Beyond "naive" factorization in exclusive radiative B–meson decays

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Abstract: We apply the QCD factorization approach to exclusive, radiative B meson decays in the region of small invariant photon mass. We calculate factorizable and non-factorizable corrections to leading order in the heavy quark mass expansion and next-to-leading order in the strong coupling constant. Phenomenological consequences for the $B \to K^*\gamma$ decay rate and the $B \to K^*\ell^+\ell^-$ forward-backward asymmetry are discussed.

Radiative B–meson decays provide an important tool to test the standard model of electroweak interactions and to constrain various models of new physics. The theoretical description of *exclusive* channels has to deal with hadronic uncertainties related to the binding of quarks in the initial and final states. For the decays $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$, that we are focusing on here, this is usually phrased as the need to know the hadronic form factors for the $B \to K^{(*)}$ transition, but there also exist "non-factorizable" strong interaction effects that do not correspond to form factors. They arise from the matrix elements of purely hadronic operators in the weak effective Hamiltonian with a photon radiated from one of the internal quarks. In Ref. [attr/Border [0 0 0] goto namebib1.9 .9 0 01] we have computed these non-factorizable corrections and demonstrated that exclusive, radiative decays can be treated in a similarly systematic manner as their inclusive counterparts. As a result we obtain the branching fractions for $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ for small invariant mass of the lepton pair to next-to-leading logarithmic (NLL) order in renormalization-group improved perturbation theory.

In the "naive" factorization approach, exclusive radiative B decays are described in terms of hadronic matrix elements of the electromagnetic penguin operator $\mathcal{O}_7$ and the semi-leptonic operators $\mathcal{O}_{9,10}$ [attr/Border [0 0 0] goto namebib2.9 .9 0 02]. These are parametrized in terms of the corresponding tensor, vector and axial-vector $B \to K^*$ transition form factors ($T_{1,2,3}(q^2)$, $V(q^2)$, $A_{0,1,2}(q^2)$). Factorizable quark–loop contributions

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†Based on work together with M. Beneke and D. Seidel [attr/Border [0 0 0] goto namebib1.9 .9 0 01].

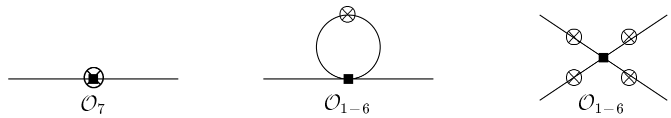

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Figure 1: LO contributions to $\langle \gamma^* \bar{K}^* | H_{\rm eff} | \bar{B} \rangle$. The circled cross marks the possible insertions of the virtual photon line. In (a) and (b) the spectator line is not shown.

(Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01b) with the four–quark operators $\mathcal{O}_{1-6}$ are taken into account by using "effective" Wilson-coefficients, $C_7 \to C_7^{\rm eff}$, $C_9 \to C_9^{\rm eff}(q^2)$, renormalized at the scale $\mu = m_b$.

In order to include non-factorizable contributions as in Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01c and Fig. attr/Border [0 0 0] goto nameref-fig2.9 .9 0 02 it is convenient to introduce generalized form factors $\mathcal{T}_i(q^2)$ for the transition into a *virtual* photon $B \to K^* \gamma^*$ as follows,

$$\langle \gamma^*(q,\mu) \bar{K}^*(p',\varepsilon^*) | H_{\rm eff} | \bar{B}(p) \rangle = - \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{i g_{\rm em} m_b}{4\pi^2} \Big\{ 2\, \mathcal{T}_1(q^2)\, \epsilon^{\mu\nu\rho\sigma} \varepsilon^*_\nu p_\rho p'_\sigma - i\, \mathcal{T}_2(q^2) \left[(M_B^2 - m_{K^*}^2)\, \varepsilon^{*\mu} - (\varepsilon^* \cdot q)\, (p^\mu + p'^\mu) \right] - i\, \mathcal{T}_3(q^2)\, (\varepsilon^* \cdot q) \left[q^\mu - \frac{q^2}{M_B^2 - m_{K^*}^2} (p^\mu + p'^\mu) \right] \Big\} .nameref-caltdeffith \quad (1)$$

In the "naive" factorization approach these new functions reduce to $\mathcal{T}_i(q^2) = C_7^{\rm eff}\, T_i(q^2) + \ldots$ Following the QCD factorization approach to exclusive B decays [attr/Border [0 0 0] goto namebib3.9 .9 0 03], factorizable and non-factorizable radiative corrections are calculable in the heavy quark mass limit and for small photon virtualities (in practice $q^2 < 4m_c^2$).

