LIMITS ON THE SIZE OF EXTRA-DIMENSIONS AND THE STRING SCALE

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We give a brief summary of present bounds on the size of possible extra-dimensions as well as the string scale from collider experiments.

1 Introduction

Although the standard model has been experimentally verified to an impressive precision level, it remains unsatisfactory in some of its theoretical aspects. The major one concerns dealing with the quantum effects of gravity. The renormalization procedure which allows to extract finite predictions for processes involving the three other fundamental forces fails when gravitational interactions are taken into account. String theory stands here as only known consistent framework to incorporate these effects. There, known fundamental particles are "point-like"only because the experimental energies are too small to excite the string oscillation modes so only the center of mass motion is perceived. In addition to these heavy oscillation modes, strings have new degrees of freedom that often take the classical geometry description of propagation in extra dimensions and thus provide a compelling reason for the latter. This raises important questions: Is it possible that our world has more dimensions than the one that we are aware of? If so, why don't we see the other dimensions? Is there a way to detect them?. Answering these questions has attracted a lot of efforts lately as it has became clear that not only these extra-dimensions might be there, but also they could be just at the border of the energy domain at reach to near future experiments. This is because, within our present knowledge, the only requirements for the sizes of compactification and string scales are to allow the correct magnitude for the strength of the gauge and gravitational couplings without falling in the already experimentally excluded regions and recent investigations [1] indicate that there are many string vacua that allow the new physics to be at reach of LHC.

2 Compactification of extra dimensions

Suppose that space-time has D extra dimensions compactified on a D-dimensional torus of volume $(2\pi)^D R_1 R_2 \cdots R_D$. The states propagating in this $(4+D)$-dimensional space are seen from the four-dimensional point of view as a having a (squared) mass (assuming periodicity of the wave functions along each compact direction):

$$M_{KK}^2 \equiv M_{\vec{n}}^2 = m_0^2 + \frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} + \cdots + \frac{n_D^2}{R_D^2}, \tag{1}$$

with m_0 the four-dimensional mass and n_i non-negative integers. The states with $\sum_i n_i \neq 0$ are called Kaluza-Klein (KK) states. An important remark is that not all states can propagate in the whole space. Some might be confined in subspaces with no KK excitations in the transverse directions. The simplest example of such a situation appears in compactification on S^1/Z_2 orbifolds obtained by gauging the Z_2 parity: $y \to -y \bmod 2\pi R$. where $y \in [-\pi R, \pi R]$ span the fifth coordinate. The spectrum of states has some intersting properties: (i) only states invariant under this Z_2 (which acts also on the gauge quantum numbers) are kept while the others are projected out; (ii) new ("twisted") states, localized at the end points have to be included. They have quantum numbers and interactions that were not present in the unorbifolded original 5-dimensional model. As they can not propagate in the extra-dimension, they have no KK excitations; (iii) The even states can have non-derivative renormalizable couplings to localized states. For instance, the couplings of massive KK excitations of even gauge bosons to localized fermions are given by:

$$g_{\vec{n}} = \sqrt{2} \sum_{\vec{n}} e^{-\ln\delta \sum_i \frac{n_i^2 l_s^2}{2R_i^2}} g_0 \tag{2}$$

where $l_s \equiv M_s^{-1}$ is the string length and $\delta = 16$ in this case of Z_2 orbifolding. The $\sqrt{2}$ comes from the relative normalization of $\cos(\frac{n_i y_i}{R_i})$ wave function with respect to the zero mode while the exponential damping is a result of tree-level string computations [2].

Another example is obtained with intersecting branes (see Figure 1 and 2). When the angle between the intersecting branes is $\pi/2$ the localized strings behave exactly as the Z_2 twisted states described above. The exponential form factor of the coupling of KK excitations can be viewed as the fact that the branes intersection has a finite thickness. In fact the interaction of the KK excitations of the gauge fields (on the big branes) $A^\mu(x,\vec{y}) = \sum_{\vec{n}} \mathcal{A}_{\vec{n}}^\mu \exp i\frac{n_i y_i}{R_i}$ with the charge density $j_\mu(x)$ associated to massless localized fermions is described by the effective Lagrangian [3]:

$$\int d^4x \sum_{\vec{n}} e^{-\ln\delta \sum_i \frac{n_i^2 l_s^2}{2R_i^2}} j_\mu(x)\, \mathcal{A}_{\vec{n}}^\mu(x), \tag{3}$$

which can be written after Fourier transform as

$$\int d^4y \int d^4x \, (\frac{1}{l_s^2 2\pi \ln\delta})^2 e^{-\frac{\vec{y}^2}{2 l_s^2 \ln\delta}} j_\mu(x)\, A^\mu(x,\vec{y}). \tag{4}$$

from which we read that the localized fermions are felt as forming a Gaussian distribution of charge $e^{-\frac{\vec{y}^2}{2\sigma^2}} j_\mu(x)$ with a width $\sigma = \sqrt{\ln\delta}\, l_s \sim 1.66\, l_s$.

