

# Proton Decay Signatures of Orbifold GUTs

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## Abstract

In grand unified theories based on orbifold constructions in higher dimensions, Higgsino-mediated proton decay is absent. However, proton decay mediated by  $X$  and  $Y$  gauge bosons is typically enhanced to levels detectable by current and future experiments. We analyse the phenomenology of proton decay induced by the minimal coupling of  $X, Y$  gauge bosons. In particular, we show that the novel realization of matter in orbifold GUTs can lead to unusual final state flavour structure, for example, the dominance of the  $p \rightarrow K^0 \mu^+$  mode. Furthermore, we discuss proton decay induced by higher-derivative brane operators, finding potentially observable rates for natural values of the operator coefficients.

# 1 Introduction

Grand unified theories (GUTs) provide an elegant explanation of the origin of the three Standard Model (SM) gauge interactions and the fermion quantum numbers [1]. In their minimal supersymmetric extension, GUTs also lead to a remarkably successful prediction of  $\sin^2 \theta_w$ . This supports both the existence of supersymmetry (SUSY), broken near the weak scale, and some form of gauge-coupling unification at a scale  $M_{\text{GUT}} \simeq 10^{16}$  GeV. The idea of GUTs can also naturally accommodate relations among Yukawa couplings, leading most notably to the quite successful prediction of  $m_b/m_\tau$ .

Among the less attractive features of GUTs are the complicated Higgs sector required for realistic breaking of the gauge group and the necessity of modifying the unsuccessful first and second generation analogues of the  $m_b/m_\tau$  mass predictions. In addition, dimension-5 proton decay operators arising from the exchange of supermassive coloured Higgsinos severely constrain minimal SUSY GUT models (see e.g. [2]).

Recently an elegant solution to these problems has been proposed in the context of SU(5) [3–7] and SO(10) [8, 9] unification. The GUT gauge symmetry is now realized in 5 or more space-time dimensions and broken to the SM group by compactification on an orbifold, utilizing boundary conditions that violate the GUT-symmetry. In the most studied case of 5 dimensions both the GUT group and 5d supersymmetry are broken by compactification on  $S^1/(Z_2 \times Z'_2)$ , leading to an N=1 SUSY model with SM gauge group. This construction allows one to avoid some unsatisfactory features of conventional GUTs with Higgs breaking, such as doublet-triplet splitting, dimension-5 proton decay, and Yukawa unification in the first two generations, while maintaining, at least at leading order, the desired gauge coupling unification [5, 6, 10].

Higgsino mediated proton decay is absent because the triplets acquire mass via the KK expansion of 5d terms of the form

$$\mathcal{L} \supset \int d^2\theta \left( H_3^c \nabla_5 H_3 + H_3^c \nabla_5 H_{\bar{3}} \right) + \text{h.c.} \quad (1)$$

Thus, the mass couples the triplet Higgsino to a state in  $H_3^c$ , and not in  $H_{\bar{3}}$ . Unlike the  $H_3$  and  $H_{\bar{3}}$  states, the  $H_3^c$  and  $H_{\bar{3}}^c$  fields do not couple directly to the quark and lepton superfields and the dangerous dimension-5 operators are absent. This absence can be viewed as a consequence of a  $U(1)_R$  symmetry of the 5d theory. An extension of the  $U(1)_R$  to matter fields localized on the branes leads to  $R$ -parity, prohibiting baryon number violation at dimension 4 as well [5].

Although there is no dimension-4 or -5 proton decay, we argue in this letter that  $X, Y$  gauge-boson mediated proton decay is now much more interesting. First, the mass scale  $M_c = 1/R$  of the  $X$  and  $Y$  gauge bosons in the orbifold GUT theories is lower than in conventional SUSY GUTs, possibly approaching  $10^{14}$  GeV.<sup>1</sup> This enhances the dimension-6 proton decay processes to a level that may be seen in current and future experiments. Second, the novel realization of the matter multiplets in orbifold GUTs

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<sup>1</sup>The reason for this is that the running of differences of gauge couplings does not stop at  $M_c$ , but continues in only a slightly modified way [5, 6]. So  $M_c$  is lower than the unification scale.

changes in a striking way the signatures of  $X, Y$  mediated proton decay and leads to a clear experimental distinction between orbifold and 4d GUT predictions.

