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Recent Result from E821 Experiment on Muon $g-2$ and Unconstrained Minimal Supersymemtric Standard Model

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Abstract

Recently, the E821 experiment at the Brookhaven National Laboratory announced their latest result of their muon $g-2$ measurement which is about 2.6-σ away from the standard model prediction. Taking this result seriously, we examine the possibility to explain this discrepancy by the supersymmetric contribution. Our analysis is performed in the framework of the *unconstrained* supersymmetric standard model which has free seven parameters relevant to muon $g-2$. We found that, in the case of large $\tan\beta$, sparticle masses are allowed to be large in the region where the SUSY contribution to the muon $g-2$ is large enough, and hence the conventional SUSY search may fail even at the LHC. On the contrary, to explain the discrepancy in the case of small $\tan\beta$, we found that (i) sleptons and $SU(2)_L$ gauginos should be light, and (ii) negative search for the Higgs boson severely constrains the model in the framework of the mSUGRA and gauge-mediated model.

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The latest result of the E821 experiment at Brookhaven National Laboratory on the muon anomalous magnetic moment $g_\mu - 2$ [1]

$$a_\mu(\text{E821}) \equiv (g_\mu - 2)/2 = 11659202(14)(6) \times 10^{-10} \tag{1}$$

indicates a possible confrontation with the standard model of particle physics. The deviation between the experimental value and the standard model prediction is

$$a_\mu(\text{E821}) - a_\mu(\text{SM}) = 43(16) \times 10^{-10}, \tag{2}$$

or 2.6-σ deviation [1]. In order to fill up the discrepancy, a new physics beyond the standard model is called for. The apparent deviation is comparable to or even larger than the contribution from the standard model electroweak sector computed as $a_\mu(\text{SMEW}) = 15.1(0.4) \times 10^{-10}$ [2]. This suggests that the energy scale of the new physics should be very close to the electroweak scale and/or it should have some enhancement mechanism to give a large contribution to a_μ.

Among other things, supersymmetry (SUSY) is the most promising candidate for such a new physics. To solve the naturalness problem in the Higgs sector of the standard model, the superparticle masses should lie below the TeV scale. The SUSY contribution to a_μ has been investigated in the literatures (see [3–10] and references therein).[1] Generically it is sizable for superparticles weighing less than 1 TeV. And as we will explain shortly, it is enhanced for large $\tan\beta$ region, where $\tan\beta$ is the ratio of the vacuum expectation values (VEVs) of the two Higgs bosons in the SUSY standard model.

The purpose of this paper is to reexamine the contribution to a_μ in the framework of the *unconstrained* minimal supersymmetric standard model (MSSM) in the light of the recently reported experimental data. The unconstrained MSSM has more than 100 SUSY breaking parameters and one usually impose some relations among the model parameters: otherwise it would be very difficult to trace all dependence of the parameters for some specific processes. The SUSY contribution to a_μ, however, depends only on seven MSSM parameters as we will list below, and thus we can leave them as free parameters to analyze their dependence. Another important point that makes the model independent analysis possible is that, unlike flavor changing neutral current (FCNC) processes, we do not rely on particular mechanisms to suppress SUSY FCNC to compute the SUSY contribution to a_μ. Conclusions we will draw are therefore very general.

The apparent deviation from the standard model prediction implies a non-vanishing SUSY contribution. We will identify the parameter region of the MSSM which is capable to account for the discrepancy (2) at 2-σ. Explicitly we require the SUSY contribution to lie in the following range,

$$11 \times 10^{-10} < a_\mu(\text{SUSY}) < 75 \times 10^{-10}. \tag{3}$$

The SUSY contribution to a_μ decreases as the superparticles become heavier. Since a non-vanishing SUSY contribution is needed, we will obtain an upper bound on the mass scale

[1] See Ref. [11,12] for scenarios of large extra dimensions.

of the superparticles. For large $\tan\beta$ region, the enhancement mechanism works and (3) is easily satisfied even for relatively large superparticle masses. We will find that the Wino as heavy as 1 TeV can be compatible with it. Another point we pay a particular attention is the case of low $\tan\beta$. We will show that for $\tan\beta \gtrsim 3$ the SUSY contribution can be large enough to explain the deviation, without confronting the present bounds on the superparticle masses obtained by negative searches of superparticles at collider experiments.