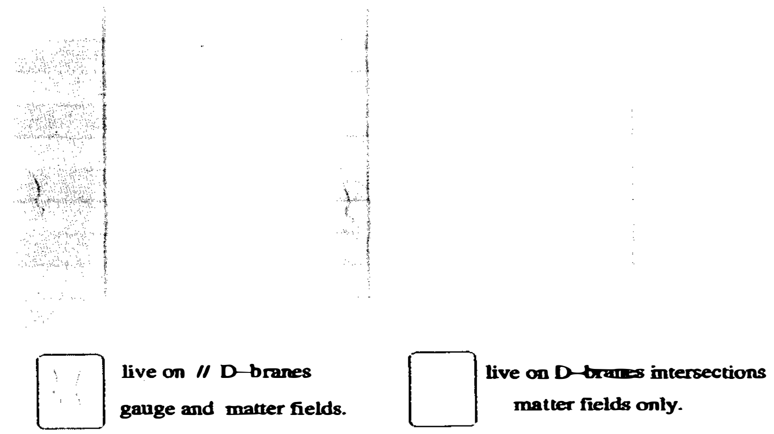

Figure 1: *Zero modes of open strings streched between two branes give rise to matter localized at their intersection. One of the branes is shown "losing" one if its longitudinal dimensions as the size of the latter shrinks. The final result is a small brane inside a bigger one*

3 Scattering of four localized fermions

The total amplitudes for the scattering of four fermions depend on the string coupling $g_s = g_{YM}^2$, the string scale $M_s \equiv 1/l_s$, the compactification radii R_i and on kinematical invariants that can be expressed in terms of the Mandelstam variables $s = -(k_1 + k_2)^2$, $t = -(k_2 + k_3)^2$ and $u = -(k_1 + k_3)^2$. The result can be decomposed as:

$$\mathcal{A} = \mathcal{A}^{(0)} + \mathcal{A}^{(KK)} + \mathcal{A}_w^{cont} + \mathcal{A}_{osc}^{cont}, \tag{5}$$

where $\mathcal{A}^{(0)}$ is the contribution of the lightest states (for example from standard model fields), $\mathcal{A}^{(KK)}$ the one from KK states of the form:

$$-\left[\bar{\psi}^{(1)}\gamma_M\psi^{(2)}\bar{\psi}^{(4)}\gamma^M\psi^{(3)}\right]\frac{g_s}{l_s^{-4}\prod_{i=1}^4 R_i}\sum_{m_i\in\mathbf{Z}-\{0\}}\frac{\delta^{-\sum_{i=1}^4\frac{m_i^2 l_s^2}{R_i^2}}}{s-\sum_{i=1}^4\frac{m_i^2}{R_i^2}}, \tag{6}$$

where $\delta = \delta(\theta)$ takes varies between $\delta = 16$ for $\theta = \pi/2$ to $\delta \to \infty$ when $\theta \to 0$. Note that the latter limit corresponds to the conservation of KK momenta in the absence of localization as seen in Figure 2. The terms $\mathcal{A}_w^{cont}$ and $\mathcal{A}_{osc}^{cont}$ contain the contribution of long string streched between the intersections while winding around the compact dimension and the ones from heavy string oscillation modes, respectively. In the large compactification radius limit $\mathcal{A}_w^{cont}$ is exponentially suppressed and we are left with[a]:

$$\mathcal{A}_{osc}^{cont} = -\left[\bar{\psi}^{(1)}\gamma_M\psi^{(2)}\bar{\psi}^{(4)}\gamma^M\psi^{(3)}\right]\left(\frac{g_s}{M_s}\right)^2\int_0^1\frac{dx}{x}\left(\frac{1}{[F_\theta(x)]^2}-1\right)$$

where θ is the angle between the branes. For $\theta \to \frac{\pi}{2}$ we have $\int_0^1 \frac{dx}{x}\left(\frac{1}{[F_\theta(x)]^2} - 1\right) \to 0.59$. For $\theta \to 0$, $F_\theta(x) \to 1$ and this contact term vanishes. There is no tree level dimension six effective