We begin in Sect. 2 by deriving the dimension-6 proton decay operators arising from the minimal couplings of  $X, Y$  gauge bosons. We integrate out the 5d  $X, Y$  states in a 4d superfield formalism. The advantage of this approach is that no ill-defined contact-interaction  $\sim \delta(0)$  (cf. [11]) appears in the intermediate stages of the calculation. Using this formalism we then discuss the effect of higher-derivative brane operators involving the  $X, Y$  gauge fields. Such interactions, which are not forbidden by the gauge symmetries of the model, can lead to sizeable proton decay rates even in scenarios which are otherwise safe due to the location of the matter multiplets. In Sect. 3, we calculate the corresponding proton decay rates for the two classes of operators. The obtained results depend crucially on the location of matter multiplets on the two branes or in the bulk. In particular, we point out that in one of the most attractive realizations of the  $SU(5)$  orbifold model, the minimal  $X, Y$  couplings favour  $p \rightarrow \mu^+ K^0$  decay. This represents a striking orbifold GUT signature. We also show that, depending on the realization of matter, the higher-derivative operators involving  $X, Y$  gauge bosons, can be the numerically dominant source of proton decay. Overall,  $X, Y$ -mediated proton decay can occur at rates observable in the current or next generation of proton decay experiments. Our conclusions are given in Sect. 4.

## 2 Integrating out the $X, Y$ gauge bosons

We begin by recalling the basic structure of the Kawamura model [3], which is based on a 5d super Yang-Mills theory on  $\mathbb{R}^4 \times S^1$ , where the  $S^1$  is parameterized by  $y \in [0, 2\pi R)$ . The field space is then restricted by imposing the two discrete  $Z_2$  symmetries,  $y \rightarrow -y$  and  $y' \rightarrow -y'$  (with  $y' = y - \pi R/2$ ). The action of the  $Z_2$ 's in field space is specified by the two gauge twists  $P$  and  $P'$ . If the original gauge group is  $SU(5)$  and the gauge twists are chosen as  $P = 1$  and  $P' = \text{diag}(1, 1, 1, -1, -1)$ , an effective low energy theory with SM gauge symmetry results.

The supersymmetric version of this model can be described in terms of 4d superfields [12]. A manifestly gauge-invariant form of the non-abelian action is given by [13]

$$S = \int d^4x \int_0^l dy \frac{1}{2g_5^2} \text{tr} \left\{ \int_{\theta^2} W^2 + \text{h.c.} + \int_{\theta^2 \bar{\theta}^2} \left( e^{-2V} \nabla_5 e^{2V} \right)^2 \right\}, \quad (2)$$

where  $V$  is a real superfield depending on the additional parameter  $y = x^5$ ,  $W$  is the corresponding field strength superfield,  $\nabla_5 = \partial_5 + \Phi$  is the covariant derivative in  $x^5$  direction, and  $\Phi$  is an  $x^5$ -dependent chiral superfield.

We will be interested in baryon-number-violating processes mediated by  $X, Y$  gauge bosons in scenarios where SM fermions are localized on the  $SU(5)$  brane at  $y = 0$ , on the SM brane at  $y = l = \pi R/2$ , or in the bulk. We begin with the simplest case of  $SU(5)$ -brane localized matter. The coupling to a chiral superfield  $\Psi$  at  $y = 0$  is described by the lagrangian

$$\mathcal{L}_\Psi = \delta(y) \int_{\theta^2 \bar{\theta}^2} \bar{\Psi} e^{2V} \Psi = \delta(y) \int_{\theta^2 \bar{\theta}^2} \left\{ \bar{\Psi} \Psi + 2V^A J^A + \mathcal{O}(V^2) \right\}, \quad (3)$$

where  $J^A = \text{tr}(\bar{\Psi} T^A \Psi)$  is the matter current. Note that the  $SU(5)$  generators  $\{T^A\}$  fall into the two subsets  $\{T^a\}$  and  $\{T^{\hat{a}}\}$ , the first one being the SM generators and the second the basis of the orthogonal complement.

