Z parameters and electroweak fits

G. Quast

Institut für Physik, Universität Mainz, 55099 Mainz, Germany
E-mail: G.Quast@uni-mainz.de

Abstract
Almost exactly ten years after the start-up of LEP and four years after the completion of the energy scans around the Z resonance the analyses of the Z parameters by the four experiments ALEPH, DELPHI, L3and OPAL are almost final. Together with other precision electroweak results these provide a stringent test of the Standard Model.

1. Introduction

This brief review presents the latest results from analyses of the Z line shape and of the leptonic forward-backward asymmetries from data taking at LEP I around the Z resonance during 1990 to 1995. These and other electroweak precision results presented at this conference [1] are used as input to fits for the parameters of the minimal Standard Model ("mSM"), which provide stringent consistency tests and significantly constrain the value of the unknown Higgs boson mass within the framework of the mSM.

2. Results on Z parameters

Between the years 1989 and 1995, the $\mathrm{e^+e^-}$ collider LEP at CERN provided interactions at centre-of-mass energies ranging from 88 to 95 GeV, *i.e.*, around the mass of the Z boson. A total of 15.5 million $\mathrm{Z} \rightarrow q\overline{q}$ and 1.7 million $\mathrm{Z} \rightarrow \ell^+\ell^-$ events have been analysed by the four experiments ALEPH, DELPHI, L3and OPAL. All results have been updated for this conference; the ALEPH results are final, while all others are still preliminary.

At various centre-of-mass energies, total production cross sections for hadrons and leptons are measured; forward-backward asymmetries are determined in lepton-pair production. These measurements of "realistic observables" allow the determination of various properties of the Z boson, such as its mass, m_Z, and total width, Γ_Z, the peak cross section, σ^0, as well as partial decay widths and coupling constants to fermions, denoted as "pseudo-observables". The LEP experiments have agreed on a common set of nine such pseudo-observables: m_Z, Γ_Z, σ^o_h, R_ℓ and $\mathrm{A_{FB}^{0,\,\ell}}$ for $\ell = e, \mu, \tau$, as defined in [2]. For the extraction of these pseudo-observables, the experiments perform so-called model-independent fits to their measured realistic observables using the latest versions of the most advanced electroweak codes [3]. The fits are based on ZFITTER [4], while TOPAZ0 [5], and in the case of ALEPH also MIZA [6], serve as cross-checks to determine theoretical errors.

Typically, the full data set of each experiment consists of about 200 individual measurements at various energies and with slightly different detector configurations, which are condensed into nine parameters in the fit. The dominating data sets are the runs in 1992 and 1994 at the peak energy, and precision energy scans at the peak energy and $\sim$1.8 GeV above and below in 1993 and 1995. The average over the four experiments is performed at the level of the nine parameters and their known correlations. Knowledge of common systematic errors like the energy scale of LEP or uncertainties arising from theoretical calculations is also required.

The high statistics is well matched by small systematic errors in the event selection procedures of the experiments. These are in the range ±0.04 % to ±0.1 % for $\mathrm{q\overline{q}}$ events, and ±0.1 % to ±0.7 % for the lepton channels. The selection uncertainties of small-angle Bhabha events, serving to determine the integrated luminosity, range between ±0.033 % and ±0.09 %. The uncertainty on the energy of the beams in LEP contributes a common error of $\pm1.7\,\mathrm{MeV}$ on m_Z and of $\pm1.2\,\mathrm{MeV}$ on Γ_Z. The theoretical error on calculations of the small-angle Bhabha cross section is 0.054 % for OPAL and 0.06 % for all other experiments. QED radiation, dominated by photon radiation from the initial state electrons, contributes an estimated common uncertainty of ±0.02 % on σ^o_h and of $\pm0.5\,\mathrm{MeV}$ on m_Z and Γ_Z, where the latter one is dominated by the uncertainty in fermion pair radiation ‡. The contribution of t-channel diagrams and the s–t interference in $\mathrm{Z} \rightarrow \mathrm{e^+e^-}$ leads to an additional theoretical uncertainty estimated to be ±0.11 % on R_e and to ±0.0013 on $\mathrm{A_{FB}^{0,\,e}}$. Uncertainties from the model-independent parameterisation of the energy dependence of the cross section are almost negligible,

‡ Progress on this issue was reported at this conference [3], but has not yet been incorporated in the experimental results presented here.

if the definitions of Reference [2] are applied. Through unavoidable Standard Model remnants, dominated by the need to fix the γ–Z interference contribution in the $\mathrm{q\overline{q}}$ channel, there is some small dependence of $\pm 0.3\,\mathrm{MeV}$ of m_{Z} on the Higgs mass, m_{H}, or the value of the electromagnetic coupling constant. Such "parametric" errors are negligible for the other pseudo-observables.

