2 + (6-4\epsilon)\text{Re}(\tilde{C}_7^{\text{eff}}\tilde{C}_9^{\text{eff}*}) \right] \\
&+ \frac{s(4s^2+10s-4)\ln(s) - 10s^3 + 7s^2 + 6s - 3}{4(1-s)^2} \left(|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2 \right) \\
&+ \frac{s(6s^2+24s-24)\ln(s) - 19s^3 + 18s^2 + 15s - 14}{3s(1-s)^2} |\tilde{C}_7^{\text{eff}}|^2 \\
&\left. - \frac{s(2s+12)\ln(s) - 9s^2 + 4s + 5}{(1-s)^2} \text{Re}(\tilde{C}_7^{\text{eff}}\tilde{C}_9^{\text{eff}*}) \right\}. \tag{4.18}
\end{aligned}$$

In order to construct an IR-safe observable, $d\Gamma_{[4]}/ds$ has to be combined with the $\mathcal{O}(\alpha_s)$ virtual corrections in the non-radiative process. The differential decay rate for the latter can be written as

$$\begin{aligned}
\frac{d\Gamma_{[3]}}{ds} &= \kappa_F^2 C_S s^{-\epsilon} (1-s)^{2-2\epsilon} \Gamma(1-\epsilon) \left(\frac{m_b}{4\pi} \right)^\epsilon \\
&\times \left\{ 2A^2[1+2s(1-\epsilon)] + 4AB(1-s) + 2B^2(1-s)^2 + (A, B \leftrightarrow A', B') \right\} \tag{4.19}
\end{aligned}$$

in terms of the reduced amplitudes (A , B) and (A' , B') defined in (4.3). Using Eqs. (4.4)–(4.5) and isolating the $\mathcal{O}(\alpha_s)$ terms we then obtain

$$\begin{aligned}
\frac{d\Gamma_{[3]}^{(1)}}{ds} &= \frac{\alpha_s}{3\pi} \kappa_F^2 C_S s^{-\epsilon} (1-s)^{2-4\epsilon} \Gamma(1+\epsilon) \Gamma(1-\epsilon) \\
&\times \left\{ -\frac{1+2s-2\epsilon s}{2} \left(|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2 \right) \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} + 4\text{Li}_2(s) + \frac{2+4s}{s} \ln(1-s) + 12 \right] \right. \\
&\quad - \frac{4+2s-4\epsilon}{s} |\tilde{C}_7^{\text{eff}}|^2 \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} + 4\text{Li}_2(s) + 4\ln(1-s) + 20 + 16\ln\left(\frac{\mu}{m_b}\right) \right] \\
&\quad - (6-4\epsilon) \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}) \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} + 4\text{Li}_2(s) + \frac{1+4s}{s} \ln(1-s) + 16 + 8\ln\left(\frac{\mu}{m_b}\right) \right] \\
&\quad \left. + \frac{\ln(1-s)}{s} \left[(1-s) \left(|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2 \right) - 4(4-s) |\tilde{C}_7^{\text{eff}}|^2 - 2(1+2s) \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}) \right] \right\} \quad (4.20)
\end{aligned}$$

It is straightforward to check that all divergences cancel in the sum of Eq. (4.18) and Eq. (4.20). Combining these two equations we can finally obtain explicit expression for the $\tau_i(s)$ and $\sigma_i(s)$ functions defined in Eq. (3.5).

The separation between universal and non-universal terms is not completely unambiguous. We find that a convenient choice for the $\sigma_i(s)$ is given by

$$\begin{aligned}
\sigma_9(s) &= \sigma(s) + \frac{3}{2}, & \sigma_7(s) &= \sigma(s) + \frac{1}{6} - \frac{8}{3} \ln\left(\frac{\mu}{m_b}\right), \\
\sigma(s) &= -\frac{4}{3} \text{Li}_2(s) - \frac{2}{3} \ln(s) \ln(1-s) - \frac{2}{9} \pi^2 - \ln(1-s) - \frac{2}{9} (1-s) \ln(1-s). \quad (4.21)
\end{aligned}$$

With this choice the $\tau_i(s)$, given by

$$\begin{aligned}
\tau_{77}(s) &= -\frac{2}{9(2+s)} \left[2(1-s)^2 \ln(1-s) + \frac{6s(2-2s-s^2)}{(1-s)^2} \ln(s) + \frac{11-7s-10s^2}{(1-s)} \right], \\
\tau_{99}(s) &= -\frac{4}{9(1+2s)} \left[2(1-s)^2 \ln(1-s) + \frac{3s(1+s)(1-2s)}{(1-s)^2} \ln(s) + \frac{3(1-3s^2)}{1-s} \right], \\
\tau_{79}(s) &= -\frac{4(1-s)^2}{9s} \ln(1-s) - \frac{4s(3-2s)}{9(1-s)^2} \ln(s) - \frac{2(5-3s)}{9(1-s)}, \quad (4.22)
\end{aligned}$$

vanish at $s=1$.