Given a current  $J$  (corresponding to an external field  $\Psi$ ), the equation of motion for  $V$  is obtained by varying the action of Eqs. (2) and (3). In the  $\Phi = 0$  gauge and to leading order in  $V$  it reads (our superspace conventions are those of [14])

$$-\frac{1}{8} D_\alpha \bar{D}^2 D^\alpha V - \partial_5^2 V + g_5^2 J \delta(y) = 0. \quad (4)$$

If  $\Psi$  is on-shell,  $D^2 \Psi = 0$ , one finds  $D^2 J = 0$ . It is self-consistent to assume that, in the presence of an  $x^\mu$ -independent current ( $\mu = 0, \dots, 3$ ), we have an  $x^\mu$  independent solution  $V$  with a superspace dependence given by  $V \sim J$ . Then the first term in Eq. (4) vanishes and we need to solve

$$-\partial_5^2 V^{\hat{a}} + g_5^2 J^{\hat{a}} \delta(y) = 0 \quad (5)$$

for the  $V$  components corresponding to  $X, Y$  gauge bosons. Given the boundary conditions  $\partial_5 V^{\hat{a}}(y=0) = V^{\hat{a}}(y=l) = 0$ , the solution is  $V^{\hat{a}} = -g_5^2(l-y)J^{\hat{a}}$  for  $y > 0$ . The function is continuous at  $y=0$  but has a discontinuous first derivative. Inserting this into the original lagrangian, the following effective operator in the low-energy 4d effective theory is generated:

$$\mathcal{O}_1 = -l g_5^2 \int_{\theta^2 \bar{\theta}^2} \sum_{\hat{a}} (J^{\hat{a}})^2. \quad (6)$$

We have checked explicitly that the four-scalar interaction contained in the superfield operator  $\mathcal{O}_1$  agrees with the result of [11], where the calculation was done by first integrating out auxiliary fields and then summing the exchanged KK modes in the component formalism. The advantage of our superfield approach is that no ill-defined contact-interaction  $\sim \delta(0)$  appears in the intermediate stages of the calculation<sup>2</sup>. This could also have been achieved by summing KK modes and making use of the cancellations between physical and auxiliary field contributions enforced by the unbroken 4d SUSY at each KK level.

The four-fermion interaction contained in  $\mathcal{O}_1$  reads

$$\mathcal{O}_1 \supset -\frac{\pi^2}{4} \frac{g_4^2}{M_c^2} (\bar{\psi}_i \bar{\psi}_j) (\psi_k \psi_l) \sum_{\hat{a}} T_{ik}^{\hat{a}} T_{jl}^{\hat{a}}, \quad (7)$$

where  $\psi$  is the fermion from  $\Psi$ ,  $g_4^2 = g_5^2/l$  is the 4d gauge coupling, and  $M_c = 1/R$  is the compactification scale which, at the same time, is the mass of the lowest-lying KK mode of  $X, Y$  gauge bosons. Note that, up to the prefactor  $\pi^2/4$ , this is precisely the result expected in a 4d GUT with gauge boson mass  $M_c$ . The prefactor can be easily understood as  $2(\pi^2/8)$ , where the 2 comes from the normalization of the KK modes due to their non-trivial bulk profile [5] and  $\pi^2/8 = \sum_{n=1}^{\infty} (2n-1)^{-2}$  accounts for exchanging the full KK tower rather than just the lowest-lying mode.

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<sup>2</sup>This observation has also been made in the context of a 3d superfield description of 4d SUSY theories with boundary [15].

We now turn to the case where SM fermions are localized on the SM brane [6].<sup>3</sup> Although in this case,  $X, Y$  mediated proton decay is no longer a generic prediction of the theory, it will occur if brane operators of the type

$$\mathcal{L}_\Psi = \frac{c}{M} \delta(y') \int_{\theta^2 \bar{\theta}^2} \bar{\Psi}_1 (\nabla_5 e^{2V}) \Psi_2 + \text{h.c.} \quad (8)$$

are included in the action [13]. Here  $\Psi_1$  and  $\Psi_2$  are SM superfields and the multiplication with  $(\nabla_5 e^{2V})$  is defined by using their standard embedding into  $SU(5)$  multiplets. Note that, even though the  $X, Y$  components of  $V$  vanish at the SM brane, their  $\partial_5$  derivatives appearing in  $(\nabla_5 e^{2V})$  are non-zero. The prefactor includes a dimensionless  $\mathcal{O}(1)$  coefficient  $c$  and the fundamental scale  $M \gg M_c$ , which is required for dimensional reasons. Parametrically,  $M$  can not be larger than  $\sim 1/g_5^2$ , since this is the scale at which the 5d gauge theory becomes strongly coupled.

