The Flavour Hierarchy and See-Saw Neutrinos from Bulk Masses in 5d Orbifold GUTs

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Abstract

In supersymmetric grand unified theories (GUTs) based on $S^1/(Z_2 \times Z_2')$ orbifold constructions in 5 dimensions, Standard Model (SM) matter and Higgs fields can be realized in terms of 5d hypermultiplets. These hypermultiplets can naturally have large bulk masses, leading to a localization of the zero modes at one of the two branes or to an exponential suppression of the mass of the lowest-lying non-zero mode. We demonstrate that these dynamical features allow for the construction of an elegant 3-generation SU(5) model in 5 dimensions that explains all the hierarchies between fermion masses and CKM matrix elements in geometrical terms. Moreover, if $U(1)_\chi$ (where $SU(5) \times U(1)_\chi \subset SO(10)$) is gauged in the bulk, but broken by the orbifold action at the SM brane, the right-handed neutrino mass scale is naturally suppressed relative to M_{GUT}. Together with our construction in the charged fermion sector this leads, via the usual see-saw mechanism, to a realistic light neutrino mass scale and large neutrino mixing angles.

1 Introduction

Supersymmetric (SUSY) grand unification provides an elegant explanation of the fermion quantum numbers and of the relative strength of the three gauge couplings of the standard model (SM). However, conventional 4d GUTs possess less attractive features such as a complicated GUT-scale Higgs sector, unsuccessful first and second generation analogues of the m_b/m_τ mass prediction, and dangerous dimension-5 proton decay operators arising from Higgsino exchange.

Recently, an elegant solution to these problems has been proposed in the context of SU(5) [1–6] and SO(10) [7] unification. The GUT gauge symmetry is now realized in 5 or more space-time dimensions and broken to the SM group by compactification on an orbifold, utilizing boundary conditions that violate the GUT-symmetry. In the most studied case of 5 dimensions both the GUT group and 5d supersymmetry are broken by compactification on $S^1/(Z_2 \times Z_2')$, leading to an N=1 SUSY model with SM gauge group. This construction provides elegant solutions to the problems of conventional GUTs with Higgs breaking, including doublet-triplet splitting, dimension-5 proton decay, and Yukawa unification in the first two generations, while maintaining, at least at leading order, the desired gauge coupling unification [3, 4, 8].

Although the hierarchy between the strong coupling scale M of the 5d gauge theory and the compactification scale $1/R$ ($MR \sim 10^2 \cdots 10^3$) can be used to generate a fermion mass hierarchy [9], the construction of a realistic three-generation model in 5 dimensions proves difficult (see also [10] and, in particular, [11], where problems very similar to the present investigation have been attacked with different tools). Furthermore, the slight discrepancy between the unification scale and the phenomenologically preferred right-handed (rhd) neutrino mass scale in see-saw models is generically enhanced in orbifold GUTs. In this letter, we demonstrate that 5d bulk masses, which are naturally present in the theories under consideration, allow for the construction of realistic models with the correct fermion mass hierarchy and rhd neutrino scale. The main ingredients we employ are the bulk-mass-driven localization of the zero mode and, in the absence of a zero-mode, the exponential suppression of the effective 4d mass in the limit where the bulk mass term becomes large.

2 Bulk masses in 5d SUSY

Recall first that the Lagrangian for a 5d hypermultiplet (written in terms of two 4d chiral superfields H and H^c [12]) is

$$\mathcal{L} = \int_{\theta^2 \bar{\theta}^2} \left(H^\dagger H + H^c H^{c\dagger} \right) + \left(\int_{\theta^2} H^c \partial_5 H + \text{h.c.} \right) . \tag{1}$$

In the general case of a 5d orbifold, the fundamental region in the direction of $x^5 = y$ is an interval, bounded by two inequivalent fixed points at $y = 0$ and $y = l$ (where $l = \pi R/2$ in the case of $S^1/(Z_2 \times Z_2')$). Each of the fixed points is invariant under a Z_2 reflection of the original 5d theory. The two superfields (H, H^c) of a singlet hypermultiplet in the