These results are in complete agreement with those recently obtained by Asatrian et al. [11]. Indeed the $\omega_i(s)$ of Ref. [11] can be written as $\omega_7 = \sigma_7 + \tau_{77}/2$, $\omega_9 = \sigma_9 + \tau_{99}/2$, $\omega_{79} = (\sigma_7 + \sigma_9 + \tau_{79})/2$.

4.3 Virtual and bremsstrahlung corrections to $A_{\text{FB}}(s)$

A use of naïve dimensional regularization is not allowed in the case of the forward–backward asymmetry because of the ambiguities arising for $d \neq 4$ in the definition of γ_5 (or in the

trace of $L_A^{\mu\nu}$); in the case of the decay rate, the problematic γ_5 contribution vanishes because of the $p_1 \leftrightarrow p_2$ permutation symmetry of the leptonic phase space [see Eq. (4.12)]. To circumvent this problem, we employ the following regularization scheme: we treat the Dirac algebra of IR-divergent pieces strictly in four dimensions (dimensional reduction). This of course simplifies the calculation of the real emission, where only IR divergences appear, but it slightly complicates the virtual corrections, where UV divergences also occur. The latter, which do not involve any γ_5 ambiguity, are still computed in naïve dimensional regularization and renormalized as discussed at the beginning of this section. We stress that, at this level of the perturbative expansion, this hybrid regularization scheme is gauge invariant.

Within the virtual corrections we therefore need to strictly separate IR and UV divergences. As a result, the on-shell wave-function-renormalization factors given in Eqs. (4.6) within naïve dimensional regularization are partially changed. While the IR part of $Z_\psi(m_b)$ is not modified, $Z_\psi(m_s = 0)$ is different within our hybrid dimensional scheme:

$$\begin{aligned} Z_\psi(m_b) &= 1 - \frac{\alpha_s}{4\pi} \frac{4}{3} \left(\frac{\mu}{m}\right)^{2\epsilon} \left(\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} + 4\right), \\ Z_\psi(m_s = 0) &= 1 - \frac{\alpha_s}{4\pi} \frac{4}{3} \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} - 1\right). \end{aligned} \quad (4.23)$$

Being of pure UV nature, the other Z factors remain unchanged. On the amplitude level, the change of $Z_\psi(m_s = 0)$ turns out to be the only change of virtual corrections. Taking this effect into account, Eqs. (4.5) are modified as follows:

$$\begin{aligned} A_{D4}^{(1)} &= A^{(1)} + \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (1 - s)^{-2\epsilon} \left(\frac{m_b^2}{4\pi}\right)^{-\epsilon} \left[\frac{1}{4} \tilde{C}_9^{\text{eff}} + \frac{1}{2s} \tilde{C}_7^{\text{eff}}\right], \\ B_{D4}^{(1)} &= B^{(1)} + \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (1 - s)^{-2\epsilon} \left(\frac{m_b^2}{4\pi}\right)^{-\epsilon} \left[-\frac{1}{2s} \tilde{C}_7^{\text{eff}}\right], \\ A'_{D4}{}^{(1)} &= A'^{(1)} + \frac{\alpha_s}{3\pi} \Gamma(1 + \epsilon) (1 - s)^{-2\epsilon} \left(\frac{m_b^2}{4\pi}\right)^{-\epsilon} \left[\frac{1}{4} \tilde{C}_9^{\text{eff}}\right], \quad B'_{D4}{}^{(1)} = B'^{(1)}. \end{aligned} \quad (4.24)$$

As a consistency check, we have explicitly verified that the finite corrections to $R(s)$, computed in the hybrid scheme, are exactly the same as in Eqs. (4.21)–(4.22). To this purpose, the bremsstrahlung term with four-dimensional Dirac algebra is obtained by neglecting all the $O(\epsilon)$ terms in Eq. (4.13).

