Universal Pion Freeze-Out in Heavy-Ion Collisions

D. Adamová,[1] G. Agakichiev,[2] H. Appelshäuser,[3] V. Belaga,[4] P. Braun-Munzinger,[2] A. Castillo,[2] A. Cherlin,[5] S. Damjanović,[3] T. Dietel,[3] L. Dietrich,[3] A. Drees,[6] S. I. Esumi,[3] K. Filimonov,[3] K. Fomenko,[4] Z. Fraenkel,[5] C. Garabatos,[2] P. Glässel,[3] G. Hering,[2] J. Holeczek,[2] V. Kushpil,[1] B. Lenkeit,[7] W. Ludolphs,[3] A. Maas,[2] A. Marín,[2] J. Milošević,[3] A. Milov,[5] D. Miśkowiec,[2] Yu. Panebrattsev,[4] O. Petchenova,[4] V. Petráček,[3] A. Pfeiffer,[7] J. Rak,[9] I. Ravinovich,[5] P. Rehak,[8] H. Sako,[2] W. Schmitz,[3] J. Schukraft,[7] S. Sedykh,[2] S. Shimansky,[4] J. Slívová,[3] H. J. Specht,[3] J. Stachel,[3] M. Šumbera,[1] H. Tilsner,[3] I. Tserruya,[5] J. P. Wessels,[2] T. Wienold,[3] B. Windelband,[3] J. P. Wurm,[9] W. Xie,[5] S. Yurevich,[3] and V. Yurevich[4]

(CERES Collaboration)

[1]*Nuclear Physics Institute ASCR, 25068 Řež, Czech Republic*
[2]*Gesellschaft für Schwerionenforschung (GSI), 64291 Darmstadt, Germany*
[3]*Physikalisches Institut der Universität Heidelberg, 69120 Heidelberg, Germany*
[4]*Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[5]*Weizmann Institute, Rehovot 76100, Israel*
[6]*Department of Physics and Astronomy, State University of New York–Stony Brook, Stony Brook, New York 11794-3800*
[7]*CERN, 1211 Geneva 23, Switzerland*
[8]*Brookhaven National Laboratory, Upton, New York 11973-5000*
[9]*Max-Planck-Institut für Kernphysik, 69117 Heidelberg, Germany*

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Based on an evaluation of data on pion interferometry and on particle yields at midrapidity, we propose a universal condition for thermal freeze-out of pions in heavy-ion collisions. We show that freeze-out occurs when the mean free path of pions λ_f reaches a value of about 1 fm, which is much smaller than the spatial extent of the system at freeze-out. This critical mean free path is independent of the centrality of the collision and beam energy from the Alternating Gradient Synchrotron to the Relativistic Heavy Ion Collider.

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A systematic study of the space-time extent and the dynamical behavior of the pion source in relativistic heavy-ion collisions at thermal freeze-out can be obtained from analysis of pion Hanbury-Brown and Twiss (HBT) interferometry data. Understanding these aspects is vital for interpretation of the data in terms of formation of the quark-gluon plasma. Indeed, recent HBT results from the Relativistic Heavy Ion Collider (RHIC) and how they fit into the systematics have been noted as a major puzzle [1]. In this Letter we present an investigation of the freeze-out conditions at beam energies from the Alternating Gradient Synchrotron (AGS) to RHIC. In particular, the HBT data at 40, 80, and 158 *A*GeV, recently published by the CERES experiment at the Super Proton Synchrotron (SPS) [2], provide an important link between the existing results from AGS, SPS, and RHIC, thereby shedding light on the RHIC puzzle.

Thermal freeze-out of pions and its connection to the mean free path has been discussed previously (see, e.g., [3–8]). The mean free path of pions at freeze-out is defined as

$$\lambda_f = \frac{1}{\rho_f \sigma} = \frac{V_f}{N\sigma}, \tag{1}$$

where σ is the total cross section of pions with the surrounding medium and ρ_f is the freeze-out density which can be replaced by the number of particles N contained in the freeze-out volume V_f.