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bulk necessarily have opposite parities under each of the Z_2's. The above Lagrangian can be supplemented by the 5d Lorentz invariant mass terms

$$\mathcal{L}_{\text{mass}} = m_o \left(\int_{\theta^2} H^c H + \text{h.c.} \right) + \frac{1}{2} m_e \left(\int_{\theta^2} \left\{ H^2 + (H^c)^2 \right\} + \text{h.c.} \right) , \qquad (2)$$

which are respectively odd and even under parity. We will mainly be interested in the case of gauged hypermultiplets, where the even mass term is forbidden ($m_e = 0$, $m_o = m$). Furthermore, we will not be concerned with the dynamical realization of the odd mass term (see, e.g., [13]), but simply include it into our effective field theory Lagrangian as a leading operator consistent with all the symmetries of the model.

The localization of fermionic zero modes in the presence of y-dependent mass terms is a familiar phenomenon [14] which has been used in the context of model building in extra dimensions by many authors (see, e.g., [15, 16]). Thus, we can content ourselves here with recalling the basic features relevant for 5d orbifold constructions. [1]

The equation for the y-dependent profile $H(y)$ in the presence of the odd mass m is

$$\left(\partial_y^2 - m^2 + m_4^2 + 2m \left(\delta(y) - \delta(y - l) \right) \right) H(y) = 0 , \qquad (3)$$

where m_4 is the effective mass of the mode in 4d, and the δ function terms arise from the discontinuity of the odd mass function at the fixed points. (A similar equation holds for $H^c(y)$.)

The superfields H and H^c can have either the same or opposite parities at $y = 0, l$. To discuss the first case, assume that H and H^c have the parities $(+, +)$ and $(-, -)$ at the two boundaries. A simple analysis of the equation in this case shows that H has a zero-mode with bulk profile

$$H(y) = e^{-ym} \qquad (4)$$

while all other KK masses, including those of H^c, are $\mathcal{O}(m)$ or larger. (Since m is naturally of the order of the UV scale M, we assume $m \gg 1/R$ in our analysis.) Thus, depending on the sign of m, the zero mode is exponentially localized at the left ($m > 0$) or right ($m < 0$) boundary of the orbifold.

In the second case, we choose H and H^c to have the parities $(+, -)$ and $(-, +)$ at the two boundaries. Although now no zero-mode exists, two 4d superfield excitations are found to have a mass much smaller than m (for $m > 0$). They can be characterised as an H and an H^c mode with bulk profiles

$$H(y) \simeq \left(e^{-ym} - e^{(y-2l)m} \right) \quad \text{and} \quad H^c(y) \simeq \left(e^{(y-l)m} - e^{-(y+l)m} \right) , \qquad (5)$$

which are linked by a 4d Dirac-type mass

$$m_4 \simeq 2m e^{-ml} . \qquad (6)$$

(for canonical 4d superfield normalization of the kinetic term). As can be seen from Eq. (5), the H and H^c mode are exponentially localized at the two opposite boundaries of the orbifold.

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[1] We would like to emphasize that, in contrast to [15] and many related papers, we do not place Gaussian zero modes at various points in the bulk but restrict ourselves to mass terms that are constant between $y = 0$ and l and only allow for a peaking of the modes at either boundary.

3 Bulk masses in orbifold GUTs

We begin by recalling the basic structure of the Kawamura model [1], which is based on a 5d super Yang-Mills theory on $I\!R^4 \times S^1$, where the S^1 is parameterised by $y \in [0, 2\pi R)$. The field space is then restricted by imposing the two discrete Z_2 symmetries, $y \to -y$ and $y' \to -y'$ (with $y' = y - \pi R/2$). The action of the Z_2's in field space is specified by the two gauge twists P and P'. If the original gauge group is SU(5) and the gauge twists are chosen as $P = 1$ and $P' = \mathrm{diag}(1, 1, 1, -1, -1)$, the full SU(5) gauge symmetry exists in the bulk and on the SU(5) brane at $y = 0$, while at $y = l$ only the SM gauge symmetry exists. As a result, the effective low energy theory is invariant under only the SM gauge symmetry.

