

Cosmic Rays as Probes of Large Extra Dimensions and TeV Gravity

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Abstract

If there are large extra dimensions and the fundamental Planck scale is at the TeV scale, then the question arises of whether ultra-high energy cosmic rays might probe them. We study the neutrino-nucleon cross section in these models. The elastic forward scattering is analyzed in some detail, hoping to clarify earlier discussions. We also estimate the black hole production rate. We study energy loss from graviton mediated interactions and conclude that they can *not* explain the cosmic ray events above the GZK energy limit. However, these interactions could start horizontal air showers with characteristic profile and at a rate higher than in the Standard Model.

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1 Introduction

In this paper we explore the possibility that the primary particles for ultrahigh-energy cosmic rays are neutrini interacting gravitationally with atmospheric nucleons. An obvious objection to this idea is that the gravitational interaction is too weak to produce any sizable cross section for this process. However, this point needs to be fully reconsidered in theories where the fundamental scale is around the TeV, as postulated in models involving large extra dimensions and a four-dimensional brane-world [1, 2]. In these scenarios not only does the cross section for elastic gravitational scattering increase at cosmic ray energies, but there is also the possibility that the collision results into the formation of microscopic black holes. Both effects can dramatically increase the cross section for scattering between neutrini and atmospheric nucleons, and hence they may play a role in explaining the most energetic cosmic ray events.

In the recent past, this possibility has been entertained in a number of papers, with differing conclusions [3, 4, 5]. There has been a controversy as to the right way to perform the calculations, and how to implement unitarity at high energies. We hope to shed some light on these issues, and show that, actually, the situation is rather simple once the appropriate point of view is taken [6]. Besides, the possibility of producing black holes in cosmic ray collisions needs to be addressed in detail. Once we have, hopefully, settled the terms for the analysis, we will turn to the actual discussion of whether the first signatures from low-scale unification and large extra dimensions might come from the study of ultrahigh-energy cosmic rays (UHECR).

While this work was in progress, detailed analyses of the possibility of black holes forming at LHC have appeared [7, 8, 9]. During the final stage of this work, a paper [10] has appeared which studies the production of black holes in cosmic rays, and also investigates their detection in horizontal air showers. Our work complements that of ref. [10]: the latter focuses on the phenomenology of the detection of black holes, whereas we address in more detail the theoretical aspects of neutrino-nucleon scattering in TeV-gravity theories.

2 Ultra-high energy scattering on the brane

An essential feature of the gravitational interaction is that at center-of-mass (c.o.m.) energies \sqrt{s} well above the fundamental scale, the coupling to gravitational coupling grows so large that graviton exchange dominates over all other interactions. This is actually the case for atmospheric nucleons being hit by neutrini of energy $E_\nu \sim 10^{11}$ GeV ($\sqrt{s} \sim 10^6$ GeV) if the fundamental scale is around 1 TeV. In particular, if the impact parameter b is sufficiently smaller than the radius of compactification, the extra-dimensions can be treated as non-

compact. In this regime one would be probing the extra dimensions purely by means of the gravitational interactions.

Another consequence of ultra-high energies in gravitational scattering is that, to leading order, its description involves only classical gravitational dynamics. In particular, this means that we do not need any detailed knowledge of quantum gravity to perform the calculations: any theory that has General Relativity as its classical limit should yield the same results¹. One can distinguish different regimes in the scattering (and we will do so below), but perhaps the most spectacular effect at such energies is the expected formation of black holes, via classical collapse, when the impact parameter is of the order of the horizon radius of the (higher dimensional) black hole[14]²,

$$R_S = \left(\frac{2^n \pi^{\frac{n-3}{2}} \Gamma\left(\frac{n+3}{2}\right)}{n+2} \right)^{\frac{1}{n+1}} \left(\frac{s}{M_D^{2n+4}} \right)^{\frac{1}{2(n+1)}}. \quad (1)$$

This implies that any dynamics at $b < R_S$ is completely shrouded by the appearance of trapped surfaces: Ultrashort distances are directly probed only for energies around the fundamental energy scale.

In the following we will assume for simplicity that no scale for new physics arises before reaching the scale for the fundamental energy M_D . In particular, we assume that scales such as the string tension, or the tension and thickness of the brane, do not appear before that scale. This prevents the possibility of additional effects arising at impact parameters larger than the ones that give rise to black hole formation. If this is the case, then the picture for ultrahigh-energy scattering that we describe here should be largely universal. Nevertheless, stringy effects below the regime where General Relativity can be trusted may be readily accommodated [9] and should not introduce large changes in our results.