The asymmetric part of the squared matrix element of the four-body process — relevant to $A_{\text{FB}}(s)$ — computed with four-dimensional Dirac algebra, is given by

$$\begin{aligned} \frac{1}{m_b^2} H_{\mu\nu}^A L_A^{\mu\nu} &= \frac{128\pi}{3} \kappa_F^2 \alpha_s \left\{ y \left[-\frac{1}{x_p^2} + \frac{1-s}{x_p x_s} - \frac{2}{x_s} + \frac{2x_p}{x_s(1-s)} \right] \text{Re} \left[\tilde{C}_{10}^{\text{eff}*} \left(s \tilde{C}_9^{\text{eff}} + 2 \tilde{C}_7^{\text{eff}} \right) \right] \right. \\ &\quad + s \left[\frac{2yx_p - x_v(1-s)}{2x_p x_s(1-s)} (1-s - 2x_p) + \frac{y}{x_p} + \frac{x_v}{x_p^2} \right] \text{Re} \left(\tilde{C}_{10}^{\text{eff}*} \tilde{C}_9^{\text{eff}} \right) \\ &\quad \left. + \frac{1}{s} \left[\frac{2yx_p - x_v(1-s)}{2x_p x_s(1-s)} (2s(1-s - 2x_p) + x_p(1-s)) + \frac{y}{x_p} (x_s + 2s - 2x_p) \right] \right\} \end{aligned}$$

$$+ \frac{x_v}{2x_p^2} \left(4s - 2x_s x_p + (1 - 3s)x_p + 2x_p^2 \right) \left. \right] \text{Re} \left(\tilde{C}_{10}^{\text{eff}*} \tilde{C}_7^{\text{eff}} \right) \Big\} . \quad (4.25)$$

As can be noted, $H_{\mu\nu}^A L_A^{\mu\nu}$ is linear in the leptonic asymmetric variables y and x_v . The latter can be written as

$$\begin{aligned} y &= -\cos\theta_\ell \left[\left(\frac{p \cdot q}{m_b^2} \right)^2 - \frac{q^2}{m_b^2} \right]^{1/2} , \\ x_v &= y \frac{p \cdot k}{m_b^2} \left[1 + (p \cdot q) \frac{(p \cdot p_s) + (p \cdot k) \cos\theta_{sk}}{(p \cdot q)^2 - q^2 m_b^2} + O(\sin\phi_\ell) \right] , \end{aligned} \quad (4.26)$$

where θ_ℓ is the angle between ℓ^+ and B momenta in the dilepton centre-of-mass frame and ϕ_ℓ the corresponding azimuthal angle. The integrals of y and x_v over the leptonic phase space, weighted by $\text{sgn}(\cos\theta_\ell)$, can be expressed in terms of s and the two integration variables x and ω defined in Eq. (4.16):

$$\begin{aligned} \int \frac{d^{d-1}p_1}{2E_1} \int \frac{d^{d-1}p_2}{2E_2} \frac{\delta^d(q - p_1 - p_2)}{(2\pi)^{d-2}} \text{sgn}(\cos\theta_\ell) \left\{ \begin{array}{c} y \\ x_v \end{array} \right\} &= \frac{(q^2)^{-\epsilon} (1 - \epsilon)^{-1}}{\pi^{1-\epsilon} 2^{4-4\epsilon} \Gamma(1 - \epsilon)} \left\{ \begin{array}{c} \bar{y}(s, x, \omega) \\ \bar{x}_v(s, x, \omega) \end{array} \right\} \\ \bar{y}(s, x, \omega) &= -\frac{(1-s)\beta(s, x, \omega)}{2[1 - x(1-s)\omega]} , \\ \bar{x}_v(s, x, \omega) &= x\bar{y}(s, x, \omega) \left[1 + \frac{(1+s)[1 - \beta^2(s, x, \omega) - x^2(1-s)\omega(1-2\omega) - 2x\omega]}{2\beta^2(s, x, \omega)} \right. \\ &\quad \left. + x(1-s)\omega[2 + x(1-s)] \frac{1 - x^2(1-s)\omega(1-2\omega) - 2x\omega}{2\beta^2(s, x, \omega)} \right] , \end{aligned} \quad (4.27)$$

where

$$\beta(s, x, \omega) = \sqrt{1 - x\omega [4(1-x) + 2x(1-s)(1-2\omega) + x^2\omega(1-s)(4-x(1-s))]} . \quad (4.28)$$

Taking into account also the phase-space integration over the hadronic variables, the regularized bremsstrahlung contribution to the FB asymmetry can be written as

$$\begin{aligned} \frac{d\Gamma_{[4]}^A}{ds} &= \int d\cos\theta_\ell \frac{d^2\Gamma_{[4]}}{ds d\cos\theta_\ell} \text{sgn}(\cos\theta_\ell) = \frac{C_A}{16\pi^2} s^{-\epsilon} (1-s)^{3-4\epsilon} \int_0^1 dx (1-x)^{1-2\epsilon} x^{1-2\epsilon} \\ &\quad \int_0^1 d\omega (1-\omega)^{-\epsilon} \omega^{-\epsilon} [1 - x\omega(1-s)]^{-2+2\epsilon} \left[\frac{1}{m_b^2} H_{\mu\nu}^A L_A^{\mu\nu} \right]_{y \rightarrow \bar{y}(s, x, \omega), x_v \rightarrow \bar{x}_v(s, x, \omega)} \end{aligned} \quad (4.29)$$

where

$$C_A = \frac{m_b^{5-6\epsilon}}{\pi^{\frac{5}{2}-3\epsilon} 2^{10-10\epsilon} (1-\epsilon) \Gamma(1-\epsilon) \Gamma(1-\epsilon) \Gamma\left(\frac{3}{2}-\epsilon\right)} . \quad (4.30)$$

