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T Violation Induced by Supersymmetry in $t\bar{t}$ and W^+W^- Physics

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Abstract

T-odd correlations of polarizations and momenta provide a promising testing ground for new physics beyond the standard model. We estimate the contribution of the minimal supersymmetric extension of the standard model to two such observables: in the production of $t\bar{t}$, we look for a term proportional to $\mathbf{J}_t \cdot \left(\mathbf{p}_q \times \mathbf{p}_t\right)$—where $\mathbf{J}_t$ is the polarization of the t quark and $\mathbf{p}_{q,t}$ are the momenta of the initial and final particles—and find that it is of the order of $10^{-1} \times (\alpha_s/\pi)$. In the production of W^+W^-, we look for a term proportional to $\mathbf{E}_W \cdot \left(\mathbf{p}_q \times \mathbf{p}_W\right) \left(\mathbf{p}_q \cdot \mathbf{E}_W\right)$—where $\mathbf{E}_W$ is the transverse polarization of W— to find that it can be as large as $10^{-1} \times (\alpha_w/\pi)$.

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1. Observables which, in a given cross section, are made out of an odd number of momenta and polarizations change sign under time reversal.

If we assume that CPT invariance holds, such a T-odd correlation can arise either because of final state interactions [1] or because of a violation of CP invariance.

The former is a consequence of the unitarity of the S matrix and carries no new dynamical information. It is a background that can be subtracted by taking the difference between the process we are interested in and its CP conjugate [2]. This way, the truly (that is, CP-odd) time-reversal-violating observable is isolated.

Such observables are negligible in the standard model—where the only possible CP-odd source of such a T-odd correlation is in the Kobayashi-Maskawa quark-mixing matrix, the effect of which is, however, suppressed by the unitarity of the matrix itself—and, for this reason, they provide a promising testing ground for physics beyond the standard model [3, 4, 5].

New physics may be unveiled either because some of the final states are made of new particles or because of its effects in the radiative corrections to the amplitude of the process. We follow this latter path and include one-loop corrections in the framework of the minimal supersymmetric extension of the standard model [6]. This model is of interest here inasmuch as time reversal invariance can be violated to a larger degree than in the standard model because of the presence of coupling strengths that cannot be made real by a suitable redefinition of the particle fields.

We consider two processes which should give rise to measurable T-odd and CP-violating observables.

The first one is the production of $t\bar{t}$ pairs in hadron $q\bar{q}$ collisions and in e^+e^- annihilation:

$$e\bar{e} \to t\bar{t}\,, \tag{1}$$

in which we look for terms in the cross section proportional to

$$\frac{\mathbf{J}_t \cdot (\mathbf{k} \times \mathbf{p})}{|\mathbf{k} \times \mathbf{p}|}\,, \tag{2}$$

where $\mathbf{J}_t$ is the polarization vector of one of the produced t quarks, $\mathbf{k}$ and $\mathbf{p}$ are two vectors characterizing the scattering plane—hereafter chosen to be $\mathbf{k}$, the center-of-mass momentum of the colliding pair, and $\mathbf{p}$, the momentum of the final t quark[1].

[1]Other CP-odd observables have been recently computed within the minimal supersymmetric extension of the standard model [7].

Because the vector product $\mathbf{k} \times \mathbf{p}$ defines a vector perpendicular to the production plane, only the component of the t quark polarization that is transverse to this plane can appear in the correlation (2); therefore, $\mathbf{J}_t$ will denote such transverse polarization only.

As pointed out before, a transverse polarization of the t-quarks can be generated either in interactions between the final-state fermions, through the imaginary part of the loop integrals the amplitude—the so-called unitarity background—or by CP-violating phases in the Lagrangian. After a CP-transformation, the transverse polarization of the t quark and the $\bar{t}$ anti-quark which originate in the final-state interactions should be equal while they should point in opposite directions in the case where they arise from CP-violating pieces in the Lagrangian.

Let us then suppose that we can measure the transverse polarizations of t and $\bar{t}$ in future collider experiments, using, for instance, the method discussed in [4]. A comparison between the two transverse polarizations would make it possible to remove the unitarity background because any difference between them would imply CP-violation in the $t\bar{t}$ production process.

It is also possible to single out the T-odd, CP-violating contribution by a direct estimate of the degree of transverse polarization due to final-state interactions. The QCD one-loop contribution, governing the leading behavior in the standard model, has been computed in [4]. An enhancement of the predicted polarization effect would then be a signal of new physics.

