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Supersymmetric CP Violations and T-Odd Observables in $t\bar{t}$ and W^+W^- Physics

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Abstract

T-odd, CP-violating correlations of polarizations and momenta provide a promising testing ground for new physics beyond the standard model. We estimate the contribution of the minimal supersymmetric extension of the standard model to two such observables: in the production of $t\bar{t}$, we look for a term proportional to $\mathbf{J}_t \cdot \left(\mathbf{p}_q \times \mathbf{p}_t\right)$—where $\mathbf{J}_t$ is the polarization of the t quark and $\mathbf{p}_{q,t}$ are the momenta of the initial and final particles—and find that it is of the order of $10^{-2} \times (\alpha_W/\pi)$. In the production of W^+W^-, we look for a term proportional to $\mathbf{E}_W \cdot \left(\mathbf{p}_q \times \mathbf{p}_W\right) \left(\mathbf{p}_q \cdot \mathbf{E}_W\right)$—where $\mathbf{E}_W$ is the transverse polarization of W— to find that it can be as large as $10^{-1} \times (\alpha_W/\pi)$.

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1. Observables which, in a given cross section, are made out of an odd number of momenta and polarizations change sign under time reversal.

If we assume that CPT invariance holds, such a T-odd correlation can arise either because of final state interactions [1] or because of a violation of CP invariance.

The former is a consequence of the unitarity of the S matrix and carries no new dynamical information. It is a background that can be subtracted by taking the difference between the process we are interested in and its CP conjugate [2]. This way, the truly (that is, CP-odd) time-reversal-violating observable is isolated.

Such observables are negligible in the standard model—where the only possible CP-odd source of such a T-odd correlation is in the Kobayashi-Maskawa quark-mixing matrix, the effect of which is, however, suppressed by the unitarity of the matrix itself—and, for this reason, they provide a promising testing ground for physics beyond the standard model [3, 4, 5].

New physics may be unveiled either because some of the final states are made of new particles or because of its effects in the radiative corrections to the amplitude of the process. We follow this latter path and include one-loop corrections in the framework of the minimal supersymmetric extension of the standard model [6]. This model is of interest here inasmuch as time reversal invariance can be violated to a larger degree than in the standard model because of the presence of coupling strengths that cannot be made real by a suitable redefinition of the particle fields.

We consider two processes which should give rise to measurable T-odd and CP-violating observables.

The first one is the production of $t\bar{t}$ pairs in hadron $q\bar{q}$ collisions and in e^+e^- annihilation:

$$e\bar{e} \to t\bar{t}\,, \tag{1}$$

in which we look for terms in the cross section proportional to

$$\frac{\mathbf{J}_t \cdot (\mathbf{k} \times \mathbf{p})}{|\mathbf{k} \times \mathbf{p}|}\,, \tag{2}$$

where $\mathbf{J}_t$ is the polarization vector of one of the produced t quarks, $\mathbf{k}$ and $\mathbf{p}$ are two vectors characterizing the scattering plane—hereafter chosen to be $\mathbf{k}$, the center-of-mass momentum of the colliding pair, and $\mathbf{p}$, the momentum of the final t quark. Because the vector product $\mathbf{k} \times \mathbf{p}$ defines a vector perpendicular to the production plane, only the component of the t quark polarization that is transverse to this plane can appear in the correlation (2); therefore, $\mathbf{J}_t$ will denote such transverse polarization only.

As pointed out before, a transverse polarization of the t-quarks can be generated either in interactions between the final-state fermions, through the imaginary part of the loop integrals the amplitude—the so-called unitarity background—or by CP-violating phases in the Lagrangian. After a CP-transformation, the transverse polarization of the t quark and the $\bar{t}$ anti-quark which originate in the final-state interactions should be equal while they should point in opposite directions in the case where they arise from CP-violating pieces in the Lagrangian.

