Preprint typeset in JHEP style. - HYPER VERSION

hep-ph/0102340
CERN-TH/2001-053
February 2001

Gluon Contributions to Parity-Violating Asymmetries in Polarized Proton-Proton Scattering

John Ellis, Stefano Moretti and Douglas A. Ross*

Division Théorique, CERN, 1211 Genève 23, Switzerland

Abstract: We report on a calculation of one-loop weak corrections to polarized quark-gluon scattering and the corresponding crossed channels. Such contributions are suppressed formally by one power of α_s relative to $W-$ or $Z-$mediated quark-quark scattering, but would enable the spin asymmetry of the gluon distribution to contribute to parity-violating asymmetries that will soon be investigated in polarized proton-proton scattering experiments at RHIC. In certain kinematic regions, gluon contributions to parity-violating asymmetries can be as large as 10% of the tree-level $W-$ and $Z-$exchanges in quark-quark scattering, but usually only where the parity-violating asymmetries are already small.

Keywords: Electroweak physics, Parity violation, Polarized pp scattering, RHIC.

*On leave of absence from: Department of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, U.K.

Contents

1. Introduction

The forthcoming generation of hadron-hadron colliders - the Tevatron with Run 2, RHIC with polarized proton beams (RHIC-Spin) and the LHC - will produce unprecedented data on high-p_T jets that merit the most accurate theoretical calculations. Next-to-leading order (NLO) jet calculations in QCD have been available for some time [1], whereas the major task of providing the complete NNLO QCD result is about to be completed [2, 3]. Formally, the latter introduce corrections relative to the leading-order predictions that are of order $(\alpha_s/\pi)^2 = \mathcal{O}(10^{-3})$, with the likelihood of parametric enhancements (or suppressions). When computing to this accuracy, one may then wonder whether NLO electroweak (EW) corrections could also be important. In fact, these are formally of order $(\alpha_W/\pi) = \mathcal{O}(10^{-2})$, again with the likelihood of parametric enhancement or suppression.

We report in this paper on a pilot one-loop calculation of purely weak corrections to high-p_T jet production. We concentrate on quark-gluon scattering and processes related to this by crossing, namely $\overset{(-)}{q} + g \rightarrow \overset{(-)}{q} + g$, $q\ +\ \bar{q} \rightarrow g\ +\ g$ and $g\ +\ g \rightarrow q\ +\bar{q}$ [1]. In order to focus on a possible distinctive experimental signal, we discuss parity-violating asymmetries in these processes that are in principle measurable at RHIC-Spin [4]. These weak corrections provide, in principle, sensitivity to the helicity-dependent component of the gluon distribution function inside a polarized proton, ΔG, that is instead absent in pure QCD [5].

Contributions to such parity-violating asymmetries already arise at tree level from $q+q \rightarrow q+q$ and $q+\bar{q} \rightarrow q+\bar{q}$ scattering processes, due to purely weak amplitudes as well as to the interference of the latter with the QCD ones. These processes are sensitive to the helicity-dependent components of the quark distributions, ΔQ_q, and their effects on parity-violating asymmetries are well known and rather small, at the level of few percent at best [6]. As we show below, the new contributions that we calculate here are even smaller, as indeed expected on the ground of a naïve estimate based on counting powers of coupling constants, without any benefit from an enhancement due to the spin asymmetry of the gluon distribution function. For plausible values of the latter, there do exist regions of the high-p_T jet phase space where the ΔG-dependent contributions to the parity-violating asymmetries are non-negligible in comparison with

[1]Hereafter, the notation q is intended to represent any possible quark flavour.

the ΔQ_q-dependent ones, but the experimental sensitivity seems unlikely to be able to disentangle the two, primarily because of the small absolute values of the observables.

Looking on the bright side, our results help stabilise the Standard Model (SM) predictions for parity-violating asymmetries at RHIC-Spin, thereby providing a solid baseline in the search for new physics. These parity-violating asymmetries have been proposed as signatures of new physics, such as a W', Z' or contact interactions that violate parity [7]. In other terms, the claimed RHIC-Spin sensitivities to these signatures are validated by our results.

A caveat to our analysis is that we have not computed the one-loop weak corrections to $q + \overset{(-)}{q} \rightarrow q + \overset{(-)}{q}$. In fact, for the QCD-mediated scattering, they are formally of the same order as those for $\overset{(-)}{q} + g \rightarrow \overset{(-)}{q} + g$, $q + \bar{q} \rightarrow g + g$ and $g + g \rightarrow q + \bar{q}$. Since such processes give rise to parity-violating asymmetries already at tree level [6], such a computation would amount to a QCD correction to their effects and is therefore unlikely to have a significant qualitative or quantitative effect on those asymmetries.