The chiral structure of the supersymmetric amplitude is such that (2) is proportional to the mass of the t quark. It is for this reason that the t quark, with its large mass [9], is such a good candidate for observing a non-vanishing value of (2). For m_t in the present experimental range, and supersymmetric masses around 200 GeV, the supersymmetrical correction is of the same order as a one-loop radiative correction within the standard model, which we can take to be typically of the order of $10^{-1} \times (\alpha/\pi)$, where α can be either $g_s^2/4\pi$ for a strong coupling correction or $g_w^2/4\pi$ for a weak correction.

The second process that we consider is the production of W^+W^- pairs in

$$q\bar{q} \to W^+W^- \quad \text{or} \quad e\bar{e} \to W^+W^- , \tag{3}$$

in which we estimate the term in the cross section proportional to the correlation

$$\frac{\mathbf{E}_W \cdot (\mathbf{k} \times \mathbf{p})}{|\mathbf{k} \times \mathbf{p}|} \frac{\mathbf{k} \cdot \mathbf{E}_W}{|\mathbf{k}|} , \tag{4}$$

where $\mathbf{E}_W$ is the transverse polarization of one of the final vector bosons; as before, $\mathbf{k}$ and $\mathbf{p}$ are, respectively, the center-of-mass momentum of the colliding pair and of the W's. $\mathbf{E}_W$ has two components, one parallel to the reaction plane which gives a non-vanishing contribution to the scalar product with $\mathbf{k}$, and one transverse to such a plane and appearing in the triple product in (4). These two components of the transverse polarization of W must both be different from zero in order for the observable (4) to be measurable.

As in the case of the t quarks, the correlation (4) takes opposite signs for the polarization of, respectively, W^+ and W^- if it originates from CP-violating phases in the Lagrangian and the same sign if it comes from the unitarity background.

The T-odd correlation (4) arising from the CP-violating part of the supersymmetric Lagrangian turns out to be of order $10^{-2} \times (\alpha_w/\pi)$. In a narrow range of scattering angles in the backward direction, it grows of one order of magnitude to become of the same order of a one-loop radiative correction within the standard model.

Both the t quark and the W boson will become copiously available as new accelerators (the LEPII, LHC and SSC) come into operation. This will make possible not only a detailed study of their properties, but also an efficient test of the possible non-vanishing of the T-odd correlations (2) and (4).

2. Let us first fix our notation by writing those parts of the minimal supersymmetric extension of the standard model we need.

We neglect generation mixing. Hence, only three terms in the supersymmetric Lagrangian can give rise to CP-violating phases which cannot be rotated away [8]: The superpotential contains a complex coefficient μ in the term bilinear in the Higgs superfields. The soft supersymmetry breaking operators introduce two further complex terms, the gaugino masses $\widetilde{M}_i$ and the left- and right-handed squark mixing term A_q. We consider only the latter two, which are carried by truly supersymmetric particles, and leave out the additional contribution of the Higgs sector.

The squark mass eigenstates $\tilde{q}^j_{\alpha,n}$ are related to the weak eigenstates $\tilde{q}^j_{\alpha,L}$ and $\tilde{q}^j_{\alpha,R}$ through the mixing matrix:

$$\tilde{q}^j_{\alpha,L} \;=\; \exp(-i\phi_{A_q}/2)\left[\cos\theta\,\tilde{q}^j_{\alpha,1} + \sin\theta\,\tilde{q}^j_{\alpha,2}\right] = \sum_m a^L_m \tilde{q}^j_{\alpha,m} \tag{5}$$

$$\tilde{q}^j_{\alpha,R} \;=\; \exp(i\phi_{A_q}/2)\left[\cos\theta\,\tilde{q}^j_{\alpha,2} - \sin\theta\,\tilde{q}^j_{\alpha,1}\right] = \sum_n a^R_n \tilde{q}^j_{\alpha,n} \tag{6}$$

where

$$\tan 2\theta = \frac{2|A_q|m_q}{(L^2 - R^2)\widetilde{m}} \tag{7}$$

and

$$A_q\widetilde{m}m_q = \xi_q v_2 + \mu^* h_q v_1 \qquad A_q = |A_q|\exp i\phi_{A_q}\,. \tag{8}$$

$\widetilde{m}L$ and $\widetilde{m}R$ are the squark mass parameters, v_i the vacuum expectation values of the Higgses, and ξ_q the coefficient in the cubic term of the soft breaking operator. The diagonalization of the squark masses gives the eigenvalues

$$\widetilde{m}^2_{1,2} = \frac{1}{2}\left\{(L^2+R^2)\widetilde{m}^2 + 2m_q^2 \mp \left[(L^2-R^2)^2\widetilde{m}^4 + 4m_q^2|A_q|^2\widetilde{m}^2\right]^{1/2}\right\}\,. \tag{9}$$

The gluino majorana mass

$$\tilde{M}_g = \tilde{m}_g \exp(i\phi_g) \tag{10}$$

gives an additional phase shift once it has been rotated into the interaction to make the masses real.

