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Testing neutrino mixing at future collider experiments

W. Porod[1], M. Hirsch[1,2], J. Romão[3], and J.W.F. Valle[1]

[1]Inst. de Física Corpuscular (IFIC), CSIC - U. de València,

Edificio de Institutos de Paterna, Apartado de Correos 22085

E-46071–València, Spain

[2]Department of Physics and Astronomy, University of Southampton,

Highfield, Southampton SO17 1BJ, England

[3] Departamento de Física, Instituto Superior Técnico

Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

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Abstract

Low energy supersymmetry with bilinear breaking of R-parity leads to a weak-scale seesaw mechanism for the atmospheric neutrino scale and a radiative mechanism for the solar neutrino scale. The model has striking implications for collider searches of supersymmetric particles. Assuming that the lightest SUSY particle is the lightest neutralino we demonstrate that (i) The neutralino decays inside the detector even for tiny neutrino masses. (ii) Measurements of the neutrino mixing angles lead to *predictions* for the ratios of various neutralino branching ratios implying an independent test of neutrino physics at future colliders, such as the Large Hadron Collider or a Linear Collider.

1 Introduction

The simplest interpretation [1] of recent solar and atmospheric neutrino data [2, 3, 4] indicates that neutrinos are massive and, in contrast to the case of quarks, at least one and possibly two of the lepton mixing angles are large. The possibility of testing for these angles at high energy colliders seems very intriguing [5, 6, 7] especially in view of the new generation of colliders such as the LHC and a new high energy linear collider.

One of the simplest ways to induce neutrino masses is if right-handed neutrinos, singlets under the $SU(2) \otimes U(1)$ SM gauge group exist, as expected in a number of extended electroweak models and grand unified theories (GUTs) [8, 9, 10]. In this case there are renormalizable neutrino Dirac masses similar to those of the charged fermions and, in addition, potentially large Majorana mass terms for the right-handed neutrinos. This leads to a neutrino mass matrix of the form

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \tag{1}$$

which has as eigenvalues

$$\lambda_{1,2} = \frac{M}{2} \mp \frac{\sqrt{M^2 + 4m^2}}{2} \tag{2}$$

where $m = h^\nu_{ij} \langle H \rangle$ with h^ν_{ij} denoting a Dirac-type Yukawa coupling for neutrinos and $\langle H \rangle$ the vacuum expectation value (vev) of the SM Higgs field. It is also assumed that right handed neutrinos have a large mass term specified by M. This so-called seesaw-mechanism [11] provides a general recipe to generate neutrino masses. For the simplest case $m/M \ll 1$ and just one generation one easily obtains that $\lambda_1 \simeq -m^2/M$ while $\lambda_2 \simeq M + m^2/M$ corresponds to a heavy right-handed state[1]. Typically the small neutrino masses required by the interpretation of present neutrino data correspond to values of M in a wide range above 10^9 GeV or so. The parameters can be adjusted in such a way that they neatly explain neutrino experiments but they are far from being predictive. They mainly 'postdict' experimental data and lead to predictions, if any, which are confined to the domain of "neutrino" experiments, performed at underground installations, reactors or accelerators and possibly neutrino factories. They hardly imply any signatures that may be tested at high energy collider experiments such as the LHC.

An alternative possibility for providing neutrino masses exists in which M is of order m_Z and that m is thus rather small. It is based on the idea of R-parity violation as origin of

[1] For detailed results on diagonalization of seesaw neutrino mass matrices see [12].

neutrino mass and mixing [5, 10, 13]. These models can have many implications for gauge and Yukawa unification, low-energy physics [14, 15, 16] as well as a variety of implications for future collider experiments at high energies [7, 17, 18]. The simplest model in which this is realized is an extension of the Minimal Supersymmetric Standard Model (MSSM) [19] with bilinear R-parity breaking terms in the superpotential [14, 15].

These bilinear terms lead to non-vanishing vevs for the sneutrinos which in turn induce a mixing between neutrinos and neutralinos [5, 13]. This leads to an effective neutrino mass matrix which is projective [5, 20] implying that only one neutrino receives mass at the tree-level. This provides for a way to account for the atmospheric neutrino problem. For the explanation of the solar neutrino puzzle one has to perform a 1-loop calculation of the neutrino/neutralino mass matrix in order to break the projectivity of the mass matrix [6, 21]. The net effect is a hybrid scheme in which the atmospheric scale is generated by a weak-scale seesaw mechanism characterized by a mass scale is of order 10^3 GeV, while the solar neutrino scale arises from genuine loop corrections [6, 21].