2. The one-loop weak corrections to gluon scattering matrix elements

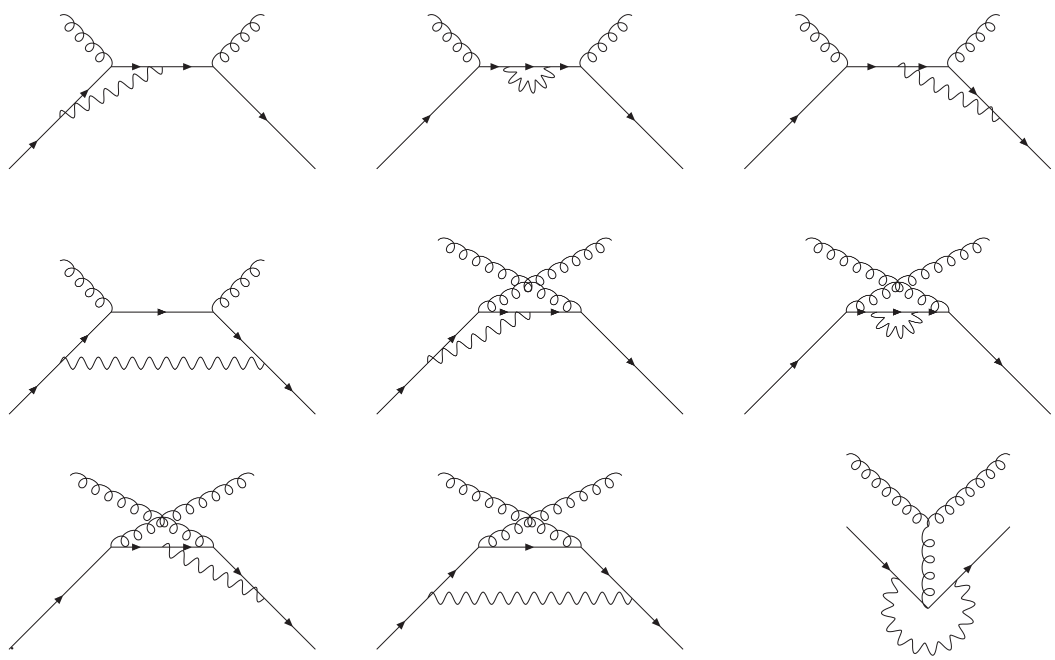

Figure 1: Diagrams contributing to the one-loop weak corrections to quark-gluon scattering. The helical lines represent gluons and the wiggly lines refer to either a $W-$ or $Z-$boson.

One-loop graphs yielding weak corrections to the process

$$q + g \rightarrow q + g$$

are shown in Fig. 1. In addition to those shown, there is a correction due to the on-shell fermion wavefunction renormalization for each tree-level graph, which we have not shown. We have calculated the graphs in Fig. 1 with the help of FeynCalc [8] and FORM [9].

The tree-level amplitudes $\mathcal{A}_{\lambda\lambda',\sigma}$, where λ and λ' are the helicities of the gluons and σ that of the (massless) quark line, can be written as

$$\mathcal{A}_{++,+} \;=\; -\,2\,g^2\sqrt{\frac{-u}{s}}\left\{\frac{s^2}{ut}\tau^a\tau^b \;+\; \frac{s}{t}\tau^b\tau^a\right\}. \tag{2.1}$$

$$\mathcal{A}_{--,+} \;=\; 2g^2\sqrt{\frac{-u}{s}}\left\{\frac{s}{t}\tau^a\tau^b \;+\; \frac{u}{t}\tau^b\tau^a\right\}, \tag{2.2}$$

where the symbols τ^a and τ^b denote the colour matrices. By parity conservation, we have $\mathcal{A}_{--,-} = \mathcal{A}_{++,+}$ and $\mathcal{A}_{++,-} = \mathcal{A}_{--,+}$.

At one-loop order, the exchanges of a vector boson of mass m and coupling g_+ to right-handed quarks make corrections to these matrix elements that can be written in the forms:

$$\begin{aligned}
\Delta\mathcal{A}_{++,+} = \frac{g^2 g_+^2}{8\pi^2}\sqrt{\frac{-u}{s}}\Bigg[&\Bigg\{2\left[1-\frac{s}{t}-\frac{t}{s}\right]\left(1+\frac{m^2}{t}\right)^2\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{t}{m^2}\right)\right]\\
&-2\,\frac{t^2}{u\,s}\left(1+\frac{m^2}{t}\right)^2\left[\ln\left(1-\frac{u}{m^2}\right)\ln\left(\frac{-t}{m^2}\right)+\mathrm{Li}_2\left(\frac{u}{m^2}\right)\right]\\
&+\left[4\frac{s}{u}+2\left(1-\frac{m^2}{u}\right)^2-\frac{s}{u}\left(1+\frac{m^2}{u}\right)^2\right]\ln\left(1-\frac{u}{m^2}\right)\\
&+\left[\frac{s}{t}-2\left(1-\frac{s}{t}\right)\left(1+\frac{m^2}{t}\right)\right]\ln\left(\frac{-t}{m^2}\right)-\frac{7\,s}{2\,t}-\frac{5\,s}{2\,u}-\frac{m^2\,s}{t^2}\left(1+\frac{s^2}{u^2}\right)\Bigg\}\,\tau^a\tau^b\\
&+\Bigg\{-2\frac{s^2}{u\,t}\left(1+\frac{m^2}{t}\right)^2\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{t}{m^2}\right)\right]\\
&-2\,\frac{s}{u}\left(1-\frac{m^2}{s}\right)^2\left[\ln\left(1-\frac{s}{m^2}\right)\ln\left(\frac{-t}{m^2}\right)+\mathrm{Li}_2\left(\frac{s}{m^2}\right)\right]\\
&+\left[2\frac{m^2}{t}-2\frac{m^2\,s}{t^2}-3\frac{s}{t}\right]\ln\left(\frac{-t}{m^2}\right)+\frac{7\,s}{2\,t}-2\frac{m^2}{t}\left(1-\frac{s}{t}\right)\Bigg\}\,\tau^b\tau^a\Bigg],
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
\Delta\mathcal{A}_{--,+} = \frac{g^2 g_+^2}{8\pi^2}\sqrt{\frac{-u}{s}}\Bigg[&\Bigg\{2\frac{u}{t}\left(1+\frac{m^2}{t}\right)^2\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{t}{m^2}\right)\right]\\
&+2\left(1-\frac{m^2}{u}\right)^2\left[\ln\left(1-\frac{u}{m^2}\right)\ln\left(\frac{-t}{m^2}\right)+\mathrm{Li}_2\left(\frac{u}{m^2}\right)\right]
\end{aligned}$$

$$
+\left[3\frac{s}{t}+2\frac{m^2}{u}-2\frac{u\,m^2}{t^2}\right]\ln\left(\frac{-t}{m^2}\right)-\frac{7}{2}\frac{s}{t}-2\frac{m^2}{u}\left(1-\frac{u^2}{t^2}\right)\Bigg\}\tau^a\tau^b
$$

$$
+\Bigg\{-2\left(\frac{u}{t}+\frac{t^2}{u^2}\right)\left(1+\frac{m^2}{t}\right)^2\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{t}{m^2}\right)\right]
$$

$$
+2\left(\frac{(t+m^2)}{u}\right)^2\left[\ln\left(1-\frac{s}{m^2}\right)\ln\left(\frac{-t}{m^2}\right)+\mathrm{Li}_2\left(\frac{s}{m^2}\right)\right]
$$

$$
-\left[4+2\frac{s}{u}\left(1-\frac{m^2}{s}\right)^2-\left(1+\frac{m^2}{s}\right)^2\right]\ln\left(1-\frac{s}{m^2}\right)
$$

$$
+\left[2\frac{s}{u}\left(1-\frac{m^2}{s}\right)-3\frac{s}{t}+2\frac{u\,m^2}{t^2}\right]\ln\left(\frac{-t}{m^2}\right)
$$

$$
+\frac{7}{2}\frac{s}{t}+\frac{5}{2}+\frac{m^2}{s}\left(1-2\frac{u\,s}{t^2}\right)\Bigg\}\tau^b\tau^a\Bigg]. \qquad (2.4)
$$

We observe that $\Delta\mathcal{A}_{--,-}$ can be obtained from $\Delta\mathcal{A}_{++,+}$ and $\Delta\mathcal{A}_{++,-}$ from $\Delta\mathcal{A}_{--,+}$ by replacing g_+ with the coupling of the gauge boson to left-handed quarks, g_-.

In the Abelian limit, $\tau^a\tau^b = \tau^b\tau^a$, and after rearranging the couplings appropriately, we recover the results of [10] for the $Z-$boson corrections to polarized Compton scattering.

3. Parton-level asymmetries

In this section, we display the parity-violating asymmetries of the cross sections for the basic partonic subprocesses

$$
q+q \;\rightarrow\; q+q, \qquad (3.1)
$$

$$
q+\bar{q}\rightarrow q+\bar{q}, \qquad (3.2)
$$

$$
q+g \;\rightarrow\; q+g, \qquad (3.3)
$$

$$
q+\bar{q}\rightarrow g+g, \qquad (3.4)
$$

$$
g+g\rightarrow q+\bar{q}. \qquad (3.5)
$$

For the sake of illustration, we choose $q = d$. The processes (3.1) and (3.2) can occur at tree level via $W-$ or $Z-$boson exchange, and all their EW and QCD ingredients have been calculated in [6]. The processes (3.3)–(3.5) occur only at the one-loop level, and are computed numerically here using the amplitudes given in the previous section.