Following the same line of reasoning as in the case of  $SU(5)$ -brane fermions, one arrives at the analogue of Eq. (5), which is

$$-\partial_5^2 V^{\hat{a}} - \frac{cg_5^2}{M} J^{\hat{a}} \partial_5 \delta(y) = 0, \quad (9)$$

with the current  $J^{\hat{a}} = \text{tr}(\bar{\Psi}_1 T^{\hat{a}} \Psi_2 + \text{h.c.})$ . The solution is given by

$$V^{\hat{a}}(y) = \frac{cg_5^2}{M} J^{\hat{a}} \int_y^l d\tilde{y} \delta(\tilde{y}). \quad (10)$$

Inserting this into the original lagrangian, the following effective operator in the low-energy 4d effective theory is obtained:

$$\mathcal{O}_2 = -\frac{c^2 g_5^2}{M^2} \int_{\theta^2 \bar{\theta}^2} \sum_{\hat{a}} (J^{\hat{a}})^2 \int_0^l dy \delta(y)^2 = -\frac{\pi b c^2 g_4^2}{2 M M_c} \int_{\theta^2 \bar{\theta}^2} \sum_{\hat{a}} (J^{\hat{a}})^2. \quad (11)$$

Here, to obtain the final expression, we have assumed the  $\delta$  function to be localized on a scale  $1/M$  and, accordingly, replaced the  $y$  integral by  $bM$ , with  $b$  an  $\mathcal{O}(1)$  coefficient depending on the specific way in which the  $\delta$  function is regularized. Thus, the operator  $\mathcal{O}_2$  depends via  $c^2$  on the a priori unknown strength of the brane-bulk coupling, Eq. (8), and via  $b$  on the UV completion of the theory. This UV sensitivity can also be understood by observing that the derivative coupling of Eq. (8) enhances the contribution of higher KK modes so that the sum diverges. Parametrically,  $\mathcal{O}_2$  is suppressed with respect to  $\mathcal{O}_1$  by a factor  $M/M_c$ , which can be as small as  $\sim 10$  (allowing at least some range of validity for the effective 5d theory) or as large as  $\sim 10^3$  (extending the 5d theory all the way to its strong coupling scale). Nevertheless, as we argue in the next section, in models where the first generation is not on the  $SU(5)$  brane,  $\mathcal{O}_2$  can be a competitive or even the dominant source of proton decay.

In models where both branes contain SM fermions, the combined effect of the couplings of Eqs. (3) and (8) will give rise to further proton decay operators involving both  $SU(5)$ - and SM-brane fermions. The calculation is a straightforward combination of the two basic cases discussed above.

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<sup>3</sup>Following [6] such realizations of SM matter have also been used in recent  $SU(3)$  models at the TeV scale [16].

### 3 Proton decay rates

In SU(5) orbifold GUTs, there are three possible locations for matter: the SU(5) brane, the bulk (which is also SU(5) symmetric), and the SM brane. The various possible models are characterized by the placement of the  $T_i$  ( $\mathbf{10}$ 's) and  $\bar{F}_i$  ( $\bar{\mathbf{5}}$ 's) that make up the three ( $i = 1, 2, 3$ ) generations of quarks and leptons. To reproduce the successful SU(5)  $m_b/m_\tau$  mass relation,  $T_3$  and  $\bar{F}_3$  must reside on the SU(5) brane. Furthermore, due to the different normalization of 4d and 5d fields, Yukawa interactions involving one or two bulk matter fields are effectively suppressed by factors  $(M/M_c)^{1/2}$  and  $(M/M_c)$  [17] (recall that  $M$  is the fundamental UV scale of the theory). Thus, both the relative lightness of the 2nd and 1st generations and the failure of the SU(5) mass predictions indicate that for  $i = 1, 2$  either  $T_i$  or  $\bar{F}_i$  or both should be in the bulk or on the SM brane.

First we consider the case of proton decay operators arising from minimal  $X, Y$  gauge boson interactions.