To be specific, in this letter we take the compactification scale to be $M_c = 1/R \sim 10^{15}$ GeV, slightly lower than the usual GUT scale $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, while the UV or cutoff scale $M \sim 10^{17}$ GeV is slightly higher. This situation is generic for the following reasons. On the one hand, the mild separation between M_c and M ensures that corrections to gauge coupling unification from brane-localized operators are under control. At the same time, this allows for a certain validity range of the 5d field theory. On the other hand, the 'differential running' of gauge couplings between M_c and M is somewhat slower than the MSSM running below M_c [3, 4, 17, 8], implying $M > M_{\mathrm{GUT}}$. Moreover, M should not be larger than the scale at which the 5d gauge theory becomes non-perturbative, $M \lesssim (12\pi/\alpha_{\mathrm{GUT}})M_c$ [3, 4]. As we will see in Sect. 4, our flavour scenario favours a value $Ml \simeq 300$, which is comfortably within the range set by the above restrictions.

The up- and down-type Higgs fields can be introduced as two hypermultiplets (H_u, H_u^c) and (H_d, H_d^c) in the bulk, transforming as a $(\mathbf{5}, \overline{\mathbf{5}})$ and $(\overline{\mathbf{5}}, \mathbf{5})$. After appropriate parity assignment, two doublet zero modes emerge. This is the celebrated solution of the doublet-triplet splitting problem [1]. Alternatively, the two required doublets can directly be introduced on the SM brane, where full SU(5) multiplets are not required [4]. In this context, bulk masses can have important effects. Firstly, they allow for an exponential localization of the doublet zero modes at either the SU(5) or SM brane. Since the 5d SUSY forbids bulk Yukawa interactions, the SM Yukawa couplings are always brane operators. This implies that localization can be used for the generation of large fermion mass hierarchies while keeping the dimensionless coefficients of all relevant operators $\mathcal{O}(1)$. Secondly, bulk masses allow for the interpretation of doublets living on the SM brane as the zero modes of bulk fields with a large mass.

Fermion fields can be introduced on the SU(5) brane [1–3], on the SM brane [4], or in the bulk [3,4]. Again, bulk masses allow for an interpretation of the brane-localized states as limiting cases of the model with bulk fields. To see this in more detail, recall that to realize a full $\overline{\mathbf{5}}$ of SU(5) in the bulk, one starts with two hypermultiplets $(\overline{F}, \overline{F}^c)$ and $(\overline{F}', \overline{F}'^c)$ and chooses parities such that, say, the $\overline{\mathbf{3}}$ from $\overline{F}$ and the $\overline{\mathbf{2}}$ from $\overline{F}'$ have a zero-mode. Clearly, introducing appropriate bulk masses for the two original hypermultiplets, these zero modes can now be localized at either of the two branes. Similarly, two $\mathbf{10}$ hypermultiplets (T, T^c) and (T', T'^c) in the bulk realize the particle content of a full $\mathbf{10}$ as zero modes, which can then be localized at either brane.

The above two paragraphs call for a number of further comments. Firstly, it is now

apparent that the 'minimal model' of [4], i.e., a model with only the gauge sector in the bulk and all other fields on the SM brane, can be viewed as a large-bulk-mass limit of a model with bulk fields only. In particular, this allows for an understanding of the quantum numbers of SM brane fermions in terms of SU(5) representations - an important GUT prediction that was previously missing in the minimal model of [4]. Thus, the introduction of bulk masses puts the minimal model, which is phenomenologically attractive because of its simplicity and its ability to accommodate gaugino mediated SUSY breaking, on a firmer conceptual ground.

Secondly, given the above discussion, it is possible to localise a SM field, e.g. a $\mathbf{2}$ zero mode from an (H, H^c) hypermultiplet, at the SU(5) brane. Should we be worried by this somewhat counterintuitive possibility? The answer is no since, in the limit of large bulk mass, the lowest-lying mode of the $\mathbf{3}$ (which is also localized at the SU(5) brane) becomes massless (cf. Eq. (6)), so that a full $\mathbf{5}$ emerges.

4 A three-generation flavour model

Let us now proceed by using the above tools to construct a 5d SU(5) model with three generations and see-saw neutrinos which explains all the mass and mixing hierarchies of the standard model.