Because of the square root in $\beta(s, x, \omega)$, the integrals on x and ω appearing in (4.29) are rather involved and cannot be performed analytically. However, divergent contributions arise only

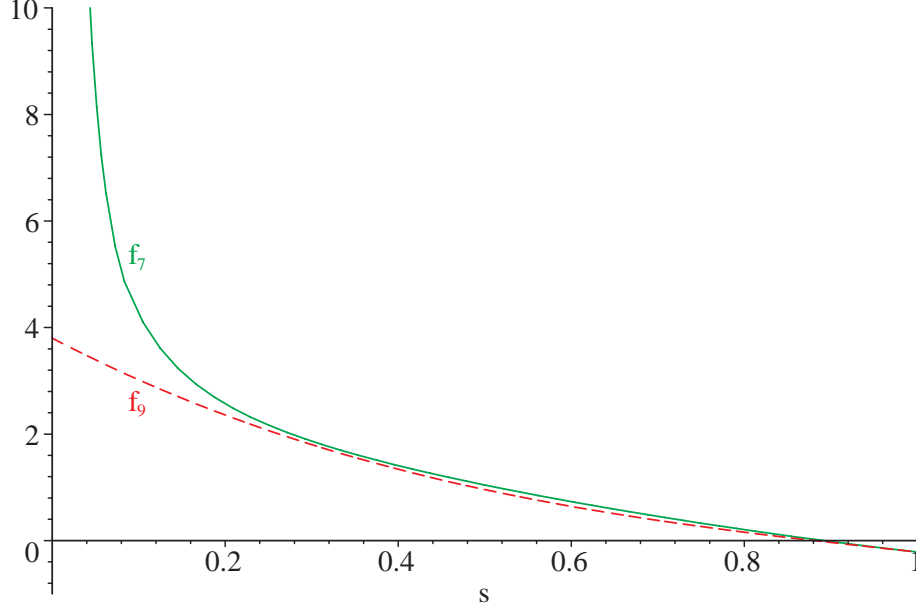


Figure 2: The functions $f_7(s)$ and $f_9(s)$.

in the limit $\bar{y} \rightarrow -(1-s)/2$ from the terms in the first line of (4.25). Computing analytically only the latter, we can write

$$\begin{aligned} \frac{d\Gamma_{[4]}^A}{ds} &= -\frac{4\alpha_s}{3\pi} \kappa_F^2 C_A s^{-\epsilon} (1-s)^{2-4\epsilon} \\ &\times \left\{ \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} - 4\text{Li}_2(s) - 4\ln(s)\ln(1-s) - \pi^2 + \frac{25}{2} + \frac{1}{1-s} - \frac{s(6-7s)}{(1-s)^2} \ln(s) \right] \right. \\ &\quad \left. \times \text{Re} \left[\tilde{C}_{10}^{\text{eff}*} \left(s\tilde{C}_9^{\text{eff}} + 2\tilde{C}_7^{\text{eff}} \right) \right] + f_9(s) \text{Re} \left(s\tilde{C}_{10}^{\text{eff}*} \tilde{C}_9^{\text{eff}} \right) + f_7(s) \text{Re} \left(2\tilde{C}_{10}^{\text{eff}*} \tilde{C}_7^{\text{eff}} \right) \right\}, \quad (4.31) \end{aligned}$$

where $f_{7,9}(s)$ denotes regular functions (for $s \neq 0$), which we evaluate by means of numerical methods.

The numerical results obtained for $f_{7,9}(s)$ are shown in Fig. 2. As can be noted, $f_7(s)$ diverges for $s \rightarrow 0$. This is a consequence of the hard-gluon emission, which allows a kinematical configuration where the hadronic system does not have spin 1/2 (as in the three-body case). This configuration, in turn, allows a non-negligible interference between \tilde{C}_7^{eff} and $\tilde{C}_{10}^{\text{eff}}$ also in the limit $s \rightarrow 0$. As a result, the photon pole ($\sim 1/s$) of the \tilde{C}_7^{eff} contribution is not cancelled and leads to a divergent term in the limit $s \rightarrow 0$. We stress that this behaviour has nothing to do with the soft-gluon emission and that is not a real divergence, since the point $s = 0$ is outside the physical kinematical region ($s \geq 4m_l^2/m_b^2$).