Ultra-high energy scattering in the Randall-Sundrum model has been addressed in [6], and the different regimes for the scattering in the present case are qualitatively the same as described there. At large impact parameters one does not expect formation of black holes, but in this case, the leading contribution to the scattering amplitude is exactly (non-perturbatively) calculable within an eikonal approach [11, 12, 16]. This is known to work particularly well for high energy gravitational scattering at large impact parameter [12, 17]³.

The eikonal resummation of ladder and crossed-ladder diagrams is achieved by computing the scattering amplitude as

$$\mathcal{M}(s, t) = \frac{2s}{i} \int d^2b e^{i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(s,b)} - 1 \right)$$

¹This has been noted often earlier, e.g., in [11, 12, 13, 6, 7, 8].

²We define M_D as in [15].

³Loops involving only momenta of internal gravitons are suppressed by factors of $1/(M_D b)^2$.

$$= \frac{4\pi s}{i} \int db b J_0(bq) \left(e^{i\chi(s,b)} - 1 \right). \quad (2)$$

This amplitude is well defined for any values of the exchanged momentum $q = \sqrt{-t}$ ($t < 0$ since the scattering is elastic). The eikonal phase $\chi(s, b)$ is obtained from the Fourier-transform to impact parameter space of the Born amplitude. Alternatively, it can be obtained from the deflection of a particle at rest when crossing the gravitational shockwave created by a second particle [11].

Note that the transforms in impact parameter space are two-dimensional, since the particles scatter in three spatial dimensions. Nevertheless, the exchanged gravitons propagate in the $4 + n$ dimensional space. Moreover, we are working at a scale where the spectrum of Kaluza-Klein modes is essentially continuous. In this case the Born amplitude comes out easily as [15]

$$i\mathcal{M}_{Born} = i\pi^{n/2} \Gamma\left(1 - \frac{n}{2}\right) \frac{s^2}{M_D^{2+n}} (-t - i\epsilon)^{\frac{n}{2}-1}. \quad (3)$$

Hence the eikonal phase,

$$\chi(s, b) = \frac{i}{2s} \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} i\mathcal{M}_{Born}, \quad (4)$$

which is finite for $b \neq 0$, is $\chi(s, b) = (b_c/b)^n$, where we have defined

$$b_c^n = \frac{(4\pi)^{\frac{n}{2}-1}}{2} \Gamma\left(\frac{n}{2}\right) \frac{s}{M_D^{2+n}}. \quad (5)$$

Having the phase $\chi(s, b)$ is sufficient for numerical evaluation of the eikonalized amplitude (2). The result in eq. 2 can be written in terms of Meijer functions. However, it is easy to get simple analytical expressions for the amplitude in both regimes of $qb_c \gg 1$ and $qb_c \ll 1$. When $q \gg b_c^{-1}$ the phase $\chi(s, b)$ yields a sharp peak for the eikonal amplitude in (2), which allows for an evaluation near the saddle point $b_s = b_c(qb_c/n)^{-1/(n+1)} \ll b_c$:

$$\begin{aligned} \mathcal{M}_{saddle} &= \frac{4\pi i e^{i\phi}}{\sqrt{n+1}} \left[(4\pi)^{\frac{n}{2}-1} \Gamma\left(\frac{n}{2} + 1\right) \left(\frac{s}{qM_D}\right)^{n+2} \right]^{\frac{1}{n+1}} \\ &\equiv Z_n \left(\frac{s}{qM_D}\right)^{\frac{n+2}{n+1}}. \end{aligned} \quad (6)$$

The phase $\phi = (n+1)(b_c/b_s)^n$ is real for $t < 0$. Observe that the amplitude is non-perturbative in the gravitational coupling $1/M_D^{2+n}$. In the limit $q \rightarrow 0$ one gets instead

$$\mathcal{M}(q=0) = 2\pi i s b_c^2 \Gamma\left(1 - \frac{2}{n}\right) e^{\frac{-i\pi}{n}}, \quad (7)$$

which is finite for $n > 2$. For $n = 2$ the real part of \mathcal{M} has a logarithmic singularity

$$\mathcal{M} \stackrel{q \rightarrow 0}{\equiv} -4\pi s b_c^2 \ln(q b_c). \quad (8)$$

Notice that also at small q the amplitude is non-analytic in the gravitational coupling. Indeed the amplitude at $q \rightarrow 0$ is effectively described by the (Born) operator \mathcal{T} defined in ref. [15] but with an effective UV cut-off $\sim b_c^{-1}$ on the mass of the exchanged KK modes. This cut-off originates from the interference with the multigraviton exchange diagrams in the eikonal series.