At one-loop level, the SUSY contribution to a_μ stems from chargino-sneutrino loops as well as from neutralino-smuon loops. Formulae of the SUSY loop contributions are given, for example, in Ref. [5]. In the (unconstrained) MSSM, the parameters involved with this process are the soft SUSY breaking mass parameters for the right-handed and left-handed smuons denoted by $m_{\tilde{\mu}_R}$ and $m_{\tilde{\mu}_L}$, respectively, the trilinear scalar coupling for muon A_μ, the $U(1)_Y$ and $SU(2)_L$ gaugino mass parameters M_1 and M_2, the higgsino mass parameter μ, and $\tan\beta$ which is the ratio of the VEVs of two Higgs bosons. Throughout this paper, we take the parameters to be real, and do not include possible CP phases which are known to be generically small due to stringent limits on electric dipole moments. (See, however, Ref. [13].) For most of the analysis we impose the GUT relation for the gaugino masses $M_1 : M_2 \approx 1 : 2$. We checked that our following results are almost unchanged even if the GUT relation is relaxed. In the MSSM analysis, we do not consider any particular SUSY breaking scenario, and thus we do not introduce specific relations among the remaining six parameters mentioned above. For comparison, we will briefly discuss the case of the minimal supergravity scenario (mSUGRA) later on. Furthermore we do not impose the lightest superparticle (LSP) in the MSSM sector to be neutral. A charged LSP would be ruled out if it were stable, but there are many ways out, including R-parity violation and light gravitino LSP.

For generic SUSY mass parameters, it is known that the chargino-sneutrino diagram gives a dominant contribution to a_μ(SUSY). Then the relevant parameters to calculate a_μ(SUSY) are $m_{\tilde{\mu}_L}$, M_2, μ and $\tan\beta$. Thus it will be reasonable to fix the other mass parameters for some specific values for the moment. Here we take $A_\mu = 0$ and assume the GUT relation to the gaugino masses.[2] The sign of the SUSY contribution to a_μ is directly correlated with the sign of $M_2\mu$ in most of the parameter regions. It is positive (negative) for $M_2\mu > 0$ ($M_2\mu < 0$). Thus, the result of the E821 experiment given in Eq. (1) suggests $M_2\mu > 0$, and hence we consider the positive sign case in the following.

Another important point is that the SUSY contribution to a_μ is enhanced for large $\tan\beta$ because, in the dominant diagrams, muon chirality is flipped by the muon Yukawa coupling $y_\mu \propto 1/\cos\beta \sim \tan\beta$, not by the muon mass itself [3–5]. Thus, for the large $\tan\beta$ case, the SUSY contribution can be large enough to explain the discrepancy even with relatively heavy superparticles. On the contrary, when $\tan\beta$ is small, Wino and slepton masses should be light to make the SUSY contribution to a_μ large enough.

In the framework of the unconstrained MSSM, we calculate an upper bound on the lighter smuon mass $m_{\tilde{\mu}1}$ as a function of the Wino mass parameter M_2. The result is shown in Fig. 1. In deriving the upper bound, we vary $m_{\tilde{\mu}_R}$ and $m_{\tilde{\mu}_L}$, and derive the largest possible

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[2]We checked that the value of a_μ(SUSY) is insensitive to the value of A_μ unless A_μ is extremely large.

value of $m_{\tilde{\mu}1}$ which can realize the 2-σ bound (3). In Fig. 1, we take $\tan\beta = 3$, 5, 10, 30, and we fixed $\mu = 500$ GeV as a typical example. We checked that the lighter smuon is mostly the left-handed one which consists of the $SU(2)_L$ doublet with the sneutrino as it is involved with the dominant chargino-sneutrino loop. We find from this figure that quite a large parameter region is in accord with the consideration of the recent data of a_μ. Thus we conclude that the SUSY is naturally able to explain the apparent discrepancy observed at the experiment.

Fig. 1 also shows that the constraint on the lighter smuon mass $m_{\tilde{\mu}1}$ is much stronger than that on the Wino mass M_2. For instance, for $\tan\beta = 10$, the smuon must be lighter than about 400 GeV while the Wino as heavy as 1 TeV is allowed at 2-σ.