At leading order (LO) in the strong coupling constant, the generalized form factors read

$$\begin{aligned} nameref-firstTfith \mathcal{T}_1(q^2) &= C_7^{\rm eff}\, T_1(q^2) + Y(q^2)\, \frac{q^2}{2m_b(M_B + m_{K^*})}\, V(q^2), \\ \mathcal{T}_2(q^2) &= C_7^{\rm eff}\, T_2(q^2) + Y(q^2)\, \frac{q^2}{2m_b(M_B - m_{K^*})}\, A_1(q^2), \\ \mathcal{T}_3(q^2) &= C_7^{\rm eff}\, T_3(q^2) + Y(q^2) \left[\frac{M_B - m_{K^*}}{2m_b}\, A_2(q^2) - \frac{M_B + m_{K^*}}{2m_b}\, A_1(q^2) \right] \\ & \quad - e_q\, (C_3 + 3C_4)\, \frac{8\pi^2 M_B f_B f_{K^*} m_{K^*}}{N_C m_b (M^2 - q^2)} \int d\omega \frac{\phi_{B,-}(\omega)}{\omega - q^2/M - i\epsilon} .nameref-lastTfith \quad (2) \end{aligned}$$

The function $Y(q^2)$, which is usually absorbed into $C_9^{\rm eff}(q^2)$, arises from the quark loop in Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01b. The last, "non-factorizable" term in $\mathcal{T}_3(q^2)$ comes from the annihilation graph in Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01c when the photon is emitted from the light spectator in the B meson (all other graphs

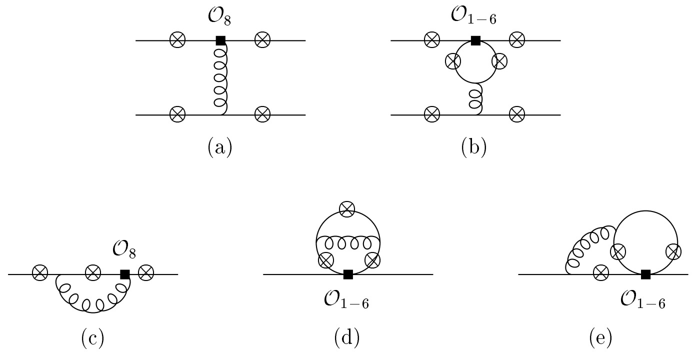

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Figure 2: Non-factorizable NLO contributions to $\langle \gamma^* \bar{K}^* | H_{\rm eff} | \bar{B} \rangle$. Diagrams that follow from (c) and (e) by symmetry are not shown.

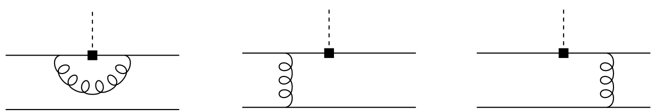
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Figure 3: Factorizable NLO corrections to the $B \to K^*$ form factors.

are sub-leading in the $1/m_b$ expansion). It introduces a new non-perturbative ingredient, namely one of the two light-cone distribution amplitudes of the B meson, $\phi_{B,\pm}(\omega)$, see [attr/Border [0 0 0] goto namebib1.9 .9 0 01, attr/Border [0 0 0] goto namebib4.9 .9 0 04] for details. Furthermore, for the considered values of q^2, the recoil-energy of the out-going K^* meson is large, and the seven independent $B \to K^*$ form factors can be described in terms of only two universal form factors [attr/Border [0 0 0] goto namebib5.9 .9 0 05], which we denote as $\xi_\perp(q^2)$ and $\xi_\parallel(q^2)$ for transversely and longitudinally polarized K^* mesons, respectively [attr/Border [0 0 0] goto namebib4.9 .9 0 04].