The parity-violating asymmetries of interest are [4]:

$$
A_L\, d\sigma \;\equiv\; d\sigma_- - d\sigma_+, \qquad (3.6)
$$

for the case where only one incident beam is polarized, and

$$
A_{PV}\, d\sigma \;\equiv\; d\sigma_{--} - d\sigma_{++}, \qquad (3.7)
$$

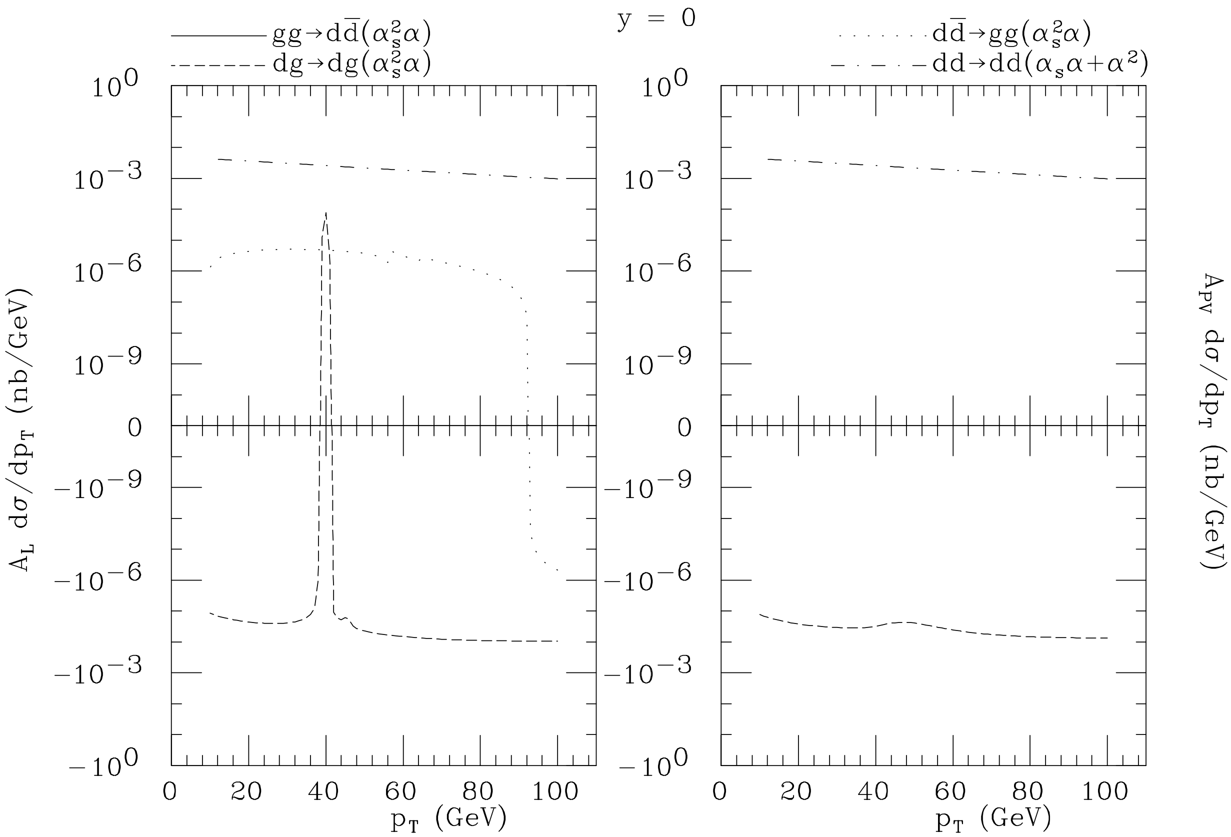

Figure 2: Parton-level asymmetries at rapidity $y = 0$, as functions of the transverse momentum p_T.

when both incident beams are polarized[2].

In Fig. 2 we show these asymmetries in the central rapidity region. The dot-dashed line is the quark-quark scattering contribution calculated in [6], and the dashed line refers to quark-gluon scattering. We see that this asymmetry has the opposite sign from the quark-quark contribution, except for A_L in a small region near the weak threshold $p_T \sim M_W/2$. The magnitude of the effect increases with transverse momentum, reaching about 10% at the upper end of the spectrum. The dotted line refers to the crossed process of quark-antiquark annihilation into two gluons and we see that in this case the asymmetry is smaller by two orders of magnitude.

In Fig. 3 we show the same plots, but now at relatively large rapidity, $y = 1$. Here we see that the contributions from quark-gluon scattering to both asymmetries can have either sign, and at sufficiently large transverse momenta the contributions to both the asymmetries are comparable in magnitude, and opposite in sign, to that from tree-level quark-quark scattering. Also shown in Fig. 3 is the other crossed process, namely gluon-gluon annihilation into a quark-antiquark pair. This was suppressed in Fig. 2, because it is negligibly small for small rapidity. Once again, we see that the asymmetry

[2]In the case of quark-gluon scattering, the first index refers to the helicity of the quark.