In the bilinear $R\!\!\!/_p$ model not only the neutrino masses but also the neutrino mixing angles are predicted in terms of the three underlying $R\!\!\!/_p$ parameters [6, 21]. These same parameters also determine the decay properties of the lightest supersymmetric particle (LSP) which we assume is the lightest neutralino. This implies the existence of simple correlations between neutrino mixing angles and neutralino decay branching ratios, as already partly observed in the literature [6, 7]. Note that in [7] mainly qualitative statements have been made considering the neutrino mass only at the tree-level. In the present paper we present a quantitative approach to this problem which takes into account the complete 1-loop calculation of neutrino/neutralino mass and couplings. This is required in order to make predictions for neutralino decays. We have taken into account also LSP decays via real and virtual Higgs bosons, which have been so far neglected in the literature. Their contribution can be more important than the one of the Z-boson if the lightest neutralino is mainly gaugino-like as preferred by SUGRA scenarios and if third-generation fermions are present in the final state. This has some remarkable implications as we are going to demonstrate.

Notice that we have ignored the results of the LSND experiment [22] which would point to the existence of four neutrinos in Nature [23] one of which is sterile. Actually it is simple to extend our $R\!\!\!/_p$ model in order to include a light $SU(2) \otimes U(1)$ singlet superfield [24]. The fermion present in this superfield is the sterile neutrino, which combines with one linear combination of $\nu_e - \nu_\mu - \nu_\tau$ to form a Dirac pair whose mass accounts for the LSND anomaly. On the other hand the sterile neutrino scalar partner can trigger the spontaneous violation of

R-parity, thereby inducing the necessary mass splittings to fit also the solar and atmospheric neutrino data. This way the model can explain all neutrino oscillation data leading to four predictions for the neutrino oscillation parameters. However, for simplicity and for definiteness we will focus here on the simpler scenario with only the standard three light neutrinos.

The paper is organized as follows: In Sec. 2 we present the model and discuss approximative formulas for some R-parity violation couplings. In Sec. 3 we discuss the phenomenology of the lightest neutralino at future accelerator experiments and in Sec. 4 we discuss in detail the relationship between neutrino-physics and neutralino-physics in this model. This includes predictions for neutralino decays before SUSY is discovered and various cross-checks after the SUSY spectrum is known. In Sec. 5 we draw our conclusions. In Appendix A we give some formulas for approximate diagonalization of the generalized Higgs matrices.

2 The model

R-parity conservation is an `ad hoc` assumption in the MSSM and $\not{R}_p$ may arise either as unification remnant [25] or through $SU(2) \otimes U(1)$ doublet left sneutrino vacuum expectation values (vevs) $\langle \widetilde{\nu}_i \rangle$ [13]. Preferably we break R-parity spontaneously through `singlet right sneutrino vevs`, either by gauging L-number, in which case there is an additional Z [26] or within the $SU(2) \otimes U(1)$ scheme, in which case there is a physical `majoron` [27]. In order to comply with LEP data on Z width we must assume that the violation of R-parity is driven by $SU(2) \otimes U(1)$ `singlet` sneutrino vevs [28] since in this case the majoron has a suppressed coupling to the Z. Spontaneous R-parity violation may lead to a successful electroweak baryogenesis [29].

As long as the breaking of R-parity is spontaneous then `only bilinear` $\not{R}_p$ `terms arise in the effective theory` below the $\not{R}_p$ violation scale. Bilinear R–parity violation may also be assumed `ab initio` as the fundamental theory. For example, it may be the only violation permitted by higher Abelian flavour symmetries [30]. Moreover the bilinear model provides a theoretically self-consistent scheme in the sense that trilinear $\not{R}_p$ implies, by renormalization group effects, that also bilinear $\not{R}_p$ is present, but `not` conversely.

The simplest $\not{R}_p$ model (we call it $\not{R}_p$ MSSM) is characterized by three independent parameters in addition to those specifying the minimal MSSM model. Using the conventions of refs. [19, 31] the model is specified by the following superpotential [14],

$$W = \varepsilon_{ab}\left[h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_u^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_d^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_d^a - \mu\widehat{H}_d^a\widehat{H}_u^b + \epsilon_i\widehat{L}_i^a\widehat{H}_u^b\right] \tag{3}$$

where the couplings h_U, h_D and h_E are 3×3 Yukawa matrices and μ and ϵ_i are parameters with units of mass. The second bilinear term in eq. (3) includes R–Parity and lepton number violation in three generations.

Supersymmetry breaking is parameterized with a set of soft supersymmetry breaking terms. In the MSSM these are given by

$$\begin{aligned}\mathcal{L}_{soft}^{MSSM} = \; & M_Q^{ij2}\widetilde{Q}_i^{a*}\widetilde{Q}_j^a + M_U^{ij2}\widetilde{U}_i\widetilde{U}_j^* + M_D^{ij2}\widetilde{D}_i\widetilde{D}_j^* + M_L^{ij2}\widetilde{L}_i^{a*}\widetilde{L}_j^a + M_R^{ij2}\widetilde{R}_i\widetilde{R}_j^* \\ & + m_{H_d}^2 H_d^{a*}H_d^a + m_{H_u}^2 H_u^{a*}H_u^a - \left[\tfrac{1}{2}M_s\lambda_s\lambda_s + \tfrac{1}{2}M\lambda\lambda + \tfrac{1}{2}M'\lambda'\lambda' + h.c.\right] \\ & + \varepsilon_{ab}\left[A_U^{ij}\widetilde{Q}_i^a\widetilde{U}_jH_u^b + A_D^{ij}\widetilde{Q}_i^b\widetilde{D}_jH_d^a + A_E^{ij}\widetilde{L}_i^b\widetilde{R}_jH_d^a - B\mu H_d^aH_u^b\right].\end{aligned} \tag{4}$$