Factorizable next-to-leading order (NLO) form factor corrections are derived from Fig. attr/Border [0 0 0] goto nameref-fig3.9 .9 0 03 after the corresponding infra-red divergent pieces are absorbed into the *soft* universal form factors $\xi_\perp$ and $\xi_\parallel$, see [attr/Border [0 0 0] goto namebib4.9 .9 for details. The non-factorizable vertex corrections (Fig. attr/Border [0 0 0] goto nameref-fig2.9 .9 0 02c-e), are similar to the NLO calculation for the *inclusive* $b \to s\gamma^*$ transition, and the result for the two-loop diagrams in Fig. attr/Border [0 0 0] goto nameref-fig2.9 .9 0 02d+e are taken from Ref. [attr/Border [0 0 0] goto namebib6.9 .9 0 06]. For the vertex corrections we chose a renormalization scale $\mu = \mathcal{O}(m_b)$. The non-factorizable hard-scattering corrections in Fig. attr/Border [0 0 0] goto nameref-fig2.9 .9 0 02a+b and Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01c involve the light-cone distribution amplitudes of both, B and K^* mesons. (For $q^2 = 0$ diagrams of this form have already been considered in [attr/Border [0 0 0] goto namebib7.9 .9 0 07], but using bound state model wave-functions, rather than light-cone distribution amplitudes.) Since in these class of diagrams the typical quark- and gluon-virtuality is of order $\sqrt{\Lambda_{\rm QCD} m_b}$ we chose a different renormalization scale μ' of that order. In principle, we also have to consider NLO order corrections to the annihilation graph in Fig. attr/Border [0 0 0] goto nameref-fig1.9 .9 0 01c. However, since this term is suppressed by small Wilson coefficients C_3 and C_4 and numerically small already at LO, we have neglected these effects. Notice however, that the annihilation topology is numerically more important for $B \to \rho\gamma$ decays [attr/Border [0 0 0] goto namebib8.9 .9 0 08, attr/Border [0 0 0] goto namebib9.9 .9 0 09].

The $B \to K^*\gamma$ decay rate is proportional to the function $|\mathcal{T}_1(0)|^2 = |\mathcal{T}_2(0)|^2$. In order to study the effect of NLO corrections it is convenient to define a generalized exclusive "Wilson" coefficient $\mathcal{C}_7 \equiv \mathcal{T}_1(0)/\xi_\perp(0)$. In Fig. attr/Border [0 0 0] goto nameref-fig4.9 .9 0 04 we have shown the μ-dependence of $|\mathcal{C}_7|^2$ at leading order (LO), including only next-to-leading order vertex corrections (NLO$_1$), and including all next-to-leading order corrections (NLO). As expected, the NLO$_1$ vertex corrections cancel the renormalization-scale dependence of the LO result to a great extent. (The hard-scattering corrections, arising at order α_s reintroduce a mild scale-dependence.) Most importantly, we observe that the NLO corrections significantly increase the theoretical prediction for $|\mathcal{C}_7|^2$. Numerically, we have $|\mathcal{C}_7|^2_{\rm NLO} \simeq 1.78 \cdot |\mathcal{C}_7|^2_{\rm LO}$. From this we predict the branching ratio as

$$\mathrm{Br}(\bar B \to \bar K^*\gamma) = (7.9^{+1.8}_{-1.6}) \cdot 10^{-5} \left(\frac{\tau_B}{1.6\mathrm{ps}}\right) \left(\frac{m_{b,\mathrm{PS}}}{4.6\,\mathrm{GeV}}\right)^2 \left(\frac{\xi_\perp(0)}{0.35}\right)^2 \tag{3}$$

Comparing with the current experimental averages [attr/Border [0 0 0] goto namebib10.9 .9 0 010] $\mathrm{Br}(\bar B^0 \to \bar K^{*0}\gamma)_{\rm exp} = (4.54\pm0.37)\cdot 10^{-5}$, $\mathrm{Br}(B^- \to \bar K^{*-}\gamma)_{\rm exp} = (3.81\pm0.68)\cdot 10^{-5}$, and using the value $\xi_\perp(0) = 0.35$ from QCD sum rules [attr/Border [0 0 0] goto namebib11.9 .9 0 011], we observe that the central value of the theoretical prediction overshoots the data by nearly a factor of two. (An equivalent analysis with similar conclusions can be found in Ref. [attr/Border [0 0 0] goto namebib9.9 .9 0 09].) Possible explanations for this discrep-

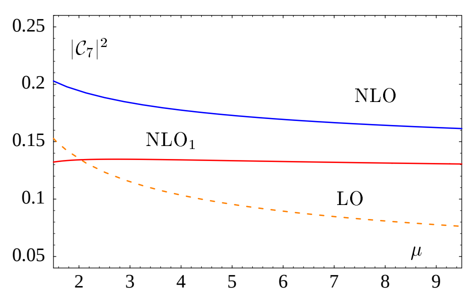

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Figure 4: $|\mathcal{C}_7|^2$ as a function of the renormalization scale μ, see text.