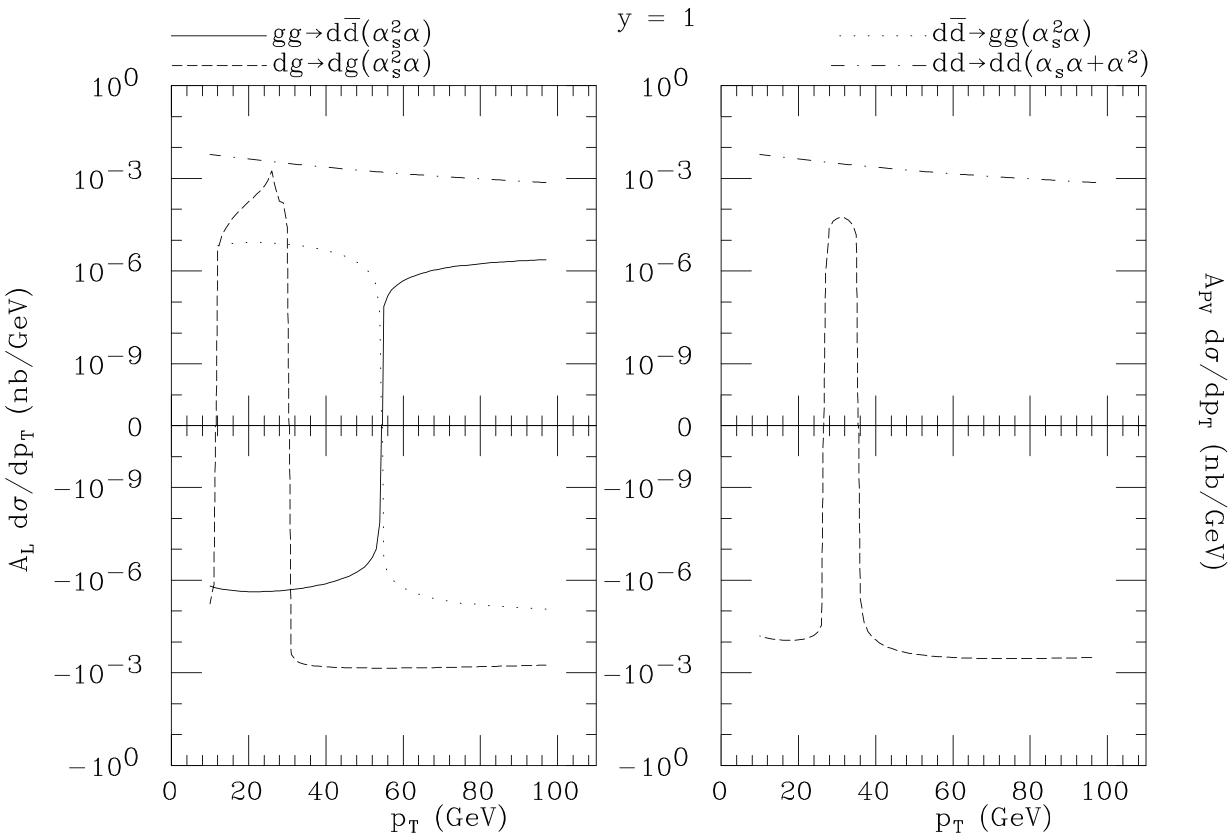

Figure 3: Parton-level asymmetries at rapidity $y = 1$, as functions of the transverse momentum p_T.

is two orders of magnitude smaller than in the quark-gluon channel.

The very strong suppression of both crossed channels (3.4)–(3.5), compared to the subprocess (3.3), irrespective of the rapidity, was not intuitively obvious to us. It is in part responsible for the smallness of the overall corrections at the hadron level, discussed below. Finally, we note the overall normalization of all the curves, namely nanobarn/GeV, which takes into account the relative weights of the various subprocesses in the integrated quantities.

4. Asymmetries in polarized proton-proton scattering

In this section we show the same asymmetries, but now in polarized proton-proton scattering, i.e., after folding the parton-level cross sections with polarized parton distribution functions. As examples, we take the latter from the LO set A of Ref. [11] (GS) (see also [12])[3] at two collider energies, $\sqrt{s}$ = 300 and 600 GeV, representative of the RHIC-Spin programme. As already mentioned, one might *a priori* have thought

[3]The parton distribution functions are here evaluated at the scale $Q = \sqrt{\hat{s}}$, i.e., the centre-of-mass (CM) energy at the parton level. The same choice is made for the argument of α_s.

that the effect at the parton level discussed in the preceding section could be enhanced by a substantial contribution due to the asymmetry in the polarized gluon distribution function. In fact, we show that this is not the case. Qualitatively, one can understand this effect from the fact that the asymmetries at the parton level were only significant at large values of transverse momentum and/or rapidity. These kinematic regions probe the large x spectrum of the distributions, where the gluon content of the proton is no longer significant.