In addition to the MSSM soft SUSY breaking terms in $\mathcal{L}_{soft}^{MSSM}$ the BRpV model contains the following extra term

$$V_{soft}^{BRpV} = B_i\epsilon_i\varepsilon_{ab}\widetilde{L}_i^aH_u^b\,, \tag{5}$$

where the B_i have units of mass. In what follows, we neglect intergenerational mixing in the soft terms in eq. (4).

The electroweak symmetry is broken when the two Higgs doublets H_d and H_u, and the neutral component of the slepton doublets $\widetilde{L}_i^1$ acquire vacuum expectation values. We introduce the notation:

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \qquad \widetilde{L}_i = \begin{pmatrix} \tilde{L}_i^0 \\ \tilde{\ell}_i^- \end{pmatrix}, \tag{6}$$

where we shift the neutral fields with non–zero vevs in the usual way

$$H_d^0 \equiv \frac{1}{\sqrt{2}}[\sigma_d^0 + v_D + i\varphi_d^0]\,, \quad H_u^0 \equiv \frac{1}{\sqrt{2}}[\sigma_u^0 + v_U + i\varphi_u^0]\,, \quad \tilde{L}_i^0 \equiv \frac{1}{\sqrt{2}}[\tilde{\nu}_i^R + v_i + i\tilde{\nu}_i^I]\,. \tag{7}$$

Note that the W boson acquires a mass $m_W^2 = g^2v^2/4$, where $v^2 \equiv v_D^2 + v_U^2 + v_1^2 + v_2^2 + v_3^2 \simeq$ (246 GeV)^2. Like in the MSSM we define $\tan\beta = v_U/v_D$.

2.1 Neutralino Mass matrix

In the following we discuss the tree level structure of neutrino masses and mixings. A complete discussion of the 1-loop mass matrix and the other mass matrices in this model can be found in [21]. We present the mass matrix for 2-component spinors and the connection to the four component Majorana neutral fermions are obtained from the two component via the relation

$$\chi_i^0 = \begin{pmatrix} F_i^0 \\ \overline{F_i^0} \end{pmatrix} . \tag{8}$$

In the basis $\psi^{0T} = (-i\lambda', -i\lambda^3, \widetilde{H}_d^1, \widetilde{H}_u^2, \nu_e, \nu_\mu, \nu_\tau)$ the neutral fermion mass matrix $\mathbf{M}_N$ is given by

$$\mathbf{M}_N = \begin{bmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0 \end{bmatrix} \tag{9}$$

where

$$\mathcal{M}_{\chi^0} = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_D & \frac{1}{2}g'v_U \\ 0 & M_2 & \frac{1}{2}gv_D & -\frac{1}{2}gv_U \\ -\frac{1}{2}g'v_D & \frac{1}{2}gv_D & 0 & -\mu \\ \frac{1}{2}g'v_U & -\frac{1}{2}gv_U & -\mu & 0 \end{bmatrix} \tag{10}$$

is the standard MSSM neutralino mass matrix and

$$m = \begin{bmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 \end{bmatrix} \tag{11}$$

characterizes the breaking of R-parity. The mass matrix $\mathbf{M}_N$ is diagonalized by

$$\mathcal{N}^* \mathbf{M}_N \mathcal{N}^{-1} = \mathrm{diag}(m_{\chi_i^0}, m_{\nu_j}) \tag{12}$$

where $(i = 1, \cdots, 4)$ for the neutralinos, and $(j = 1, \cdots, 3)$ for the neutrinos.

We are interested in the case where the neutrino mass which is chosen at tree level is small, since it will be determined in order to account for the atmospheric neutrino anomaly. Under this assumption one can perform a perturbative diagonalization of the neutrino/neutralino mass matrix [12], by defining [20]

$$\xi = m \cdot \mathcal{M}_{\chi^0}^{-1} \tag{13}$$

If the elements of this matrix satisfy $\xi_{ij} \ll 1 \;\forall ij$ then one can use it as expansion parameter in order to find an approximate solution for the mixing matrix $\mathcal{N}$. Explicitly we have

$$\begin{aligned} \xi_{i1} &= \frac{g' M_2 \mu}{2det(\mathcal{M}_{\chi^0})} \Lambda_i \\ \xi_{i2} &= -\frac{g M_1 \mu}{2det(\mathcal{M}_{\chi^0})} \Lambda_i \\ \xi_{i3} &= -\frac{\epsilon_i}{\mu} + \frac{(g^2 M_1 + g'^2 M_2) v_U}{4det(\mathcal{M}_{\chi^0})} \Lambda_i \end{aligned}$$