Let us now closely look at the large $\tan\beta$ case. In this case, the SUSY contribution is enhanced as $\tan\beta$ increases. Thus one can expect that even heavy superparticles can be compatible with the lower bound $a_\mu(\text{SUSY}) = 11 \times 10^{-10}$. In Fig. 2, we show a plot of the 2-σ upper bounds on $m_{\tilde{\mu}1}$ as a function of the μ parameter. Here we take several values of $\tan\beta$ and $M_2 = 1$ TeV. Even with such a large Wino mass, we find that the 2-σ constraint can be satisfied with $\tan\beta$ as small as 10 when the slepton mass is lighter than about 200 GeV. Applying the GUT relation of the gaugino masses to the gluino mass as well, the Wino mass $M_2 = 1$ TeV corresponds to the gluino mass of about 3 TeV. Namely the accuracy of the present a_μ data still allows the possibility of such a heavy gluino in the framework of the unconstrained MSSM. The Large Hadron Collider (LHC) experiment will not be able to reach such a heavy gluino, and thus SUSY searches at the LHC would require unconventional approaches. A future analysis of a_μ with more statistics will further reduce the error in the measurement, which may constrain the superparticle masses within the reach of the LHC.

Let us turn to the case of low $\tan\beta$. In this case the enhancement mechanism operated at large $\tan\beta$ is not effective and thus one may expect that the SUSY contribution is rather small. The case of $\tan\beta = 3$ in Fig. 1 shows, however, that the SUSY contribution can explain the deviation at 2-σ level when the SUSY mass parameters are close to their experimental bounds.[3]

Next we would like to discuss the mass of the lightest scalar Higgs boson m_h. It is known that small $\tan\beta$ tends to give relatively light m_h because the tree-level contribution to m_h^2 is roughly given as $m_Z^2 \cos^2 2\beta$. To survive the Higgs mass bound obtained at LEP 200, which approaches 113.5 GeV as the pseudo-scalar Higgs mass increases [15], large radiative correction from top and stop loops is required [16]. Thus the stop masses should be large enough. Roughly speaking, the mass bound is satisfied for small $\tan\beta \approx 3$ if the stop masses exceed about 1 TeV. The relation between the slepton masses which are constrained by the a_μ analysis and the squark masses is highly model dependent.

To give more quantitative arguments on the light Higgs mass, we consider the case of the mSUGRA. In Figs. 3 and 4, we plotted contours of 0-σ, 1-σ, and 2-σ preferred values of

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[3]Existence of light superparticles may affect the fit to the electroweak precision data, which was considered in Ref. [14] in the parameter region where $a_\mu(\text{SUSY})$ is sizable. According to Ref. [14], there is some region of light higgsino dominant LSP case in which the inclusion of SUSY particles gives a better fit to the electroweak precision data than the standard model alone.

a_μ(SUSY) on m_0 vs. M_2 plane, where m_0 is the universal scalar mass at the GUT scale and $M_{1/2}$ is the universal gaugino mass at the same scale. (The Wino mass at the electroweak scale is related to $M_{1/2}$ as $M_2 \approx 0.83 M_{1/2}$.) In this framework, we also calculated the lightest Higgs mass m_h, and plotted the contours of constant m_h on the same figures. In the figures the trilinear scalar coupling is taken to be zero. As one can see, for the case of $\tan\beta = 5$, the constraint $m_h \geq 113.5$ GeV severely constrains the parameter region which gives preferred value of a_μ(SUSY). Taking account of the Higgs mass constraint, however, the 2-σ constraint (3) can be realized with $\tan\beta \gtrsim 5$. It is interesting to see that the region with small universal scalar mass is favored. Although the region with $m_0 < 100$ GeV is not shown in the figures, we checked that 2-σ constraint and the Higgs mass constraint can be simultaneously satisfied in the limit of $m_0 \to 0$ (i.e., with the no-scale type boundary condition). For smaller $\tan\beta$, a_μ(SUSY) cannot be large enough to explain the discrepancy at the 2-σ level. For large $\tan\beta$, on the contrary, the Higgs mass can be easily large enough in the region with sufficient a_μ(SUSY).