The cancellation of IR divergences is obtained by combining $d\Gamma_{[4]}^A/ds$ with the $\mathcal{O}(\alpha_s)$ terms

in

$$\frac{d\Gamma_{[3]}^A}{ds} = -\kappa_F^2 C_A s^{1-\epsilon} (1-s)^{2-2\epsilon} \Gamma(1-\epsilon) \left(\frac{m_b}{4\pi}\right)^\epsilon [16A_{D4}A'_{D4}] . \quad (4.32)$$

Notice that, as explicitly indicated, in this case one should use the one-loop expressions of A and A' computed in the hybrid scheme, given in Eqs. (4.24). Isolating the $\mathcal{O}(\alpha_s)$ terms we get

$$\begin{aligned} \frac{d\Gamma_{[3]}^{A(1)}}{ds} &= \frac{4\alpha_s}{3\pi} \kappa_F^2 C_A s^{-\epsilon} (1-s)^{2-4\epsilon} \Gamma(1+\epsilon) \Gamma(1-\epsilon) \\ &\times \left\{ \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} + 4\text{Li}_2(s) + \frac{2+4s}{s} \ln(1-s) + 11 \right] \text{Re} \left(s \tilde{C}_{10}^{\text{eff}*} \tilde{C}_9^{\text{eff}} \right) \right. \\ &\left. + \left[\frac{2}{\epsilon^2} + \frac{5}{\epsilon} + 4\text{Li}_2(s) + \frac{1+5s}{s} \ln(1-s) + 15 + 8 \ln \left(\frac{\mu}{m_b} \right) \right] \text{Re} \left(2 \tilde{C}_{10}^{\text{eff}*} \tilde{C}_7^{\text{eff}} \right) \right\} \quad (4.33) \end{aligned}$$

and by means of Eqs. (4.31) and (4.33) we finally obtain

$$\begin{aligned} \tau_{910}(s) &= -\frac{5}{2} + \frac{1}{3(1-s)} - \frac{s(6-7s)}{3(1-s)^2} \ln(s) - \frac{2}{9s} (3-5s+2s^2) \ln(1-s) + \frac{1}{3} f_9(s) , \\ \tau_{710}(s) &= -\frac{5}{2} + \frac{1}{3(1-s)} - \frac{s(6-7s)}{3(1-s)^2} \ln(s) - \frac{1}{9s} (3-7s+4s^2) \ln(1-s) + \frac{1}{3} f_7(s) . \quad (4.34) \end{aligned}$$

5 Phenomenological analysis

The results for $\sigma_i(s)$ and $\tau_i(s)$ functions, which encode bremsstrahlung and virtual IR corrections to $R(s)$ and $A_{\text{FB}}(s)$, and which we have determined in the previous section, are summarized in Fig. 3. As anticipated, in all cases, except for very small values of s , the universal contribution of the $\sigma_i(s)$ is largely dominant with respect to the non-universal contribution of the τ_i terms.³ The natural scale of these corrections is $\sigma_i \alpha(m_b)/\pi = \mathcal{O}(10\%)$, and they thus represent a numerically important effect both in the rate and in the FB asymmetry.

A detailed phenomenological analysis of $R(s)$ and $A_{\text{FB}}(s)$, including all NNLL effects, will be presented elsewhere, together with an independent calculation of the two-loop matrix-element corrections [21], which extends existing calculations also to the high-dilepton-mass region. Here we shall limit ourselves to analysing how the zero of the FB asymmetry (s_0) is modified by the inclusion of bremsstrahlung and virtual IR corrections. As is well known, this quantity, defined by $A_{\text{FB}}(s_0) = 0$, is particularly interesting to determine relative sign and magnitude of the Wilson coefficients C_7 and C_9 .

Employing the counting rule of Ref. [11], i.e. treating the formally $\mathcal{O}(1/\alpha_s)$ term of \tilde{C}_9^{eff} as $\mathcal{O}(1)$ (see discussion at the end of Section 3), the lowest-order value of s_0 (formally derived by the NLL expression of A_{FB}) is determined by the solution of

$$s_0 \tilde{C}_9^{\text{eff}}(s_0) + 2\tilde{C}_7^{\text{eff}} = 0 . \quad (5.35)$$

³ Note that the $\sigma_i(s)$ enter linearly in the effective coefficients, while the $\tau_i(s)$ enter directly in the rates: for this reason, to facilitate the comparison, the $\sigma_i(s)$ have been multiplied by a factor of 2 in Fig. 3.