Both for matter on the SM brane and in the SU(5)-symmetric bulk, minimally coupled  $X, Y$  gauge bosons do *not* lead to dimension-6 baryon number violating operators. For matter on the SM brane this is simply because the orbifold boundary conditions imply the vanishing of the  $X, Y$  gauge boson wavefunctions on the SM brane. For bulk matter the situation is slightly more involved. Although 5d fields come in complete SU(5) multiplets, the orbifold projections  $P$  and  $P'$  imply that the zero modes under this action do not fill out full SU(5) multiplets. From a  $\mathbf{10}$  we just get  $\bar{U}$  and  $\bar{E}$  zero modes, while from a  $\bar{\mathbf{5}}$  we get  $\bar{D}$ . The remaining components of a full SM generation are realized at the zero mode level by taking another copy of  $\mathbf{10}$  and  $\bar{\mathbf{5}}$  in the bulk and flipping the overall sign of the action of  $P'$  on these multiplets. As a result, we have zero modes which fill out the full matter content of  $T$  or  $\bar{F}$  at the zero mode level [5, 6]. However, since the components of  $T$  or  $\bar{F}$  arise from different 5d SU(5) parent multiplets, the interaction of  $X$  and  $Y$  gauge bosons with these ‘split’ multiplets do not convert SM quarks into SM leptons.<sup>4</sup> Rather, they convert SM fermions into superheavy ( $m \sim 1/R$ ) exotic partner states. Only with multiple  $X, Y$  exchange can a baryon-number-violating operator involving only SM fields be produced. Such operators are at minimum of dimension 8 and are irrelevant for proton decay experiments in the foreseeable future.<sup>5</sup> Thus, we now focus on proton decay due to interactions of  $X, Y$  gauge bosons with SU(5)-brane matter.

The only  $\Delta B = 1$  operators induced by single  $X$  or  $Y$  gauge boson exchange are

$$\mathcal{O}_{TF} = \frac{\pi^2}{4} \frac{g_4^2}{M_c^2} \sum_{i,j} a_i b_j \bar{d}_{Ri} \bar{u}_{Rj} L_{Li} Q_{Lj} \quad (12)$$

$$\mathcal{O}_{TT} = \frac{\pi^2}{4} \frac{g_4^2}{M_c^2} \sum_{i,j} b_i b_j \bar{e}_{Ri} \bar{u}_{Rj} Q_{Li} Q_{Lj}. \quad (13)$$

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<sup>4</sup>Similarly, SU(5) mass relations do not apply in the split multiplet case.

<sup>5</sup>If some or all of the SM fermions come from split multiplets localized in the bulk,  $X, Y$ -boson-mediated proton decay via dimension-6 operators will occur if couplings analogous to Eqs. (3) and (8) mixing different bulk SU(5) multiplets are present. One can think of such terms as non-diagonal brane-localized kinetic terms (in a basis chosen to make the bulk kinetic term diagonal). We do not consider this case in detail here.

Here and below our Weyl spinor notation is such that  $\psi_{\text{Dirac}} = (\psi_L, \bar{\psi}_R)^T$ . The operators  $\mathcal{O}_{TF}$  and  $\mathcal{O}_{TT}$  arise from gauge exchange between  $T$  and  $\bar{F}$  and between two  $T$  multiplets respectively. The generation indices  $i, j$  label the matter fields in the ‘locality basis,’ [17] (in which the fields are defined by their location) so that  $a_i = 1$  or  $a_i = 0$  depending on whether  $\bar{F}_i$  is on the SU(5) brane or not. Similarly  $b_i = 1, 0$  depending on whether  $T_i$  is localized on the SU(5) brane, or not.

The fields in the locality basis (which is also the gauge basis) get mass via Yukawa interactions characterized by the matrices  $\lambda_u, \lambda_d$ , and  $\lambda_e$ . Low-energy experiments determine only their diagonal form,  $\lambda_u^{\text{diag}}, \lambda_d^{\text{diag}}, \lambda_e^{\text{diag}}$ , and the CKM matrix,  $V_{\text{CKM}}$ :

$$\lambda_u^{\text{diag}} = L_u^\dagger \lambda_u R_u, \quad \lambda_d^{\text{diag}} = L_d^\dagger \lambda_d R_d, \quad \lambda_e^{\text{diag}} = L_e^\dagger \lambda_e R_e, \quad V_{\text{CKM}} = L_u^\dagger L_d. \quad (14)$$

The diagonalizing unitary rotations of type  $L$  and  $R$  act in generation space and connect the gauge and mass eigenstate bases. Denoting the latter by primes, the above  $\Delta B = 1$  operators take the form

$$\mathcal{O}_{TF} = \frac{\pi^2 g_4^2}{4M_c^2} \sum_{i,j} a_i b_j \left( (\bar{d}'_R R_d^\dagger)_i (\bar{u}'_R R_u^\dagger)_j (\nu'_L)_i (L_d d'_L)_j - (\bar{d}'_R R_d^\dagger)_i (\bar{u}'_R R_u^\dagger)_j (L_e e'_L)_i (L_u u'_L)_j \right) \quad (15)$$

