QCD DESCRIPTION OF ANGULAR CORRELATIONS

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We review the predictions on angular correlations and their recent experimental tests at LEP and HERA. Power behaviour of correlation functions appear for large angles and reflect the underlying fractal structure in jet evolution. Asymptotic scaling laws work rather well for the low momentum particles. For angular correlations the predictions work at least qualitatively, sometimes quantitatively, at LEP and the higher HERA energies. Some limitations of the approach and possible improvements are discussed.

1 Introduction

There has been much interest in the last decade in the study of correlation functions and their possible power dependence on resolution scale. Such behaviour is expected, in particular, from the fractal structure of the selfsimilar parton cascade.[1] Specific predictions, mainly within the Double Logarithmic approximation (DLA) of pQCD, have been derived for angular correlations and multiplicity moments.[2,3,4] In recent years these predictions have been tested in various experimental works by the L3[5], DELPHI[6] and ZEUS[7] Collaborations, see also talk at this conference[8], so there is now a good time for a critical assessment on what has been learned from these studies.

A major aim is the test of "Local Parton Hadron Duality" (LPHD [9]) which suggests comparing directly the multi-hadron with the multi-parton final state without the interface of a "hadronization model." This approach has been proposed originally for inclusive spectra but has been applied subsequently to many other problems (recent reviews[10]). Here we adress its application to multi-particle correlations beyond single inclusive phenomena.

2 Correlations in Full and Restricted Angular Range

Particle multiplicity distributions can be characterized by their factorial moments $F_q = \langle n(n-1)\ldots(n-q+1)\rangle/\langle n\rangle^q$ which relate to integrals of the densities $(dn/dp_1 \ldots dp_q)/\langle n\rangle^q$. Perturbative QCD predictions for the moments in full phase space (event or jet) improve strongly with increasing accuracy of the logarithmic approximations (see review[11]) and are entirely satisfactory for the exact numerical solution of the pertaining evolution equations.[12] The

predictions depend on the QCD scale Λ and one non-perturbative parameter ($k_\perp$ cut-off Q_0); they can be determined from a fit to the global event multiplicities.

For limited phase space different observables have been considered:

Distributions in relative angle ϑ_{12} between two particles, both inside the forward cone of a jet with half opening angle Θ and momentum P. The distribution is normalized either by the full multiplicity inside this cone $\hat{r}(\vartheta_{12}) = (dn/\vartheta_{12})/\overline{n}(\Theta)$ or by the corresponding distribution of uncorrelated particles $r(\vartheta_{12}) = (dn/\vartheta_{12})/(dn/\vartheta_{12})_{uncorr}$.

Multiplicity moments F_q, from particles inside an angular ring of size 2δ at polar angle Θ to the jet axis or from inside a cone of half angle δ again at polar angle Θ; they correspond to dimensions $D = 1$ and $D = 2$ respectively.

In the theory with fixed coupling α_s all these observables (generically denoted by h_q) show a universal power behaviour ($\vartheta_{12} \to \delta$)

$$h_q(\delta, \Theta, P) \sim (\Theta/\delta)^{\varphi_q} \tag{1}$$

$$\text{for } F_q : \quad \varphi_q = (q-1)D - (q - q^{-1})\gamma_0; \qquad \text{for } \hat{r} : \quad \varphi_2 = -3\gamma_0/2 \tag{2}$$

corresponding to the selfsimilar structure of the parton cascade. The power is given in terms of the "QCD anomalous dimension" $\gamma_0 = \sqrt{6\alpha_s/\pi}$.

For running coupling this result is retained only at the large angles $\delta \sim \mathcal{O}(\Theta)$ where the coupling varies as $\alpha_s(P\Theta/\Lambda)$. However, with decreasing angles δ there is an increasing deviation from the power law because of the logarithmic growth of the coupling for small angular scales $k_\perp \sim P\delta$. The authors[2,3,4] obtain slightly different approximations, for example,[2]

$$h_q(\delta, \Theta, P) \sim \exp(2q\gamma_0(P\Theta/\Lambda)\omega(\epsilon, q)), \qquad \varepsilon = \frac{\ln(\Theta/\delta)}{\ln(P\Theta/\Lambda)} \tag{3}$$

where $\omega(\epsilon, q)$ is the solution of an algebraic equation and reproduces the limit (1) for large angles.

3 Discussion of Experimental Results

Angular dependence (on δ, ϑ_{12}). In all measurements at LEP and HERA the data behave qualitatively as predicted: for large angles (small ε) there is an approximate power behaviour and a deviation at small angles according to the prediction. The deviation between predictions and experiment varies between 10-20% for $\ln F_q$, $q \geq 3$ and a bit more for F_2, the differential correlations r, $\hat{r}$ at the higher HERA energies ($Q^2 > 2000$ GeV^2) agree rather well with the prediction but deviate considerably at low Q^2, according to what is expected for an asymptotic prediction.
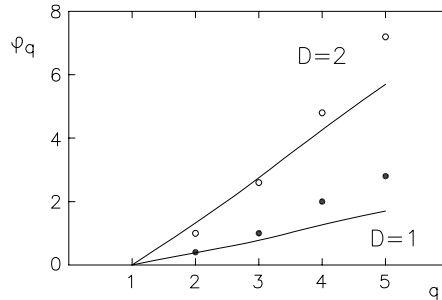
Figure 1. Initial slope φ_q of factorial moments from DELPHI[6] in comparison with asymptotic predictions (1,2) for angular ring ($D = 1$) and cone ($D = 2$) at order q, curves connect results at integer q (parameters $n_f = 3$, $\Lambda = 0.04$ GeV).