Here, let us comment on the allowed parameter region in the mSUGRA case. We find that the allowed parameter region is rather tighten in the mSUGRA scenario. To illustrate this point, let us consider the case of $\tan\beta = 5$. In the unconstrained MSSM, the 2-σ bound constrains $M_2 \lesssim 600$ GeV. On the other hand, in the mSUGRA case, larger value of $M_{1/2}$ results in larger slepton mass through renormalization group effect. As a result, the 2-σ bound in the mSUGRA case is $M_{1/2} \lesssim 300$ GeV which corresponds to $M_2 \lesssim 250$ GeV.[4] The difference will be crucial when one considers the discovery potential of superparticles at hadron colliders.

To make a comparison, we also consider a case of the gauge-mediated SUSY breaking (GMSB) scenario whose free parameters are the overall scale of the soft-breaking parameters $\Lambda_{\rm GM}$, the messenger scale $M_{\rm mess}$, $\tan\beta$, and the number of the $\mathbf{5} + \bar{\mathbf{5}}$ messenger multiplets $N_{\rm mess}$ [17]. The overall scale is related to the Wino mass as

$$M_2 = \frac{N_{\rm mess} g_2^2}{16\pi^2} \Lambda_{\rm GM}, \tag{4}$$

with g_2 being the $SU(2)_L$ gauge coupling constant. In our analysis, we use M_2 instead of $\Lambda_{\rm GM}$ using this relation. In Fig. 5, we show the region which is consistent with the 1-σ and 2-σ constraint on M_2 vs. $\tan\beta$ plane. Here, we take $M_{\rm mess} = 10^6$ GeV and $N_{\rm mess} = 1$. In addition, we also plot the constant m_h contour. We can see that a_μ(SUSY) can be within the 2-σ bound with $\tan\beta \gtrsim 5$ even with the Higgs mass constraint $m_h \geq 113.5$ GeV. We checked that this result is insensitive to the choices of $M_{\rm mess}$ and $N_{\rm mess}$.

Finally, let us briefly comment on the branching ratio of $b \to s\gamma$. To this process, the SUSY contribution is dominated by the stop-chargino loops. It is well-known that it can interfere with the standard model and charged Higgs contribution constructively or destructively, depending on the relative sign of the μ parameter and the trilinear scalar coupling of stop A_t. Furthermore, in most cases the sign of A_t is essentially determined

[4] We expect that a similar tight bound on M_2 can be obtained in a wide class of models where the supersymmetry breaking effect is mediated at high energy scale, and thus the left-handed smuon mass suffers from the renormalization effect of $SU(2)_L$ gaugino.

by the sign of the gluino mass M_3 via large renormalization group effect. It is found that for $M_3\mu < 0$ the SUSY contribution is constructive, which is in fact disfavored because the branching ratio $B(b \to s\gamma)$ tends to be predicted too large compared to the experimental value. Thus the $b \to s\gamma$ consideration favors the $M_3\mu > 0$ case. On the other hand, the positive contribution to the a_μ is obtained when the sign of $M_2\mu$ is positive, provided that the chargino loop dominates over the neutralino loops. Thus the case that the Wino and gluino have masses with the same sign is favored by the combined consideration of a_μ and $B(b \to s\gamma)$. This is the case in many models of SUSY breaking. In particular, models with the GUT relation of the gaugino masses fall into this category, including the mSUGRA and the GMSB. On the other hand, the case with the opposite sign of the Wino and gluino masses is disfavored, which is in fact the case in a simple class of anomaly mediated supersymmetry breaking models [7].

In this paper, we considered implications of the recently reported a_μ measurement at the E821 experiment to supersymmetric standard models. We made the analysis mainly based on the unconstrained MSSM. Since the SUSY contribution should be non-vanishing at 2-σ level, the upper bounds on the superparticle masses were obtained. For large $\tan\beta$, the superparticles can be quite heavy, which may be escaped from the LHC gluino reach. We observed that the allowed region of the superparticles masses is significantly larger than the case of constrained models such as the mSUGRA model. On the contrary for smaller $\tan\beta$, the Wino mass as well as the lighter slepton mass should be light. In this case, the bound on the Higgs boson mass obtained at LEP200 gives stringent constraints. We illustrated this point in the mSUGRA model as well as the GMSB model, yielding $\tan\beta \gtrsim 5$ for both models.