$$\xi_{i4} = -\frac{(g^2 M_1 + g'^2 M_2) v_D}{4 det(\mathcal{M}_{\chi^0})} \Lambda_i \tag{14}$$

where

$$\Lambda_i = \mu v_i + v_D \epsilon_i \tag{15}$$

are the "alignment" parameters. From eq. (14) and eq. (15) one can see that $\xi = 0$ in the MSSM limit where $\epsilon_i = 0$ and $v_i = 0$. In leading order in ξ the mixing matrix $\mathcal{N}$ is given by,

$$\mathcal{N}^* = \begin{pmatrix} N^* & 0 \\ 0 & V_\nu^T \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2}\xi^\dagger \xi & \xi^\dagger \\ -\xi & 1 - \frac{1}{2}\xi \xi^\dagger \end{pmatrix} \tag{16}$$

The second matrix above block-diagonalizes the mass matrix $\mathbf{M}_N$ approximately to the form diag$(\mathcal{M}_{\chi^0}, m_{eff})$, where

$$\begin{aligned} m_{eff} &= -m \cdot \mathcal{M}_{\chi^0}^{-1} m^T \\ &= \frac{M_1 g^2 + M_2 g'^2}{4\, det(\mathcal{M}_{\chi^0})} \begin{pmatrix} \Lambda_e^2 & \Lambda_e \Lambda_\mu & \Lambda_e \Lambda_\tau \\ \Lambda_e \Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu \Lambda_\tau \\ \Lambda_e \Lambda_\tau & \Lambda_\mu \Lambda_\tau & \Lambda_\tau^2 \end{pmatrix} \end{aligned} \tag{17}$$

One sees that the effective neutrino mass matrix m_{eff} clearly has a projective nature, a feature often encountered in $\not{R}_p$ models [5]. The sub-matrices N and V_ν diagonalize $\mathcal{M}_{\chi^0}$ and m_{eff}

$$N^* \mathcal{M}_{\chi^0} N^\dagger = \text{diag}(m_{\chi_i^0}), \tag{18}$$

$$V_\nu^T m_{eff} V_\nu = \text{diag}(0, 0, m_\nu), \tag{19}$$

where

$$m_\nu = Tr(m_{eff}) = \frac{M_1 g^2 + M_2 g'^2}{4\, det(\mathcal{M}_{\chi^0})} |\vec{\Lambda}|^2. \tag{20}$$

Due to the projective nature of m_{eff}, only one neutrino acquires mass [5]. As a result one can rotate away one of the three angles in the matrix $V_{\nu,\text{tree}}$, leading to [32]

$$V_{\nu,\text{tree}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}, \tag{21}$$

where the mixing angles can be expressed in terms of the *alignment vector* $\vec{\Lambda}$ as

$$\tan\theta_{13} = -\frac{\Lambda_e}{(\Lambda_\mu^2 + \Lambda_\tau^2)^{\frac{1}{2}}}, \tag{22}$$

$$\tan\theta_{23} = -\frac{\Lambda_\mu}{\Lambda_\tau}. \tag{23}$$

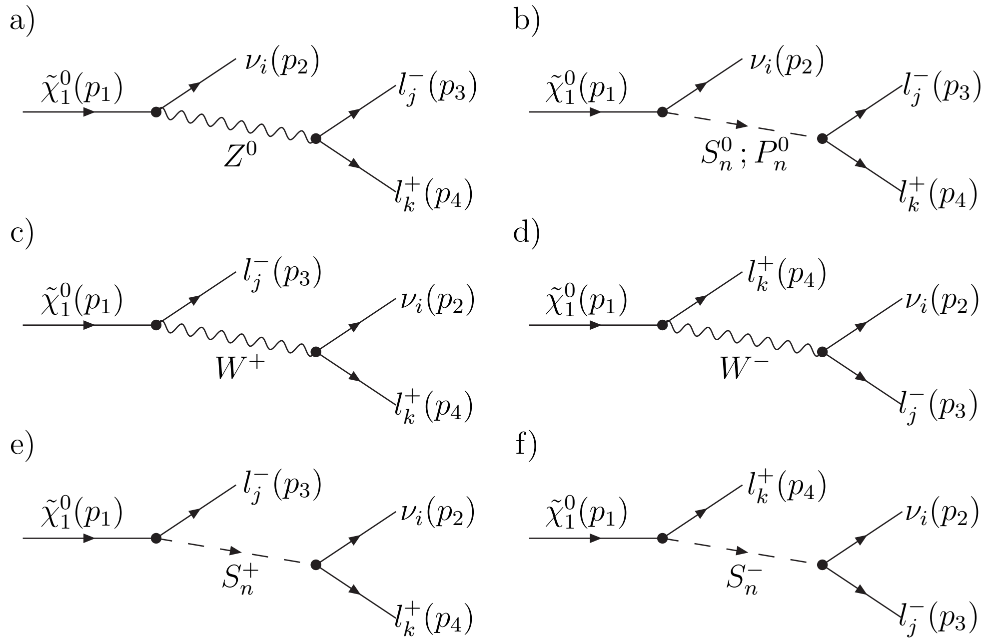


Figure 1: Feynman graphs for the decay $\tilde{\chi}^0_1 \to \nu_i\, l^-_j\, l^+_k$.

