BOSE-EINSTEIN CORRELATIONS AT THE Z^0 PEAK

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Presented here is an overview of studies of the manifestations of the Bose-Einstein correlations in the hadronic decays of Z^0 boson, performed by the LEP experiments. The scope of the overview is confined to studies of two- and three-particle correlation functions for charged pions, with emphasis on analyses of transverse mass m_T and directional dependence of the two-particle correlation function parameters.

1 Introduction

Studies of Bose-Einstein correlations between particles produced in hadronic decays of a virtual γ^* or Z^0 boson : $e^+e^- \rightarrow \gamma^*, Z^0 \rightarrow hadrons$, have been performed by almost all e^+e^- annihilation experiments [1] at different center-of-mass energies. Hadronic decays of the Z^0 boson are of particular interest because they allow profound studies of the hadronization process and of various related fragmentation models, providing a clean sample of basically uniform events. Data collected at LEP are unique from the point of view of amount of registered and analyzed events. The fact that the measurements were done by four different detectors gives an unmatched possibility to estimate systematic uncertainties.

This review will give an insight into the specifics of the Bose-Einstein correlations (BEC in what follows) studies at the Z^0 peak. The summary of the measurements of the two-particle correlation function in terms of the invariant four-momentum difference will be presented, as well as the latest achievements in the multidimensional analysis of this correlation function. Results of the studies of the genuine three-particle BEC will be overviewed as well.

2 BEC in Z^0 hadronic decays

BEC is a quantum mechanical phenomenon, which reflects the fact that the wave functions of bosons are symmetric. It manifest itself, in particular, as an enhanced probability for identical bosons to be emitted from a common

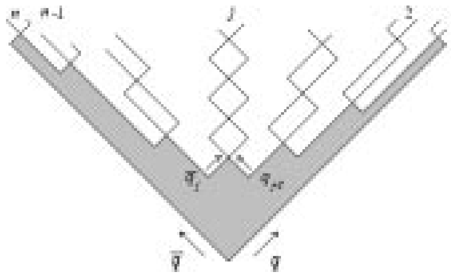

Figure 1: Scheme of a string decay in the Lund Model.

boson source with small relative momenta. The width of this enhancement is directly connected to the size of the source [2]. Therefore, measurements of momenta of the produced in the Z^0 hadronic decays bosons and consecutive analysis of correlations between them is the way of estimating the size of the hadronization region. It is a particularly appealing task, as long as there is no analytical description for the hadronization process, and only semi-classical models are available.

In case of the Z^0 decay, the most appropriate description of the hadron source appears to be the Lund Model [3], incorporated into the JETSET [4] event generator, used by all the LEP experiments. In the framework of this model, bosons are produced in the process of a string fragmentation (see Fig. 1). There might be several strings in a given event of the Z^0 decay, but in the simplest case of the so-called 2-jet events, when Z^0 decays into a quark-antiquark pair, without a hard gluon radiation, one can talk about a single string, which is stretched between quark and antiquark, being a source of the observed bosons.

Due to the kinematics of the process, strings are very much elongated, and can be regarded as a basically one-dimensional objects, with a typical length of about 1 fm. Here comes the most obvious difference between the boson production in case of heavy ion collision and in e^+e^- annihilation : while in the former case one can approximate the boson source with a sphere, in the latter, the longitudinal size of the source has to be bigger than the transverse one [5].

To test these and other model predictions, LEP experiments conducted various studies of the multiparticle correlations. However, due to the specifics of the experimental installations, there are many limitations imposed. One of the most important obstacle to a full-scale analysis is the low efficiency of neutral particles reconstruction in e^+e^- experiments. Therefore, studies of the BEC at LEP are mostly performed for charged particles. Another difficulty is the particle identification : due to its poor efficiency and purity, especially in the low momentum region, all registered particles are usually considered to be pions. It is quite a valid approximation, taking into account that charged pions constitute about 80% of the all measured charged particles at the Z^0 peak.

Studies reviewed in the following sections are all performed using unidentified charged particles, assumed to be pions, unless stated otherwise.