As input we assume a small separation between the scale of the bulk masses and M (e.g., $m/M \sim 0.1$). This is justified since, on the one hand, M is the fundamental scale of the bulk theory and, on the other hand, $m \sim M$ would imply the localization of zero modes on length scales $\sim 1/M$ – a situation outside the realm of our effective field theory approach. Given the hierarchy between M_c and M, this implies that ml is large (e.g., $ml \sim 10$). Such a situation is phenomenologically attractive since the localization of zero modes is sufficiently strong to produce a large fermion mass hierarchy.

We define the SU(5) gauge sector of our model as explained at the beginning of Sect. 3 and introduce two Higgs doublets (H_u, H_u^c), (H_d, H_d^c) as well as three $\overline{\mathbf{5}}$'s $(\overline{F}_i, \overline{F}_i^c)$ as hypermultiplets in the bulk. Furthermore, we distribute the three $\mathbf{10}$'s of the SM at the three distinct locations of our model, namely, T_3 at the SU(5) brane, T_2 in the bulk, and T_1 at the SM brane. More precisely [4,9], for T_2 we have to introduce (T_2, T_2^c) and $(T_2', T_2'^c)$ choosing opposite P' action between the two, so that a full $\mathbf{10}$ of zero modes emerges. For T_1 we mean that states with quantum numbers of a full $\mathbf{10}$ are located on the SM brane. (As discussed above the correct quantum numbers for T_1 automatically follow if these states are understood as localized bulk fields. T_3 can also be thought of as the limit of a bulk field.) The location of fields is shown in Fig. 1.

We allow all Yukawa couplings consistent with gauge symmetry and R parity (see, e.g., [3]) at both branes with $\mathcal{O}(1)$ dimensionless coefficients. The hierarchical structure of the effective 4d Yukawas will be entirely due to the different normalization of bulk vs. brane fields and to the bulk-mass-driven localization. To begin, let us denote a Yukawa coupling between 3 brane superfields by λ. If one of the three fields is replaced by a bulk field with y-independent zero mode, the effective 4d Yukawa coupling is rescaled

according to $\lambda \to \lambda/\sqrt{Ml}$ [18, 9]. Here the factor $M^{-1/2}$ arises because of the mass dimension of the coefficient of the original brane-bulk interaction of the 5d theory (the natural scale being M) and the factor $l^{-1/2}$ comes from the different normalization of the kinetic term for brane and bulk fields. If the bulk field has a mass term m, so that the zero mode is localized as in Eq. (4), the corresponding rescaling reads

$$\lambda \to \frac{\lambda}{c(-ml)\sqrt{Ml}} \qquad \text{or} \qquad \lambda \to \frac{\lambda}{c(ml)\sqrt{Ml}} \tag{7}$$

depending on whether the original 5d interaction is localized at the SU(5) or the SM brane. Here the coefficient function

$$c(ml) = \sqrt{\frac{e^{2ml} - 1}{2ml}} \tag{8}$$

takes into account the proper normalization of the 5d vs. 4d kinetic terms and the value of the zero mode at the respective branes.

Taking $\lambda \sim 1$ for all interactions, introducing bulk masses m_u and m_d for the two Higgs hypermultiplets (with $m_u l$, $m_d l \gg 1$), and keeping all other bulk masses zero for simplicity, we arrive at the following Yukawa matrix structure for the two effective 4d interactions $H_u T^T \lambda_{TT} T$ and $H_d T^T \lambda_{TF} \overline{F}$:

$$\lambda_{TT} = \lambda_t \begin{pmatrix} \delta_u & \varepsilon \delta_u & 0 \\ \varepsilon \delta_u & \varepsilon^2 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix}, \qquad \lambda_{TF} = \lambda_b \begin{pmatrix} \delta_d & \delta_d & \delta_d \\ \varepsilon & \varepsilon & \varepsilon \\ 1 & 1 & 1 \end{pmatrix}. \tag{9}$$

Here we have used the definitions

$$\lambda_t = \sqrt{\frac{2m_u}{M}}, \quad \lambda_b = \frac{1}{\sqrt{Ml}}\sqrt{\frac{2m_d}{M}}, \quad \varepsilon = \frac{1}{\sqrt{Ml}}, \quad \delta_u = e^{-m_u l}, \quad \delta_d = e^{-m_d l}. \tag{10}$$

We recall that we are only attempting to generate the correct hierarchical Yukawa structure and that unknown $\mathcal{O}(1)$ coefficients (including complex phases) multiply each of the entries of the above matrices.