The eikonal amplitude will be used in the next section to compute the differential cross section for neutrino-nucleon scattering. At the partonic level we have

$$\frac{d\sigma}{dq^2} = \frac{1}{16\pi s^2} |\mathcal{M}|^2. \quad (9)$$

We can also derive the total elastic cross section from the optical theorem:

$$\sigma_{el} = \frac{\text{Im}\mathcal{M}(q=0)}{s} = 2\pi b_c^2 \Gamma\left(1 - \frac{2}{n}\right) \cos\frac{\pi}{n}, \quad (10)$$

i.e., it is essentially given by the area of a disk of radius $\sim b_c$.

Observe that

$$\sigma_{el} \sim s^{2/n}. \quad (11)$$

This growth of the cross section at high energy is slower than the perturbative result $\sigma \sim s^2$, and also slower (for $n > 2$) than the linear dependence $\sigma \sim s$ postulated (apparently for all n) in [4]. Unitarity in impact parameter space is manifest in the eikonal amplitude (2)⁴. For large impact parameter this implies as well unitarity for high partial waves. Partial wave unitarity at shorter impact parameter is a harder problem, and indeed, corrections to the eikonal amplitude are expected to become crucial. As t grows, graviton self-interactions, which carry factors of t associated to the vertices, increase the attraction among the scattered particles, and it is expected that, eventually, gravitational collapse to a black hole will take place. Hence the initial state is expected to be completely absorbed, but in such a way that any short distance effects will be screened by the appearance of a horizon. Indeed as shown in ref.[12] the effects of the non-linearity of gravity are suppressed by a power of R_S/b , so our eikonal approximation should be valid for $b \gg R_S$ and its breakdown be associated to the formation of black-holes. This relation between eikonal breakdown and black-hole formation can also be established as follows. In the region $b \ll b_c$, there is a one to one correspondence between the transferred momentum q and the saddle point impact parameter b_s . The case

⁴The Froissart bound [18], generalized to higher dimensions in [19], does not apply since the exchanged particle is massless.

$q \sim \sqrt{s}$, where the (small angle) eikonal approximation breaks down, corresponds precisely to $b_s \sim R_S$. Notice in passing that we can also write eq. (9) as $d\sigma = 2\pi b_s db_s$, as expected for a classical trajectory with impact parameter b_s .

At present, the cross section for black hole production can only be estimated as the geometric cross section,

$$\sigma_{bh} \sim \pi R_S^2. \quad (12)$$

with R_S as in (1). In this case $\sigma_{bh} \sim s^{1/(n+1)}$, again slower than linear.

Clearly this result cannot be very accurate. Radiation is expected to be emitted during the collapse, and the amount of energy that is expected to be radiated in the process can be a sizable fraction of the total energy (perhaps around 15 – 30%, from four-dimensional estimates [20]), but at large enough energies it will not be able to prevent the collapse. This effect will tend to reduce the above value for the cross section. However, there are also factors which increase it, such as the fact that a black hole acts as a somewhat larger scatterer (40 – 75% larger radius [21]). It seems reasonable to expect that the above expression is not off by any large factors.

3 Neutrino-nucleon scattering and black hole production

These results can now be readily applied to neutrino-nucleon scattering at ultra-high energies. At impact parameters $b < 1 \text{ GeV}^{-1}$ the neutrino interacts essentially with the partons, and if $b > R_S$ the eikonal approximation gives a good description of the scattering. At smaller distances, trapped surfaces are expected to form and the neutrino and the parton will collapse to form a black hole.

In order to numerically evaluate the amplitude (2), we proceed as follows. First, we write it as

$$i\mathcal{M} = 4\pi s b_c^2 \int x dx J(x q b_c) (e^{i/x^n} - 1) = 4\pi s b_c^2 \hat{\mathcal{M}}(q b_c). \quad (13)$$

At large values of $q b_c$ we know this is well described by the simple result (6). It is convenient to extract this behavior, and write the squared amplitude $|\hat{\mathcal{M}}|^2$ as

$$|\hat{\mathcal{M}}|^2 = \left(1 + (q b_c)^2\right)^{-\frac{n+2}{n+1}} \frac{n^{\frac{2}{n+1}}}{n+1} F(q b_c). \quad (14)$$

The prefactors have been chosen in such a way that for $q b_c \rightarrow \infty$ the function F goes to 1. Apart for the case $n = 2$ where it has a mild logarithmic singularity at $q b_c \rightarrow 0$ (see eq. (8)), F is $O(1)$ over the full range of $q b_c$.