(note that, for our purposes, neutrinos can be treated as massless) and

$$\mathcal{O}_{TT} = \frac{\pi^2 g_4^2}{2M_c^2} \sum_{i,j} b_i b_j (\bar{u}'_R R_u^\dagger)_j (\bar{e}'_R R_e^\dagger)_i (L_u u'_L)_j (L_d d'_L)_i. \quad (16)$$

Here, in contrast to the minimal version of 4d GUTs, the coefficients of the operators involve combinations of elements of  $L$  and  $R$  matrices beyond those determined by  $V_{\text{CKM}}$ .

We start our discussion with the case considered originally in [3]: all generations located at the SU(5) brane. We use the approximation  $L \simeq R \simeq \mathbf{1}$  and focus on the mode  $p \rightarrow \pi^0 e^+$ . The relevant operator reads

$$\mathcal{O} = \frac{\pi^2 g_4^2}{4M_c^2} \left( 2 \bar{u}'_R e'_L u'_L d'_L - \bar{d}'_R u'_R e'_L u'_L \right), \quad (17)$$

with only first generation fields. This gives rise to the decay rate (see, e.g., [18])

$$\Gamma(p \rightarrow \pi^0 e^+) = \left( \frac{\pi^2}{4} \right)^2 \frac{5\alpha^2 m_p}{64\pi f_\pi^2} (1 + D + F)^2 \left( \frac{g_4^2 A_R}{M_c^2} \right)^2. \quad (18)$$

Taking the hadronic parameter  $\alpha = 0.015 \text{ GeV}^3$  [19], the pion decay constant  $f_\pi = 0.13 \text{ GeV}$ , the chiral perturbation theory parameters  $D = 0.80$  and  $F = 0.47$ , the unified gauge coupling  $g_4^2/(4\pi) = 1/25$ , and the renormalization coefficient  $A_R = 2.5$ , the resulting life time is

$$1/\Gamma(p \rightarrow \pi^0 e^+) = 1.4 \times 10^{34} \text{ years} \times \left( \frac{M_c}{10^{16} \text{ GeV}} \right)^4. \quad (19)$$

Using the Super-Kamiokande limit of  $5.3 \times 10^{33}$  years [20, 21], this leads to the bound  $M_c \geq 0.8 \times 10^{16} \text{ GeV}$  [5]. Given the usual unifications scale  $M_{\text{GUT}} \simeq 2 \times 10^{16} \text{ GeV}$ , almost no validity range for the higher-dimensional field theory is left.

Let us now turn to a better motivated case where only the third generation is located on the SU(5) brane. This scenario predicts the large mass of the third generation,  $b - \tau$  unification, and the absence of a similar mass relation in the first two generations. Given the observed structure of  $V_{CKM}$ , we assume that all matrices  $L$  and  $R$  are near the unit matrix. Thus, proton decay will be suppressed by the small off-diagonal elements of these matrices. To minimize the number of such elements, is advantageous to have an anti-neutrino and a strange quark in the final state. The relevant operator reads

$$\mathcal{O} = \frac{\pi^2 g_4^2}{4M_c^2} \overline{s'_R u'_R} \nu'_{\tau L} d'_L (R_d^\dagger)_{23} (R_u^\dagger)_{13} (L_d)_{31}. \quad (20)$$

The resulting decay rate, calculated using the analysis of [19] and including the phase space suppression due to the mass of the  $K^+$ , is

$$\Gamma(p \rightarrow K^+ \bar{\nu}_\tau) = \left(\frac{\pi^2}{4}\right)^2 \frac{\alpha^2 m_p}{72\pi f_\pi^2} \left(D \frac{m_p}{m_\Lambda}\right)^2 \left(\frac{g_4^2 A_R}{M_c^2}\right)^2 \left(1 - \frac{m_K^2}{m_p^2}\right)^2 \left|(R_d^\dagger)_{23} (R_u^\dagger)_{13} (L_d)_{31}\right|^2, \quad (21)$$

with  $m_\Lambda = 1.1$  GeV and  $m_K = 0.5$  GeV. If, guided by  $V_{CKM}$ , one estimates the 2-3 and 1-3 mixing elements by 0.05 and 0.01 respectively, one finds a life time

$$1/\Gamma(p \rightarrow K^+ \bar{\nu}_\tau) \simeq 6.6 \times 10^{38} \text{years} \times \left(\frac{M_c}{10^{14} \text{GeV}}\right)^4. \quad (22)$$

Thus, to observe an effect, one requires either experimental progress beyond the currently considered multi-megaton detectors (see e.g. [21]), or a very low compactification scale, or off-diagonal elements of the unmeasured mixing matrices larger than the naive CKM-based estimate.