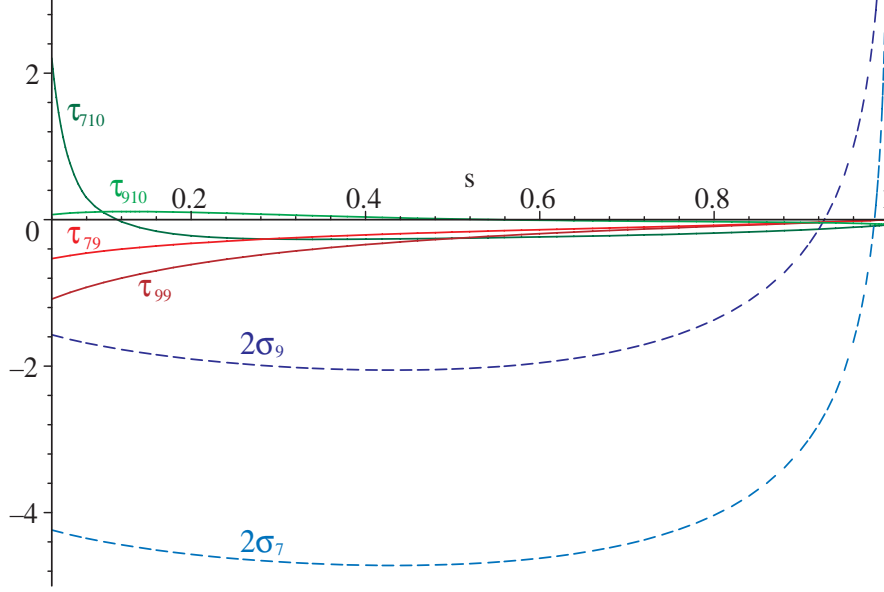


Figure 3: The functions $\sigma_i(s)$ and $\tau_i(s)$; $\tau_{77}(s)$ is not explicitly shown here, since it is almost indistinguishable from $\tau_{99}(s)$.

Using the values of the effective coefficients at $\mu = 5$ GeV reported in Ref. [11], this leads to

$$s_0^{\text{NLL}} = 0.14 \pm 0.02 , \quad (5.36)$$

where the error is determined by the scale dependence ($2.5 \text{ GeV} \leq \mu \leq 10 \text{ GeV}$).

Neglecting the small contribution of δ_{FB} [see Eq. (3.6)], the next-to-leading order equation for s_0 reads

$$s_0 C_9^{\text{new}}(s_0) \left(1 + \frac{\alpha_s}{\pi} \tau_{910}(s_0) \right) + 2C_7^{\text{new}}(s_0) \left(1 + \frac{\alpha_s}{\pi} \tau_{710}(s_0) \right) = 0 , \quad (5.37)$$

which leads to

$$s_0^{\text{NNLL}} = 0.16 \pm 0.01 . \quad (5.38)$$

Also in this case the error, of about 6%, is determined by the residual scale dependence. In particular, this does not include the parametric uncertainty induced by the ratio m_c/m_b , which we fixed to $m_c/m_b = 0.29$.

We recall that the *new* effective Wilson coefficients defined in (3.7) include not only purely UV virtual terms, but also universal bremsstrahlung and corresponding IR virtual corrections (parametrized in σ_i). The global effect of the bremsstrahlung and virtual IR corrections we have evaluated is that of reducing by $\sim 11\%$ the large ($\sim 27\%$) shift between s_0^{NNLL} and s_0^{NLL}