As discussed in detail in [21] the inclusion of 1-loop corrections to the mass matrix lifts the degeneracy between these states. Only after including these corrections one obtains a meaningful angle in the $1-2$ sector. Both features are required to account for the solar neutrino data.

2.2 Approximate Formulas for neutralino couplings

The set of Feynman diagrams involved in neutralino decays is indicated in Figs. 1, 2 and 3. Most of the relevant couplings involved have been given in appendix B of ref. [21] and the remaining ones will be included in appendix B of the present paper. Even though these are sufficient for our calculation of neutralino production and decay properties, it is very useful to have approximate formulas for the neutralino couplings, since this allows some qualitative understanding of the correlations we are going to discuss. To achieve this we make use of the expansions for the neutralino mass matrix and also a corresponding one for the charginos as given in [20][2]. For this purpose we will confine ourselves to the tree-level neutralino/neutrino mass matrix and we refer to Sec. 4.1 for a short discussion of the necessary changes once the 1-loop corrections to the mass matrix are included. However, we have used exact numerical diagonalizations and loop effects in the calculation of all resulting

[2]Note that one has to reverse the sign of the ϵ_i in [20] to be consistent with our present notation.

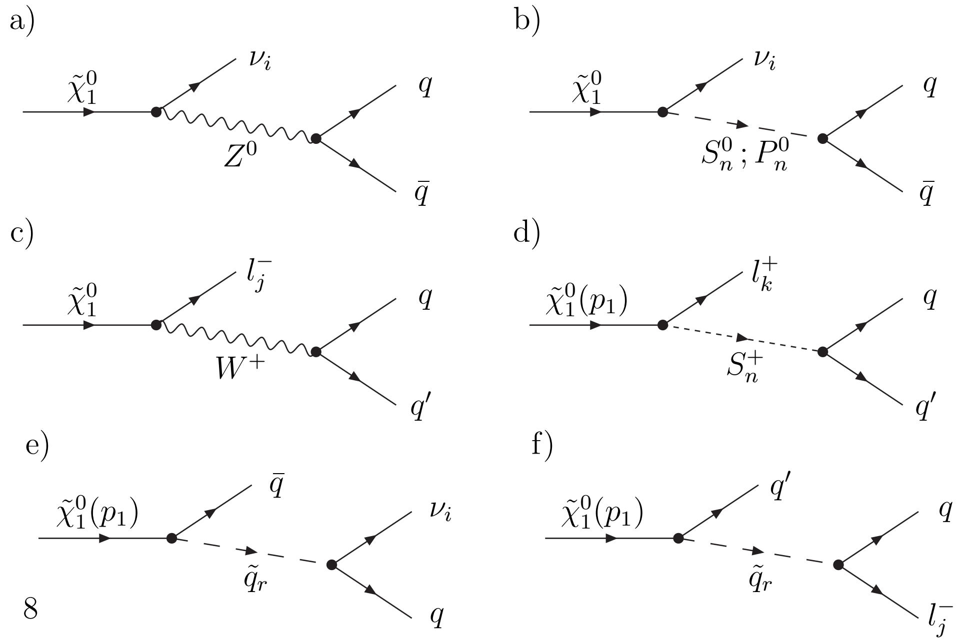


Figure 2: Generic Feynman graphs for semi-leptonic neutralino decays.

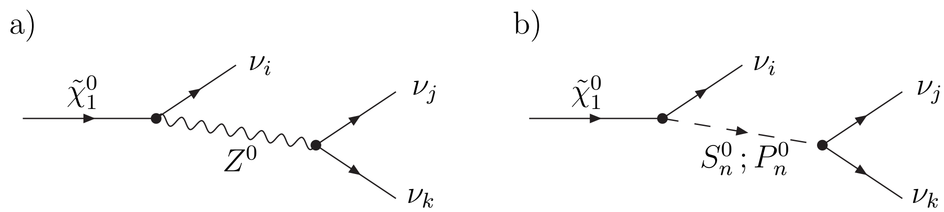


Figure 3: Generic Feynman graphs for invisible neutralino decays.

physical quantities presented in Secs. 3 and 4.