3 Definitions and early analyses

The most straightforward approach to study BEC is to measure a two-particle correlation function. Statistical physics defines this correlation function as

$$C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P_0(p_1, p_2)} , \tag{1}$$

where $P(p_1, p_2)$ is the two-particle probability density, and $P_0(p_1, p_2)$ is the similar probability density in absence of correlations, with p_1 and p_2 denoting four-momenta of particles. Strictly speaking, in the denominator should be the product of single-particle probability densities : $P(p_1)P(p_2)$. However, due to the phase space limitations, e^+e^- annihilation experiments are unable to measure this product, which is thus being replaced by the more convenient observable, P_0.

In terms of the four-momenta difference Q which is defined as

$$Q = \sqrt{(\vec{p}_1 - \vec{p}_2)^2 - (E_1 - E_2)^2} , \tag{2}$$

($\vec{p}_i$ and E_i are momenta and energies of particles), the correlation function $C_2(Q)$ is measured as

$$C_2(Q) = \frac{d\sigma^{\pm\pm}/dQ}{d\sigma^{ref}/dQ} . \tag{3}$$

Here $d\sigma^{\pm\pm}/dQ$ refers to the Q-distribution of like charge bosons, and $d\sigma^{ref}/dQ$ is the reference Q-distribution, i.e., one which is not affected by BEC. There are generally two kinds of the reference samples used by LEP experiments : either one constructed from the unlike charge pions, or the "mixed sample", which consists of pairs of like charge bosons, each being picked up from a different event. Usually, the measured correlation function (3) is being normalized to the analogous quantity, obtained from the Monte Carlo generated events without BEC.

The most commonly used parameterization for $C_2(Q)$ is the Gaussian form, which stems from the model for a source to be a sphere of emitters distributed according to the Gaussian probability density :

$$C_2(Q) = N(1 + \lambda e^{-R^2Q^2}) . \tag{4}$$

Experiment	Unlike ref. λ	Unlike ref. R, fm	Mixed ref. λ	Mixed ref. R, fm
ALEPH	0.48 ± 0.03	0.81 ± 0.04	0.30 ± 0.01	0.51 ± 0.02
DELPHI	0.31 ± 0.02	0.83 ± 0.03	0.24 ± 0.02	0.47 ± 0.03
L3	0.30 ± 0.01	0.94 ± 0.04	0.20 ± 0.02	0.58 ± 0.05
OPAL	0.87 ± 0.03	0.93 ± 0.02		

Table 1: Collated results for $C_2(Q)$ parameters. Results are shown for both unlike charge reference sample and the mixed one. Errors include both statistical and systematic ones.

Parameter R is interpreted as the correlation width and is associated with the size of a boson source. Parameter λ basically describes the strength of the correlation. N is the overall normalization. Sometimes a linear term is being added as another multiplier in (4), to account for a background.

All four LEP experiments measured $C_2(Q)$ using different reference samples in (3). Table 1 shows the summary of their results [6,7,8,9] on R and λ. There is an evident systematic difference between results obtained using different reference samples. An enormous divergence of parameters measured with the unlike charge reference sample suggests that this reference sample is inappropriate for the BEC studies. Usage of the mixed reference sample gives more uniform results, especially for the radius parameter R.

Summarizing results on parameters of $C_2(Q)$ obtained by the LEP experiments at Z^0 peak, one comes to the conclusion that the observed size of the boson source is likely to be about 0.5 fm, which is twice smaller then the supposed string size.

4 Multidimensional analysis

Next step towards more profound studies of BEC is to conduct the correlation function analysis in a multidimensional space. A convenient coordinate system was suggested for this purpose by Csörgő and Pratt [10] : the Longitudinal Centre-of-Mass System, also referred sometimes to as the Longitudinal Co-moving System (LCMS). For each pair of particles it is the system where the sum of the two particle momenta is transverse to the jet direction, which is basically the primary parton direction (see Fig. 2).

In LCMS, Q is resolved into three orthogonal components : $Q_{t,out}$, $Q_{t,side}$ and Q_{long}. The longitudinal component Q_{long} is aligned with the jet direction, hence in the two-jet configuration it is aligned with the color flow in the string. Considering the Lund Model, and also aiming at the comparison with hydrodynamical models used for the heavy ion collisions, another interesting

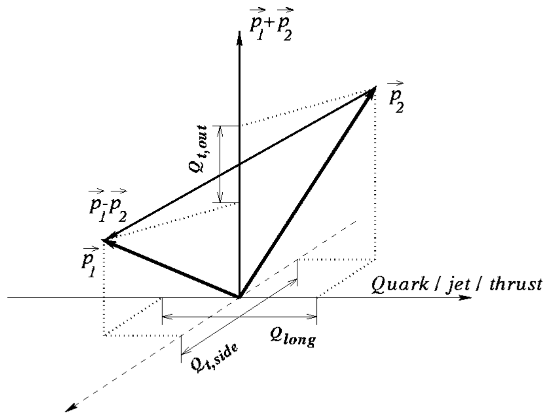

Figure 2: LCMS.