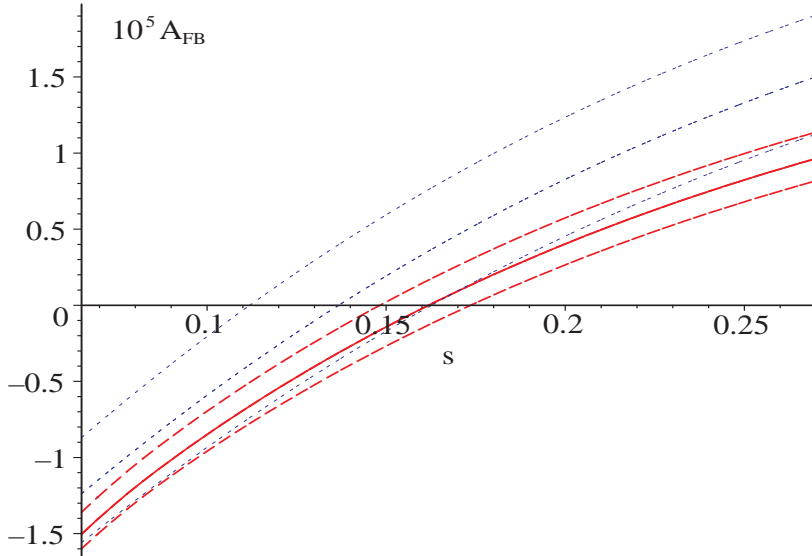


Figure 4: Comparison between NNLL and NLL results for $A_{\text{FB}}(s)$ in the low s region. The three thick (red) lines are the NNLL predictions for $\mu = 5$ GeV (full), and $\mu = 2.5$ and 10 GeV (dashed); the dotted (blue) curves are the corresponding NLL results. All curves have been obtained for $m_c/m_b = 0.29$.

induced by the NNLL UV virtual terms:⁴

$$\left. \frac{\delta s_0}{s_0} \right|_{\sigma_i, \tau_i} = \frac{\alpha_s}{\pi} [\sigma_7(s_0) - \sigma_9(s_0) + \tau_{710}(s_0) - \tau_{910}(s_0)] = -0.11 . \quad (5.39)$$

On the other hand, it is clear that the reduction of the error (or the reduction of the scale dependence) in Eq. (5.38) is only due to the inclusion of the UV virtual corrections computed in Ref. [11]. An illustration of the shift of the central value and the reduced scale dependence between NNL and NNLL expressions of $A_{\text{FB}}(s)$, in the low s region, is presented in Fig. 4. We note that in this region the nonperturbative $1/m_b^2$ and $1/m_c^2$ corrections to $A_{\text{FB}}(s)$ are very small [8, 9, 10] and can safely neglected compared to the scale-dependence uncertainty.

Interestingly, at this level of precision, the result obtained using the ordinary LL expansion is very similar to the one obtained using the phenomenological counting rule of Ref. [11]. Indeed, at a pure NNLL level, i.e. neglecting all $O(\alpha_s)$ terms in C_7^{new} and retaining the $O(\alpha_s^{-1} \times \alpha_s)$ terms of C_9^{new} , we also find as central value $s_0^{\text{NNLL}} = 0.16$.

⁴A similar increase of the position of the zero in the FB asymmetry at the $\sim 30\%$ level has been found in the exclusive $B \rightarrow V$ channel [25]. In the inclusive channel the effect turns out to be somehow reduced by the inclusion of the bremsstrahlung corrections, which are of course absent in the exclusive mode.

6 Summary

The inclusive $B \rightarrow X_s \ell^+ \ell^-$ transition, which is starting to be accessible at B factories [5], represents a new source of theoretically clean observables, complementary to the $B \rightarrow X_s \gamma$ rate. In particular, kinematic observables such as the invariant-dilepton-mass spectrum and the lepton forward–backward asymmetry in $B \rightarrow X_s \ell^+ \ell^-$, provide a clean information on short-distance couplings not accessible in $B \rightarrow X_s \gamma$.

In the present paper we calculated bremsstrahlung and corresponding virtual corrections in $B \rightarrow X_s \ell^+ \ell^-$. We used a full dimensional regularization scheme of infrared divergences (both soft and collinear ones), which also avoids any ambiguity related to the definition of γ_5 .

For the dilepton-invariant-mass spectrum, these contributions have already been evaluated in Refs. [11, 12], using a different technique. Our results are in complete agreement with those. We also presented the first computation of the soft-gluon (and corresponding virtual) corrections in the FB asymmetry, with the help of which we evaluated this observable systematically to NNLL precision for the first time.

The new contributions are rather important and significantly improve the sensitivity of the inclusive $B \rightarrow X_s \ell^+ \ell^-$ decay in testing extensions of the Standard Model in the sector of flavour dynamics. In particular, the corrections we have computed shift by about 10% the position of the zero of the FB asymmetry. The complete effect of NNLL contributions to this interesting observable adds up to a 16% shift compared with the NLL result, with a residual error due to the scale dependence reduced at the 6% level.