For our applications it is useful to study the cross section as a function of the fraction y of energy transferred to the nucleon:

$$y = \frac{E_\nu - E'_\nu}{E_\nu} = \frac{q^2}{xs}. \quad (15)$$

where x is the fraction of proton momentum carried by the parton. Summing over partons we have

$$\frac{d\sigma}{dy} = \int_0^1 dx \frac{1}{16\pi xs} \left(\sum_i f_i(x, \mu) \right) |\mathcal{M}(x, y, \sqrt{s}/M_D)|^2. \quad (16)$$

Here $f_i(x, \mu)$ are the parton distribution functions (PDFs) (we use the CTEQ5 set extended to $x < 10^{-5}$ with the methods in [22]). Notice that quarks and gluons interact in the same way. The scale μ should be chosen in order to minimize the higher order QCD corrections to our process. A simple, but naive, choice would be $\mu = q$. However $1/q$ does not really represent the typical time or length scale of the interaction. As we have seen, in the stationary phase regime, the neutrino is truly probing a distance $b_s \gg 1/q$ from the parton. Heuristically: the total exchanged momentum can be large, but through the exchange of many soft gravitons. So we believe that a better normalization is to take $\mu = b_s^{-1}$ when $q > b_c^{-1}$ and $\mu = q$ if $q < b_c^{-1}$. The latter choice is effectively equivalent to choosing $\mu = b_c^{-1}$ as at small q the eikonal corresponds to a pointlike interaction. Our choice of μ is consistent with the fact that gravity at ultra-Planckian energies is dominated by long distance classical physics. Choosing $\mu = q$ would also make little sense. q can be as big as $\sim \sqrt{s} \gg M_D$, but the evolution of the PDF's at $Q^2 > M_D^2$ cannot be simply performed withing QCD, as truly quantum gravitational effects (string theory) would come into play. Instead as \sqrt{s} grows above M_D , and t/s is kept fixed but small, the impact parameter b_s grows and we are less sensitive to short distance physics. As a matter of fact, for large enough s the total $\sigma_{\nu N}$ will be bigger than the proton area $\sim (\text{GeV})^2$: at higher energies the parton picture breaks down, the proton interacts gravitationally as a pointlike particle, and the neutrino scatters elastically on it.

A useful quantity to study is the cross section integrated for $y > y_0$. In Fig. 1 and Fig. 2 we plot this quantity for $M_D = 1$ TeV and $M_D = 5$ TeV, respectively. We include the cases with $n = (2, 3, 6)$ and $E_\nu = (10^{10}, 10^{12}, 10^{14})$ GeV.

Finally, to estimate the total cross section to produce a black hole in a neutrino-nucleon scattering we compute

$$\sigma = \int_{M^2/s}^1 dx \left(\sum_i f_i(x, \mu) \right) \pi R_S^2, \quad (17)$$

where R_S is given in Eq. (1) and $\mu = R_S^{-1}$. Again for the choice of scale in the PDF's the previous discussion applies: the Schwarzschild radius rather than the black-hole mass sets

the time scale of gravitational collapse. Notice that in a more standard case of, say, neutrino-quark fusion into an elementary lepto-quark the right choice would be μ of the order of the lepto-quark mass. The crucial difference is that the black-hole is not an elementary object: its physical size is much bigger than its Compton wavelength.

We plot in Fig. 3 this cross section versus the energy of the incoming neutrino for $n = (2, 3, 6)$ and $M = (1, 5)$ TeV. We include plots with $xs > M_D^2$ (solid) and $xs > (10M_D)^2$ (dots). These correspond to the cross sections for producing black holes with a mass larger than M_D or $10M_D$, respectively.

4 Discussion

We are now ready to discuss the implications of our results on the phenomenology of ultra-high energy cosmic rays. The first question is whether neutrino nucleon scattering at super-Planckian energies can explain the observed cosmic ray events with energy $E > E_{GZK} = 5 \times 10^{10}$ GeV. It is known since long ago that cosmic protons with energy above E_{GZK} are damped by inelastic scattering with the microwave background photons. The relevant reaction is $p + \gamma \rightarrow p + \pi$, and E_{GZK} is the threshold proton energy given the photon temperature. Because of this reaction, ultra energetic cosmic protons are brought down to $E \simeq E_{GZK}$ within a few Mpc. Since there are good reasons to believe that the cosmic protons have extra-galactic origin, we should observe a sharp drop in the observed event rate at $E > E_{GZK}$. However, various experiments do not observe this drop at all. There have been several suggestions to explain that. One idea is that the primary particles for the UHECR are neutrini [23, 24, 4, 5], as these particles interact negligibly with the microwave background and are essentially undamped. However, any of these suggestions has to face the fact that, within the Standard Model (SM), the neutrini interact too weakly also with the nucleons in the atmosphere. In order to explain the ultra-GZK events by cosmic neutrini one needs new physics enhancing their cross section with nucleons at high energy. In ref. [4] it was suggested that, in models with TeV scale gravity, the eikonalized cross section could be of the right order of magnitude. However ref. [4] did not investigate the rate of energy loss in the eikonalized process, and, in particular, did not pay attention to its “softness”. The production of black holes was also neglected in ref. [4].