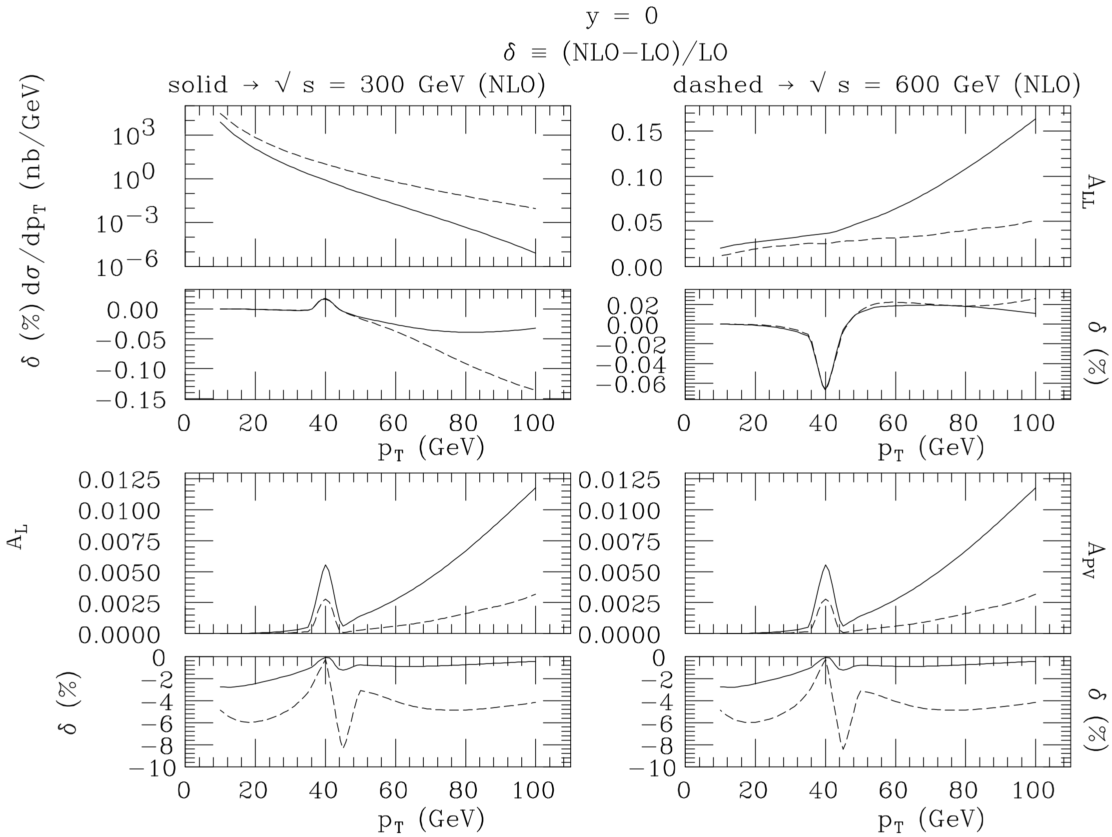

Figure 4: Differential cross sections and asymmetries in polarized proton-proton scattering at rapidity $y = 0$, as functions of the transverse momentum p_T.

In Fig. 4 we present the differential cross sections and asymmetries for zero rapidity. Beneath each plot, we also show the percentage contribution due to the one-loop weak corrections, normalised to the LO rate. Each graph has two sets of curves, corresponding to the two mentioned CM energies, as indicated.

The top-left graph is the total differential cross-section, which is dominated by LO QCD. Here, as expected, the one-loop weak corrections are of the order of 0.1%. The top-right graph shows another asymmetry, A_{LL}, defined as

$$A_{LL}\, d\sigma \;\equiv\; d\sigma_{++} - d\sigma_{+-} + d\sigma_{--} - d\sigma_{-+}\,, \tag{4.1}$$

which is also present in QCD and depends entirely on the asymmetry of the parton distributions. Here we note that the contributions from the one-loop weak corrections are even smaller than for the differential cross-section itself, at both energies.

The lower two graphs show the parity-violating asymmetries, normalized to the total cross section, so that the dimensionless quantities A_L and A_{PV} are shown. We have normalised A_{LL} in the same way. We see that the effect of the weak corrections to quark-gluon scattering is generally to reduce the two asymmetries by a few percent, i.e., of the same order as was found at the parton level.