Let us then suppose that we can measure the transverse polarizations of t and $\bar{t}$ in future collider experiments, using, for instance, the method discussed in [3]. A comparison between the two transverse polarizations would make it possible to remove the unitarity background because any difference between them would imply CP-violation in the $t\bar{t}$ production process.

It is also possible to single out the T-odd, CP-violating contribution by a direct estimate of the degree of transverse polarization due to final-state interactions. The QCD one-loop contribution, governing the leading behavior in the standard model, has been computed in [3]. An enhancement of the predicted polarization effect would then be a signal of new physics.

The chiral structure of the supersymmetric amplitude is such that (2) is proportional to the mass of the t quark. It is for this reason that the t quark, with its large mass [7], is such a good candidate for observing a non-vanishing value of (2). For m_t in the present experimental range, and supersymmetric masses around 200 GeV, the supersymmetrical correction is of one order of magnitude smaller than a one-loop radiative correction within the standard model, which we can take to be typically of the order of $10^{-1} \times (\alpha_W/\pi)$, where $\alpha_W \equiv g^2/4\pi$.

The second process that we consider is the production of W^+W^- pairs in

$$q\bar{q} \to W^+W^- \quad \text{or} \quad e\bar{e} \to W^+W^- \,, \tag{3}$$

in which we estimate the term in the cross section proportional to the correlation

$$\frac{\mathbf{E}_W \cdot (\mathbf{k} \times \mathbf{p})}{|\mathbf{k} \times \mathbf{p}|} \frac{\mathbf{k} \cdot \mathbf{E}_W}{|\mathbf{k}|} \,, \tag{4}$$

where $\mathbf{E}_W$ is the transverse polarization of one of the final vector bosons; as before, $\mathbf{k}$ and $\mathbf{p}$ are, respectively, the center-of-mass momentum of the colliding pair and of the W's. $\mathbf{E}_W$ has two components, one parallel to the reaction plane which gives a non-vanishing contribution to the scalar product with $\mathbf{k}$, and one transverse to such a plane and appearing in the triple product in (4). These two components of the transverse polarization of W must both be different from zero in order for the observable (4) to be measurable.

As in the case of the t quarks, the correlation (4) takes opposite signs for the polarization of, respectively, W^+ and W^- if it originates from CP-violating phases in the Lagrangian and the same sign if it comes from the unitarity background.

The T-odd correlation (4) arising from the CP-violating part of the Lagrangian turns out to be of the same order as the previous one in the $t\bar{t}$ production—except in a narrow range of scattering angles in the backward direction, where it grows to become of the same order of a one-loop radiative correction within the standard model.

Both the t quark and the W boson will become copiously available as new accelerators (the LEPII, LHC and SSC) come into operation. This will make possible not only a detailed study of their properties, but also an efficient test of the possible non-vanishing of the T-odd correlations (2) and (4).

2. Let us first fix our notation by writing those parts of the minimal supersymmetric extension of the standard model we need.

The imaginary phases relevant for us arise from the mixing matrices of the neutralinos and the charginos. The neutralino mass eigenstates χ_i^0 are related to the weak eigenstates

$$\psi_j^0 = \left(\widetilde{W}^3, \widetilde{B}, \widetilde{H}_1^0, \widetilde{H}_2^0\right) \tag{5}$$

through the neutralino mixing matrix N:

$$\chi_{iL}^0 = N_{ij}\, \psi_{jL}^0 \,, \qquad i = 1,2,3,4 \,, \tag{6}$$

where the subscript L denotes the left-handed components of the field. The chargino mass eigenstates $\omega_i^\pm$ are defined by

$$\omega_i^+ = V_{ij}\psi_j^+ \qquad \text{and} \qquad \omega_i^- = U_{ij}\psi_j^- \,, \qquad i = 1,2 \,, \tag{7}$$

where

$$\psi_j^+ = \left(-i\lambda^+, \widetilde{H}_2^+\right) \qquad \text{and} \qquad \psi_j^- = \left(-i\lambda^-, \widetilde{H}_1^-\right) \tag{8}$$