variable is the transverse mass of the pair of particles :

$$m_T = \frac{\sqrt{m_1^2 + p_{t,1}^2} + \sqrt{m_2^2 + p_{t,2}^2}}{2} , \tag{5}$$

where m_1, m_2 are masses of the particles and $p_{t,1}$, $p_{t,2}$ – their transverse momenta with respect to LCMS. Particular interest in studying the dependence of the two-particle correlation function parameters on m_T arises from the results reported by the heavy ion collision experiments, which show the decrease of the correlation radii with increasing m_T. This decrease is well described by hydrodynamical models, which can not be valid in the e^+e^- annihilation case.

The three-dimensional two-particle correlation function (4) will in the assumption of absence of cross-talk between directional components turn into the following form :

$$C_2(Q_{t,out}, Q_{t,side}, Q_{long}) = N(1 + \lambda e^{-Q_{t,out}^2 R_{t,out}^2 - Q_{t,side}^2 R_{t,side}^2 - Q_{long}^2 R_{long}^2}) \tag{6}$$

Even further simplification can be achieved by reducing the tree-dimensional picture to the two-dimensional one, considering the single transverse component $Q_\perp = \sqrt{Q_{t,out}^2 + Q_{t,side}^2}$. Using the notation $Q_\parallel$ for the longitudinal component, Eq.(6) can be rewritten as

$$C_2(Q_\perp, Q_\parallel) = N(1 + \lambda e^{-Q_\perp^2 R_\perp^2 - Q_\parallel^2 R_\parallel^2}) \tag{7}$$

The two-dimensional analysis is more convenient because it gives smaller statistical errors as compared to the three-dimensional one, and also because

λ	0.249 ± 0.004
$R_{t,out}$, fm	0.550 ± 0.010
$R_{t,side}$, fm	0.420 ± 0.010
R_{long}, fm	0.850 ± 0.020

Table 2: Three-dimensional two-particle correlation function parameters as measured by the DELPHI experiment. Only statistical errors are shown.

the theoretical predictions [5] were done in this particular system. However, the three-dimensional analysis allows to separate the boosted component $R_{t,out}$, which is also the only one coupled to the temporal difference of the two bosons emission.

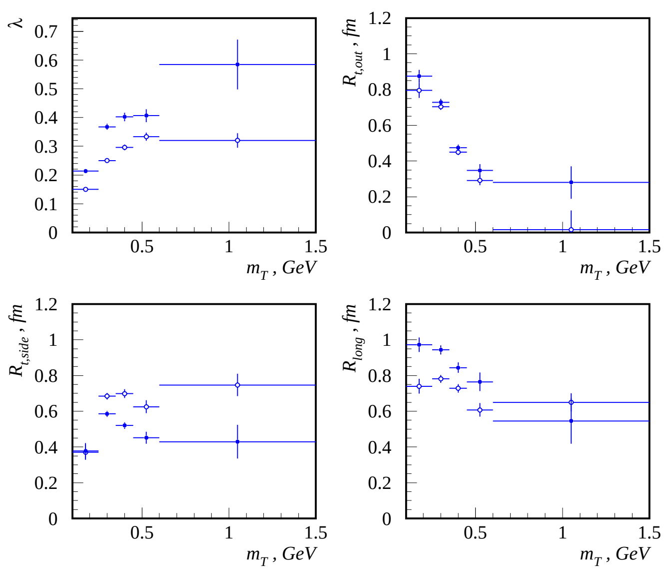

Figure 3: Transverse mass dependence of the correlation function parameters. Closed points: DELPHI data. Open points : JETSET with BEC simulation and with the detector simulation applied. Errors are only statistical.

The three-dimensional BEC analysis in hadronic decays of Z^0 is being conducted within the DELPHI collaboration [11]. The preliminary results are shown in Table 2 and Fig. 3. No acceptance correction were made, since they rely totally on the JETSET simulation of the Z^0 hadronic decays, which includes only a very limited model for BEC [12]. The fit was performed in the region of $|Q_{t,out}|$, $|Q_{t,side}|$, $|Q_{long}| < 0.5\,GeV$.