The pion freeze-out volume V_f can be accessed experimentally by pion interferometry. Midrapidity pion HBT data have been published from central collisions of lead and gold nuclei over a wide range of beam energies. Here, we focus on recent HBT results from three experiments which have kinematical access to the region of low transverse pair momentum $k_t = \frac{1}{2}|\vec{p}_{t,1} + \vec{p}_{t,2}|$: Experiment E895 at the AGS [9], the CERES experiment at the SPS [2,10], and the STAR experiment at RHIC [11]. All three experiments employ large volume time projection chambers, thereby applying comparable analysis methods with similar sources of systematic uncertainties.

For the calculation of the freeze-out volume V_f, we use the following expression:

$$V_f = (2\pi)^{3/2} R_{\mathrm{long}} R_{\mathrm{side}}^2, \tag{2}$$

assuming a density distribution of Gaussian shape in all three dimensions. The longitudinal and sideward radius parameters R_{long} and R_{side} are measured in the longitudinal comoving system of the pion pair, using the Cartesian decomposition of the three-momentum difference vector $\vec{q} = (q_{\mathrm{long}}, q_{\mathrm{side}}, q_{\mathrm{out}})$ as proposed in [12].

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The definition of a freeze-out volume in heavy-ion collisions has to be taken with some care because of the strong collective expansion of the system. Collective expansion leads to space-momentum correlations of the emitted pions, resulting in a k_t-dependent reduction of the observed HBT radius parameters as compared to the "true" geometric dimensions of the source [13–16]. We prefer to use a definition containing only measured quantities rather than model dependent parameters that may not be appropriate at all beam energies. But to avoid strong bias of the extracted beam energy dependence due to expansion, we have selected and compared data measured at similar $\langle k_t \rangle$ values of about 0.16 GeV/c: The E895 data were taken at $\langle k_t \rangle = 0.148$ GeV/c, while the STAR data are at $\langle k_t \rangle = 0.170$ GeV/c. From the CERES data we have calculated the average of the results in the k_t bins at $\langle k_t \rangle = 0.125$ and 0.195 GeV/c.

At all beam energies under investigation, the observed strong decrease of $R_{\rm long}$ as a function of k_t is consistent with a boost-invariant expansion in the longitudinal direction [14]. In this limiting case, the geometric size of the system in the longitudinal direction is much larger than the measured homogeneity length $R_{\rm long}$. The corresponding homogeneity scale in velocity space is given by the average thermal velocity $\langle \beta_{\rm th} \rangle$ of the pions. The thermal velocity $\langle \beta_{\rm th} \rangle$ can be calculated relativistically using the expression

$$\langle \gamma_{\rm th} \rangle - 1 = \frac{1}{3}\left(\frac{K_1(m_t/T_f)}{K_2(m_t/T_f)} - 1\right) + \frac{T_f}{m_t}, \qquad (3)$$

with $\langle \gamma_{\rm th} \rangle = (1 - \langle \beta_{\rm th} \rangle^2)^{-1/2}$, $m_t = (m_\pi^2 + k_t^2)^{1/2}$, the thermal freeze-out temperature T_f, and Bessel functions K_1 and K_2. Using Eq. (3), we obtain $\langle \beta_{\rm th} \rangle = 0.7$ at $k_t = 0.16$ GeV/c, assuming $T_f = 120$ MeV. Turning this into a rapidity results in $y_{\rm th} = {\rm arctanh}(0.7) = 0.87$. We conclude that a source element of longitudinal size $R_{\rm long}$ corresponds to 0.87 units of rapidity at $k_t = 0.16$ GeV/c. Note that $R_{\rm long}$ is the rms of the length of homogeneity and so is the corresponding width in rapidity. As for the spatial distribution, we assume a Gaussian shape also in rapidity.

Also, the transverse radius parameter $R_{\rm side}$ is reduced in the presence of collective transverse flow. Model dependent approximations [17–19] predict a reduction by about 20%–25% as compared to the true geometric transverse source size at $k_t = 0.16$ GeV/c for $T_f = 120$ MeV and a typical average transverse flow velocity $\langle v_t \rangle = 0.5c$ [2,20,21]. We prefer not to use this reduction in the analysis but rather include the corresponding model dependent underestimate of the volume in the systematic errors below. The observed beam energy dependence of T_f and $\langle v_t \rangle$ [22] imposes an additional systematic uncertainty. Within reasonable limits of T_f and $\langle v_t \rangle$, the relative change of $R_{\rm side}$, however, is small and even partially compensated, as both T_f and $\langle v_t \rangle$ increase with beam energy.