As a matter of fact, in order to determine the signal it is important to establish quantitatively which is the process that dominates energy loss – whether elastic gravitational scattering or black hole production. It turns out that energy loss is mostly determined by black hole production and by scattering at $y \sim 1$. (As we already pointed out the gravitational cross section at $y \sim 1$ becomes comparable to σ_{bh} , though its precise value is not

calculable within our linearized gravity approximation.) To see this, consider a neutrino travelling through a medium of density ρ . The mean free path for black hole production, at which all energy is lost to the shower, is $L_{bh} = (\sigma_{bh}\rho)^{-1}$. While travelling through the medium the neutrino also loses energy through the softer, but more frequent, eikonized scattering. After travelling a distance L_{bh} , the energy fraction lost to soft scatterings with $y < y_0$ is controlled by the quantity

$$\eta(y_0) = \int_0^{y_0} y \frac{d\sigma}{dy} \rho L_{bh} = \frac{1}{\sigma_{bh}} \int_0^{y_0} y \frac{d\sigma}{dy}. \quad (18)$$

When η is less than 1 the soft scatterings play a negligible role in the transfer of energy to the atmosphere. In Fig. 4 we plot η for several cases: they all show that black hole formation and scattering at large y dominate energy loss. This is a direct consequence of the scaling with q^2 of the amplitude in eq. (6). This leads roughly to $yd\sigma/dy \sim y^{-1/(n+1)}$, implying that energy loss is dominated by scattering at large y , and by black hole formation.

The observed showers above the GZK cut-off are all consistent with an incoming particle that loses all its energy to the shower already in the high atmosphere. From the above discussion, low scale gravity could explain these events if the mean free path L_{bh} for black hole production were somewhat smaller than the vertical depth of the atmosphere. In standard units, the vertical depth x_v is measured as the number of nucleons per unit area $x_v = 1033 \times N_A/\text{cm}^{-2} = \text{mb}^{-1}$ (where N_A is the Avogadro number), so the requirement is $\sigma_{bh} > x_v^{-1} = \text{mb}$. From Fig. 3 one can see that, at the relevant energies, the black hole cross section, however large, falls short of this requirement. In order to satisfy $\sigma_{bh} > \text{mb}$ the gravity scale M_D should be well below a TeV, which would contradict collider limits.

Hence, we conclude that neutrino-nucleon interactions in TeV-gravity models are *not* sufficient to explain the showers above the GZK limit. Also, at present, cosmic rays do not appear to place any significant bounds on such scenarios.

Nevertheless, neutrino-nucleon cross sections $\sigma_{\nu N}$ in the range 10^{-5} mb to 1 mb, like in our scenario, can still lead to interesting new phenomena in cosmic ray physics, which may be observed in upcoming experiments. Cosmic primaries with cross section below 1 mb can travel deep into the atmosphere before starting a shower. In particular they can cross the atmosphere at a large zenith angle and start characteristic horizontal air showers. The horizontal depth of the atmosphere is x_h is about 36 times the vertical one, so that for $\sigma_{\nu N} \lesssim .1$ mb a neutrino can travel horizontally down to the interaction point. In the Standard Model the charged-current cross section is $\sigma_{\nu N} \sim 10^{-5}(E/10^{10}\text{GeV})^{0.363}$ mb. No horizontal air shower has been detected so far. However, conservative estimates of the flux of ultra energetic cosmic neutrini [24] suggest that the next generation of experiments should be barely sensitive to neutrino cross sections of the order of the SM one. In our scenario $\sigma_{\nu N}$

can be considerably bigger, so there is the interesting possibility that gravitational scattering and black hole production will lead to a sizeable event rate, higher than in the SM.