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Appendix: Auxiliary definitions

- The function $h(z)$ describing next-to-leading order QCD corrections to the semileptonic decay [see Eq. (3.2)] is given by [22]:

$$\begin{aligned}
 h(z) = & -(1 - z^2) \left(\frac{25}{4} - \frac{239}{3} z + \frac{25}{4} z^2 \right) + z \ln(z) \left(20 + 90 z - \frac{4}{3} z^2 + \frac{17}{3} z^3 \right) \\
 & + z^2 \ln^2(z) (36 + z^2) + (1 - z^2) \left(\frac{17}{3} - \frac{64}{3} z + \frac{17}{3} z^2 \right) \ln(1 - z) \\
 & - 4(1 + 30 z^2 + z^4) \ln(z) \ln(1 - z) - (1 + 16 z^2 + z^4) (6 \operatorname{Li}(z) - \pi^2) \\
 & - 32 z^{3/2} (1 + z) \left[\pi^2 - 4 \operatorname{Li}(\sqrt{z}) + 4 \operatorname{Li}(-\sqrt{z}) - 2 \ln(z) \ln \left(\frac{1 - \sqrt{z}}{1 + \sqrt{z}} \right) \right].
 \end{aligned}$$

- The effective coefficients $\tilde{C}_{7-10}^{\text{eff}}$ appearing in Eq. (3.7) are defined as:

$$\begin{aligned}
 \tilde{C}_7^{\text{eff}} &= \frac{4\pi}{\alpha_s(\mu)} C_7(\mu) - \frac{1}{3} C_3(\mu) - \frac{4}{9} C_4(\mu) - \frac{20}{3} C_5(\mu) - \frac{80}{9} C_6(\mu) \\
 \tilde{C}_8^{\text{eff}} &= \frac{4\pi}{\alpha_s(\mu)} C_8(\mu) + C_3(\mu) - \frac{1}{6} C_4(\mu) + 20 C_5(\mu) - \frac{10}{3} C_6(\mu)
 \end{aligned}$$

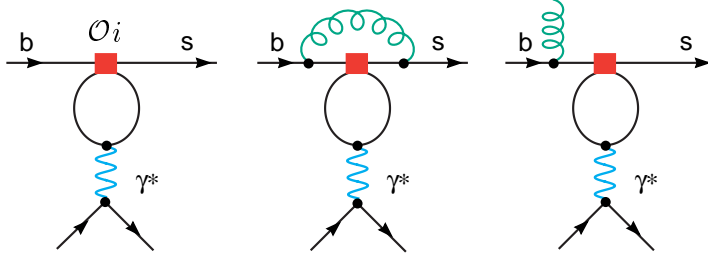


Figure A1: Examples of virtual and bremsstrahlung contributions of the four-quark operators $\mathcal{O}_{1\dots 6}$ that are automatically taken into account by the redefinition of the Wilson coefficients in Eq. (A.1).

$$\begin{aligned}
\tilde{C}_9^{\text{eff}}(s) &= \frac{4\pi}{\alpha_s(\mu)} C_9(\mu) + \sum_{i=1}^6 C_i(\mu) \gamma_{i9}^{(0)} \ln\left(\frac{m_b}{\mu}\right) \\
&+ h(z, s) \left(\frac{4}{3} C_1(\mu) + C_2(\mu) + 6C_3(\mu) + 60C_5(\mu) \right) \\
&+ h(1, s) \left(-\frac{7}{2} C_3(\mu) - \frac{2}{3} C_4(\mu) - 38C_5(\mu) - \frac{32}{3} C_6(\mu) \right) \\
&+ h(0, s) \left(-\frac{1}{2} C_3(\mu) - \frac{2}{3} C_4(\mu) - 8C_5(\mu) - \frac{32}{3} C_6(\mu) \right) \\
&+ \frac{4}{3} C_3(\mu) + \frac{64}{9} C_5(\mu) + \frac{64}{27} C_6(\mu) \\
\tilde{C}_{10}^{\text{eff}} &= \frac{4\pi}{\alpha_s(\mu)} C_{10}(\mu) , \tag{A.1}
\end{aligned}$$

where

$$\begin{aligned}
h(z, s) &= -\frac{4}{9} \ln(z) + \frac{8}{27} + \frac{16}{9} \frac{z}{s} - \frac{2}{9} \left(2 + \frac{4z}{s} \right) \sqrt{\left| \frac{4z-s}{s} \right|} \times \\
&\times \begin{cases} 2 \arctan \sqrt{\frac{s}{4z-s}} & \text{for } s < 4z , \\ \ln \left(\frac{\sqrt{s} + \sqrt{s-4z}}{\sqrt{s} - \sqrt{s-4z}} \right) - i\pi & \text{for } s > 4z . \end{cases} \tag{A.2}
\end{aligned}$$

Note that specific one- and two-loop and matrix-element contributions of the four-quark operators \mathcal{O}_{1-6} (including the corresponding bremsstrahlung contributions) such as the one shown in Fig. A1 are automatically included by the redefinition of the Wilson coefficients C_7 , C_9 and C_{10} given in (A.1). In fact, using this redefinition, the bremsstrahlung and virtual corrections that were shown in Fig. 1 automatically take these effects into account. The Wilson coefficients C_i in (A.1), which are needed to NNLL precision, are given in [16].

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