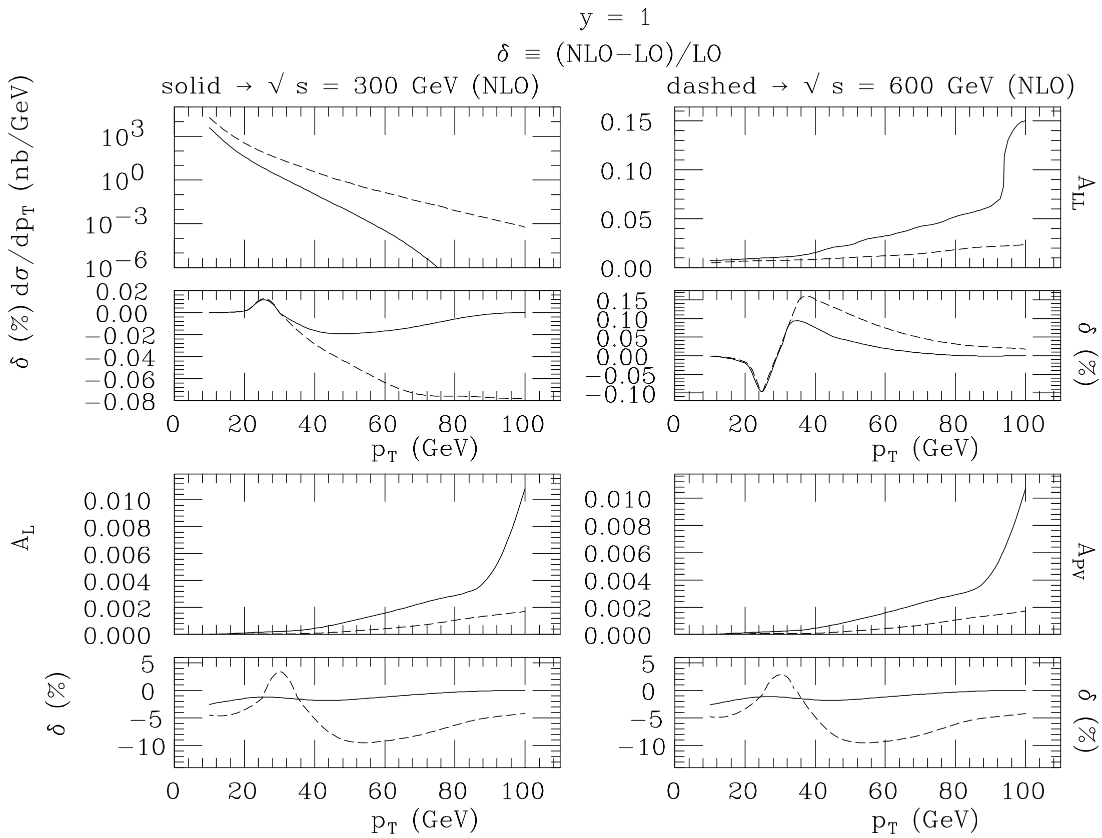

Figure 5: Differential cross sections and asymmetries in polarized proton-proton scattering at rapidity $y = 1$, as functions of the transverse momentum p_T.

Fig. 5 plots the same quantities as in Fig. 4, but now at rapidity $y = 1$. Here is where one gets the largest effects at the one-loop level, as the weak corrections can reduce the parity-violating asymmetries by up to about 10–12%. The effects on the total rate and on A_{LL} are of the same order as at lower rapidity.

Finally, in Fig. 6, we display the usual selection of curves, but now integrated over rapidity, from $y = 0$ to $|y| = 1$. The shapes modulate between the extreme trends seen in the two previous figures. The salient features already appreciated at fixed rapidities, that one-loop weak corrections are larger at higher energy and carry