The experimental results are given in Tab. 1. In the combination procedure the full $(4\times 9)\times(4\times 9)$ error correlation matrix is constructed from the independent experimental errors (statistics plus detector systematics) and from the sources of common errors discussed above, and the average is performed, yielding the results of Tab. 2. The value of χ^2 per degree of freedom of the combination is 32.5 / 27, corresponding to a χ^2-probability of 21.5 %. If lepton universality is assumed, the six lepton parameters can be combined into two; this is shown in the second part of the table. Significant correlations ($\geq$10 %) are -28 % between Γ_{Z} and σ^o_{h}, about $+12$% between σ^o_{h} and R_{e}, R_μ and R_τ ($+19$ % between σ^o_{h} and R_ℓ), and finally -36 % between R_{e} and $\mathrm{A_{FB}^{0,\,e}}$.

By parameter transformation some more familiar pseudo-observables than the experimentally-motivated set of Tab. 2 may be obtained. The partial Z decay widths are summarised in Tab. 3. A limit on the invisible width not originating from $\mathrm{Z}\rightarrow\nu\overline{\nu}$ is obtained by taking the difference between the value given in the table and the Standard Model expectation of $(\Gamma_{\mathrm{inv}})_{\mathrm{SM}} = 501.7^{+0.1}_{-0.9}\,\mathrm{MeV}$, $\Gamma^x_{\mathrm{inv}} = -2.9^{+1.7}_{-1.5}\,\mathrm{MeV}$, or expressed as a limit, $\Delta\Gamma^x_{\mathrm{inv}} < 2.0\,\mathrm{MeV}$ @ 95% CL; here, the limit was conservatively calculated allowing only positive values of Γ^x_{inv}.

Table 1. Results on Z parameters.

	ALEPH	DELPHI
m_{Z} [GeV]	91.1886 ± 0.0031	91.1864 ± 0.0029
Γ_{Z} [GeV]	2.4952 ± 0.0043	2.4870 ± 0.0041
σ^o_{h} [nb]	41.558 ± 0.057	41.580± 0.069
R_{e}	20.683 ± 0.075	20.88 ± 0.12
R_μ	20.800 ± 0.056	20.650± 0.076
R_τ	20.707 ± 0.062	20.84 ± 0.13
$\mathrm{A_{FB}^{0,\,e}}$	0.0184 ± 0.0034	0.0173 ± 0.0049
$\mathrm{A_{FB}^{0,\,\mu}}$	0.0171 ± 0.0024	0.0165 ± 0.0025
$\mathrm{A_{FB}^{0,\,\tau}}$	0.0170 ± 0.0028	0.0241 ± 0.0037
	L3	OPAL
m_{Z} [GeV]	91.1893 ± 0.0030	91.1852 ± 0.0029
Γ_{Z} [GeV]	2.5017 ± 0.0041	2.4941 ± 0.0041
σ^o_{h} [nb]	41.536 ± 0.055	41.508 ± 0.055
R_{e}	20.814 ± 0.089	20.905 ± 0.085
R_μ	20.860 ± 0.097	20.813 ± 0.058
R_τ	20.79 ± 0.14	20.834 ± 0.091
$\mathrm{A_{FB}^{0,\,e}}$	0.0106 ± 0.0058	0.0090 ± 0.0044
$\mathrm{A_{FB}^{0,\,\mu}}$	0.0188 ± 0.0033	0.0154 ± 0.0023
$\mathrm{A_{FB}^{0,\,\tau}}$	0.0260 ± 0.0047	0.0145 ± 0.0030

Table 2. Combined results

without lepton universality	
m_{Z} [GeV]	91.1872± 0.0021
Γ_{Z} [GeV]	2.4944 ± 0.0024
σ^o_{h} [nb]	41.544 ± 0.037
R_{e}	20.803 ± 0.049
R_μ	20.786 ± 0.033
R_τ	20.764 ± 0.045
$\mathrm{A_{FB}^{0,\,e}}$	0.0145 ± 0.0024
$\mathrm{A_{FB}^{0,\,\mu}}$	0.0167 ± 0.0013
$\mathrm{A_{FB}^{0,\,\tau}}$	0.0188 ± 0.0017
with lepton universality	
m_{Z} [GeV]	91.1871± 0.0021
Γ_{Z} [GeV]	2.4944 ± 0.0024
σ^o_{h} [nb]	41.544 ± 0.037
R_ℓ	20.768 ± 0.024
$\mathrm{A_{FB}^{0,\,\ell}}$	0.01701 ± 0.00095