The experimentally determined freeze-out volume V_f, calculated according to Eq. (2) as a function of $\sqrt{s}$, is shown in Fig. 1(a). We observe a steep decrease of V_f at AGS energies and an increase throughout the SPS energy regime towards RHIC. The data indicate the existence of a minimum between AGS and SPS energies. The origin of this nonmonotonic behavior cannot be understood in terms of previously presented freeze-out scenarios, where constant particle density in coordinate space or constant pion phase space density have been proposed as possible universal freeze-out conditions [23,24]. This is demonstrated in Fig. 1(b), where a compilation of the midrapidity densities $dN/dy|_{y_{\rm mid}}$ of pions and protons as a function of $\sqrt{s}$ is shown [25–31]. The number of pions and the sum of pions and nucleons are monotonically increasing with $\sqrt{s}$, without indication of a minimum.

For the determination of the critical mean free path λ_f, we need to evaluate the denominator in Eq. (1). In the presence of different particle species i, we replace $N\sigma$ by a sum:

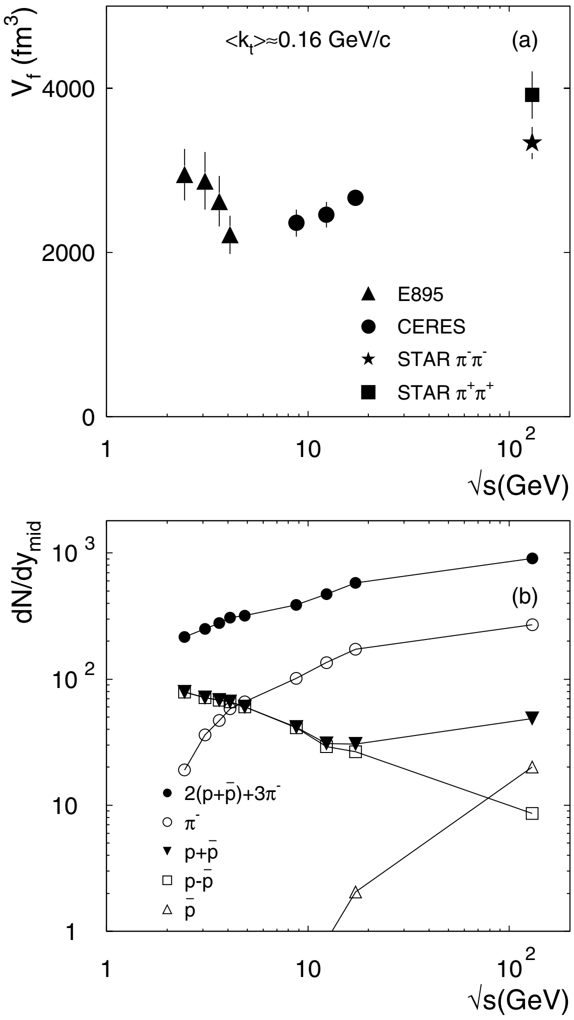

FIG. 1. (a) The freeze-out volume V_f as a function of $\sqrt{s}$. (b) Midrapidity densities of negative pions, protons, and antiprotons vs $\sqrt{s}$.

$$N\sigma = \sum_i N_i\sigma_{\pi i} = N_N\sigma_{\pi N} + N_\pi\sigma_{\pi\pi}. \qquad (4)$$

For simplicity, we neglect less abundant particle species such as kaons and deuterons. Following our previous argument, we integrate over the rapidity distribution with an rms of $y_{\rm th} = 0.87$ and account for the different isospin states, leading to:

$$\begin{aligned} N_N &= 2\sqrt{2\pi} \times 0.87 dN_{p+\bar{p}}/dy|_{y_{\rm mid}}, \quad \text{and} \\ N_\pi &= 3\sqrt{2\pi} \times 0.87 dN_{\pi^-}/dy|_{y_{\rm mid}}. \end{aligned} \qquad (5)$$

For the cross sections we use $\sigma_{\pi\pi} = 13$ mb and a thermal and isospin averaged value $\sigma_{\pi N} = 72$ mb [7]. The latter assumes a thermal freeze-out temperature $T_f = 120$ MeV independent of beam energy and ignoring possible changes due to the nonequilibrium nature of thermal freeze-out.