The neutralino mass eigenstates $\tilde{\chi}^0_i$ are 4-component Majorana spinors, whose left-handed components are related to the weak interacting two-component spinor fields

$$\psi^0_j = \left(-i\lambda', -i\lambda^3, \psi^0_{H_1}, \psi^0_{H_2}\right) \tag{11}$$

through the 4×4 neutralino mixing matrix N:

$$\tilde{\chi}^0_{iL} = N_{ij}\,\psi^0_{jL}\,, \qquad i = 1,2,3,4\,. \tag{12}$$

The chargino two-component mass eigenstates $\chi^\pm_i$ are defined by

$$\chi^+_i = V_{ij}\psi^+_j \qquad \text{and} \qquad \chi^-_i = U_{ij}\psi^-_j\,, \qquad i = 1,2\,, \tag{13}$$

where

$$\psi^+_j = \left(-i\lambda^+, \psi^+_{H_2}\right) \qquad \text{and} \qquad \psi^-_j = \left(-i\lambda^-, \psi^-_{H_1}\right) \tag{14}$$

are the weak interacting spinor fields. Here, V and U are 2×2 unitary matrices that diagonalize the wino-higgsino mass matrix. The 4-component chargino field is

$$\tilde{\chi}^+_k = \left(\chi^+_k, \bar{\chi}^-_k\right), \tag{15}$$

whose left and right-handed components are expressed in terms of the two actually independent matrices U and V. Both the neutralino N and the chargino V and U mixing matrices are determined by the supersymmetry breaking mechanism and contain in general CP-violating phases [6].

We can construct the diagrams of Figs. 1 and 2 by means of the supersymmetric Lagrangian [6]. In the computation we present in this letter, we make use, in addition to those of the standard model, of the following six terms:

$$
\begin{aligned}
L_{\tilde{t}\tilde{t}V^i} &= i g_i V^i_\mu \left(\epsilon^i_L \tilde{t}^*_L \overleftrightarrow{\partial^\mu}\, \tilde{t}_L + \epsilon^i_R \tilde{t}^*_R \overleftrightarrow{\partial^\mu}\, \tilde{t}_R \right) , && (16)\\
L_{Z\tilde{\chi}^+\tilde{\chi}^-} &= g_Z Z_\alpha \sum_{i,j} \bar{\tilde{\chi}}^+_i \gamma^\alpha \left[Q^L_{ij}(1-\gamma_5) + Q^R_{ij}(1+\gamma_5) \right] \tilde{\chi}^+_j + h.c. , && (17)\\
L_{\gamma\tilde{\chi}^+\tilde{\chi}^-} &= -e A_\alpha \bar{\tilde{\chi}}^+_i \gamma^\alpha \tilde{\chi}^+_j \, \delta_{ij} && (18)\\
L_{W^-\tilde{\chi}^+\tilde{\chi}^0} &= \frac{g}{2} W_\alpha \sum_{k,i} \bar{\tilde{\chi}}^0_k \gamma^\alpha \left[O^L_{ki}(1-\gamma_5) + O^R_{ki}(1+\gamma_5) \right] \tilde{\chi}^+_i + h.c. , && (19)\\
L_{\tilde{q}q\tilde{g}} &= \frac{g_s}{\sqrt{2}} T^a_{jk} \sum_{\alpha=t,b} \Big[\bar{\tilde{g}}_a (1-\gamma_5) q^k_\alpha \Gamma^m_{L*} \tilde{q}^{j*}_{\alpha,m} + \bar{q}^j_\alpha (1+\gamma_5) \tilde{g}_a \Gamma^m_L \tilde{q}^k_{\alpha,m} \\
&\quad - \bar{\tilde{g}}_a (1+\gamma_5) q^k_\alpha \Gamma^n_{R*} \tilde{q}^{j*}_{\alpha,n} - \bar{q}^j_\alpha (1-\gamma_5) \tilde{g}_a \Gamma^n_R \tilde{q}^k_{\alpha,n} \Big] + h.c. && (20)\\
L_{Z\chi^0\chi^0} &= \frac{g_Z}{2} Z_\alpha \sum_{j,i} \bar{\tilde{\chi}}^0_i \gamma^\alpha \left[R^L_{ij}(1-\gamma_5) + R^R_{ij}(1+\gamma_5) \right] \chi^0_j + h.c. . && (21)
\end{aligned}
$$