Given the upper bounds on the masses of the sleptons and the charginos/neutralinos, lepton flavor violation such as $\mu \to e\gamma$ as well as μ-e conversion may be observed in near future experiments [18]. Another implication is to proton decay. As we discussed, a model dependent lower bound on $\tan\beta$ is obtained by combining analyses of $a_\mu(\mathrm{SUSY})$ and m_h. This tightens proton decay constraints in SUSY GUT models, though details are quite model dependent. Result of detailed study along this line will be presented elsewhere.

To conclude, we should emphasize the importance of the further reduction of error of the a_μ measurement which is expected to be done in near future, as well as the further study of the uncertainty coming from the hadronic vacuum polarization. We hope that they may sharpen the confrontation with the standard model more clearly in near future and confirm the necessity of the physics beyond the standard model.

While preparing the manuscript, we were aware of the papers: A. Czarnecki and W.J. Marciano, hep-ph/0102122, L. Everett, G.L. Kane, S. Rigolin and L.-T. Wang, hep-ph/0102145, J.L. Feng and K.T. Matchev, hep-ph/0102146, E.A. Baltz and P. Gondolo, hep-ph/0102147, U. Chattopadhyay and P. Nath, hep-ph/0102157, which have some overlap with our analyses.

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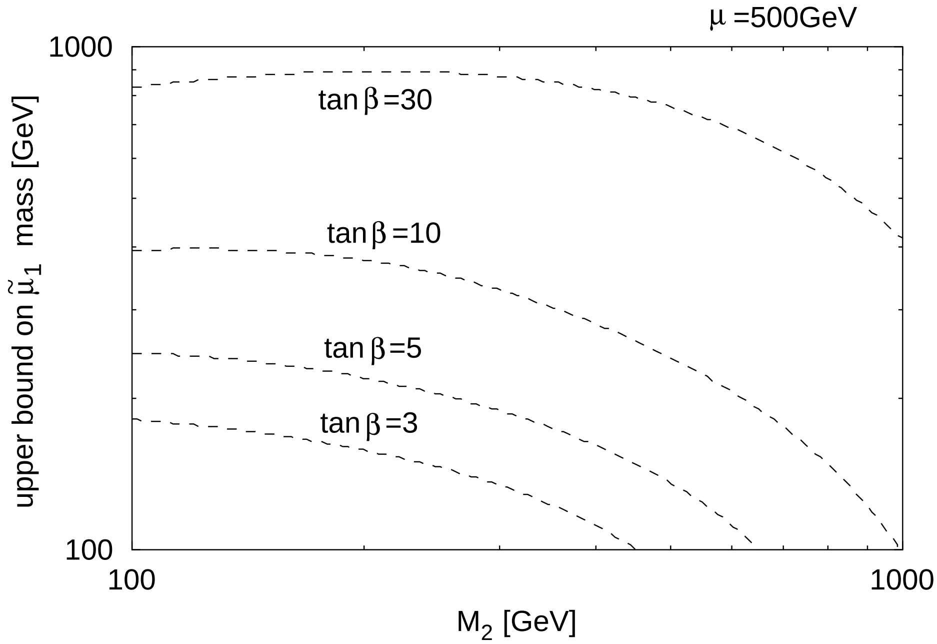

FIG. 1. The upper bounds on the lighter smuon mass $m_{\tilde{\mu}1}$. The horizontal line is the Wino mass parameter M_2, and we take $\mu = 500$ GeV and $A_\mu = 0$. Here, $\tan\beta$ is taken to be 3, 5, 10, and 30 from below.