Table 3. Partial Z decay widths.

	without lepton universality	with lepton universality
Γ_{had} [MeV]	1745.3±2.7	1743.9±2.0
Γ_{inv} [MeV]	497.2±2.5	498.8±1.5
$\Gamma_{\mathrm{inv}}/\Gamma_{\ell\ell}$	-	5.941±0.016
$\Gamma_{\ell\ell}$ [MeV]	-	83.959±0.089
Γ_{ee} [MeV]	83.90 ±0.12	-
$\Gamma_{\mu\mu}$ [MeV]	83.96 ±0.18	-
$\Gamma_{\tau\tau}$ [MeV]	84.05 ±0.22	-

The results in Tab 2 may also be expressed in terms of vector and axial-vector couplings of the Z to leptons. The most stringent test of universality among the couplings of the Z is obtained if the results presented so far are combined with the result from τ polarisation and the left-right polarised asymmetries of the hadronic cross section and the leptonic forward-backward asymmetries at SLD [7]. This is shown in Table 4.

3. Standard Model fits

The parameters of the mSM are the electromagnetic and strong coupling constants, $\alpha(m_{\mathrm{Z}})$ and α_s, and the Z and Higgs boson and top quark masses, m_{Z}, m_{H} and m_{t}. Among these, only m_{H} and α_s are truly free in the χ^2 fits shown below, the others are "constrained" by also specifying them as input to the fit. Other parameters of the mSM, *i.e.* fermion masses and the Fermi constant, are taken as fixed values.

The full set of electroweak precision data considered here is summarised in Tab. 5. The last column labelled

Table 4. Z couplings to leptons, from LEP and SLD data.

g_e^v	=	-0.03809 ± 0.00047
g_μ^v	=	-0.0360 ± 0.0024
$g_\mu^v\,/\,g_e^v$	=	0.946 ± 0.065
g_τ^v	=	-0.0364 ± 0.0010
$g_\tau^v\,/\,g_e^v$	=	0.955 ± 0.030
g_e^a	=	-0.50105 ± 0.00036
g_μ^a	=	-0.50117 ± 0.00054
$g_\mu^a\,/\,g_e^a$	=	1.0002 ± 0.0013
g_τ^a	=	-0.50198 ± 0.00064
$g_\tau^a\,/\,g_e^a$	=	1.0019 ± 0.0015
g_l^v	=	-0.03772 ± 0.00041
g_l^a	=	-0.50117 ± 0.00027
g_ν	=	0.50058 ± 0.00075

Table 5. Summary of electroweak observables.

	Measurement	Pull
LEP I		
$m_{\rm Z}$ [GeV]	91.1871 ± 0.0021	0.1
$\Gamma_{\rm Z}$ [GeV]	2.4944 ± 0.0024	-0.6
$\sigma_{\rm h}^o$ [nb]	41.544 ± 0.037	1.7
R_ℓ	20.768 ± 0.024	1.2
$A_{\rm FB}^{0,\,\ell}$	0.01701 ± 0.00095	0.8
τ pol.:		
$\mathcal{A}_\tau$	0.1425 ± 0.0044	-1.1
$\mathcal{A}_{\rm e}$	0.1483 ± 0.0051	0.2
b & c quarks:		
$R_{\rm b}$ (incl. SLD)	0.21642 ± 0.00073	0.8
$R_{\rm c}$ (incl. SLD)	0.1674 ± 0.0038	-1.3
$A_{\rm FB}^{0,\,\rm b}$	0.0988 ± 0.0020	-2.2
$A_{\rm FB}^{0,\,\rm c}$	0.0692 ± 0.0037	-1.2
$\rm q\overline{q}$ charge asym.:		
$\sin^2\theta_{eff}^{\rm lept}$ ($\langle {\rm Q_{fb}}\rangle$)	0.2321 ± 0.0010	0.6
LEP II		
$m_{\rm W}$ [GeV]	80.350 ± 0.056	-0.6
SLD		
$\sin^2\theta_{eff}^{\rm lept}$ ($A_{\rm lr}$)	0.23099 ± 0.00026	-2.0
$\mathcal{A}_{\rm b}$	0.911 ± 0.025	-0.9
$\mathcal{A}_{\rm c}$	0.630 ± 0.026	-1.5
$\rm p\overline{p}$ **colliders**		
$m_{\rm W}$ [GeV]	80.448 ± 0.062	1.0
$m_{\rm t}$ [GeV]	174.3 ± 5.1	0.2
ν**N scattering**		
$\sin^2\theta_{\rm W}$	0.2255 ± 0.0021	1.1
$\alpha(m_{\rm Z})^{-1\;(a)}$	128.886 ± 0.090	0.05