are the two-component weak interacting spinor fields of the winos $\lambda^\pm$ and the charged higgsinos $\widetilde{H}_1^-$ and $\widetilde{H}_2^+$. Here, V and U are two-by-two unitary matrices that diagonalize the wino-higgsino mass matrix. Both the neutralino N and the chargino V and U mixing matrices are determined by the supersymmetry breaking mechanism and are in general complex numbers [6] that cannot be made real by a redefinition of the phases of the spinor fields. These are the only terms we are going to consider. The additional phases that could come from the ordinary Higgses give no contribution to the correlations we are interested in.

We can construct the diagrams of Figs. 1 and 2 by means of the supersymmetric Lagrangian [6]. In the computation we present in this letter, we make use, in addition to those of the standard model, of only the following five terms:

$$\begin{aligned}
L_{\tilde{t}\tilde{t}V^i} &= i g_i V^i_\mu \left(\epsilon^i_L \tilde{t}^*_L \overleftrightarrow{\partial^\mu} \tilde{t}_L + \epsilon^i_R \tilde{t}^*_R \overleftrightarrow{\partial^\mu} \tilde{t}_R \right) , && (9)\\
L_{Z\tilde{\chi}^+\tilde{\chi}^-} &= g_Z Z_\alpha \sum_{i,j} \bar{\tilde{\chi}}^+_i \gamma^\alpha \left[Q^L_{ij}(1-\gamma_5) + Q^R_{ij}(1+\gamma_5) \right] \tilde{\chi}^+_j + h.c. , && (10)\\
L_{\gamma\tilde{\chi}^+\tilde{\chi}^-} &= -e A_\alpha \bar{\tilde{\chi}}^+_i \gamma^\alpha \tilde{\chi}^+_j \, \delta_{ij} && (11)\\
L_{W^-\tilde{\chi}^+\tilde{\chi}^0} &= \frac{g}{2} W_\alpha \sum_{k,i} \bar{\tilde{\chi}}^0_k \gamma^\alpha \left[O^L_{ki}(1-\gamma_5) + O^R_{ki}(1+\gamma_5) \right] \tilde{\chi}^+_i + h.c. , && (12)\\
L_{\tilde{t}t\tilde{\chi}^0} &= \frac{g}{2} \sum_k \left\{ \tilde{t}_L \bar{t} \left[f_k (1+\gamma_5) + \frac{m_t}{M_W} N^*_{k4} (1-\gamma_5) \right] \right. \\
&\quad \left. + \tilde{t}_R \bar{t} \left[h_k (1-\gamma_5) + \frac{m_t}{M_W} N_{k4} (1+\gamma_5) \right] \right\} \tilde{\chi}^0_k + h.c. . && (13)
\end{aligned}$$

A few comments on the notation are in order. The index i in (9) keeps track of whether the scalar-quark vertex is coupled to γ or Z. We have

$$\begin{aligned}
g_\gamma &= g \sin\theta_W = e , & g_Z &= g/2\cos\theta_W ,\\
\epsilon^\gamma_L &= \epsilon^\gamma_R = 2/3 , & &\\
\epsilon^Z_L &= 1 - (4/3)\sin^2\theta_W , & \epsilon^Z_R &= -(4/3)\sin^2\theta_W . && (14)
\end{aligned}$$

Moreover, in (10)-(13) we have used the notation

$$\begin{aligned}
f_k &\equiv -\frac{1}{\sqrt{2}} \left[N_{k2} + \frac{1}{3} \tan\theta_W N_{k1} \right] , && (15)\\
h_k &\equiv \frac{2\sqrt{2}}{3} \tan\theta_W N^*_{k2} , && (16)\\
O^L_{ki} &\equiv -\frac{1}{\sqrt{2}} N_{k4} V^*_{i2} + N_{k1} V^*_{i1} , & Q^L_{ij} &\equiv -V_{i1} V^*_{j1} - \frac{1}{2} V_{i2} V^*_{j2} + \sin^2\theta_W ,\\
O^R_{ki} &\equiv -\frac{1}{\sqrt{2}} N^*_{k3} U_{i2} + N^*_{k1} U_{i1} , & Q^R_{ij} &\equiv -U_{i1} U^*_{j1} - \frac{1}{2} U_{i2} U^*_{j2} + \sin^2\theta_W .
\end{aligned}$$