A few comments on the notation: The index i in (16) keeps track of whether the scalar-quark vertex is coupled to γ or Z. We have

$$
\begin{aligned}
g_\gamma &= g \sin\theta_W = e , \qquad\qquad g_Z = g/2\cos\theta_W , \\
\epsilon^\gamma_L &= \epsilon^\gamma_R = 2/3 , \\
\epsilon^Z_L &= 1 - (4/3)\sin^2\theta_W , \qquad\qquad \epsilon^Z_R = -(4/3)\sin^2\theta_W . && (22)
\end{aligned}
$$

Moreover, in (17)-(20) we have used the notation

$$
\begin{aligned}
\Gamma^m_L &= a^{L*}_m \exp(-i\phi_g) \\
\Gamma^n_R &= a^R_n \exp(-i\phi_g) \\
O^L_{ki} &\equiv -\frac{1}{\sqrt{2}} N_{k4} V^*_{i2} + N_{k1} V^*_{i1} , \qquad Q^L_{ij} \equiv -V_{i1}V^*_{j1} - \frac{1}{2} V_{i2} V^*_{j2} + \sin^2\theta_W , \\
O^R_{ki} &\equiv -\frac{1}{\sqrt{2}} N^*_{k3} U_{i2} + N^*_{k1} U_{i1} , \qquad Q^R_{ij} \equiv -U_{i1}U^*_{j1} - \frac{1}{2} U_{i2} U^*_{j2} + \sin^2\theta_W \\
R^L_{ij} &\equiv \frac{1}{2} N_{i3} N^*_{j3} + \frac{1}{2} N_{i4} N^*_{j4} , \qquad R^R_{ij} = R^{L*}_{ij} . && (23)
\end{aligned}
$$

In (16) and below, $\tilde{g}_a$ are the gluinos and $\tilde{t}_{L,R}$ are the scalar left- and right-handed partners of the t quark field.

3. We turn now to estimating the observable (2) in the production of $t\bar{t}$ in the minimal supersymmetrical extension of the standard model. It can receive a contribution from several diagrams (Fig.1). However, as long as we are not committed to a particular model of supersymmetry breaking, it is sufficient to consider just a class of them. We take only the gluino Penguin diagram, depicted in Fig. 1(a) which is slightly enhanced by a factor α_s/α_w with respect to the neutralino and chargino diagrams.

The gluino Penguin diagram gives a contribution to the following CP-violating and helicity flipping term:

$$\bar{u}(p)\Gamma^i_\alpha u(-p') = ig_i\bar{u}(p)\,P_\alpha\,\gamma_5 u(-p')\,\mathcal{B}^i \qquad\qquad i=\gamma, Z\,, \tag{24}$$

where p and p' are the momenta of, respectively, t and $\bar{t}$ and $P = p - p'$. The $\mathcal{B}^i$ are real functions given by:

$$\begin{aligned}\mathcal{B}^i \;&=\; -2g_s^2\widetilde{m}_g\left(\epsilon^L_i + \epsilon^R_i\right)\\ &\quad\times\sum_{n,m}\mathrm{Im}\left\{\left(\Gamma^{R*}_n\Gamma^L_n - \Gamma^{L*}_m\Gamma^R_m\right)\left(\Gamma^{R*}_m\Gamma^R_n + \Gamma^{L*}_m\Gamma^L_n\right)\right\}\left[2a^{m,n} + I^{m,n}\right].\end{aligned} \tag{25}$$

In eq. (25), $I^{m,n}$ and $a^{m,n}$ are defined by means of the loop integrals as follows:

$$I^{m,n} = \int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-(\widetilde{m}_g)^2}\frac{1}{(k-p')^2-(\widetilde{m}_n)^2}\frac{1}{(k+p)^2-(\widetilde{m}_m)^2}\,, \tag{26}$$

and

$$\begin{aligned}I^{m,n}_\alpha \;&=\; \int\frac{\mathrm{d}^4k}{(2\pi)^4}\,k_\alpha\,\frac{1}{k^2-(\widetilde{m}_g)^2}\frac{1}{(k-p')^2-(\widetilde{m}_n)^2}\frac{1}{(k+p)^2-(\widetilde{m}_m)^2}\\ &=\; a^{m,n}P_\alpha + b^{m,n}q_\alpha\,,\end{aligned} \tag{27}$$

where $q = p + p'$.