Initial slope φ_q. The powers φ_q in (1) have been extracted by DELPHI[6] from the F_q moments at small ε for $q \leq 5$ and $D = 1, 2$, see Fig. 1. The overall variation of the slopes by a factor ~ 20 is roughly met by the prediction (2) and the deviations grow up to $\sim 30\%$ at high q. Furthermore it is found,[6] that the initial slope decreases with increasing Θ as expected from the running coupling $\alpha_s(P\Theta/\Lambda)$, best results are obtained for $\Lambda = 0.04$ GeV which is a bit small but still acceptable for a DLA calculation, also $n_f = 3$ is taken.

Dependence on approximation scheme. The approximations (DLA) are justified only at asymptotic energies and could be responsible for the partial disagreement with experiment. The ZEUS collaboration also compared their results on F_q with the predictions of a parton level MC (ARIADNE[13]) using a small cut-off parameter Q_0 as in the analytic calculations. Then agreement with data improves considerably, with results comparable, for example, to the LEPTO MC with full hadronization.

4 Asymptotic scaling laws

The DLA results correspond to the high energy asymptotics. These results can often be written in scaling form. A well known example is the KNO scaling, which says, that the rescaled probability $\langle n \rangle P_n$ approaches an energy independent (scaling) limit as function of the rescaled multiplicity $n/\langle n \rangle$. In a similar way one finds an asymptotic limit for the energy spectra in appropriate rescaled variables ("ζ-scaling"[14]). The data follow this scaling prediction only for the low momentum particles, a phenomenon understood as a consequence of soft gluon coherence.[10] Typically, at all available primary energies

the deviations from the scaling limit increase with momentum (they can be taken into account by the higher order (MLLA) corrections).[15]

For angular correlations Eq. (3) can be rewritten in scaling form

$$\ln h_q(\delta, \Theta, P)/\gamma_0(\Theta, P) = f(\epsilon, q) \qquad (4)$$

with known f, so the rescaled correlations in the variable ε become independent of both the jet energy P and the large angle Θ ("ε-scaling"[2]).

Remarkably, the correlation functions r, $\hat{r}$ approach the predicted scaling limit within the HERA Q^2 range, furthermore, they show the independence of angle Θ in the range $\Theta = 30° \ldots 90°$, also at LEP. We relate this good agreement already at present energies to the good scaling properties of low momentum particles which dominate the correlation measurements.

5   Limitations and Improvements

A problem has recently been encountered with moments for particles in the cylinder $p_\perp < p_\perp^{cut}$. A recent measurement by ZEUS[7,8] has revealed a rise of these moments for small cut-off $p_\perp^{cut}$ contrary to the prediction[16] of a decrease $F_q \to 1$ for $p_\perp^{cut} \to 0$ (or $p_\perp^{cut} \to Q_0$ in the calculation) corresponding to a Poissonian multiplicity distribution; this prediction was also confirmed by a parton MC calculation.[16]

We take this observation as showing a limitation of the duality approach. For sure, there cannot be a perfect matching between parton and hadron final states, as already obvious from the existence of hadronic resonances. The correlations of particles at low $p_\perp$ seem to involve additional non-perturbative effects. Perhaps one can find a better treatment of particles near the kinematic cut-off $p_\perp \sim Q_0$ as it was possible for single inclusive spectra in this limit where similar problems occur. Beyond the original discussion,[16] we would expect the perturbative prediction of the Poissonian limit to apply for jets of not too low virtuality $y_{cut} \gg (Q_0/Q)^2$ where the dependence on Q_0 disappears.

On the other hand, the perturbative DLA predictions for correlations in angle are in overall agreement with measurements. A considerable improvement of the calculations is not obtained by including MLLA corrections.[3] We would expect a major improvement in accuracy, especially for the large angle behaviour, if full matrix element calculations were carried out as it has been done for azimuthal angle correlations with good success.[17] An easier path towards higher accuracy is available by running a parton MC (ARIADNE) with non-standard parameters $Q_0 \sim \Lambda$ as discussed previously.[16]

6 Summary

The dual connection between parton and hadron final states is not restricted to single particle distributions but extends to multi-particle correlations; care has to be taken with particles near the kinematic border at $p_\perp \sim Q_0$. The simple asymptotic formulae from DLA for angular correlations describe all data at least at the qualitative level. Near quantitative results follow for global observables or from MC calculations. The fixed coupling results correspond to a selfsimilar cascade and fractal structure. The realistic QCD cascade with running coupling corresponds to this scaling picture approximately but shows characteristic deviations in angular distributions and with primary energy.

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