In (9) and below, $\tilde{\chi}_i^+ = (\omega_i^+, \bar{\omega}_i^-)$ are four-component spinors, $\tilde{\chi}_i^{+c}$ are their charge-conjugate states, t is the quark field and $\tilde{t}_{L,R}$ are its scalar left- and right-handed partners.

3. We turn now to estimating the observable (2) in the production of $t\bar{t}$ in the minimal supersymmetrical extension of the standard model.

The term in the $t\bar{t}V^i$ vertices which does not flip the helicities of t and $\bar{t}$ and is CP-violating is of the form:

$$\bar{u}(p)\Gamma^i_\alpha u(-p') = ig_i \bar{u}(p)\, P_\alpha\, \gamma_5 u(-p')\, \mathcal{B}^i \qquad i = \gamma, Z\,, \tag{17}$$

where p and p' are the momenta of, respectively, t and $\bar{t}$ and $P = p - p'$. The real functions $\mathcal{B}^i$ depend on the specific diagram one considers.

There are five types of one-loop supersymmetric diagrams that can give a contribution; they are depicted in Fig. 1. As we do not commit ourselves to any definite scheme for the breaking of supersymmetry, and therefore leave arbitrary mixing parameters in the Lagrangian, it is not important to compute all of them. Here we consider the contribution of diagonal terms of the supersymmetric Lagrangian. In this case, only the two diagrams of Figs. 1(a) and (b), in which scalar (left and right handed) quarks are exchanged, give a contribution. The other diagrams have no CP-violating phase. These diagrams of Figs. 1(a) and (b) assure the required helicities of t and $\bar{t}$ in the amplitude through the Higgsino coupling in (13). The CP-violating phases appear entirely via the neutralino mixing matrix.

We thus obtain the following expression for $\mathcal{B}^i$:

$$\mathcal{B}^i = -g^2 \frac{m_t}{M_W} \widetilde{m}^0_k \left[\epsilon^L_i \, Im(N_{4k} f_k)(I^L_k + 2a^L_k) + \epsilon^R_i \, Im(N_{4k} h^*_k)(I^R_k + 2a^R_k) \right] . \tag{18}$$

In eq. (18), $I^{L,R}_k$ and $a^{L,R}_k$ are defined by means of the loop integrals as follows:

$$I^{L,R}_k = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - (\widetilde{m}^k_0)^2} \frac{1}{(k-p')^2 - (\widetilde{m}_{L,R})^2} \frac{1}{(k+p)^2 - (\widetilde{m}_{L,R})^2}\,, \tag{19}$$

and

$$\begin{aligned} (I_\alpha)^{L,R}_k &= \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, k_\alpha \, \frac{1}{k^2 - (\widetilde{m}^k_0)^2} \frac{1}{(k-p')^2 - (\widetilde{m}_{L,R})^2} \frac{1}{(k+p)^2 - (\widetilde{m}_{L,R})^2} \\ &= a^{L,R}_k P_\alpha + b^{L,R}_k q_\alpha\,, \end{aligned} \tag{20}$$

where $q = p + p'$, $\widetilde{m}_{L,R}$ and $\widetilde{m}_0^k$ are the masses of $\tilde{t}_{L,R}$ and χ_k^0, respectively.