The matrix element for the process contains, beside the tree-level standard model term, the following CP-violating amplitude:

$$i\mathcal{N}\left[\frac{e^2}{s}\,\bar{u}(-k')\gamma_\alpha u(k)\;\bar{u}(p)\,iP^\alpha\mathcal{B}^\gamma\gamma_5 u(-p')\right.$$

$$+\ \frac{g_Z^2}{s-M_Z^2}\bar{u}(-k')\gamma_\alpha\,(c_V+c_A\gamma_5)\,u(-k)\ \bar{u}(p)\,iP^\alpha\mathcal{B}^Z\gamma_5 u(-p')\Big]$$
$$\times\,(2\pi)^4\delta(p+p'-k-k')\,. \qquad (28)$$

In eq. (28), $c_V \equiv -(1/2)+2\sin^2\theta_W$ and $c_A \equiv 1/2$, $\mathcal{N}$ is the usual factor containing the normalization of the states.

The cross section can now be computed. It contains the standard model tree-level part $\mathrm{d}\sigma_0^{t\bar{t}}/\mathrm{d}\Omega$ [10] together with the observable (2) which arises in the interference with the one-loop supersymmetric amplitude (28). It can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mathrm{d}\sigma_0^{t\bar{t}}}{\mathrm{d}\Omega}\left(1+D_t\,\frac{(\mathbf{J}\cdot\mathbf{p}\times\mathbf{k})}{|\mathbf{p}\times\mathbf{k}|}\right)\,. \qquad (29)$$

Here $\mathbf{J}$ is the unit polarization vector perpendicular to the production plane, defined by $\mathbf{J}_t = D_t\cdot\ \mathbf{J}$. The degree of transverse polarization D_t, which gives the magnitude of the T-odd, CP-violating effect we are after is in evidence. Our computation yields:

$$\begin{aligned} D_t \;=\; & -\left(\frac{1}{\mathcal{T}_0}\right)\sqrt{s}\beta\sin\vartheta\tan^2\theta_W\frac{s}{s-M_Z^2} \qquad (30)\\ & \times\left\{c_A\left[g_V\,\mathcal{B}^\gamma+\left(\frac{2}{3}-\frac{c_Vg_V}{2}\frac{1}{\sin^2\theta_W\cos^2\theta_W}\frac{s}{s-M_Z^2}\right)\mathcal{B}^Z\right]\right.\\ & \left.-\,\beta\cos\vartheta g_A\left[c_V\,\mathcal{B}^\gamma-\left(\frac{c_V^2+c_A^2}{4}\frac{1}{\sin^2\theta_W\cos^2\theta_W}\frac{s}{s-M_Z^2}\right)\mathcal{B}^Z\right]\right\}\,, \end{aligned}$$

where

$$\mathcal{T}_0 = 16s\frac{\mathrm{d}\sigma_0^{t\bar{t}}/\mathrm{d}\Omega}{\alpha_W^2\beta}\,. \qquad (31)$$

In (31) $g_V = 1/2-(4/3)\sin^2\theta_W$, $g_A = 1/2$ and $\beta = |\mathbf{p}|/E$. The scattering angle is denoted by ϑ.

The masses of the squarks can be taken as equal once we have factorized out the overall dependence on them of the integrals, so that the sum in (25) does not vanish by the orthogonality of the mixing matrices $\Gamma^{L,R}$. This factorization gives a factor

$$\frac{\widetilde{m}_2^2-\widetilde{m}_1^2}{\widetilde{m}_1^2\widetilde{m}_2^2}\simeq\frac{2|A_t|m_t}{\widetilde{m}_1^2\widetilde{m}_2^2}\widetilde{m}\,, \qquad (32)$$

where we have assumed for simplicity in (9) $L=R$. In the same approximation $\sin\theta\cos\theta = 1/2$. All supersymmetrical masses can now be assumed to be of the same order, and their value denoted by $\widetilde{M}$. We are thus left with only two arbitrary supersymmetric parameters: one CP-violating phase and one supersymmetric mass. Moreover, even though expression (31) is correct both below and above the threshold for production of super particles, we are more interested in the case in which no superparticles are produced, i.e. to the regime