ancy are: i) new physics contributions (this is rather unlikely because of the good agreement between NLO theory and experiment for the *inclusive* counterpart, $B \to X_s\gamma$), ii) sizeable $1/m_b$ power-corrections ("chirally enhanced" corrections play a role for decays into light pseudoscalars [attr/Border [0 0 0] goto namebib12.9 .9 0 012]; in our case, however, we expect a less dramatic effect), iii) an insufficient understanding of the $B \to K^*$ form factors

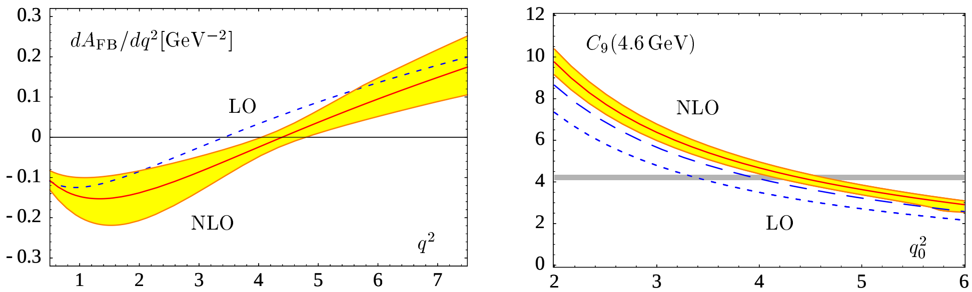

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Figure 5: The FB asymmetry as a function of q^2 (left). The Wilson-coefficient C_9 as a function of the FB asymmetry zero (right). The error band refers to a variation of all input parameters and changing the renormalization scale between $m_b/2$ and $2m_b$. The dashed line is obtained from using the complete form factors from [attr/Border [0 0 0] goto namebib11.9 .9 0 011], see text. The grey band indicates the standard model value.

(a fit to the experimental data on the basis of our formalism yields a somewhat smaller value, $\xi_\perp(0) = 0.24 \pm 0.06$).

A quantity that is less sensitive to the precise value of $\xi_\perp(q^2)$ is provided by the $B \to K^* \ell^+ \ell^-$ forward-backward asymmetry $\mathcal{A}_{\rm FB}$. At LO the position of the asymmetry zero q_0^2 is determined by the implicit relation

$$C_9 + {\rm Re}(Y(q_0^2)) = -\frac{2 M_B m_b}{q_0^2}\, C_7^{\rm eff} \ , \tag{4}$$

and does not depend on form factors at all [attr/Border [0 0 0] goto namebib13.9 .9 0 013]. As illustrated in Fig. attr/Border [0 0 0] goto nameref-fig5.9 .9 0 05 NLO corrections shift the position of the asymmetry zero from $q_0^2 = 3.4^{+0.6}_{-0.5}$ GeV^2 at LO to $q_0^2 = 4.39^{+0.38}_{-0.35}$ GeV^2. (A slightly different value $q_0^2 = 3.94$ GeV^2 is found if one takes the complete form factors from QCD sum rules [attr/Border [0 0 0] goto namebib11.9 .9 0 011], instead of $\xi_\perp$ and the factorizable NLO corrections from [attr/Border [0 0 0] goto namebib4.9 .9 0 04]). In any case, a measurement of the forward-backward asymmetry zero provide a clean test of the Wilson-coefficient C_9 in the standard model with a rather small theoretical uncertainty of about 10%.

In summary, we have shown that a systematic improvement of the theoretical description of exclusive radiative B meson decays is possible. This is because in the heavy quark limit decay amplitudes factorize into perturbatively calculable hard-scattering kernels and universal soft form factors or light-cone distribution amplitudes, respectively. The next-to-leading order corrections increase the branching ratio for the decay $B \to K^*\gamma$ by almost a factor of two (which is at variance with the current experimental data if "standard" values for the soft form factors are used). They also shift the position of the forward-backward asymmetry in the decay $B \to K^* \ell^+ \ell^-$ towards $q_0^2 = 4.2 \pm 0.6$ GeV^2 in the standard model.

In this case the precision of the prediction is sufficient to test the Wilson coefficient C_9 with only 10% theoretical uncertainty.

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