The matrix element for the process contains, beside the tree-level standard model term, the following CP-violating amplitude:

$$\begin{aligned} i \;\; \mathcal{N} \Bigg[& \frac{e^2}{s}\, \bar{u}(-k')\gamma_\alpha u(k)\;\, \bar{u}(p)\, iP^\alpha \mathcal{B}^\gamma \gamma_5 u(-p') \\ + & \;\frac{g_Z^2}{s - M_Z^2} \bar{u}(-k')\gamma_\alpha \left(c_V + c_A\gamma_5\right) u(-k)\;\, \bar{u}(p)\, iP^\alpha \mathcal{B}^Z \gamma_5 u(-p') \Bigg] \\ \times & \;(2\pi)^4 \delta(p + p' - k - k')\,. \end{aligned} \tag{21}$$

In eq. (21), $c_V \equiv -(1/2) + 2\sin^2\theta_W$ and $c_A \equiv 1/2$, $\mathcal{N}$ is the usual factor containing the normalization of the states.

The cross section can now be computed. It contains the standard model tree-level part $\mathrm{d}\sigma_0^{t\bar{t}}/\mathrm{d}\Omega$ [8] together with the observable (2) which arises in the interference with the one-loop supersymmetric amplitude (21). It can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0^{t\bar{t}}}{\mathrm{d}\Omega} \left(1 + D_t\, \frac{(\mathbf{J} \cdot \mathbf{p} \times \mathbf{k})}{|\mathbf{p} \times \mathbf{k}|} \right)\,. \tag{22}$$

Here $\mathbf{J}$ is the unit polarization vector perpendicular to the production plane, defined by $J_t = D_t \cdot\, \mathbf{J}$. The degree of transverse polarization D_t, which gives the magnitude of the T-odd, CP-violating effect we are after is in evidence. Our computation yields:

$$\begin{aligned} D_t \;\; = \;\; & -\left(\frac{1}{\mathcal{T}_0}\right) \sqrt{s}\beta \sin\vartheta \tan^2\theta_W \frac{s}{s - M_Z^2} \qquad (23) \\ \times \;\; & \Bigg\{ c_A \left[g_V\, \mathcal{B}^\gamma + \left(\frac{2}{3} - \frac{c_V g_V}{2} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_Z^2} \right) \mathcal{B}^Z \right] \\ & - \beta\cos\vartheta\, g_A \left[c_V\, \mathcal{B}^\gamma - \left(\frac{c_V^2 + c_A^2}{4} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_Z^2} \right) \mathcal{B}^Z \right] \Bigg\}\,, \end{aligned}$$

where

$$\mathcal{T}_0 = 16s \frac{\mathrm{d}\sigma_0^{t\bar{t}}/\mathrm{d}\Omega}{\alpha_W^2 \beta}\,. \tag{24}$$

In (24) $g_V = 1/2 - (4/3)\sin^2\theta_W$, $g_A = 1/2$ and $\beta = |\mathbf{p}|/E$. The scattering angle is denoted by ϑ.

Even though (24) contains the result we are after, the presence in it of too many parameters, coming from the neutralino-mixing matrix elements N_{ki} and the masses of the super particles in the loop, produces a rather cumbersome expression not suitable for numerical estimates. To bring it into a more intelligible form, one that will allow us to obtain a reliable estimate of the effect, we make some simplifications.

First of all, we consider only one CP-violating phase, δ_{CP}, such as

$$\mathrm{Im}\, N_{k4} f_k = \mathrm{Im}\, N_{k4} h_k^* \simeq \frac{1}{2} \sin \delta_{CP} \,. \tag{25}$$