These results confirm the theoretical predictions that the transverse size of the boson source ought to be almost twice smaller than the longitudinal one. They also show existence of the transverse mass dependence of the correlation function parameters. Increase of λ with increasing m_T corresponds to the relative increase of the promptly produced pions at high m_T. Fast decrease of $R_{t,out}$ comes in a big part from the boost to the LCMS. The clear decrease of $R_{t,side}$ and R_{long} with growing m_T, however, can not be explained so far. Surprising is also the similar behaviour of R_{long} manifested in the JETSET generated events.

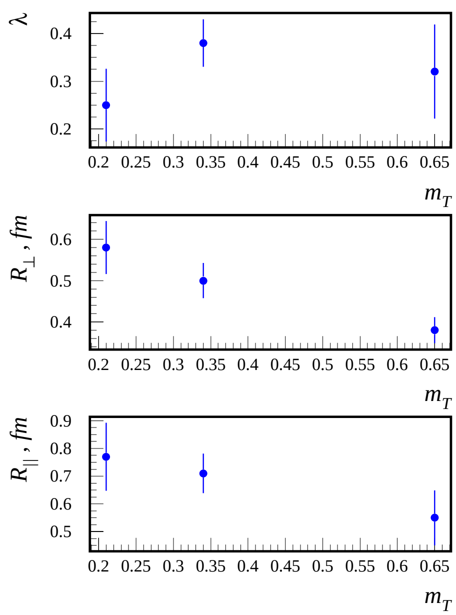

Figure 4: Transverse mass dependence of the correlation function parameters, L3 data.

The complementary two-dimensional analysis of the two-particle correlation function is being done by L3 [13] and DELPHI groups. Fig. 4 shows the transverse mass dependence of the two-dimensional correlation function parameters as measured by L3. Table 3 compares preliminary results obtained by two experiments. DELPHI results are presented without acceptance corrections, while L3 results include both acceptance and generator corrections. DELPHI fit was performed in the region of $Q_\perp < 0.6\,GeV$ and $|Q_\parallel| < 0.8\,GeV$.

	L3	DELPHI
λ	0.28 ± 0.09	0.27 ± 0.04
$R_\perp$, fm	0.49 ± 0.05	0.42 ± 0.08
$R_\parallel$, fm	0.72 ± 0.07	0.74 ± 0.04

Table 3: Two-dimensional two-particle correlation function parameters as measured by the L3 and DELPHI experiments (both preliminary). Errors include both statistical and estimated systematic uncertainties.

The two-dimensional analysis done by both experiments shows the same basic tendencies as in the three-dimensional case : the transverse radius appears to be significantly smaller then the longitudinal one, and there is a clear indication of the m_T-dependence of the correlation radii. Also it is worth noticing that results of both experiments agree very well within the errors.

5 Three-particle correlations

The BEC phenomenon is not limited to the two-particle correlations, but applies to any number of bosons emitted with close space-time characteristics. Therefore, given a sufficiently high multiplicity of hadroproduction events, one must be able to observe three-particle Bose-Einstein correlations as well[a].

Analogously to C_2 of Eq.(1), the correlation function for three particles is defined as

$$C_3(p_1, p_2, p_3) = \frac{P(p_1, p_2, p_3)}{P(p_1)P(p_2)P(p_3)} \ . \tag{8}$$

However, knowing that there are two-particle correlations, one has to be aware that C_3 includes both "genuine" three-particle correlations, R_3, and those arising from two-particle correlations within a triplet, R_3'. The main challenge of the analysis is therefore to extract the component R_3.

The genuine 3-particle correlation function $R_3 = C_3 - R_3'$ is given in terms of probability densities by the form

$$R_3 = \frac{P(p_1, p_2, p_3) + 3P(p_1)P(p_2)P(p_3) - \sum\limits_{i \neq j \neq k} P(p_i, p_j)P(p_k)}{P(p_1)P(p_2)P(p_3)} \tag{9}$$

There are different methods used by different experiments to calculate terms containing products of probability densities. The DELPHI experiment used the mixing technique [14], similar to that used for the C_2 analysis. The

[a]It is rather surprising that no genuine three-particle correlations were observed by the heavy ion collision experiments so far