$^{(a)}$ The electroweak libraries require as input the value of the hadronic vacuum polarisation for five flavours, $\Delta\alpha_{\rm had}^{(5)}(m_{\rm Z})$ $=0.02804\pm0.00065$; small top-dependent parts and the other well-known contributions to the running of α are added internally.

"Pull" gives the deviations of the measurements from the mSM expectation in units of the experimental error. The expected value is calculated using the parameters in the last column of Tab. 6. The most sizeable values and hence the dominant contribution to the overall χ^2 are those from the hadronic pole cross section ($1.7\,\sigma$), from the $A_{\rm FB}^{0,\,\rm b}$ measurements at LEP ($2.2\,\sigma$) and from the measurements of the weak mixing angle from left-right polarised asymmetries at the SLC ($2.0\,\sigma$).

The mSM in the fits is represented by the most advanced electroweak calculational tools, namely ZFITTER [4] and TOPAZ0 [5], which include the complete set of presently known higher order electroweak corrections [3]. Comparison of two independent implementations with different renormalisation and factorisation schemes in addition to variations of the options within each of the codes gives a handle on the theoretical errors involved. Unless stated otherwise, the numerical results and figures in this write-up are based on ZFITTER.

With the presently available set of electroweak precision measurements the mSM is over-constrained, and useful consistency checks can be made by comparing the directly measured values of the top quark and W boson mass with predictions from electroweak corrections. For this purpose, $m_{\rm t}$ or $m_{\rm W}$ are removed from the set of input data, and their fitted values are compared with the direct measurements. This is shown in columns two and three of Tab. 6 and in Fig. 1 and 2. The 1-σ contour lines are calculated using both ZFITTER and TOPAZ0; there is very good agreement indicating that theoretical uncertainties are small and well under control. Note also that the error on the indirect determination of the W mass is still about two times smaller than the error from the direct measurement; by the end of the LEP II program, these are expected to become equal.

The fit in the fourth column shows the indirect values for both the top quark and the W boson mass, to be compared with the direct measurements in the first column of Tab. 5. There is a substantial correlation coefficient between the fit results for $m_{\rm t}$ and $m_{\rm W}$ of 82 %.

Information on the strong coupling constant in these fits comes exclusively from the LEP I measurements of $\Gamma_{\rm Z}$, R_ℓ and $\sigma_{\rm h}^o$ and their correlations. There is a non-negligible QCD related error on the value of the strong coupling constant of about ±0.002, which is not included in Tab. 6.

Taking all results and fitting for the only really unknown parameter of the mSM, the Higgs boson mass, yields $\log_{10}(m_{\rm H}/{\rm GeV}) = 1.88 \pm 0.029$ or $m_{\rm H} = 77^{+69}_{-39}\,{\rm GeV}$, with an estimated theoretical uncertainty of about $\pm10\,\%$ of the error on $\log_{10}(m_{\rm H}/{\rm GeV})$. The central value is lower than the present lower limit on

Table 6. Fits to data [8] with ZFITTER.

	all data wo. $m_{\rm t}$	all data wo. $m_{\rm W}$	all data w.o. $m_{\rm W}$ & $m_{\rm t}$	all data
χ^2 / DoF (prob.)	22.7/14 (6.5 %)	21.5/13 (6.4 %)	21.1/12 (4.9 %)	22.9/15 (8.6 %)
$m_{\rm t}$ [GeV]	$169.7^{+9.8}_{-7.0}$	172.9 ± 4.7	$167.3^{+10.5}_{-8.3}$	173.2 ± 4.5
$m_{\rm H}$ [GeV]	57^{+93}_{-30}	81^{+77}_{-42}	55^{+84}_{-27}	77^{+69}_{-39}
α_s	0.1183 ± 0.0026	0.1185 ± 0.0026	0.1183 ± 0.00026	0.1184 ± 0.0026
$\sin^2\theta_W^{eff}$	0.23148 ± 0.00016	0.23152 ± 0.00017	0.23151 ± 0.00017	0.23150 ± 0.00016
$m_{\rm W}$	80.378 ± 0.027	80.381 ± 0.026	80.366 ± 0.035	80.385 ± 0.022