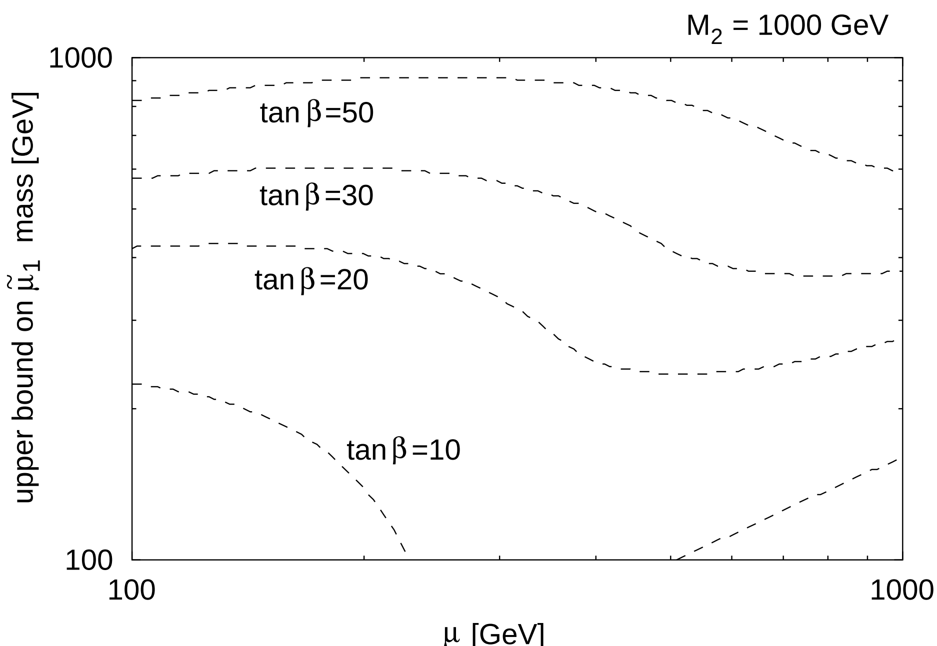

FIG. 2. The 2-σ upper bounds on the lighter smuon mass $m_{\tilde{\mu}1}$. The horizontal line is the Higgsino mass parameter μ, and we take $M_2 = 1$ TeV, $A_\mu = 0$, and $\tan\beta = 10$, 20, 30, 50.

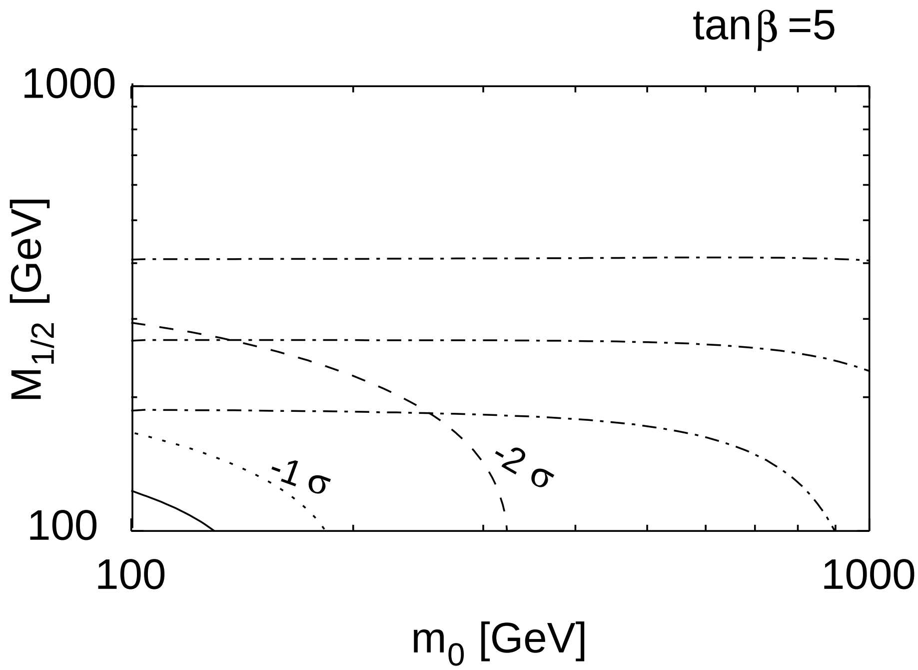

FIG. 3. Contours of the constant $a_\mu(\mathrm{SUSY})$ on m_0 vs. M_2 plane in the minimal supergravity model. The solid (dotted, dashed) line is for the center (± 1-σ, ± 2-σ) value of $a_\mu(\mathrm{SUSY})$. We take $A_\mu = 0$ and $\tan\beta = 5$. We also plot the constant m_h contours in the dash-dotted lines ($m_h = 110$, 115, and 120 GeV from below).