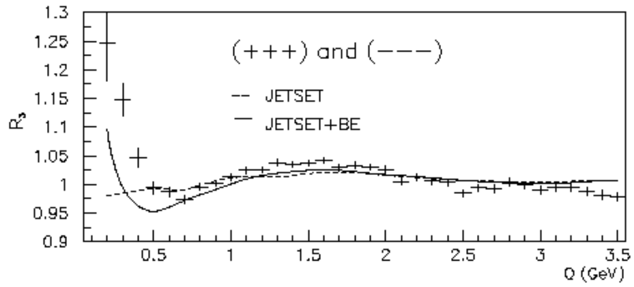

Figure 5: Genuine three-particle correlation function as measured by DELPHI. Comparison to JETSET with and without BEC simulation is also shown.

OPAL collaboration evaluated those terms using the JETSET generated events and also utilizing mixed-charged combinations from the data, corrected to the Coulomb effects [15].

After being extracted, R_3 is parameterized in terms of the summary four-momenta difference for all particles in the triplet, Q_3 :

$$Q_3 = \sqrt{Q_{12}^2 + Q_{23}^2 + Q_{31}^2} \ . \tag{10}$$

The parameterization is analogous to the Gaussian form of (4), with an extra factor of 2, arising from the presence of the two possible pion exchange diagrams :

$$R_3(Q_3) = N(1 + 2\lambda_3 \, e^{-r_3^2 Q_3^2}) \ . \tag{11}$$

Both DELPHI and OPAL observe a clear presence of the genuine three-particle correlations signal (see Fig. 5 and Fig. 6). Their results are shown in Table 4. While the amplitude of the effect is strikingly different, the width of the correlation appears to be very much the same within the errors.

	DELPHI	OPAL
λ_3	0.28 ± 0.09	0.50 ± 0.04
r_3, fm	0.66 ± 0.07	0.58 ± 0.03

Table 4: Parameters of the genuine three-particle correlation function as measured by DELPHI (mixed reference sample) and OPAL (Monte Carlo reference sample). Errors include both statistical and systematic uncertainties.

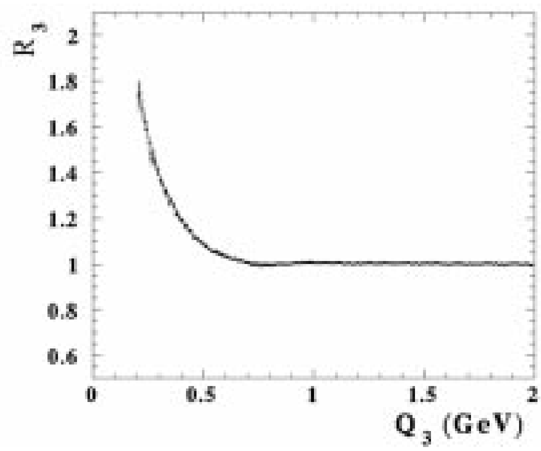

Figure 6: Genuine three-particle correlation function as measured by OPAL.

Summary

Investigations into BEC at the Z^0 peak produced wide variety of results, being of great use for studies of the multiparticle production and for tests of corresponding models in particular.

Width of the two-particle correlation function $C_2(Q)$ at Z^0 peak is found to be $R \approx 0.5\,fm$ using the mixed reference sample and $R \approx 0.9\,fm$ using the unlike charge one. Measured values of parameters appear to depend strongly on the analysis method.

Multidimensional analysis of the two-particle correlation function shows that the longitudinal size of the boson source is almost twice as big as the transversal one. Also, all components of R exhibit *decrease with increasing transverse mass* of a boson pair : a similar effect was previously observed in the heavy ion data analysis.

There is a clear presence of the genuine three-particle correlations in hadronic decays of the Z^0 boson. The width of the correlation function was found to be $r_3 \approx 0.6\,fm$. This kind of correlations has been observed so far only in $p\bar{p}$ and e^+e^- collisions.

Not reviewed here, some other studies of two-particle BEC at the Z^0 peak were done in past. Correlations between K^0_S were studied by ALEPH, DELPHI and OPAL [16], giving $\lambda \approx 1$ and $R = 0.65 \div 0.90\,fm$. L3 reported studies of BEC for neutral pions [9], giving $R \approx 0.5\,fm$. OPAL collaboration showed that the measured correlation width R *slightly increases with multiplicity* [17], similarly to the results found for the heavy ion collisions.

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