In Fig. 2(a) $N\sigma$ is shown as a function of $\sqrt{s}$. At AGS energies $N\sigma$ is dominated by the contribution from nucleons, while at RHIC the contribution from pions exceeds that from nucleons, due to the change in chemical fireball composition. It is the large ratio of $\sigma_{\pi N}/\sigma_{\pi\pi}$ combined with the steep increase in pion multiplicity with beam energy that produces a nonmonotonic beam energy dependence of $N\sigma$, leading to a minimum in the transition region between AGS and SPS energies, in striking similarity with the observed behavior of V_f in Fig. 1(a). To demonstrate this quantitatively, we have superimposed V_f and $N\sigma$ in Fig. 2(b). According to Eq. (1) we conclude that thermal pion freeze-out occurs at a constant mean free path of $\lambda_f \approx 1.0$ fm. We note that λ_f is much smaller than the system size: Thermal freeze-out is determined by the product of cross section times local density. A similar local pion freeze-out criterion was discussed in [6,7] albeit only for fireballs with comparable nucleon and pion numbers. This behavior indicates a significant degree of opaqueness of the pion source [5], as it arises naturally for a collectively expanding system [32]. Considering systematic effects on the evaluation of λ_f such as the usage of $R_{\rm side}$ for the volume, the Gaussian assumption for the spatial and rapidity distributions, the uncertainty in temperature (assumed as 120 MeV) and thereby in cross sections, and the uncertainty in evaluating the average thermal velocity, we find that the first two completely dominate, leading to $0.7 \le \lambda_f \le 1.4$ fm.

Pion freeze-out at constant λ_f implies that the freeze-out density ρ_f is constant, as long as the chemical composition of the fireball does not change. This is indeed demonstrated by recent results of the CERES collaboration for the freeze-out volume V_f in 40, 80, and 158 *A*GeV Pb + Au collisions [2] shown in Fig. 3(a). Since, at fixed energy, particle abundances scale approximately linearly with the number of participating nucleons $\langle N_{\rm part}\rangle$ [33–35], the observation of a linear increase of V_f with centrality is consistent with freeze-out at constant density.

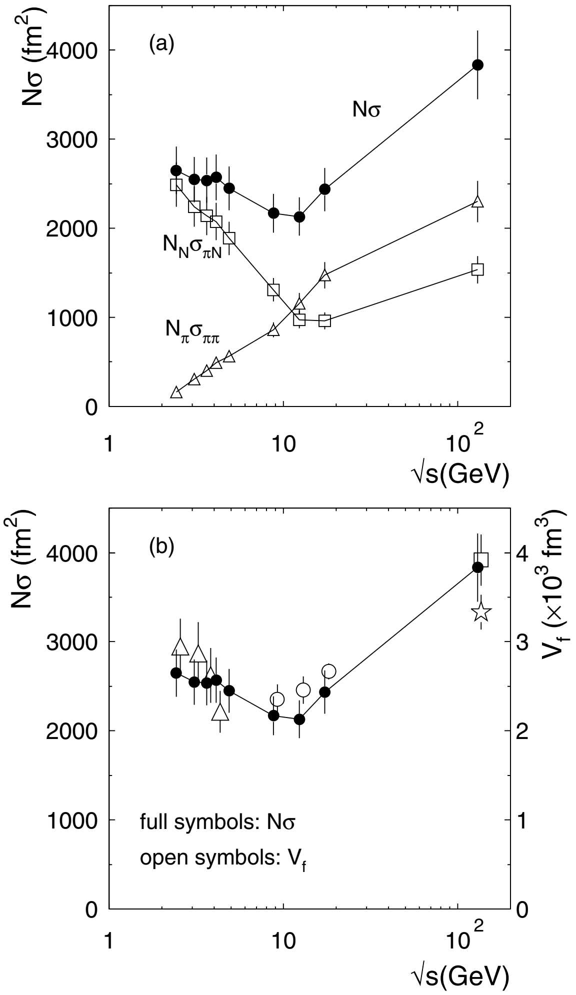

FIG. 2. (a) Beam energy dependence of $N\sigma$ (for explanation, see text). (b) Comparison of $N\sigma$ and V_f.