Moreover, the masses of the left and right squarks can be taken as equal. By the same token, all supersymmetrical masses can be assumed to be of the same order, and their value denoted by $\widetilde{M}$. We are thus left with only two arbitrary supersymmetric parameters: one CP-violating phase and one supersymmetric mass. Moreover, even though expression (24) is correct both below and above the threshold for production of super particles, we are more interested in the case in which no superparticles are produced, i.e. we are restricted to the regime

$$2m_t < \sqrt{s} < 2\widetilde{M} \,. \tag{26}$$

Under these assumptions, the integrals (19) and (20) can be computed to yield

$$I_k^{L,R} = I \equiv \frac{1}{4\pi^2 s} \widetilde{I} \tag{27}$$

where

$$\widetilde{I} \equiv \int_0^1 \mathrm{d}x \frac{1}{\sqrt{\Delta}} \arctan \frac{(1-v)\,(1-x)}{\sqrt{\Delta}} \,, \tag{28}$$

and

$$\Delta \equiv (1-v) \left[(1-x)^2 - v(1-x) + u \right] \,. \tag{29}$$

The two parameters

$$v \equiv 4m_t^2/s \quad \text{and} \quad u \equiv 4\widetilde{M}^2/s \,. \tag{30}$$

are such that $v < 1$ and $u > 1$ because of (26).

Similarly,

$$a_k^{L,R} = a \equiv \frac{1}{4\pi^2 s} \left(-\frac{1}{2} \widetilde{a} \right) \,, \tag{31}$$

where

$$\widetilde{a} \equiv \int_0^1 \mathrm{d}x \frac{1-x}{\sqrt{\Delta}} \arctan \frac{(1-v)\,(1-x)}{\sqrt{\Delta}} \,, \tag{32}$$

Hence, we obtain that

$$\mathcal{B}^i \simeq -\frac{\alpha_W}{\pi} \sin\delta_{CP} \frac{m_t}{\widetilde{M}_W} \cdot \frac{\tilde{M}(\tilde{I}-\tilde{a})}{s} \frac{\epsilon_L^i + \epsilon_R^i}{2} \,. \tag{33}$$

The value of $\tilde{\mathcal{B}}^i$ determines the order of magnitude of the effect. By assuming maximum CP-violation ($\sin\delta_{CP} = 1$), the effect can be made of the order of magnitude of a one-loop radiative correction (the factor α_W/π) times a factor m_t/M_W coming from the helicity-flip in the supersymmetric coupling; a suppression factor comes from $\tilde{I} - \tilde{a}$ which contains the dependence on $\widetilde{M}$.

We have evaluated the integrals (28) and (32) numerically to obtain the curves depicted in Figs. 3(a)-(c) for different choices of $\sqrt{s}$, m_t and $\widetilde{M}$.

Figs. 3(a)-(b) show that the size of D_t scales as $m_t/\widetilde{M}$, becoming larger for larger energies. In Fig. 3(c) we have also included the case of energies above the threshold for the production of the supersymmetrical particles to show that the D_t becomes one order of magnitude larger.

In the kinematical regime (26), at $\sqrt{s} = 130$ GeV and for $\widetilde{M} = 100 - 200$ GeV, the effect is of order

$$10^{-2} \times \left(\frac{\alpha_W}{\pi}\right) , \tag{34}$$

for maximal CP violation and $-0.9 \leq \cos\vartheta \leq 0$. It becomes smaller for larger supersymmetric masses (~ 1 TeV)

4. Our second example, the observable (4) in the production of W^+W^-, can be estimated as follows.

The most general Lorentz invariant coupling of a neutral current Γ^μ to a pair of conjugate vector bosons $W_\alpha^+ W_\beta^-$ can be parameterized in terms of ten form factors f_i (see, for instance, [9], the notation of which we follow). Four of them are CP-violating and potentially give rise to the correlation (4). However, as we want to obtain just an estimate of the effect, we only need consider one, for example, f_6 that is defined by the WWV^i vertices as follows:

$$\Gamma_\mu^i = \tilde{g}_i \, f_6^i \epsilon_{\mu\sigma\alpha\beta} \, q^\sigma E_-^\alpha E_+^\beta \qquad\qquad i = \gamma,\, Z \tag{35}$$

where $\tilde{g}_\gamma = e$ and $\tilde{g}_Z = e\cot\theta_W$. In (35) the $E_\pm$ are the $W^\pm$-polarization 4-vectors, $q = p + p'$; p and p' being the momenta of, respectively, W^- and W^+.