One class of decays which is important are those involving a W-boson, either virtual or real. The $\tilde{\chi}^0_1$-$W^\pm$-l_i couplings are approximatively given by:

$$
\begin{aligned}
O^{cnw}_{Ri1} &= \frac{g h_E^{ii} v_D}{2\mathrm{Det}_+}\left[\frac{g v_D N_{12} + M_2 N_{14}}{\mu}\epsilon_i \right. \\
&\qquad \left. + g\frac{\left(2\mu^2 + g^2 v_D v_U\right) N_{12} + \left(\mu + M_2\right) g v_U N_{14}}{2\mu \mathrm{Det}_+}\Lambda_i\right] \qquad (24) \\
O^{cnw}_{Li1} &= \frac{g\Lambda_i}{\sqrt{2}}\left[-\frac{g' M_2 \mu}{2\mathrm{Det}_0}N_{11} + g\left(\frac{1}{\mathrm{Det}_+} + \frac{M_1\mu}{2\mathrm{Det}_0}\right)N_{12}\right. \\
&\qquad \left. -\frac{v_U}{2}\left(\frac{g^2 M_1 + g'^2 M_2}{2\mathrm{Det}_0} + \frac{g^2}{\mu \mathrm{Det}_+}\right)N_{13} + \frac{v_D(g^2 M_1 + g'^2 M_2)}{4\mathrm{Det}_0}N_{14}\right] \qquad (25)
\end{aligned}
$$

Here Det_+ and Det_0 denote the determinant of the MSSM chargino and neutralino mass matrix, respectively. N_{ij} are the elements of the mixing matrix which diagonalizes the

MSSM neutralino mass matrix.

For the coupling Z-$\tilde{\chi}^0_1$-ν_i we find

$$O^{nnz}_{L\chi^0_1\nu_1} = O^{nnz}_{L\chi^0_1\nu_2} = 0 \tag{26}$$

$$O^{nnz}_{L\chi^0_1\nu_3} = \left(\frac{g\left(gM_1N_{12} - g'M_2N_{11}\right)\mu}{4\cos\theta_W \mathrm{Det}_0} + \frac{g\left(g^2M_1 + g'^2M_2\right)v_DN_{14}}{4\cos\theta_W \mathrm{Det}_0}\right)|\vec{\Lambda}| \; . \tag{27}$$

As already mentioned, the tree-level the states ν_1 and ν_2 are not well defined. Therefore one has to consider the complete 1-loop mass matrix as it will be done in the numerical part in sections 3 and 4. However, as one cannot detect single neutrino flavours, in experiments one observes the decay of $\tilde{\chi}^0_1 \to X + \nu_i$ summing over all neutrinos ν_i. Therefore, for the Z-mediated decay the interesting quantity is $\sum_{i=1,3}|O^{nnz}_{L\chi^0_1\nu_i}|^2$ and, in contrast to the individual $\chi^0_1 \to Z\,\nu_i$ decay rates, this only gets small radiative corrections.

For the coupling $\tilde{\chi}^0_1$-ν_i-$(S^0_1 \simeq h^0)$ we get

$$\begin{aligned} O^{nnh}_{111} &= E_{\tilde{\chi}^0_1}\left(\sin\alpha\, c_2\, c_4\, c_6 \frac{-\epsilon_e\left(\Lambda_\mu^2 + \Lambda_\tau^2\right) + \Lambda_e\left(\epsilon_\mu\Lambda_\mu + \epsilon_\tau\Lambda_\tau\right)}{\mu\sqrt{\Lambda_e^2 + \Lambda_\tau^2}|\vec{\Lambda}|}\right. \\ &\qquad \left. + \frac{-s_2\left(\Lambda_\mu^2 + \Lambda_\tau^2\right) + \Lambda_e\left(s_4\Lambda_\mu + s_6\Lambda_\tau\right)}{\sqrt{\Lambda_e^2 + \Lambda_\tau^2}|\vec{\Lambda}|}\right) \end{aligned} \tag{28}$$

$$O^{nnh}_{121} = E_{\tilde{\chi}^0_1}\left(\sin\alpha\, c_2\, c_4\, c_6 \frac{\epsilon_\tau\Lambda_\mu - \epsilon_\mu\Lambda_\tau}{\mu\sqrt{\Lambda_e^2 + \Lambda_\tau^2}} + \frac{s_6\Lambda_\mu - s_4\Lambda_\tau}{\sqrt{\Lambda_e^2 + \Lambda_\tau^2}}\right) \tag{29}$$

$$O^{nnh}_{131} = E_{\tilde{\chi}^0_1}\left(\sin\alpha\, c_2\, c_4\, c_6 \frac{(\vec{\epsilon}.\vec{\Lambda})}{|\vec{\Lambda}|} + \frac{(\vec{s}.\vec{\Lambda})}{|\vec{\Lambda}|}\right) - D_{\tilde{\chi}^0_1}\, c_2\, c_4\, c_6|\vec{\Lambda}| \tag{30}$$

with

$$\vec{s} = (s_2, s_4, s_6) \; , \tag{31}$$

$$E_{\tilde{\chi}^0_1} = \frac{(g'N_{11} - gN_{12})}{2} \;, \text{and} \tag{32}$$

$$D_{\tilde{\chi}^0_1} = \frac{\left(g^2M_1 + g'^2M_2\right)\left[(\cos\alpha\, v_D + \sin\alpha\, v_U)\left(g'N_{11} - gN_{12}\right) + 2\mu\left(\sin\alpha N_{13} - \cos\alpha N_{14}\right)\right]}{8\mathrm{Det}_0} \tag{33}$$

The quantities s_i and c_i are parts of the mixing matrix for the neutral scalars, which is discussed in Appendix A .