However, as a function of beam energy, relative particle abundances do change, and therefore the freeze-out density ρ_f is in general not constant. In Fig. 3(b) we show the pion and nucleon freeze-out densities $\rho_{f,\pi} = N_\pi/V_f$ and $\rho_{f,N} = N_N/{\rm V}_f$ as a function of $\sqrt{s}$. The freeze-out densities change drastically with $\sqrt{s}$ at AGS and SPS energies; however, only little change is observed from top SPS energy to RHIC. This indicates that the freeze-out densities reach asymptotic values at high beam energy.

In conclusion, we have derived a universal condition for pion freeze-out from pion interferometry data and single particle yields. Thermal pion freeze-out occurs at a critical mean free path $\lambda_f \approx 1$ fm, independent of beam energy. We observe a transition from nucleon to pion dominated freeze-out between AGS and SPS energies, characterized by a minimum of the freeze-out volume V_f. The existence of this minimum appears as a consequence of the relative change of particle abundances with beam energy, under consideration of their different cross sections with pions. In this picture, the overall weak $\sqrt{s}$ dependence of HBT radii up to RHIC energies finds a

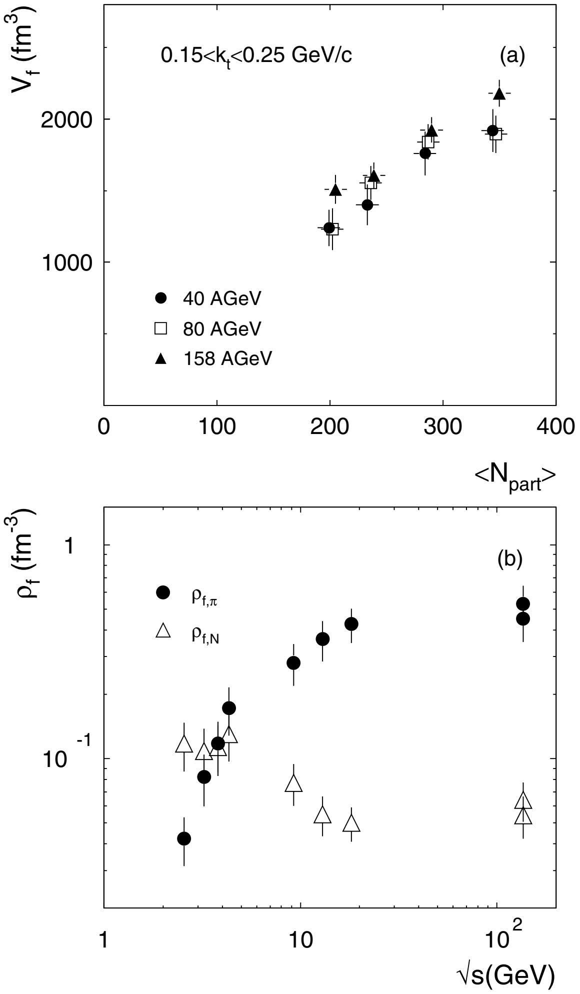

FIG. 3. (a) Freeze-out volume as a function of $\langle N_{\text{part}} \rangle$ in Pb + Au collisions (from [2]). (b) Pion and nucleon freeze-out densities $\rho_{f,\pi}$ and $\rho_{f,N}$ as a function of $\sqrt{s}$. At $\sqrt{s} =$ 130 GeV, results from $\pi^+\pi^+$ and $\pi^-\pi^-$ interferometry are shown.

simple interpretation. The surprisingly small value of λ_f points to a considerable opaqueness of the source [5,32]. This opaqueness may be at the root of the small observed values for R_{out} at RHIC [5,32,36].

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