The eigenvalues of the matrices multiplying λ_t and λ_b in Eq. (10) are $(1, \varepsilon^2, \delta_u)$ and $(1, \varepsilon, \delta_d)$. Successful phenomenology requires the GUT scale relations (see, e.g., [19]) $\lambda_t \sim 0.6$, $(\lambda_t \tan \beta)/\lambda_b \sim 110$, $m_t/m_c \sim \varepsilon^{-2} \sim 300$, $m_b/m_s \sim \varepsilon^{-1} \sim 30$, $m_t/m_u \sim \delta_u^{-1} \sim 10^5$ and $m_b/m_d \sim \delta_d^{-1} \sim 10^3$. For a moderate value of $\tan \beta \simeq 5$, and up to $\mathcal{O}(1)$ factors that depend on the precise brane Yukawa couplings, this set of flavour hierarchies is realized by taking

$$Ml \simeq 300, \qquad m_u l \simeq 11.5, \qquad m_d l \simeq 6.9. \tag{11}$$

These values are within the favoured parameter range for Ml and for the ratio of bulk and brane masses (see Sect. 3 and the beginning of this section). An illustration of the essential features of our complete setup is given in Fig. 1.

To derive the resulting structure of the CKM matrix, recall that λ_{TT} and λ_{TF} are diagonalized by the bi-unitary transformations $\lambda_{TT}^{\text{diag}} = L_T^\dagger \lambda_{TT} R_T$ and $\lambda_{TF}^{\text{diag}} = L_F^\dagger \lambda_{TF} R_F$.
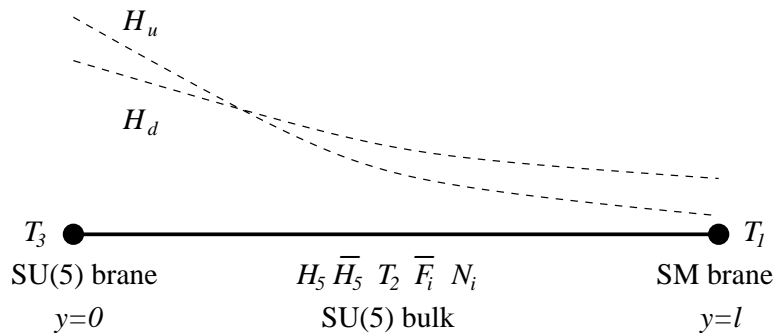
Figure 1: The location of the **5** and $\overline{\mathbf{5}}$ Higgs and 3 generations of matter. The dotted lines schematically illustrate the bulk profile of the massless doublet-Higgs H_u and H_d states. The N_i are three SM singlet rhd neutrino states as discussed in Sect. 5.

(Note that, in our approach, SU(5) breaking effects in the Yukawa couplings arise only from unknown $\mathcal{O}(1)$ coefficients.) Using the fact that, in our model, we approximately have $\delta_u \sim \delta_d^2 \sim \varepsilon^4$, the following structure results:

$$
V_{\mathrm{CKM}} = L_T^\dagger L_F \sim \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon & 1 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} . \tag{12}
$$

With our choice of $\varepsilon = 1/\sqrt{Ml} \simeq 1/17$, this compares favourably with the data as far as 2-3 and 1-3 mixings are concerned. However, we underestimate 1-2 mixing by a factor ~ 4. Although we have to admit that this is arguably the weakest point of our model, we would like to emphasize that a very modest enhancement of one of the off-diagonal entries in λ_{TF} is sufficient to explain the large observed Cabibbo angle.

