

LASER-DRIVEN CYCLOTRON AUTO-RESONANCE ACCELERATOR*

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Abstract

Theory is developed for the cyclotron autoresonance acceleration of electrons in a gently-focused Gaussian laser beam and a gently tapered axial magnetic field. Numerical simulation shows, for example, acceleration from 50 MeV to 178 MeV over a distance of 148 cm, using a 10.6 μm CO₂ laser with a minimum spot size of 0.1 cm.

1 INTRODUCTION

Electron acceleration using intense lasers has engendered significant attention within the accelerator research community. This interest stems from the enormous optical electric field strengths E_0 that can be obtained with a focused laser, i.e. of the order of $E_0 = 3 \times 10^{-9} \sqrt{I}$ TV/m, where the intensity I is in W/cm^2 [1]. Since compact terawatt focused lasers can have $I > 10^{18} \text{W}/\text{cm}^2$, field strengths of the order of teravolts/m are possible. Of course, since this field is transversely polarized, it can not give much net acceleration to a charged particle directly, so a number of indirect means have been devised to achieve net acceleration. For example, in the laser wake field accelerator [2] an intense laser pulse is used to locally polarize a plasma, thus creating a strong longitudinally-polarized plasma wake field for acceleration. In another example, the vacuum beat-wave accelerator [3], two laser pulses of differing frequencies are combined to create a slow optical ponderomotive beat wave that can exert a strong force for acceleration.

Originally a microwave interaction, the cyclotron autoresonance acceleration (CARA) of a low energy electron beam has been studied and demonstrated experimentally to operate with efficiencies exceeding 95% for transforming microwave energy into directed beam energy [4]. For acceleration of a high energy beam, however, the magnetic field required in the microwave CARA becomes so strong that it is not practical.

An alternative CARA is laser-driven cyclotron autoresonance accelerator (LACARA) [5]. The refractive index in LACARA is so close to unity that the upper energy limit for acceleration [6] is removed, and for acceleration of a high energy electron beam the magnetic field required is realizable. Furthermore, the group velocity in LACARA exceeds the axial particle velocity, so operation with strong pump depletion is possible without causing energy spread for the beam. The acceleration gradient in LACARA can be as high as $E_0 (v_{\perp} / v_z)_{\text{max}}$, with v_{\perp} and v_z the particle transverse

and axial velocities respectively.

Recently, by simulation we have studied LACARA for acceleration of electron beams. Here we present the theory and preliminary numerical results.

2 OUTLINE OF LACARA THEORY

The preliminary analysis presented here is for a traveling Gaussian laser beam focused by two spherical mirrors. The laser fields in cylindrical coordinates (r, θ, z) for the lowest-order mode with circular polarization are [7]

$$E_r = cB_{\theta} = E_0 \frac{w_0}{w} \exp\left(-\frac{r^2}{w^2}\right) \cos(\psi - \theta), \quad (1)$$

$$E_{\theta} = -cB_r = E_0 \frac{w_0}{w} \exp\left(-\frac{r^2}{w^2}\right) \sin(\psi - \theta), \quad (2)$$

$$E_z \approx -\frac{2}{kw} \frac{r}{w} \left(E_{\theta} + \frac{z}{z_R} E_r \right), \quad (3)$$

$$B_z \approx -\frac{2}{kw} \frac{r}{w} \left(B_{\theta} + \frac{z}{z_R} B_r \right), \quad (4)$$

where c is the vacuum light speed, $w = w_0(1 + z^2/z_R^2)^{1/2}$ is the spot size, w_0 is the waist radius (minimum spot size), and z_R is the Rayleigh distance. The waist radius and Rayleigh distance are related by $w_0 = (\lambda z_R / \pi)^{1/2}$, with λ the laser wavelength. The laser phase is $\psi = \omega t - kz + \tan^{-1}(z/z_R) - kr^2/2R$, with ω the laser angular frequency, $k = \omega/c$, and $R = z + z_R^2/z$ the radius of curvature of the ray normals. The axial and radial effective refractive indices (group velocities normalized to c) are given by

$$n = n_z = 1 - \frac{1}{2} \frac{w_0^2}{Rz} + \frac{1}{2} \frac{(z_R^2 - z^2)r^2}{(Rz)^2} \quad \text{and} \quad n_r = \frac{r}{R}. \quad (5)$$