The shape of the shower is probably one of the better ways to characterize these processes. In the SM charged-current process, a significant fraction of the neutrino energy is released to just one or a few hadrons from the breakdown of the target proton. The shower then builds up from the cascading hadronic interactions of these few hadrons. In the scenarios we are considering, the production of a black hole of mass $M_{bh} \sim \sqrt{s} = \sqrt{2M_\nu m_p}$ is followed by its very quick evaporation by emission on the brane [21] of a number of particles of the order of $M_{bh}/T_{bh} \sim (\sqrt{s}/M_D)^{(n+2)/(n+1)}$. For the energies we are considering this number can be bigger than 100. Then the shower builds more quickly than for SM processes. It is reasonable to expect that the shapes will differ, very much in the way that a shower formed by a primary iron nucleus differs from the shower formed by a primary proton. To investigate the difference in the case at hand requires a more detailed study. Note that the BH cross section plotted in Fig. 3 is inclusive over the mass of the BH. A significant portion of that cross section is due to the production of not so heavy BH's, through scattering with partons with small x . Moreover, as discussed above, the cross section σ_{bh} is of the same order as the elastic gravitational scattering at $y \sim 1$. In the latter processes a significant fraction of the neutrino energy is transferred to a few proton fragments. We then expect the resulting shower to resemble those induced by SM physics. In order to assess how well one can distinguish gravity induced showers from SM showers requires to take into account all these facts. This is an important point: if an excess of horizontal shower is observed, the shower shape information will be crucial to secure that the excess is not due to an underestimate of the (unknown) neutrino flux.

Finally, in order to establish which process (gravitational elastic scattering, or black hole production) dominates the signal, one needs some knowledge about the energy dependence of the incoming neutrino flux. We have already established that BH production dominates energy loss. However, as the eikonized cross section grows with E , if the neutrino flux $J(E)$ decreases with E slowly enough, the number of BH events at energy E may be overshadowed by soft scattering events due to neutrini with energy $\gg E$. The signal is the number $dN(E)$ of showers with energy between E and $E + dE$. In terms of the neutrino flux $J(E)$ and the differential cross section we can write

$$\frac{dN(E)}{dE} \propto \int_E^{E_{max}} \frac{dE'}{E'} J(E') \frac{d\sigma}{dy}(E', y \equiv \frac{E}{E'}). \quad (19)$$

E_{max} represents the energy at which σ_{bh} becomes larger than the inverse horizontal depth x_h^{-1} . Neutrini with $E > E_{max}$ interact right away and cannot generate horizontal showers. We have studied the above integrand by assuming $J(E) \sim E^\alpha$. We found that, in the cases of interest, already for $\alpha < -2$ the signal is dominated by events with large y , and then by

black holes. This condition is satisfied for the cosmogenic neutrino flux in fig. 1 of ref. [24] for which $\alpha \simeq -3$. If the cosmogenic neutrini dominate the flux, then black hole production and gravitational scattering at $y \sim 1$, and not the softer processes, will dominate the signal in horizontal air showers.

To conclude, we hope to have convincingly established that neither higher-dimensional graviton-mediated neutrino-nucleon scattering nor black hole production in TeV-gravity models can explain the observed cosmic ray showers above the GZK limit. Nevertheless, horizontal air showers may probe these scenarios. In this case, black hole production and gravitational deflection by a large angle will be the processes that dominate the signal.

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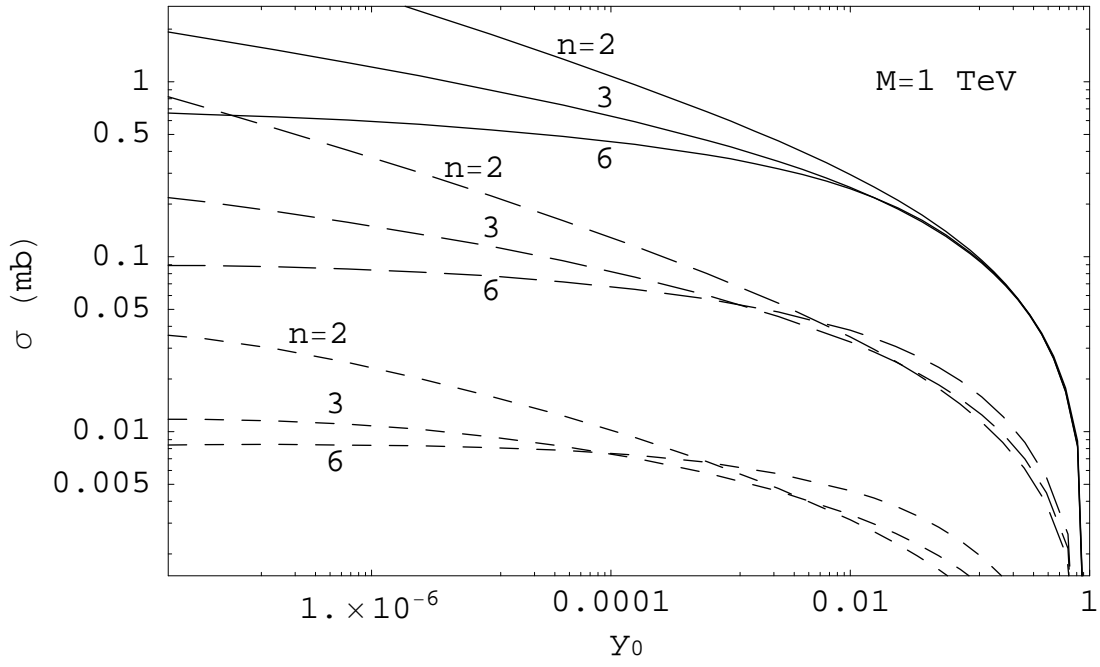