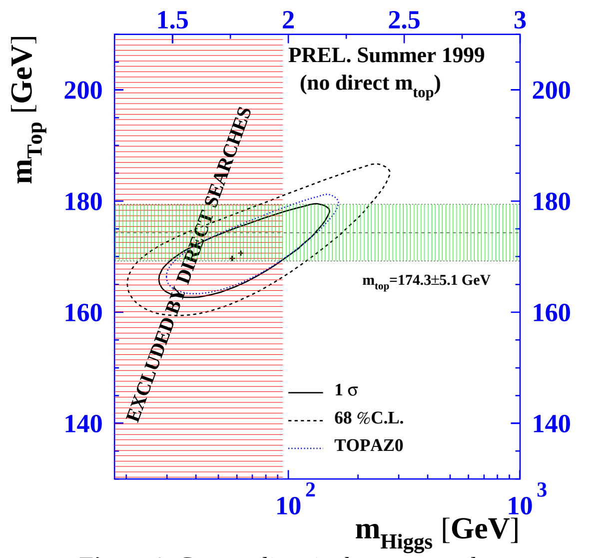

Figure 1. Contour lines in the $m_{\rm H}$–$m_{\rm t}$ plane.

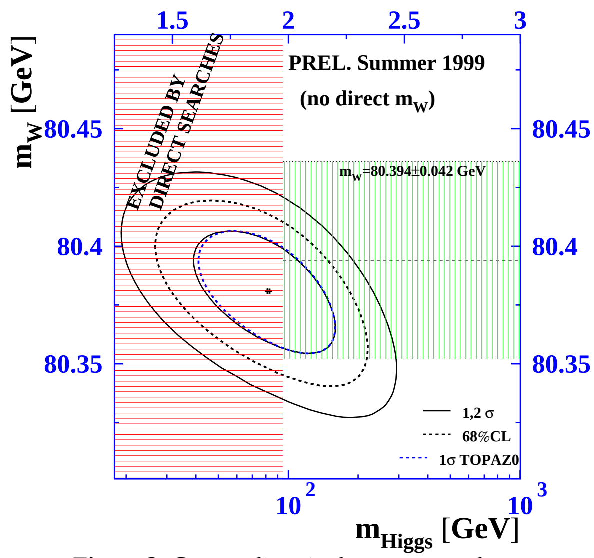

Figure 2. Contour lines in the $m_{\rm H}$–$m_{\rm W}$ plane.

the Higgs boson mass of 95 GeV [9], but well consistent within the error. Taking a purely probabilistic viewpoint and neglecting the direct lower limit leads to an upper limit of $m_{\rm H} < 215$ GeV @95 % CL.

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References

[1] Talks by A. Barczyk, J. Ellison, B. Carithers, L. Mir, R. Chierici, K. McFarland, S. Fahey and J. Brau at this conference.

[2] D. Bardin *et al.*, hep-ph/9902452 .

[3] D. Bardin, talk at this conference.

[4] Fortran program ZFITTER vers. 6.21, D. Bardin *et al.*, Z. Phys. **C44** (1989) 493; Comp. Phys. Comm. **59** (1990) 303; Nucl. Phys. **B351**(1991) 1; Phys. Lett. **B255** (1991) 290; CERN-TH 6443/92 (May 1992) and hep-ph/9412201; *ZFITTER v.6.10 A Semi-Analytical Program for Fermion Pair Production in* e^+e^- *Annihilation*, DESY 99-070, June 1999.

[5] Fortran Program TOPAZ0 vers. 4.4, G. Montagna *et al.*, Nucl. Phys. **B401** (1993) 3; Comp. Phys. Comm. **76** (1993) 328; **93** (1996) 120; G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini, hep-ph/9804211.

[6] L. Garrido *et al.*, Z. Phys. **C49** (1991) 645; S. Jadach, B. Pietrzyk and M. Skrzypek, Phys. Lett. **B456** (1999) 77.

[7] J. Brau, talk at this conference.

[8] The LEP electroweak working group, accessible via the internet, *http://www.cern.ch/LEPEWWG/*.

[9] Z. Szillasi, talk at this conference.