For the couplings $\tilde{d}_{Li} - d_i - \nu_j$ one finds

$$O^{dnL}_{Li1} = h^{ii}_D \frac{-\epsilon_e\left(\Lambda_\mu^2 + \Lambda_\tau^2\right) + \Lambda_e\left(\epsilon_\mu\Lambda_\mu + \epsilon_\tau\Lambda_\tau\right)}{\mu\sqrt{\Lambda_e^2 + \Lambda_\tau^2}|\vec{\Lambda}|} \tag{34}$$

$$O^{dnL}_{Ri1} = 0 \tag{35}$$

$$O^{dnL}_{Li2} = h^{ii}_D \frac{\epsilon_\tau \Lambda_\mu - \epsilon_\mu \Lambda_\tau}{\mu\sqrt{\Lambda_e^2+\Lambda_\tau^2}} \tag{36}$$

$$O^{dnL}_{Ri2} = 0 \tag{37}$$

$$O^{dnL}_{Li3} = h^{ii}_D \left(G_{\tilde{\chi}^0_1} |\vec{\Lambda}| - \frac{(\vec{\epsilon},\vec{\Lambda})}{\mu|\vec{\Lambda}|} \right) \tag{38}$$

$$O^{dnL}_{Ri3} = H_{\tilde{\chi}^0_1} |\vec{\Lambda}| \tag{39}$$

with $G_{\tilde{\chi}^0_1} = (g^2 M_1 + g'^2 M_2) v_U/(4\mathrm{Det}_0)$ and $H_{\tilde{\chi}^0_1} = (3g^2 M_1 + g'^2 M_2)\mu/(6\sqrt{2}\mathrm{Det}_0)$. For the couplings $\tilde{d}_{Ri} - d_i - \nu_j$ one finds that $O^{dnR}_{Rij} = O^{dnL}_{Lij}$ and $O^{dnR}_{Lij} = O^{dnL}_{Rij}$ as above but with $H_{\tilde{\chi}^0_1} \to g'^2 M_2 \mu/(3\sqrt{2}\mathrm{Det}_0)$.

One can obtain the couplings between $\tilde{l}_{Li}$-l_i-ν_j by replacing $h_D \to h_E$ and $H_{\tilde{\chi}^0_1} \to (g^2 M_1 + g'^2 M_2)\mu/(2\sqrt{2}\mathrm{Det}_0)$ in the above equations. For the case of $\tilde{l}_{Ri}$-l_i-ν_j one finds the couplings by replacing $h_D \to h_E$ and $H_{\tilde{\chi}^0_1} \to g'^2 M_2\mu/(\sqrt{2}\mathrm{Det}_0)$.

For the couplings $\tilde{u}_{Li} - u_i - \nu_j$ one finds

$$O^{unL}_{Li1} = O^{unL}_{Ri1} = O^{unL}_{Li2} = O^{unL}_{Ri2} = 0 \tag{40}$$

$$O^{unL}_{Li3} = -h^{ii}_U I_{\tilde{\chi}^0_1} |\vec{\Lambda}| \tag{41}$$

$$O^{unR}_{Li3} = J_{\tilde{\chi}^0_1} |\vec{\Lambda}| \tag{42}$$

with $I_{\tilde{\chi}^0_1} = (g^2 M_1 + g'^2 M_2) v_D/(2\mathrm{Det}_0)$ and $J_{\tilde{\chi}^0_1} = (-3g^2 M_1 + g'^2 M_2)\mu/(6\sqrt{2}\mathrm{Det}_0)$. For the couplings $\tilde{u}_{Ri}$-u_i-ν_j one finds that $O^{unR}_{Rij} = O^{unL}_{Lij}$ and $O^{unR}_{Lij} = O^{unL}_{Rij}$ as above but with $J_{\tilde{\chi}^0_1} \to -\sqrt{2} g'^2 M_2 \mu/(3\mathrm{Det}_0)$.

For the couplings $\tilde{u}_j - d_k - l_i$ one finds

$$C^{\tilde{u}}_{Lkl_i} = h^{kk}_D R^{\tilde{u}}_{j1} \left(\frac{\epsilon_i}{\mu} + \frac{g^2 v_U}{2\mu} \frac{\Lambda_i}{\mathrm{Det}_+} \right) \tag{43}$$

$$C^{\tilde{u}}_{Rl_i} = \frac{h^{ii}_E v_D}{\sqrt{2}\mathrm{Det}_+} \left\{ \left(\frac{g^2 v_D R^{\tilde{u}}_{j1}}{\sqrt{2}} + h^{kk}_U M_2 R^{\tilde{u}}_{j2} \right) \frac{\epsilon_i}{\mu} \right. \tag{44}$$

$$\left. + \left[\frac{g^2 \mu R^{\tilde{u}}_{j1}}{\sqrt{2}} \left(1 + \frac{g^2 v_D v_U}{2\mu^2} \right) + \frac{g^2 h^{kk}_U v_U R^{\tilde{u}}_{j2}}{2} \left(1 + \frac{M_2}{\mu} \right) \right] \frac{\Lambda_i}{\mathrm{Det}_+} \right\} \tag{45}$$