Figure 1: Elastic cross section vs. minimum fraction of energy lost by the neutrino for $M_D = 1$ TeV and $n = 2, 3, 6$ large extra dimensions. Solid, long-dashed and short-dashed lines correspond respectively to $E_\nu = 10^{14}$ GeV, $E_\nu = 10^{12}$ GeV and $E_\nu = 10^{10}$ GeV.

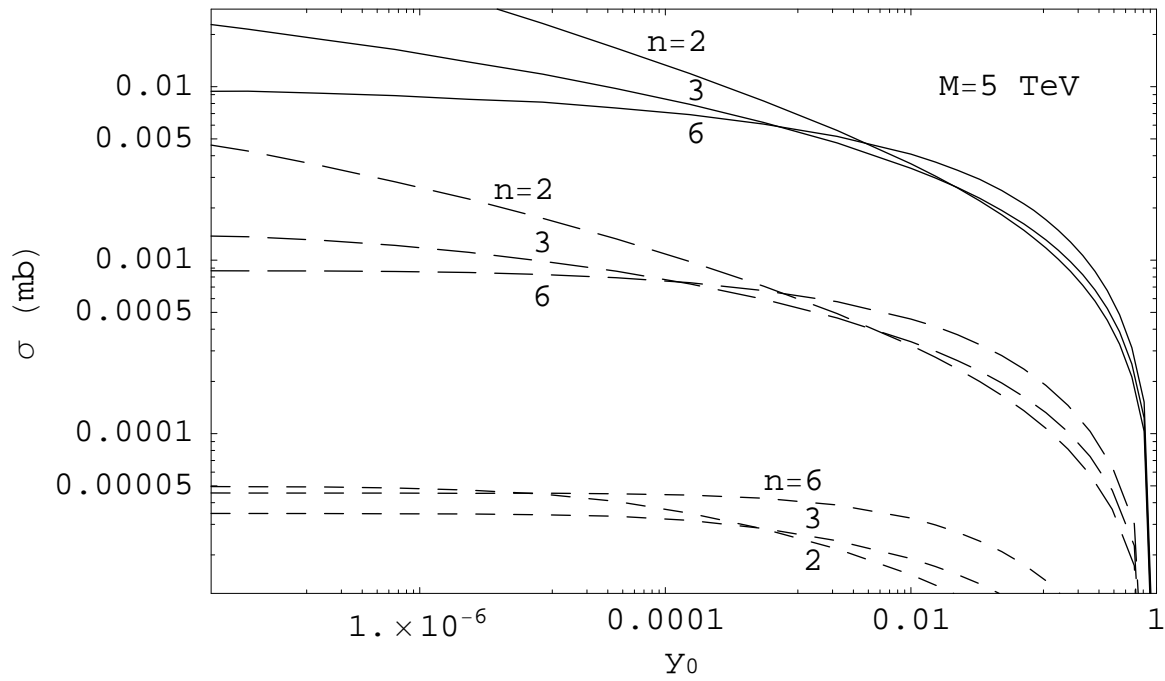


Figure 2: As in Fig. 1 but for $M_D = 5$ TeV.