There are four types of loop diagrams which can give a contribution to (35)—see Fig. 2. However, as long as we are not committed to a particular model of supersymmetry breaking, to obtain an estimate of D_W, it is sufficient to consider just a class of them, as we did for the diagrams for the t-quark production. We take only the vertex diagrams, depicted in Figs. 2(a) and 2(b), neglecting the box-diagrams.

Since both the supersymmetric masses and the imaginary phases have no definite value, we can also make the further simplification of considering the case $i = j$. This leaves us with only one diagram, the one in Fig. 2(a), the other one, in Fig. 2(b), being real under these assumptions. The CP-violating phases of the remaining diagram arise only from the $\tilde{\chi}^0_k \tilde{\chi}^+_i W^-$-vertex, the diagonal couplings Q^L_{ii} and Q^R_{ii} in the $\tilde{\chi}_i \tilde{\chi}_i V$ vertex being real.

A straightforward algebraic manipulation gives

$$
\begin{aligned}
f_6^Z &= -\frac{2g^2}{\cos^2\theta_W} \sum_{k,i} \widetilde{m}^0_k \widetilde{m}_i \mathrm{Im}\,(O^L_{ki} O^{R*}_{ki})(Q^L_{ii} + Q^R_{ii}) I^L_k \\
f_6^\gamma &= 4\, g^2 \sum_{k,i} \widetilde{m}^0_k \widetilde{m}_i \mathrm{Im}\,(O^L_{ki} O^{R*}_{ki}) I^L_k \,, \qquad (36)
\end{aligned}
$$

where I^L_k is the same integral as the one defined in (19) but for the masses of the squarks being replaced by the masses $\tilde{m}_i$ of the charginos.

Correlation (4) in the relevant cross section arises from the interference of the one-loop amplitude

$$
\begin{aligned}
& i \ \mathcal{N} \left[\frac{g_\gamma}{s} \, \Gamma^\gamma_\mu \, \bar{u}(-k') \gamma^\mu u(k) \right. \qquad (37) \\
& \left. + \frac{g_Z}{s - M_Z^2} \, \Gamma^Z_\mu \, \bar{u}(-k') \gamma^\mu \left(c_V + c_A \gamma_5\right) u(k) \right] (2\pi)^4 \delta(p + p' - k - k') \,,
\end{aligned}
$$

where Γ^i are given by (35), with the tree-level amplitude corresponding to diagrams with Z, photon and neutrino being exchanged.

We assume the polarization of W^- to be fixed and we take it to be transverse to the momentum of W^-. In this case E_W can be chosen to have space components only, $E_W = (0, E_W)$. E_W can be decomposed into two real polarization vectors: one perpendicular to the momentum of W^- and parallel to the reaction plane and the other transverse to the reaction plane. In the center-of-mass system we have also $(E_W \cdot p) = 0$ and $(E_W \cdot q) = 0$. This way, after summing over the polarizations of the other vector boson W^+, we obtain the dependence of the cross section on the transverse polarization of W^-. It can be written in the form:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0^{W^+W^-}}{d\Omega}\left(1 + D_W\,\frac{(\mathbf{E}_{W^-}\cdot\mathbf{k}\times\mathbf{p})}{|\mathbf{k}\times\mathbf{p}|}\,\frac{\mathbf{E}_{W^-}\cdot\mathbf{k}}{|\mathbf{k}|}\right), \tag{38}$$

where the tree-level standard model contribution $d\sigma_0^{W^+W^-}/d\Omega$ [10] has been factorized out to put in evidence the T-odd, CP-violating coefficient D_W, which determines the magnitude of the effect.