5 Neutrino masses and mixings

An important aspect of flavour physics in orbifold GUTs is the generation of large neutrino mixing angles and the overall neutrino mass scale. In a straightforward approach, one could introduce 3 rhd neutrino singlets N_i with $\mathcal{O}(1)$ Yukawa couplings and Majorana masses $\sim M$ on the SU(5) brane. However, given that M tends to be larger than 10^{16} GeV and the effective 4d Yukawas are suppressed because H_u is a bulk field, the resulting light neutrino masses generated by the see-saw mechanism come out too small. In the following, we discuss two possibilities for obtaining a realistic neutrino mass scale in 5d orbifold GUTs, both of which make essential use of bulk mass terms.[2]

In our first scenario, we introduce three bulk hypermultiplets (N_i, N_i^c), which are singlets under SU(5), with parity assignments $(+, +)$ and $(-, -)$ for the chiral components

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[2]The use of bulk singlet states as rhd neutrinos was previously analysed for large extra dimensions [18], and mentioned in Ref. [9] in the context of orbifold GUTs. Our analysis differs from these previous discussions.

N and N^c, respectively. In addition, we gauge $U(1)_\chi$ (named following [20]) in the bulk, where $SU(5) \times U(1)_\chi \subset SO(10)$, with orbifold boundary conditions that break $U(1)_\chi$ at the SM brane. (Specifically this is achieved by $(+, -)$ and $(-, +)$ parities for A_μ^χ and A_5^χ, and opposite assignments for the gaugino partners. This leaves no $U(1)_\chi$ zero modes.) Under the bulk $U(1)_\chi$, the charges of the various states are $\chi(T_i) = -1$, $\chi(\overline{F_i}) = 3$, and $\chi(N_i) = -5$, and opposite for the conjugate chiral superfield in each hypermultiplet.[3]

At leading order the most general 5d superpotential for the N_i's consistent with the above symmetries takes the form

$$N^{cT}(\partial_5 + m_N)N + H_u L^T \lambda N \delta(yM) + H_u L^T \lambda' N \delta([y-l]M) + M N^T \kappa N \delta([y-l]M) , \quad (13)$$

where λ, λ', m_N and κ are 3×3 matrices in generation space. Note that the N_i^c can not have a similar Majorana mass term since they vanish at the SM brane. Because the H_u Higgs is highly peaked towards the $SU(5)$ brane, the λ'-term can be neglected.

Appealing to a complete flavour symmetry of the 5d bulk which acts on the $\overline{F}_i$ and N_i, we take the matrix m_N to be of the form $m_N \mathbf{1}_3$, where $m_N > 0$. In the absence of the Majorana mass term, one would find 3 zero modes of the superfields N_i with bulk profile $\exp(-m_N y)$. The Majorana mass couples these zero modes and, given the exponential suppression of the bulk profile at the $SU(5)$ brane and properly normalising the effective 4d kinetic term, leads to the effective 4d rhd neutrino mass matrix

$$M_R \simeq 4\kappa m_N e^{-2m_N l} . \quad (14)$$

It is a simple exercise to check that this result also follows from first deriving the exact solution of the equations of motion, including the $N^T \kappa N \delta(y-l)$ interaction, and then expanding the expression for the light-mode mass-matrix to leading order in $e^{-2m_N l}$. For moderate values of $m_N l$, the mass eigenvalues in M_R are still super-heavy, but they are parametrically lighter than M. Integrating out these modes, the usual see-saw mechanism now generates the light Majorana neutrino mass matrix. The relevant term in the low-energy 4d superpotential is

$$\frac{(\lambda^T M_R^{-1} \lambda)_{ij}}{2(Ml)^3 c(-m_u l)^2 c(-m_N l)^2}(L_i H_u)(L_j H_u) , \quad (15)$$

where L_i are the lepton doublets contained in $\overline{F}_i$.

Because all N_i and $\overline{F}_i$ are treated on an equal footing, the resulting form of the light neutrino mass matrix in generation space is non-hierarchical,

$$\frac{\langle H_u \rangle^2 (\lambda^T M_R^{-1} \lambda)}{(Ml)^3 c(-m_u l)^2 c(-m_N l)^2} \simeq \frac{m_u l \, v^2 \, e^{2m_N l}}{2M(Ml)^2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \quad (16)$$

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[3] To ensure anomaly freedom and the absence FI terms, the sum of the charges of the brane localized fields plus half the sum of the charges of bulk fields with even boundary conditions at that brane have to vanish [13]. Given the T_i, T_i', $\overline{F}_i$ and $\overline{F}_i'$ of Sect. 4 and the anomaly freedom of $SO(10)$, this is easily realized by adding partner hypermultiplets N_i'. They will not interfere with the neutrino mass generation described below if they are peaked at the SM brane.