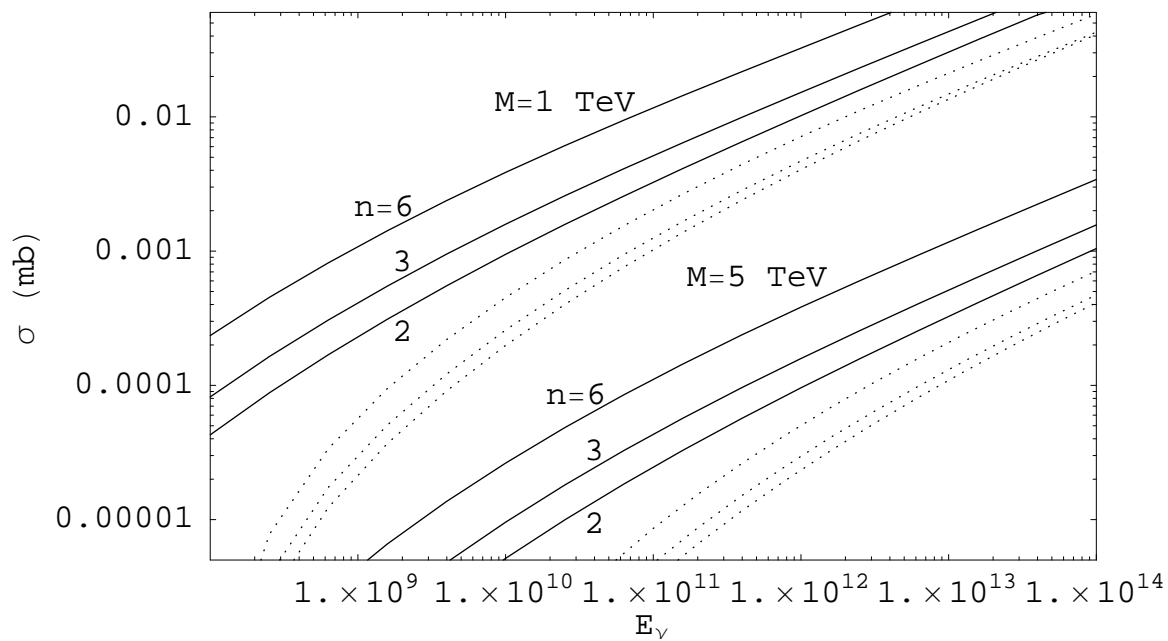


Figure 3: Cross section for black hole production as a function of E_ν , for $M_D = 1, 5$ TeV and $n = 2, 3, 6$. Solid and dotted lines correspond to $xs > M_D^2$ and $xs > (10M_D)^2$ respectively.

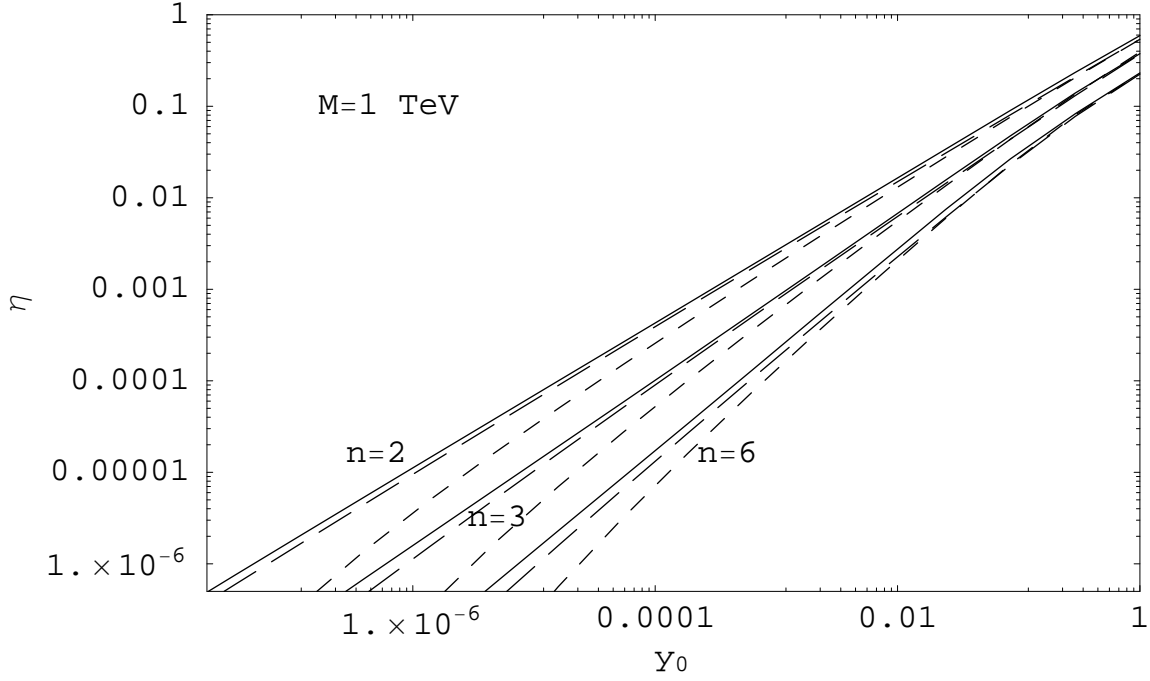


Figure 4: Fraction η of neutrino energy lost to soft scatterings. Solid, long-dashed, and short-dashed lines correspond to $E_\nu = 10^{14}$, 10^{12} and 10^{10} GeV, respectively.