The non-zero n_r means that there is diffraction loss for the Gaussian beam. The condition for resonance between wave and particles is $\Omega_0 = \gamma\omega(1 - n_z\beta_z - n_r\beta_r)$, where the non-relativistic gyrofrequency is $\Omega_0 = eB_0/m$ with e the electron charge in magnitude and m the rest mass, the relativistic energy factor is $\gamma = (1 - \beta_{\perp}^2 - \beta_z^2)^{-1/2}$ with $\beta_{\perp} = v_{\perp}/c$ and $\beta_z = v_z/c$, and β_r is the radial velocity normalized to c . Usually, the radial dimension of electron motion in LACARA is much less than the Rayleigh distance. Thus the resonance condition can be written as $\Omega_0 \approx \gamma\omega(1 - n\beta_z)$, with $n \approx 1 - w_0^2/(2Rz)$.

The laser power is related to the electric field amplitude by $P_L = 0.5\pi w_0^2 E_0^2 / \eta_0$, with $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$ the wave impedance of a plane wave in free space.

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Eqs. (1)-(4), together with the relativistic equations of motion for the electrons

$$\frac{d}{dt}(\gamma\mathbf{v}) = -\frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (6)$$

allow solutions to be found for single-particle orbits. In the results of computations to be shown below, iterative solutions for the position, velocity and energy of the particles are found at each computational stage by specifying the change in guide magnetic field value necessary to maintain resonance.

3 SIMULATION RESULTS

The simulation results presented below are for three LACARA examples. The laser power is from a CO₂ laser at a wavelength of 10.6 μm . Example 1 is for a 50 MeV cold electron beam driven by a laser power of 4 TW. The interaction length or mirror separation is 148 cm (5 Rayleigh distances). The simulation parameters are given in Table I and the results are shown in Figs. 1-4. In Example 2, the interaction length and the mirror radius are both 59.28 cm (2 Rayleigh distances), and all the other parameters are the same as those in Example 1. The result is shown in Fig. 5. Example 3 is for a 0.5 GeV cold beam driven by a laser power of 4 PW. Simulation parameters are given in Table II and the result is shown in Fig. 6. In all simulations, 8 computational particles initially uniformly distributed over one laser period are taken, and all electrons experience the same acceleration history. Beam loading is neglected.

Table I: Parameters in simulation for Example 1

Input cold beam energy	50 MeV
Laser power P_L	4.0 TW
Waist radius w_0	0.1 cm
Field amplitude E_0	31.0 GV/m
Rayleigh distance z_R	29.64 cm
Mirror radius	85.95 cm
Mirror separation	148 cm

Table II: Parameters in simulation for Example 3

Input cold beam energy	0.5 GeV
Laser power P_L	4.0 PW
Waist radius w_0	0.3 cm
Field amplitude E_0	326.5 GV/m
Rayleigh distance z_R	267 cm
Mirror radius	889 cm
Mirror separation	1600 cm

Fig. 1 shows average axial and transverse normalized velocities versus axial distance. The maximum $\langle\beta_{\perp}\rangle$ is 0.0063; the initial $\langle\beta_z\rangle$ is 0.999949 (minimum) and the final $\langle\beta_z\rangle$ is 0.999984 (maximum), only 0.0035% change. From this, it is seen that the velocity ratio β_{\perp}/β_z is quite small for a high energy beam in the LACARA, compared with a low energy beam in the microwave CARA [8]. This insures that all the electrons

have very small transverse displacements, as shown in Fig. 2. The laser waist radius is 0.1 cm; so all the orbits of the 8 particles are within the minimum spot size.