$$2m_t < \sqrt{s} < 2\widetilde{M}\,. \tag{33}$$

Under these assumptions, the integrals (26) and (27) can be computed to yield

$$I^{m,n} = I \equiv \frac{1}{4\pi^2 s}\widetilde{I} \tag{34}$$

where

$$\widetilde{I} \equiv \int_0^1 \mathrm{d}x \frac{1}{\sqrt{\Delta}} \arctan \frac{(1-v)\,(1-x)}{\sqrt{\Delta}}\,, \tag{35}$$

and

$$\Delta \equiv (1-v)\left[(1-x)^2 - v(1-x) + u\right]\,. \tag{36}$$

The two parameters

$$v \equiv 4m_t^2/s \quad \text{and} \quad u \equiv 4\widetilde{M}^2/s\,. \tag{37}$$

are such that $v < 1$ and $u > 1$ because of (33).

Similarly,

$$a^{m,n} = a \equiv \frac{1}{4\pi^2 s}\left(-\frac{1}{2}\widetilde{a}\right)\,, \tag{38}$$

where

$$\widetilde{a} \equiv \int_0^1 \mathrm{d}x \frac{1-x}{\sqrt{\Delta}} \arctan \frac{(1-v)\,(1-x)}{\sqrt{\Delta}}\,, \tag{39}$$

Hence, we obtain that

$$\mathcal{B}^i \simeq -8(\epsilon_L^i + \epsilon_R^i)\frac{\alpha_s}{\pi} \sin\delta_{CP} \frac{m_t}{s}\,(\widetilde{I} - \widetilde{a})\,, \tag{40}$$

where we have denoted by $\sin\delta_{CP}$ the phase $|A_t|\sin(\phi_{A_t} - \phi_g)$.

The value of $\widetilde{\mathcal{B}}^i$ determines the order of magnitude of the effect. By assuming maximum CP-violation ($\sin\delta_{CP} = 1$), the effect can be made of the order of magnitude of a one-loop radiative correction.

We have evaluated the integrals (35) and (39) numerically to obtain the dependence of the T-violating coefficient D in

$$D_t = \left(\frac{\alpha_s}{\pi}\right) D \, \sin\delta_{CP} \tag{41}$$

on the scattering angle $\cos\vartheta$, for different choices of $\sqrt{s}$, m_t and $\widetilde{M}$ (see Fig. 3).

Figs. 3(a)-(b) show that the size of D is proportional to m_t, becoming smaller for a larger supersymmetric mass scale $\widetilde{M}$.

The energy dependence has a kinematic origin. D_t is normalized by the tree-level cross section $\mathcal{T}_0$. The enhancement in the backward direction (see Fig. 3) is due to the fact that $\mathcal{T}_0$ is smaller there. In the same backward direction, $\mathcal{T}_0$ decreases as the center-of-mass energy $\sqrt{s}$ is increased, and therefore D is made bigger (see Fig. 3(c)).

For $\sqrt{s} = 280$ GeV, $\widetilde{M} = 150$ GeV and $m_t = 130$ GeV we have $D \simeq .1$, the value that gives the estimate quoted in the abstract.

4. Our second example, the observable (4) in the production of W^+W^-, can be estimated as follows.

The most general Lorentz invariant coupling of a neutral current Γ^μ to a pair of conjugate vector bosons $W^+_\alpha W^-_\beta$ can be parameterized in terms of ten form factors f_i (see, for instance, [11], the notation of which we follow). Four of them are CP-violating and potentially give rise to the correlation (4). However, as we want to obtain just an estimate of the effect, we only need consider one, for example, f_6 that is defined by the WWV^i vertices as follows:

$$\Gamma^i_\mu = \tilde{g}_i \, f^i_6 \epsilon_{\mu\sigma\alpha\beta} \, q^\sigma E^\alpha_- E^\beta_+ \qquad\qquad i = \gamma, \, Z \, , \tag{42}$$

where $\tilde{g}_\gamma = e$ and $\tilde{g}_Z = e \cot\theta_W$. In (42) the $E_\pm$ are the $W^\pm$-polarization 4-vectors, $q = p + p'$; p and p' being the momenta of, respectively, W^- and W^+.

