

Small x Physics, High Gluon Density Nuclei

A Scale:

$$\Lambda^2 = \frac{dN/dy}{\pi R^2}$$

GLR

$$y = \ln 1/x \quad \text{DIS}$$

$$y = \frac{1}{2} \ln \frac{P^+}{p_i^+} = \ln \frac{P^+}{M_1^+} \sim \ln \frac{1}{x} \quad \text{in scattering}$$

$$y = \ln \frac{P^+}{p^+} \sim \ln P^+ x^-$$

Short range correlations in y

⇓

Λ^2 is a local variable

⇓

$\frac{dN}{dy} / \pi R^2$ a local density

MV

⇓

$$\alpha_s(\Lambda) \ll 1$$

(2)

Gluon Density is Large



Phase space density is Large



Classical gluon field

$$\frac{dN}{dy} \sim \frac{1}{\alpha_s}$$

$$A_{gc} \sim \frac{1}{g}$$

Theory:

Effective Action

MV,
JKLW

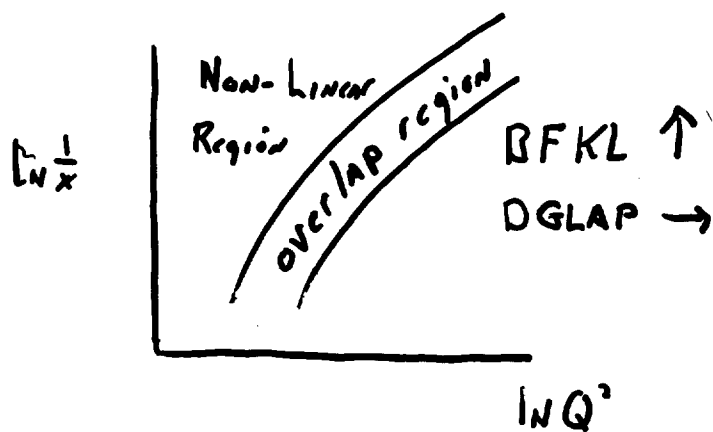
$$\int [d\rho] e^{-F[\rho]} \int [dA] e^{iS[A,\rho]}$$

$$p^+ \lesssim \Lambda^+$$

p^+ not $\ll \Lambda^+ \Rightarrow A$ is
classical

Reduce Λ^+ by renormalization
group

JKMW
JKLW



Why nuclei?

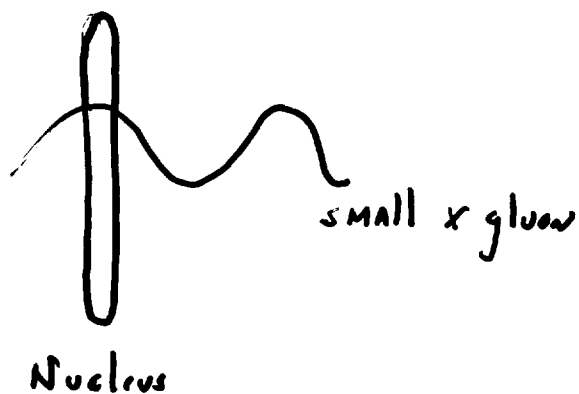
$$\Lambda^2 = \frac{dN/dy}{\pi R^2} \sim A^{1/2} / x^\delta$$

$$\delta \sim .2 - .4$$

$$A \sim 10^2 \Rightarrow x \rightarrow 10^{-2} - 10^{-4} x_0$$

Large A allows gluon densities
to increase

Nuclear structure
unimportant
All hadrons the
same.