[a]The generic cases with finite radii can be found in [3]

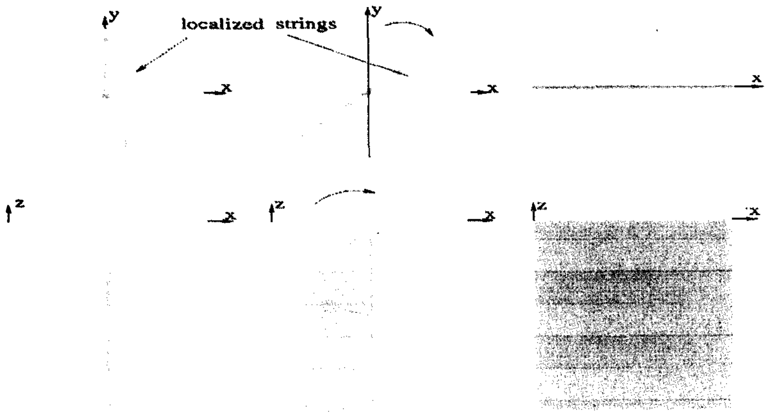

Figure 2: *Rotating two branes from orthogonal position $\theta = \pi/2$ to parallel one $\theta = 0$.*

operator in the case of open strings ending on parallel branes but the final amplitude can be written as:

$$\mathcal{A}(s,t) = \mathcal{A}_{point}(s,t) \cdot \frac{\Gamma(1-l_s^2 s)\Gamma(1-l_s^2 t)}{\Gamma(1-l_s^2 s - l_s^2 t)} = \mathcal{A}_{point}(s,t) \cdot \left[1 - \frac{\pi^2}{6}\frac{st}{M_S^4} + \cdots\right] \tag{7}$$

where $\mathcal{A}_{point}$ is the result usually derived from the (up-to-two-derivatives) low energy effective Lagrangian, while the dimension-eight operator here proportional to $\frac{st}{M_S^4}$ represents the tree-level lowest order correction and originates from the form factor due to the string-like structure.

4 Experimental constraints on extra dimensions

4.1 The scenario:

In order to pursue further, we need to provide the quantum numbers and couplings of the relevant light states. We consider (see figure 3):

- Closed strings correpond to gravitons which describe fluctuations of the metric propagate in the whole space.
- The gauge bosons propagate on a $(3+d)$-branes. They coresond on figure 3 to the open strings with both ends on the big brane.
- The matter fermions, quarks and leptons, are localized on 3-branes (the small branes inside bigger one on figure 3) and have no KK excitations. Our results strongly depend on this assumption. Instead, the possible localization of the Higgs scalar, as well as the possible existence of supersymmetric partners do not lead to major modifications for most of the obtained bounds.

4.2 Extra-dimensions along the world brane: KK excitations of gauge bosons

The experimental signatures of extra-dimensions are of two types:

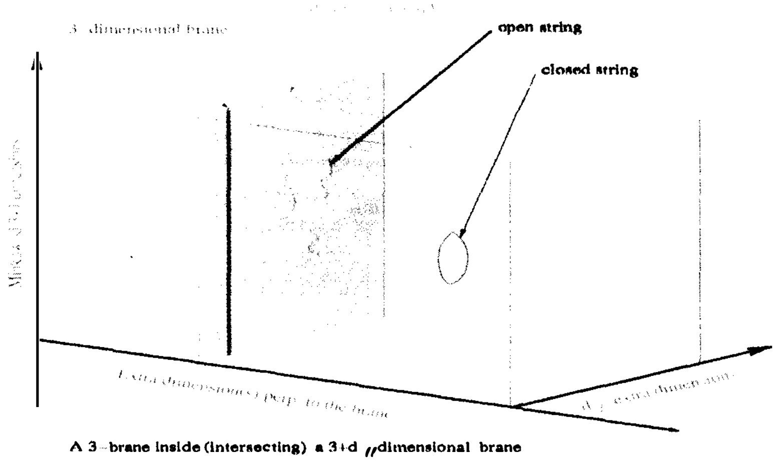

Figure 3: *The geometrical set-up of our scenario for experimental bounds.*

- Observation of resonances due to KK excitations. This needs a collider energy $\sqrt{s} \gtrsim 1/R_{\parallel}$ at LHC. The discovery limits in the case of one extra-dimension are given in table 1.