The vertex (42) corresponds to an effective gauge non-invariant operator

$$i\kappa^i W^\dagger_\mu W_\nu \tilde{V}^{\mu\nu}_i + \frac{i\lambda^i}{M^2_W} W^\dagger_{\lambda\mu} W^\mu_\nu \tilde{V}^{\nu\lambda}_i \, , \tag{43}$$

where $\tilde{V}^{\mu\nu}_i$ stands for either the photon or the Z dual field strength and $f^i_6 = \kappa^i - \lambda^i$. This operator originates by spontaneous symmetry breaking from an higher dimensional gauge invariant operator [12]. It gives rise to an electric dipole moment for the W's [13].

There are four types of loop diagrams which can give a contribution to (42)—see Fig. 2. However, as we did for the t quark case, it is sufficient to consider just a class of them. We take only the vertex diagrams, depicted in Figs. 2(a) and 2(b), neglecting the box-diagrams.

Since both the supersymmetric masses and the imaginary phases have no definite value, we can also make the further simplification of considering the case $i = j$. This leaves us with only one diagram, the one in Fig. 2(a), the other one, in Fig. 2(b), being real under these assumptions. The CP-violating phases of the remaining diagram arise only from the $\tilde{\chi}^0_k \tilde{\chi}^+_i W^-$-vertex, the diagonal couplings Q^L_{ii} and Q^R_{ii} in the $\tilde{\chi}_i \tilde{\chi}_i V$ vertex being real.

A straightforward algebraic manipulation gives

$$\begin{aligned} f_6^Z &= -\frac{2g^2}{\cos^2\theta_W} \sum_{k,i} \widetilde{m}^0_k \widetilde{m}_i \mathrm{Im}\,(O^L_{ki} O^{R*}_{ki})(Q^L_{ii} + Q^R_{ii}) I_k \\ f_6^\gamma &= 4\,g^2 \sum_{k,i} \widetilde{m}^0_k \widetilde{m}_i \mathrm{Im}\,(O^L_{ki} O^{R*}_{ki}) I_k\,, \end{aligned} \qquad (44)$$

where I_k is the same integral as the one defined in (26) but for the masses of the squarks being replaced by the masses $\tilde{m}_i$ of the charginos and the mass of the gluino by the masses $\widetilde{m}^k_0$ of the neutralino.

Correlation (4) in the relevant cross section arises from the interference of the one-loop amplitude

$$\begin{aligned} & i\mathcal{N} \left[\frac{g_\gamma}{s}\, \Gamma^\gamma_\mu\, \bar{u}(-k')\gamma^\mu u(k) \right. && (45) \\ & \left. + \frac{g_Z}{s - M_Z^2}\, \Gamma^Z_\mu\, \bar{u}(-k')\gamma^\mu \left(c_V + c_A\gamma_5\right) u(k) \right] (2\pi)^4 \delta(p + p' - k - k')\,, \end{aligned}$$

where Γ^i are given by (42), with the tree-level amplitude corresponding to diagrams with Z, photon and neutrino being exchanged.

We assume the polarization of W^- to be fixed and we take it to be transverse to the momentum of W^-. In this case E_W can be chosen to have space components only, $E_W = (0, \mathbf{E}_W)$. $\mathbf{E}_W$ can be decomposed into two real polarization vectors: one perpendicular to the momentum of W^- and parallel to the reaction plane and the other transverse to the reaction plane. In the center-of-mass system we have also $(E_W \cdot p) = 0$ and $(E_W \cdot q) = 0$. This way, after summing over the polarizations of the other vector boson W^+, we obtain the dependence of the cross section on the transverse polarization of W^-. It can be written in the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0^{W^+W^-}}{\mathrm{d}\Omega}\left(1 + D_W\,\frac{(\mathbf{E}_{W^-}\cdot\mathbf{k}\times\mathbf{p})}{|\mathbf{k}\times\mathbf{p}|}\,\frac{\mathbf{E}_{W^-}\cdot\mathbf{k}}{|\mathbf{k}|}\right), \tag{46}$$

where the tree-level standard model contribution $\mathrm{d}\sigma_0^{W^+W^-}/\mathrm{d}\Omega$ [14] has been factorized out to put in evidence the T-odd, CP-violating coefficient D_W, which determines the magnitude of the effect.