Fig. 3 shows normalized axial group velocity and average axial electron velocity, plotted as $1-n$ and $1-\langle\beta_z\rangle$, versus axial distance. It can be seen that the group velocity everywhere exceeds the axial particle velocity, allowing for rapid replenishment of laser energy that is given to the beam. This fact bodes well for achievement of an accelerated beam with low energy spread, even with significant beam loading. This allows efficient use of laser power without loss of beam quality.

Fig. 4 shows the dependence of average relativistic energy factor and axial magnetic field on axial distance. It is seen that the beam energy rises monotonically from 50 MeV to 178 MeV in a distance of 148 cm, corresponding to an average acceleration gradient of 86.6 MeV/m. The resonance magnetic field required rises from 52 kG to about 80 kG near the laser focus, and then falls back to about 60 kG. It is the fall in magnetic field that allows continuous acceleration without stalling; the fall in field can be traced to the fall in $(1-n)$ after the focus. This demonstrates that LACARA is not limited to be a γ -doubler, as in the microwave CARA.

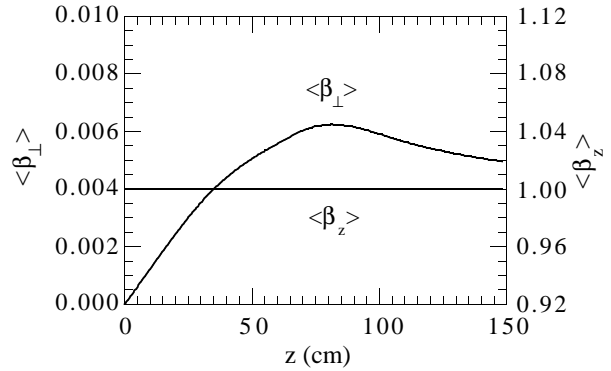


Figure 1: Average axial and transverse normalized velocities for Example 1 with a $5z_R$ interaction length. Initial beam energy is 50 MeV, $z_R = 29.64$ cm, $w_0 = 0.10$ cm, and $P_L = 4.0$ TW.

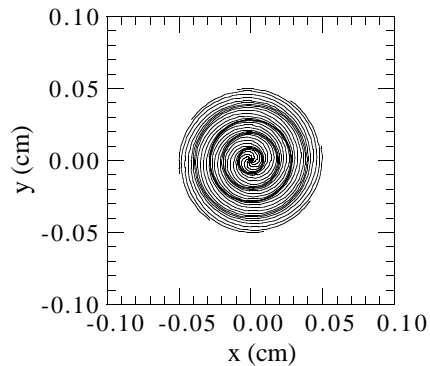


Figure 2: Projection in $x-y$ plane of orbits of 8 computational particles for Example 1. It is seen that the maximum transverse excursion is not greater than $w_0/2$.

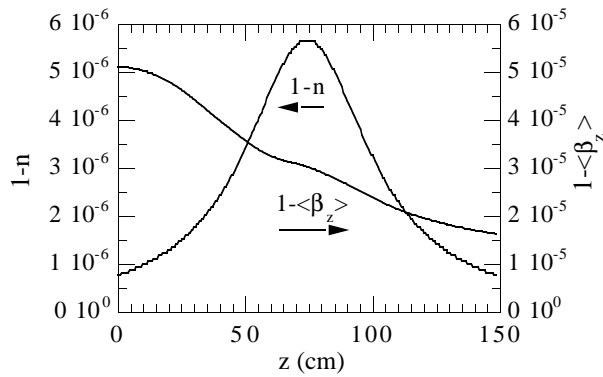


Figure 3: Variation in normalized axial group velocity plotted as $1-n$ (left scale), and normalized average axial velocity plotted as $1-\langle\beta_z\rangle$ (right scale), for parameters of Example 1. It is seen that n exceeds $\langle\beta_z\rangle$ throughout the interaction.