(4)

Saturation \leftrightarrow Unitarity

$$\sigma_{\text{gluon}} \sim \frac{1}{E_{\text{sat}}^2} \quad \text{IR dominance}$$

$$P \sim (\sigma_{\text{gluon}} N) \lambda \quad \text{B-M}$$

$$\sim \sigma_{\text{gluon}} \Lambda^2 \sim 1$$

Saturation scale is energy transfer
scale for glue collisions where

$$P \sim 1$$

E-K, L-M

Consequence:

$$\frac{dN}{dy} = 2 A G(x_0; Q^2 = E_{\text{sat}}^2) \quad \text{Balasubramanian-Mueller}$$

$$\frac{1}{x_0} \sim \frac{\sqrt{s}}{E_{\text{sat}}}$$

($N_{gc} \sim N_{\pi}$ in any
reasonable cascade-hydro scenario)

Comment on Blaizot-Mueller relation:

In full classical treatment

$$\frac{1}{\pi R^2} \frac{dN}{dy} = \# \text{ produced gluons/area}$$

$$\Lambda^2 = \frac{\chi G(x)}{\pi R^2} = \text{intrinsic glue/area}$$

Dimensionally

$$\frac{dN}{dy} = K \alpha A G(x_0, Q^2 = E_{\text{sat}}^2)$$

K being computed by Krasnitz + Venugopalan;
Bass, Muller + Poschl

Expect $K \sim 1$

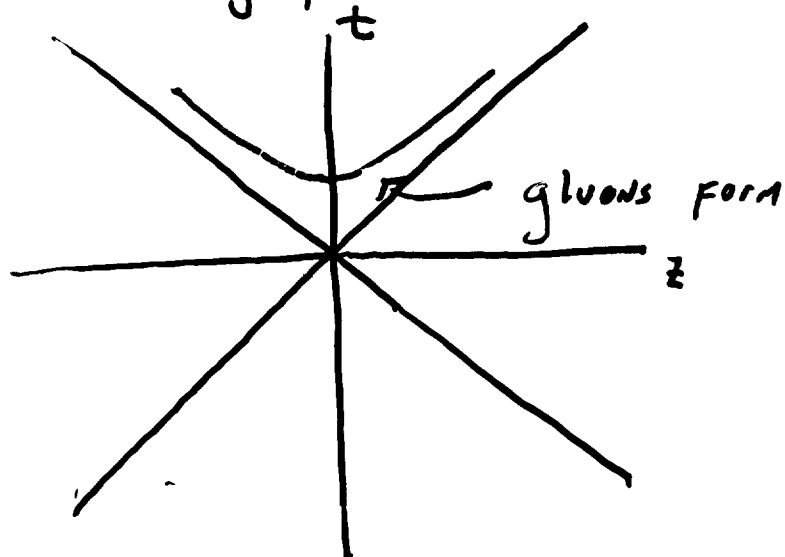
(5)

$$\frac{dN}{dE_T} \sim \exp - \frac{(E - c_1 E_{SAT} N)^2}{4c_2 E_{SAT}^2 N} \quad E-K$$

E_T multiplicity dominated by jets

Early space-time dynamics:

KMW
BMP
KV



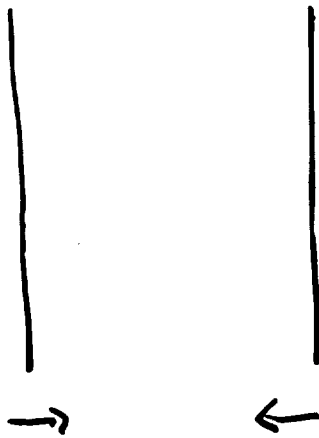
Classical Fields:

$$A^i = \epsilon(x^-) \alpha_1^i(x_1) + \epsilon(x^+) \alpha_2^i(x_2)$$

$$(E^i \cdot B^i \sim \delta(x^-) B_1^i + \delta(x^+) B_2^i)$$

$$\Upsilon_{\text{formation}} \sim \bar{\Lambda}^{-1} \sim \left(\frac{\pi R^2}{dN/dy} \right)^{1/2}$$