- Virtual exchange of the KK excitations which lead to measurable deviations in cross-sections compared to the standard model prediction. The exchange of KK states gives rise to an effective operator discussed above in section 3. For $d > 1$ the result depends then on both parameters $R_{\parallel}$ and M_s. Example of analysis for $d = 2$ can be found in Ref. [4]. The simpler case of $d = 1$ has been studied in detail. Possible reaches of colliders experiments [5,4] are summarized in table 1.

Provided with good statistics, there are some ways to help distinguish the corresponding signals from other possible origin of new physics, such as models with new gauge bosons: (i) the observation of resonances located practically at the same mass value; (ii) the heights and widths of the resonances are directly related to those of standard model gauge bosons in the corresponding channels; (iii) the size of virtual effects do not reproduce a tail of Bright-Wigner resonance and a deep is expected just before the resonance of the photon+Z, due to the interference between the two.

4.3 Extra-dimensions transverse to the brane world: KK excitations of gravitons

During a collision of center of mass energy $\sqrt{s}$, there are $(\sqrt{s}R_{\perp})^{d_{\perp}}$ KK excitations of gravitons with mass $m_{KK\perp} < \sqrt{s} < M_s$, which can be emitted to the bulk. Each of these states looks from the four-dimensional point of view as a massive, quasi-stable, extremely weakly coupled (s/M_{pl}^2 suppressed) particle that escapes from the detector. The total effect is a missing-energy cross section roughly of order $\frac{(\sqrt{s}R_{\perp})^n}{M_{pl}^2} \sim \frac{1}{s}(\frac{\sqrt{s}}{M_s})^{n+2}$. Explicit computation of these effects leads to the bounds given in table 2 [6] while astrophysical bounds [7,8] arise from the requirement that the radiation of gravitons should not carry on too much of the gravitational binding energy released during core collapse of supernovae. The best cosmological bound [9] is obtained from requiring that decay of bulk gravitons to photons do not generate a spike in the energy spectrum of the photon background measured by the COMPTEL instrument. The bulk gravitons are themselves expected to be produced just before nucleosynthesis due to thermal radiation from the brane.

Table 1: Limits on $R_{\parallel}^{-1}$ in TeV at present and future colliders. The luminosity is given in fb^{-1}.

Collider	Luminosity	Gluons	$W^{\pm}$	$\gamma + Z$
Discovery of Resonances				
LHC	100	5	6	6
Observation of Deviation				
LEP 200	4×200	-	-	1.9
TevatronI	0.11	-	-	0.9
TevatronII	2	-	-	1.2
TevatronII	20	4	-	1.3
LHC	10	15	8.2	6.7
LHC	100	20	14	12
NLC500	75	-	-	8
NLC1000	200	-	-	13

Table 2: Limits on $R_{\perp}$ in mm from missing-energy processes.

Experiment	$R_{\perp}(n=2)$	$R_{\perp}(n=4)$	$R_{\perp}(n=6)$
Collider bounds			
LEP 2	4.8×10^{-1}	1.9×10^{-8}	6.8×10^{-11}
Tevatron	5.5×10^{-1}	1.4×10^{-8}	4.1×10^{-11}
LHC	4.5×10^{-3}	5.6×10^{-10}	2.7×10^{-12}
NLC	1.2×10^{-2}	1.2×10^{-9}	6.5×10^{-12}
Present non-collider bounds			
SN1987A	3×10^{-4}	1×10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	-	-

The limits assume that the temperature was at most 1 MeV as nucleosynthesis begins, and become stronger if this temperature is increased. While the obtained bounds for $R_{\perp}^{-1}$ are smaller than those that could be checked in table-top experiments probing macroscopic gravity at small distances, one should keep in mind that larger radii are allowed if one relaxes the assumption of isotropy, by taking for instance two large dimensions with different radii.

5 Dimension-Six Effective Operators:

The dimension-six effective operators are generically parametrized as [10]:

$$\Lambda_{eff} = \frac{4\pi}{(1+\varepsilon)\Lambda^2} \sum_{a,b=L,R} \eta_{ab} \bar{\psi}_a \gamma^{\mu} \psi_a \bar{\psi}'_b \gamma_{\mu} \psi'_b \tag{8}$$

with $\varepsilon = 1$ (0) for $\psi = \psi'$ ($\psi \neq \psi'$), where ψ_a and ψ'_b are left (L) or right (R) handed spinors. Λ is the scale of contact interactions and η_{ab} parametrize the relative strengths of various helicity combinations. The generic analysis of these operators can be found in [3]. We summarize here some of the results.