This coefficient is now

$$\begin{aligned} D_W &= \frac{\beta\sin\vartheta}{\mathcal{C}_0}\left\{f_6^\gamma\sin^2\theta_W\left[\frac{4s}{M_W^2}\left(\sin^2\theta_W + \frac{c_V}{2}\frac{s}{s-M_Z^2}\right) - \frac{s}{2t}\left(1-\frac{2t}{M_W^2}\right)\right]\right.\\ &\quad + \frac{1}{2}f_6^Z\frac{s}{s-M_Z^2}\left[\frac{4s}{M_W^2}\left(c_V\,\sin^2\theta_W + \frac{c_V^2+c_A^2}{2}\frac{s}{s-M_Z^2}\right)\right.\\ &\quad \left.\left. -\frac{s}{2t}(c_V-c_A)\left(1-\frac{2t}{M_W^2}\right)\right]\right\}, \end{aligned} \tag{39}$$

where

$$\mathcal{C}_0 = 16\,s\,\frac{d\sigma_0^{W^+W^-}/d\Omega}{\alpha_W^2\beta}, \tag{40}$$

and

$$t = M_W^2 - \frac{s}{2}(1-\beta\cos\vartheta)\,. \tag{41}$$

Again we have a complicated expression that we want to simplify. As we have done in the previous case, we take the supersymmetric masses at about the same values $\widetilde{M}$ and assume a unique CP-violating phase:

$$\mathrm{Im}\,(O_{ki}^L O_{ki}^{R*}) = \sin\delta_{CP}/2\,, \tag{42}$$

as well as

$$Q_{ii}^L = Q_{ii}^R \simeq 1/2\,. \tag{43}$$

We consider the energy range below the threshold for the production of the superparticles.

Under these assumptions we obtain the following expressions for f_6^γ and f_6^Z:

$$f_6^\gamma = \frac{\alpha_W}{\pi}\sin\delta_{CP}\,\frac{2\tilde{M}^2}{s}\widetilde{I} \tag{44}$$

$$f_6^Z = -\frac{\alpha_W}{\pi}\sin\delta_{CP}\,\frac{\tilde{M}^2}{s\cos^2\theta_W}\widetilde{I}, \tag{45}$$

where the integral $\tilde{I}$ has already been defined in (28), except that now

$$v \equiv 4M_W^2/s\,. \tag{46}$$

Here, as compared to the $t\bar{t}$-production, the only dependence on the mass of the produced particles is in the loop integral $\tilde{I}$, because of the form of the supersymmetric couplings.

We can now use eqs.(44), (45) and (39) to obtain an estimate of the effect. A numerical evaluation of D_W for the same choice of energies and supersymmetrical mass as before is given in Figs. 4(a)-(b). This time, D_W is very much independent of the supersymmetric masses and smaller at larger energies (there is no enhancement above the threshold for the production of the superparticles). For maximal CP violation and at $\sqrt{s} = 280$ GeV, the size of D_W in most of the backward direction is about of the same order as the one of D_t discussed above. It, however, grows quite steeply in the narrow region where $0.90 \leq \cos\vartheta \leq 0.95$ to become of the order of

$$10^{-1} \times \left(\frac{\alpha_W}{\pi}\right)\,. \tag{47}$$

5. The two T-odd and CP-violating observables we have computed within the minimal supersymmetric extension of the standard model turn out to be rather large. They are of the same order of magnitude as, or only one order of magnitude smaller than, a one-loop radiative correction within the standard model itself. They may provide independent bounds on the supersymmetric parameters (for present limits see, for example, [11]) and clues on new physics if they are measured and found to be different from zero.

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Fig. 1: The supersymmetric loop diagrams that give a contribution to the T-odd, CP-violating observable in the production of $t\bar{t}$.

Fig. 2: The supersymmetric loop diagrams that give a contribution to the T-odd, CP-violating observable in the production of W^+W^-.

Fig. 3: The T-violating coefficient D_t in the production of $t\bar{t}$.

Fig. 4: The T-violating coefficient D_W in the production of W^+W^-.