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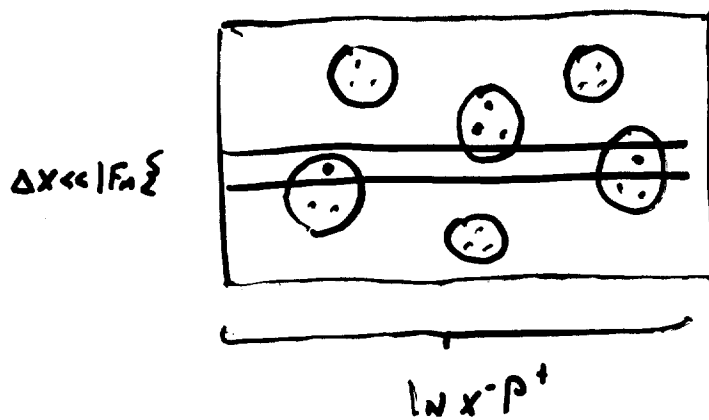


$$\vec{E} \perp \vec{B} \perp \hat{z}$$

$$\vec{E}, \vec{B} \sim \delta(z) \text{ or } \delta(z')$$

On scale $\sim l_{Fn}$ $\vec{E}, \vec{B} = 0$

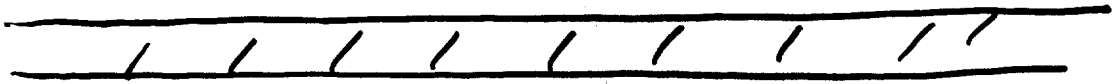
On smaller scales \vec{E}, \vec{B} randomly oriented in sheet



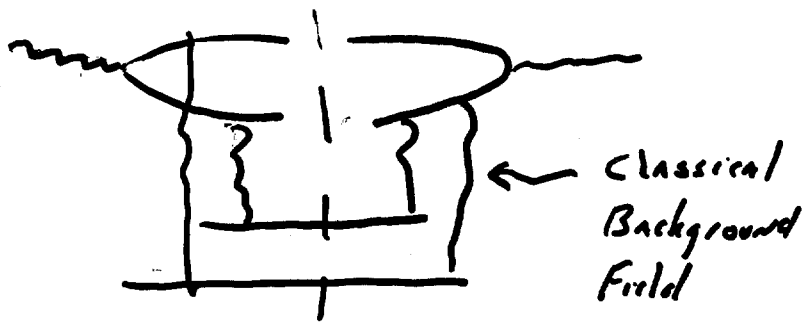
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$$E_{\text{FORM}} \sim \left(\frac{dN}{dy} / \pi R^2 \right)^2 \sim A^{2/3}$$

$$\begin{aligned} \Upsilon_{\text{decoupling}} &\sim \Upsilon_{\text{pom}} \Lambda^3 / \Lambda_{\text{decoupling}}^3 \\ &\sim \left(\frac{\Lambda^2}{\Lambda_{\text{decoop}}^2} \right) \frac{1}{\Lambda_{\text{decoupling}}} \end{aligned}$$

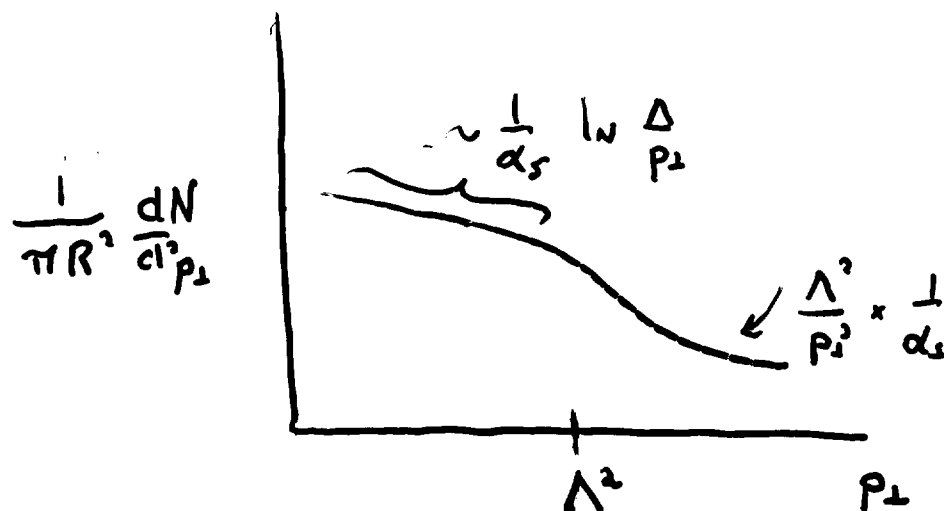


Deep Inelastic Scattering KMV



⑦

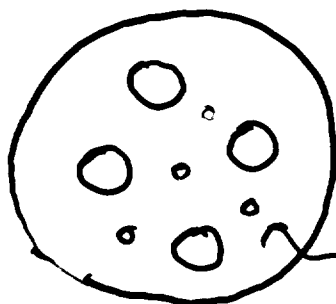
Intrinsic p_{\perp} distribution
of glue:



Tail rises as Λ increases

$$\Lambda \sim \left(\frac{1}{x}\right)^{K d_s}$$

Plateau stays flat $\sim 1/d_s$



smaller x gluons
are smaller

No problem with unitarity
in DIS!

$$G \sim \pi R^2 \int_0^{Q^2} d^2 p_\perp \frac{d}{\pi R^2} \frac{dN_g}{d^2 p_\perp}$$

$$\sim \pi R^2 Q^2, \quad Q \ll \Lambda$$

$$\sim \pi R^2 \Lambda^2, \quad Q \gg \Lambda$$

Λ^2 small Q

Λ large Q

? How to best measure saturation
in deep inelastic?

Problem: $Q \lesssim E_{SAT}$, $\frac{dF_2}{d \ln Q^2}$ is ($x \ll 1$)

not proportional to gluon
density as usually assumed

(8)

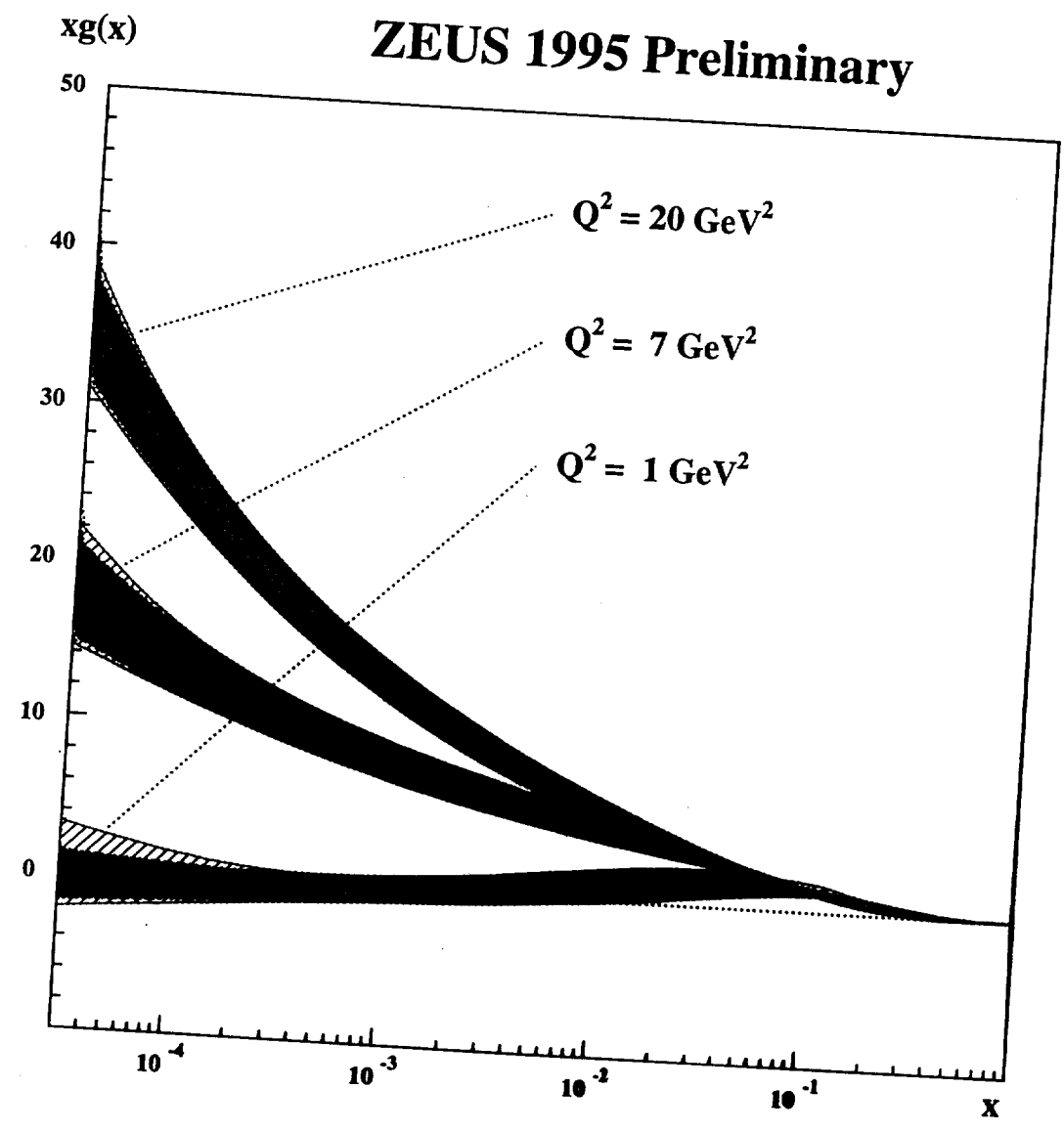


Figure 10: The gluon distribution $xg(x)$ as a function of x at fixed values of $Q^2 = 1, 7$ and 20 GeV^2 . The inner shaded bands show the 'HERA standard errors' of Sec. 6. The outer hatched bands indicate the quadratic sum of the 'HERA standard' and the 'parameterisation' errors.

ZEUS 1995 Preliminary

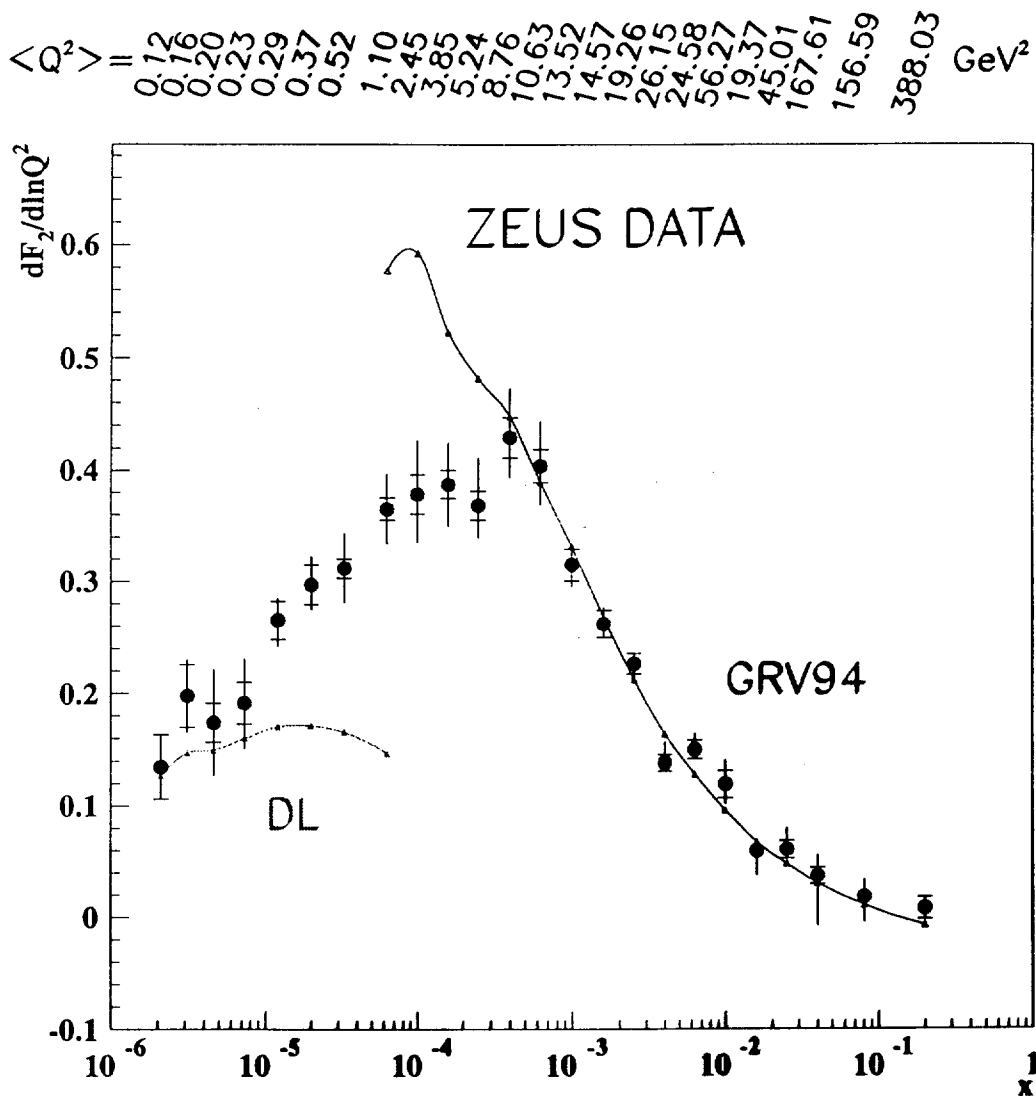
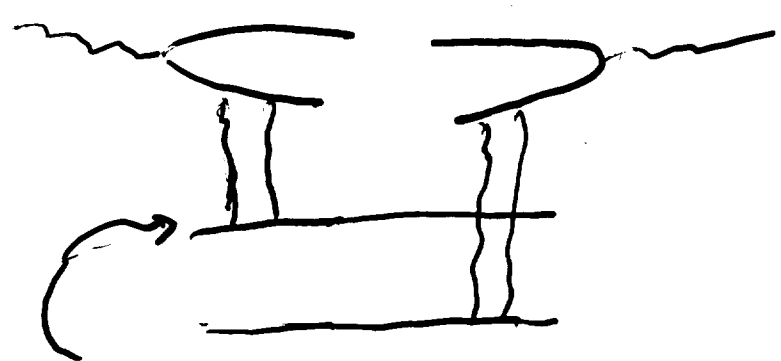