For $\psi \neq \psi'$ the contributions only from the exchange of the massive open string states on the small brane lead to parameters in eq. (8) as:

$$\eta_{LL} = \eta_{RR} = \eta_{LR} = \eta_{RL} = 1\,, \quad \Lambda \simeq \sqrt{\frac{4\pi}{0.59 g_s}} M_s \tag{9}$$

The signs and relative ratios of the different terms in (8) correspond to what is usually refered to as Λ^{+}_{VV}. The present bounds from LEP [11] are of the order of $\Lambda^{+}_{VV} \gtrsim 16$ TeV which for $g_s = g^2_{YM} \sim 1/2$, with g_{YM} the gauge coupling, leads to $M_s \gtrsim 2.5$ TeV. A stronger bound can be obtained from the analysis of high precision low energy data in the presence of effective four-fermion operators that modify the μ-decay amplitude. Using the results of ref. [12], we obtain $M_s \gtrsim 3.1$ TeV.

In the case $\psi = \psi'$ as for Bhabha scattering in e^+e^- there is an additional contribution to the effective operator coming from the operators that are associated with the exchange of other massives oscillation modes leading instead to $0.75\eta_{LL} = 0.75\,\eta_{RR} \simeq \eta_{LR} = \eta_{RL} = 1$.

On the other hand, the contact interactions due to exchange of KK excitations give rise (for $d_{\parallel} = 1$ to [2]:

$$\Lambda^{KK}_{eff} \simeq -\frac{\pi^2}{3(1+\varepsilon)} R^2 g_s \sum_{a,b=L,R} \eta_{ab} \bar{\psi}_a \gamma^{\mu} \psi_a \bar{\psi}'_b \gamma_{\mu} \psi'_b \tag{10}$$

Experimental constraints on such operators translate into lower bounds on the scale of compactification. For instance exchanges of KK excitations of photon corresponds to $\eta_{ab} = 1$ and $g_s/4\pi = 1/128$ from which we obtain a bound $R^{-1} \gtrsim 2.2$ TeV, using LEP bounds [11] $\Lambda^{-}_{VV} \gtrsim 14$ TeV. Low energy precision electroweak data lead instead to $R^{-1} \gtrsim 3.5$ TeV [13].

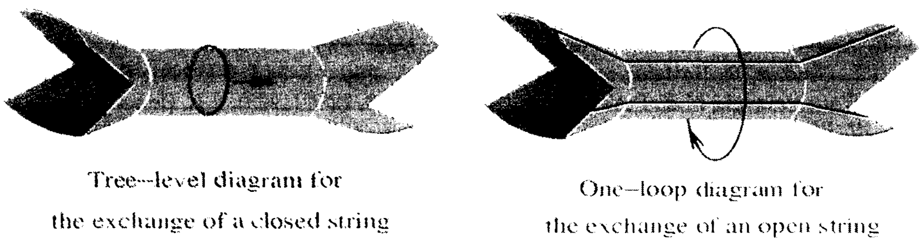

Figure 4: *The exchange of virtual gravitons.*

5.1 Dimension-Eight Effective Operators:

We consider two generic sources for dimension-eight operators: (i) Form factors due to the extended nature of strings eq. (7) (ii) exchange of virtual KK excitations of bulk fields (gravitons,...).

The limit obtained from dimension-eight operators (i) is of order $M_s \gtrsim 0.63$ TeV [14,15]. Instead (ii) can not provide reliable model dependent results. The exchange of virtual KK excitations of bulk gravitons is described in the effective field theory by an amplitude involving the sum $\frac{1}{M_p^2} \sum_n \frac{1}{s - \frac{n^2}{R_\perp^2}}$. For $n > 1$, this sum diverges. This means it is sensitive to the UV cut-off physics thus cannot be compute reliably in field theory. In string models it reflects the ultraviolet behavior of open string one-loop diagrams which are compactification dependent.

In order to understand better this issue, it is important to remember that gravitons and other bulk particles correspond to excitations of closed strings. Their tree-level exchange of order g_s^2 is described by a cylinder which can also be seen as an annulus corresponding to an open string describing a loop (see figure 4). First, the result of such one-loop diagrams are compactification dependent. Second, they correspond to box diagrams in a gauge theory which are of order g_{YM}^4 thus samller by a factor $g_s = g_{YM}^2$ compare to the ones in (i).

Acknowledgments

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