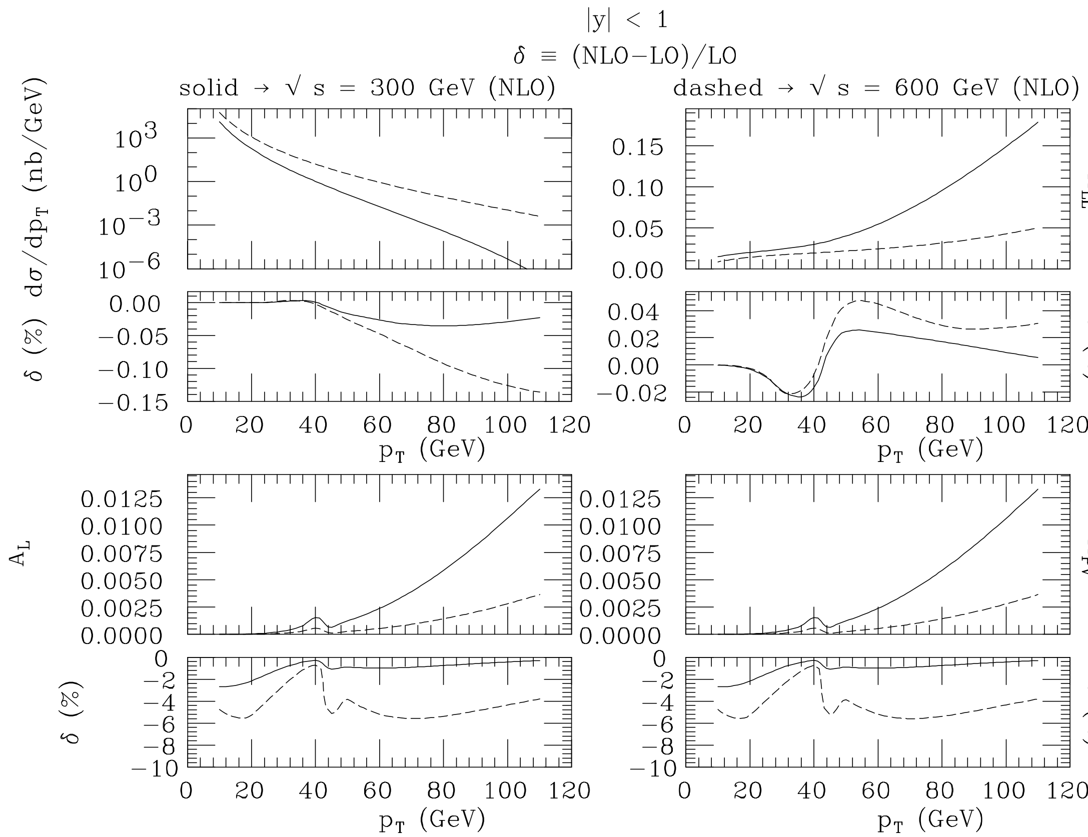

Figure 6: Differential cross-sections and asymmetries in polarized proton-proton scattering integrated over the rapidity range $|y| < 1$, as functions of the transverse momentum p_T.

some visible structure induced by the $W-$ or $Z-$mass threshold effects, remain clearly visible.

5. Summary and conclusions

We have calculated the one-loop weak corrections to the quark-gluon scattering differential cross section, as a function of the transverse momentum and for selected values of rapidity, and crossed channels. We have evaluated the corresponding contributions to parity-violating asymmetries in polarized proton-proton scattering at energies relevant to RHIC-Spin. We have found that in certain kinematic regions these higher-order effects can be as large as 10% of the tree-level contributions to some parity-violating asymmetries. We recall that the study of these asymmetries has gathered particular attention lately [4], both because they are sensitive to the helicity-dependent components of the parton distribution functions, and because they can carry the distinctive hallmark of new physics.

It turns out that, the larger the rapidity and the collider energy, the bigger the effect of the one-loop weak corrections, with some resonant enhancement in the transverse momentum spectrum in the vicinity of the $W-$ and $Z-$mass thresholds. These corrections are dominated by the contributions induced by quark-gluon scattering, as both quark-antiquark annihilation into two gluons and gluon-gluon scattering to a quark-antiquark pair are considerably suppressed in comparison. In fact, at the parton level, one-loop corrections via quark-gluon scattering can be comparable to the tree-level contribution due to electroweak processes involving four external (anti)quarks.

These parton-level effects are not enhanced when one folds the parton-level cross sections with polarized distribution functions. The main reason is that the effects on the parity-violating asymmetries are only large for large values of the Bjorken x variable, where the gluon distribution function is generally small.

In order to check whether our results depend on the particular polarized distribution functions we used, we have repeated our exercises using different sets [13], and find no significant variations. We find that the central qualitative result, namely that the contribution from quark-gluon scattering contributes around 10% to the parity-violating asymmetries A_L and A_{PV}, is unchanged. Nevertheless, we note that at the higher CM energy of 600 GeV, there is some sensitivity to the choice of polarized parton distributions. For the central rapidity case, the magnitude of the quark-gluon contribution is diminished by around 2% if one uses the polarized distribution functions of GRSV [13] or set C of GS [11]. For larger rapidity, $y = 1$, the GSRV distributions and sets A,B of the GS distributions give very similar results, whereas set C of GS leads to substantial enhancements of the parity-violating asymmetries, namely up to -25%, in the region of the mass threshold $p_T \sim M_W$. Set C of [11] differs qualitatively from the other sets in that the contribution to the proton spin from the gluons is *negative* over a substantial range of x.

Our calculations can readily be extended to evaluate similar corrections to direct photon production, $\overset{(-)}{q} + g \to \overset{(-)}{q} + \gamma$ and $\bar{q} + q \to g + \gamma$. Besides these processes of potential interest to RHIC-Spin, the generalization to the case of three-jet production in e^+e^- collisions, namely the one-loop weak corrections to $\gamma^*/Z^* \to q\ +\ \bar{q}\ +\ g$, is also relatively straightforward. This is especially worthy of attention, given that the NNLO QCD corrections to this process are also expected to become available soon, and in view of the copious three-jet data samples furnished by LEP. Presumably though, the one-loop weak corrections will be suppressed at the Z^0 peak by a factor $G_F Q^2$, where Q is a typical virtuality scale of order M_{Z^0}. Hence, they would be relatively more important at LEP 2 or at a higher-energy e^+e^- linear collider.

Acknowledgements

We are grateful to Dima Bardin, Stefano Catani, Stefan Dittmaier and Thomas Gehrmann for several fruitful discussions.

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