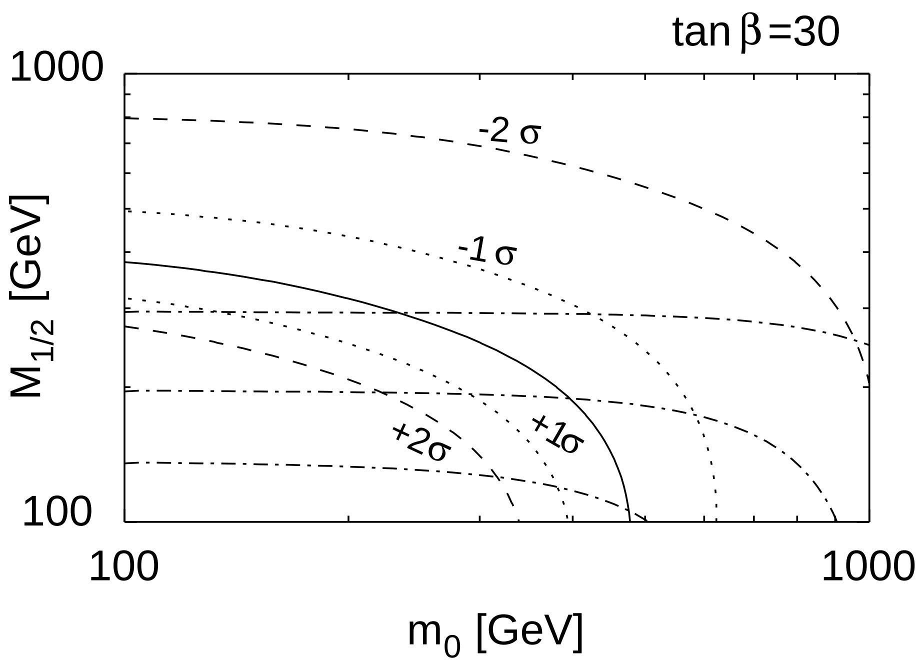

FIG. 4. Same as Fig. 3, except $\tan\beta = 30$.

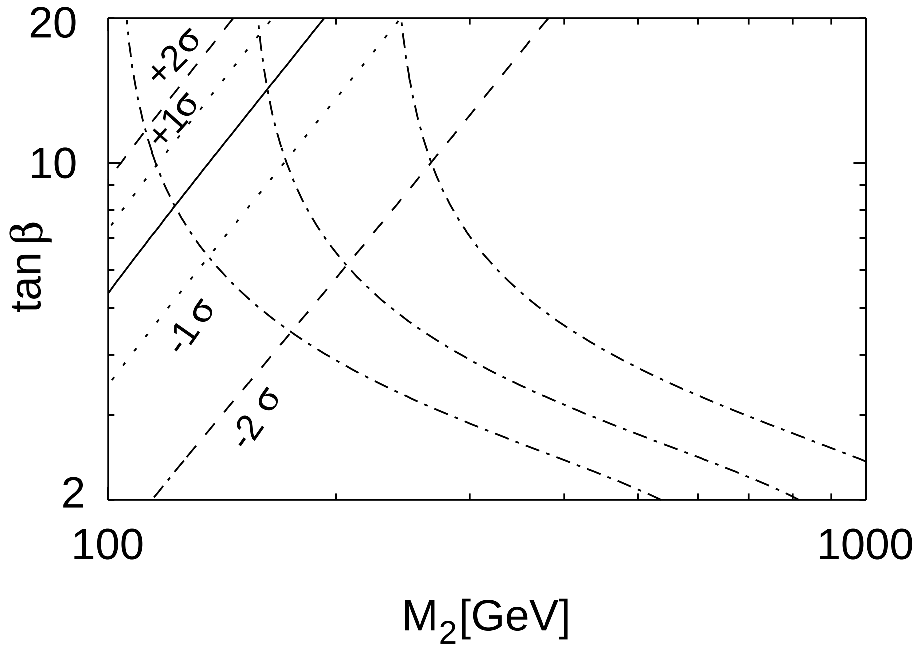

FIG. 5. Contours of the constant a_μ(SUSY) on M_2 vs. $\tan\beta$ plane in the gauge-mediated SUSY breaking scenario. The solid (dotted, dashed) line is for the center (± 1-σ, ± 2-σ) value of a_μ(SUSY). We take $M_{\rm mess} = 10^6$ GeV and $N_{\rm mess} = 1$. We also plot the constant m_h contours in the dash-dotted lines. ($m_h = 110$, 115, and 120 GeV from below).