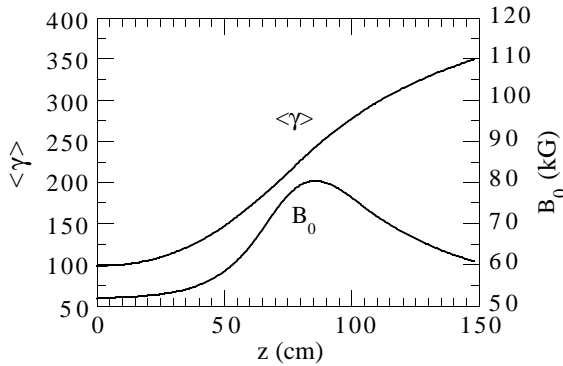


Figure 4: Variation in average relativistic energy factor $\langle\gamma\rangle$ and axial magnetic field B_0 along LACARA, for parameters of Example 1.

Fig. 5 shows the dependence of average relativistic energy factor and axial magnetic field on axial distance for Example 2. It is seen that the energy rises monotonically from 50 MeV to 110 MeV in a distance of 59.28 cm ($2z_R$), corresponding to an average acceleration gradient of 100 MeV/m. The resonance magnetic field required rises from 54 kG to about 67 kG near the laser focus, and then falls back to about 65 kG. This example shows that a shorter interaction region has a larger average acceleration gradient, since the laser field is weaker in the region far away from the focus.

Fig. 6 shows the dependence of average relativistic energy factor and axial magnetic field on axial distance for Example 3. It is seen that the energy rises from 0.5 GeV to 1.53 GeV in a distance of 16 m ($6z_R$), corresponding to an average acceleration gradient of 64.4 MeV/m. The orbit in this case was seen in the computations to execute only about one full gyration over its 16-m length, with a maximum displacement from the axis of less than 3.0 mm. The magnetic field is seen to vary from 6 kG, up to 24 kG, then down to 13 kG. This example shows that a lower magnetic field is required for a higher energy beam, as expected, in rough proportion to $1/\gamma$.

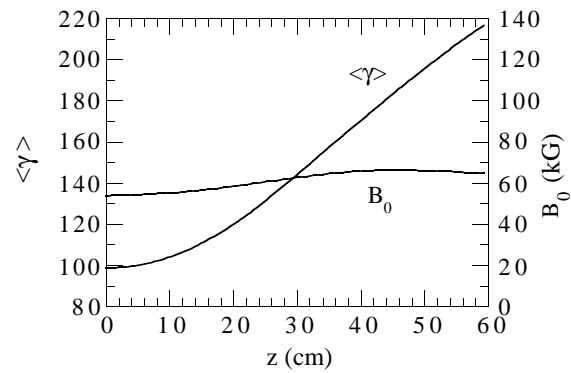


Figure 5: Variation in average relativistic energy factor $\langle\gamma\rangle$ and axial magnetic field B_0 along LACARA for Example 2. Parameters are the same as Example 1, except for a LACARA of length $2z_R$ with a mirror radius of curvature 59.28 cm.

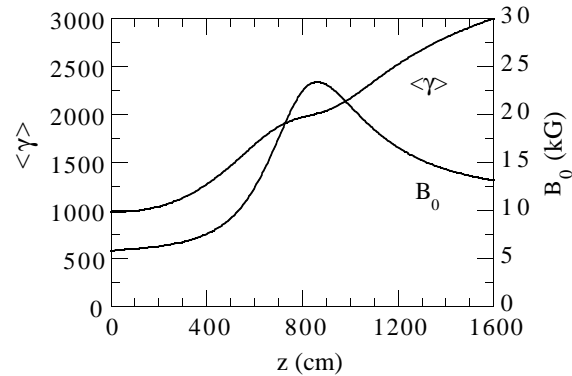


Figure 6: Variation in average relativistic energy factor $\langle\gamma\rangle$ and axial magnetic field B_0 along LACARA for Example 3. The beam is accelerated from 0.50 GeV to 1.53 GeV within a $6z_R$ interaction length.

4 CONCLUSIONS

It is shown by simulation that in a LACARA driven by a power of 4 TW at $10.6 \mu\text{m}$, a 50 MeV electron beam is accelerated to 178 MeV in a distance of 148 cm, corresponding to an average acceleration gradient of 87 MeV/m. The magnetic field varies from 52 kG to 80 kG.

5 REFERENCES

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