where, of course, unknown $\mathcal{O}(1)$ factors multiply the different entries, and $v = 246\,\text{GeV}$. As a result the neutrino mixing angles are naturally large, and the super-light neutrino mass-differences are not strongly hierarchical. One may in principle be worried about the CHOOZ constraint $\theta_{e3} < 0.16$, but an analysis of the same texture structure in "neutrino mass anarchy" models shows that no particular fine tuning is necessary for there to be one accidentally small mixing angle [21] (see, however, [22] for a recent 6d orbifold model addressing this issue). Taking the parameters of Eq.(11) and assuming $M \simeq 10^{17}\,\text{GeV}$, we find that a reasonable value of $m_N l \simeq 6.8$ leads to a phenomenologically viable light neutrino mass scale of $m_\nu \sim 0.03\,\text{eV}$.

A second scenario is even simpler. We again have N_i and N_i^c chiral superfields, but choose the orbifold action to be $(+, -)$ and $(-, +)$ respectively. We further take the superpotential to be of the form

$$N^{cT}(\partial_5 + m_N)N + H_u L^T \lambda N \delta(yM) + H_u L^T \lambda' N^c \delta([y - l]M)\,. \tag{17}$$

Note that this superpotential is not the most general that can written, as possible brane-localized masses $M_N NN\delta(yM)$ and $M'_N N^c N^c \delta([y-l]M)$ have been set to zero. (This is technically natural due to supersymmetry.) If one is willing to accept this, then the odd bulk mass m_N leads to an exponential suppression of the 4d mass connecting N and N^c. Integrating out this mode then gives an $(LH_u)^2$ operator with a coefficient that can easily accommodate the correct light neutrino mass scale. The required value of m_N is larger than in our first scenario since the resulting exponential factor has to compensate for the weak coupling of the H_u zero mode at the SM brane. Once again, because of the symmetrical treatment of the N_i and $\overline{F}_i$ bulk modes, which is only broken by brane Yukawa interactions, large neutrino mixing angles are natural.

6 Conclusions

In this letter we have argued that there exists, in the context of a 5-dimensional orbifold SU(5)-GUT model, an appealing explanation of the observed hierarchical structure of the quark and lepton masses and mixing angles. Our model uses only ingredients intrinsic to orbifold GUT constructions. These are the existence of branes fixed by the orbifold action on which gauge and other symmetries can be violated, and the presence of bulk mass terms for the bulk hypermultiplets. Our model has the attractive feature that it does not invoke high-scale Higgs breaking. Flavour hierarchies arise from two effects: first, the geometrical suppression of the couplings of bulk fields, as compared to the couplings of brane fields; second, bulk masses leading to partial localization, or 4d mass scale suppression. Our model provides a simple and concrete demonstration that the observed flavour hierarchies (dimensionless ratios $\gtrsim 4$ or larger) can be explained in geometrical terms within the elegant framework of a 5d orbifold GUT.

Concerning neutrinos, we have shown that there are two attractive higher-dimensional variations of the traditional see-saw mechanism. Both take the rhd neutrino states to be modes of SU(5)-singlet bulk hypermultiplets, N_i. The first involves the gauging of an additional $U(1)_\chi$ in the bulk which is broken on the SM brane by the orbifold action.

The allowed masses for the rhd neutrino states are then a large Majorana mass on the SM brane and a bulk NN^c mass. The latter leads to a suppression of the effective 4d Majorana masses of the lightest 4d rhd states and thus to a suitably enhanced coefficient of the $(LH_u)^2$ operator compared to the naive $1/M \simeq (10^{17}\,\text{GeV})^{-1}$. The second model does not involve any additional bulk gauge symmetry or brane-localized Majorana mass, but flips the sign of the orbifold action to forbid zero modes of N and N^c. In this case the bulk mass suppresses the 4d mass of the lightest KK modes. Both mechanisms naturally lead to large mixing angles as the bulk structure of the $\overline{F}_i$'s and N_i's is independent of generation, with this symmetry being only weakly broken by brane interactions.

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