Finally, we consider what is arguably the best-motivated scenario, namely, a model with only  $T_3$ ,  $\overline{F}_3$  and  $\overline{F}_2$  on the SU(5) brane. Following the discussion of [17], one finds that this model provides an explanation of all fermion mass hierarchies except for the lightness of the first-generation up-quark and the neutrinos in terms of the single parameter  $(M/M_c)^{1/2}$ . In this case, one of the dominant decay modes is  $p \rightarrow K^0 \mu^+$ , the relevant operator being

$$\mathcal{O} = -\frac{\pi^2 g_4^2}{4M_c^2} \overline{s'_R u'_R} \mu'_L u'_L (R_u^\dagger)_{13} (L_u)_{31}. \quad (23)$$

This gives rise to the decay rate

$$\Gamma(p \rightarrow K^0 \mu^+) = \left(\frac{\pi^2}{4}\right)^2 \frac{\alpha^2 m_p}{32\pi f_\pi^2} \left(1 + (D-F) \frac{m_p}{m_\Lambda}\right)^2 \left(\frac{g_4^2 A_R}{M_c^2}\right)^2 \left(1 - \frac{m_K^2}{m_p^2}\right)^2 \left|(R_u^\dagger)_{13} (L_u)_{31}\right|^2, \quad (24)$$

and the life time

$$1/\Gamma(p \rightarrow K^0 \mu^+) \simeq 2.1 \times 10^{35} \text{years} \times \left(\frac{M_c}{10^{14} \text{GeV}}\right)^4. \quad (25)$$

Thus, in a phenomenologically well-motivated realization of this scenario, with  $M \simeq 10^{17}$  GeV and  $(M/M_c)^{1/2} \simeq 30$ , the next generation of proton decay experiments could ‘discover’ orbifold GUTs via the striking signal of a  $K^0 \mu^+$  final state.



We now turn to the non-minimal case where proton decay arises from the higher-derivative interactions of the  $X, Y$  bosons with matter on the SM brane. This is of interest especially in the minimal GUT models where all matter is localized on the SM brane [6]. The proton decay rate that results from the operator Eq.(11) is

$$1/\Gamma(p \rightarrow \pi^0 e^+) = 3.5 \times 10^{34} \text{years} \times \frac{1}{b^2 c^4} \left( \frac{M_c}{10^{15} \text{GeV}} \right)^2 \left( \frac{M}{10^{17} \text{GeV}} \right)^2. \quad (26)$$

Assuming natural  $\mathcal{O}(1)$  values for the unknown coefficients  $b$  and  $c$ , this leads to a potentially observable rate. This same mechanism can also apply to split multiplets, but in this case there is a further suppression of  $M_c/M$  in the rate for each bulk field.

## 4 Conclusions

Proton decay is one of the most characteristic and striking signatures of conventional 4d GUTs. In orbifold GUTs, although dimension 5 operators are naturally absent, proton decay induced by the exchange of  $X, Y$  boson states is enhanced. Moreover, the minimal gauge interactions of  $X, Y$  gauge bosons only induce  $\Delta B = \Delta L = 1$  operators for those matter multiplets localized to the SU(5)-invariant brane. For example, in models with the most realistic flavour structure, the 1st generation is not localized to the SU(5) brane, and proton decay arises from the mixing of brane and bulk matter to form mass eigenstates. As a result, minimal  $X, Y$  gauge interactions favour unusual final states, e.g.  $p \rightarrow K^0 \mu^+$ , at a potentially observable rate. Thus the rates and flavour structure of proton decay processes provide important probes of orbifold GUTs. In addition to minimal  $X, Y$  couplings, there can exist higher-derivative brane-localized couplings of  $X, Y$  gauge bosons directly to 1st generation states. Although formally suppressed by  $M_c/M$ , these new operators do not rely on mixing and can be the numerically dominant source of proton decay, with rates  $\Gamma(p \rightarrow \pi^0 e^+)$  observable in current and future experiments.

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