Figure 8: $dF_2/d\ln Q^2$ as a function of x calculated by fitting ZEUS F_2 data in bins of x to the functional form $a + b \ln Q^2$ as described in the text. The inner error bar shows the statistical error and the outer the total statistical and systematic error added in quadrature. In an x bin $\langle Q^2 \rangle$ is calculated from the weighted mean of $\ln Q^2$ as described in the text. The linked points labelled DL and GRV are from the Donnachie-Landshoff Regge fit [21] and the GRV94 NLO QCD fit [7]. In both cases the points are obtained using the same weighted range of Q^2 as for the experimental data.

Diffraction:

H
H-B
K-M



Averaging over color \Rightarrow no p_{\perp} transfer

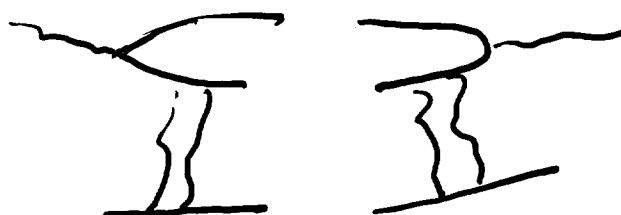
Large p'' difference \Rightarrow no p'' transfer



Hadron wave function unchanged



How can this be? Diffraction
is leading twist



Two gluon exchange ~~model~~ model

$$\frac{dG}{d^2k_2} \sim G^2 \frac{1}{k_2^4}$$

$$\int_{\Lambda^2}^{Q^2} d^2k_2 \frac{dG}{d^2k_2} \sim \frac{1}{\Lambda^2} \sim \frac{1}{G}$$

$G \sim G$ is leading twist
and $\sim G$

Non-leading twist sum to
leading twist!!

Outlook:

Experiment:

Gross properties of RHIC events?

What is E_{sat} ?

Tests of space-time picture?

eA at HERA or RHIC?

Distributions + saturation?

Diffraction + saturation?

Theory:

Is it really true?

Explicit renormalization group
at high density?

Computing in saturation region?

G_{tot} at high E ?

AGK rules?