This coefficient is now

$$\begin{aligned} D_W \;=\; & \frac{\beta\sin\vartheta}{\mathcal{C}_0}\left\{ f_6^\gamma \sin^2\theta_W\left[\frac{4s}{M_W^2}\left(\sin^2\theta_W + \frac{c_V}{2}\frac{s}{s-M_Z^2}\right) - \frac{s}{2t}\left(1-\frac{2t}{M_W^2}\right)\right]\right. \\ & + \frac{1}{2}f_6^Z\frac{s}{s-M_Z^2}\left[\frac{4s}{M_W^2}\left(c_V\,\sin^2\theta_W + \frac{c_V^2+c_A^2}{2}\frac{s}{s-M_Z^2}\right)\right. \\ & \left.\left. -\frac{s}{2t}(c_V-c_A)\left(1-\frac{2t}{M_W^2}\right)\right]\right\}, \end{aligned} \tag{47}$$

where

$$\mathcal{C}_0 = 16\,s\,\frac{\mathrm{d}\sigma_0^{W^+W^-}/\mathrm{d}\Omega}{\alpha_W^2\beta}, \tag{48}$$

and

$$t = M_W^2 - \frac{s}{2}\left(1-\beta\cos\vartheta\right). \tag{49}$$

Again we have a complicated expression that we want to simplify. As we have done in the previous case, we take the supersymmetric masses at about the same values $\widetilde{M}$ and assume a unique CP-violating phase:

$$\mathrm{Im}\,(O_{ki}^L O_{ki}^{R*}) = \sin\delta_{CP}/2\,, \tag{50}$$

as well as

$$Q_{ii}^L = Q_{ii}^R \simeq 1/2\,. \tag{51}$$

The sum over the neutralino states is non-zero only if their masses are different. This introduces a factor

$$\frac{(\widetilde{m}_j^0)^2-(\widetilde{m}_k^0)^2}{(\widetilde{m}_j^0)^2(\widetilde{m}_k^0)^2} \simeq \frac{M_Z\widetilde{M}}{(\widetilde{m}_j^0)^2(\widetilde{m}_k^0)^2} \tag{52}$$

in the simplest case of mixing of only two neutralinos [6]. Notice that this factor suppresses the CP-odd vertices as we take the supersymmetric masses to infinity for a fixed neutralino mass difference and gives zero if instead we take all masses to be degenerate.

Under these assumptions we obtain the following expressions for f_6^γ and f_6^Z:

$$f_6^\gamma = \frac{\alpha_W}{\pi} \sin\delta_{CP} \frac{2M_Z\widetilde{M}}{s}\widetilde{I} \tag{53}$$

$$f_6^Z = -\frac{\alpha_W}{\pi} \sin\delta_{CP} \frac{M_Z\widetilde{M}}{s\cos^2\theta_W}\widetilde{I}, \tag{54}$$

where the integral $\widetilde{I}$ has already been defined in (35), except that now

$$v \equiv 4M_W^2/s\,. \tag{55}$$

We consider the energy range below the threshold for the production of the superparticles.

We can now use eqs.(53), (54) and (47) to obtain an estimate of the effect. A numerical evaluation of D_W for the same choice of energies and supersymmetrical mass as before is given in Figs. 4(a)-(b), where we plot the coefficient D defined by

$$D_W = \left(\frac{\alpha_w}{\pi}\right) D\, \sin\delta_{CP}\,. \tag{56}$$

For maximal CP violation, and at $\sqrt{s} = 280$ GeV, the size of D in most of the backward direction is about 10^{-2}. It, however, increases quite steeply in the narrow region where $0.90 \leq \cos\vartheta \leq 0.95$ to become one order of magnitude bigger, that is, the value quoted in the abstract. This is also true for the tree-level cross section $\mathcal{C}_0$ which has a peak in the forward direction because of the neutrino-exchange diagram. Since this peak in $\mathcal{C}_0$ is made bigger by increasing the center-of-mass energy, the size of D_W is accordingly made smaller. As the supersymmetrical scale $\widetilde{M}$ grows, the effect becomes smaller.

Such a maximal CP breaking is consistent with bounds coming from the induced electric dipole moment of the neutron [15].

5. The two T-odd and CP-violating observables we have computed within the minimal supersymmetric extension of the standard model turn out to be rather large. They are of the same order of magnitude as, or only one order of magnitude smaller than, a one-loop radiative correction within the standard model itself. They may provide independent bounds on the CP-violating supersymmetric parameters (for present limits see, for example, [16]) and clues on new physics if they are measured and found to be different from zero.

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Fig. 1: The supersymmetric loop diagrams that give a contribution to the T-odd, CP-violating observable in the production of $t\bar{t}$.

Fig. 2: The supersymmetric loop diagrams that give a contribution to the T-odd, CP-violating observable in the production of W^+W^-.

Fig. 3: The T-violating coefficient D_t in the production of $t\bar{t}$.

Fig. 4: The T-violating coefficient D_W in the production of W^+W^-.