For the couplings $\tilde{d}_j - u_k - l_i$ one finds

$$C^{\tilde{d}}_{Lkl_i} = \frac{h^{ii}_E h^{kk}_U v_D R^{\tilde{d}}_{j1}}{\sqrt{2}\mathrm{Det}_+} \left[M_2 \frac{\epsilon_i}{\mu} + \frac{g^2 v_U}{2} \left(1 + \frac{M_2}{\mu} \right) \frac{\Lambda_i}{\mathrm{Det}_+} \right] \tag{46}$$

$$C^{\tilde{d}}_{Rkl_i} = h^{kk}_D R^{\tilde{d}}_{j2} \frac{\epsilon_i}{\mu} + \left(\frac{g^2 R^{\tilde{d}}_{j1}}{\sqrt{2}} + \frac{g^2 h^{kk}_D v_U R^{\tilde{d}}_{j2}}{2\mu} \right) \frac{\Lambda_i}{\mathrm{Det}_+} \tag{47}$$

In eq. (43) - (47) we have assumed that there is no generation mixing between the squarks implying that $j = 1, 2$.

Data from reactor experiments [33] indicate that the mixing element U_{e3} must be small [1]. This implies that $|\Lambda_e| \ll |\Lambda_{2,3}|$. In the limit $\Lambda_e/\Lambda_{2,3} \to 0$ some of the above formulas simplify to

$$O^{nnh}_{111} = -E_{\tilde{\chi}^0_1} \left(\frac{c_2\, c_4\, c_6\, \sin\alpha\, \epsilon_e}{\mu} + s_2 \right) \quad (48)$$

$$O^{nnh}_{121} = E_{\tilde{\chi}^0_1} \left(\sin\alpha\, c_2\, c_4\, c_6 \frac{\epsilon_\tau \Lambda_\mu - \epsilon_\mu \Lambda_\tau}{\mu |\Lambda_\tau|} + \frac{s_6 \Lambda_\mu - s_4 \Lambda_\tau}{|\Lambda_\tau|} \right) \quad (49)$$

$$O^{nnh}_{131} = E_{\tilde{\chi}^0_1} \left(\sin\alpha\, c_2\, c_4\, c_6 \frac{\epsilon_\mu \Lambda_\mu + \epsilon_\tau \Lambda_\tau}{\mu \sqrt{\Lambda_2^2 + \Lambda_3^2}} + \frac{s_4 \Lambda_\mu + s_6 \Lambda_\tau}{\sqrt{\Lambda_2^2 + \Lambda_3^2}} \right) - D_{\tilde{\chi}^0_1}\, c_2\, c_4\, c_6 \sqrt{\Lambda_2^2 + \Lambda_3^2} \quad (50)$$

$$O^{dnL}_{Li\nu_1} = O^{dnR}_{Ri\nu_1} = -\frac{h^{ii}_D \epsilon_e}{\mu} \quad (51)$$

$$O^{dnL}_{Li\nu_2} = O^{dnR}_{Ri\nu_2} = \frac{h^{ii}_D \left(\epsilon_\mu \Lambda_\tau - \epsilon_\tau \Lambda_\mu\right)}{\mu |\Lambda_\tau|} \quad (52)$$

$$O^{dnL}_{Li\nu_3} = O^{dnR}_{Ri\nu_3} = h^{ii}_D \left(G_{\tilde{\chi}^0_1} \sqrt{\Lambda_\mu^2 + \Lambda_3^2} - \frac{\epsilon_\mu \Lambda_\mu + \epsilon_\tau \Lambda_\tau}{\mu \sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}} \right) \quad (53)$$

Later on we will also use the so-called sign-condition [21], defined by

$$\frac{\epsilon_\mu}{\epsilon_\tau} \frac{\Lambda_\mu}{\Lambda_\tau} < 0\ . \quad (54)$$

Its origin can be traced back to the above eq. (48) - (53). Assuming $\epsilon_\mu \simeq \epsilon_\tau$ as indicated by unification and $|\Lambda_\mu| \simeq |\Lambda_\tau|$ as required by the atmospheric neutrino problem one sees easily from the above equations that either the ϵ part[3] of the couplings to the second or the third neutrino state is very small depending on the relative sign between Λ_μ and Λ_τ. If $\Lambda_\mu \simeq -\Lambda_\tau$ one can show, after a lengthy calculation [34], that the resulting effective neutrino mixing matrix is given by

$$V_{\nu,\mathrm{loop}} = \begin{pmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times V_{\nu,\mathrm{tree}} \quad (55)$$

with nearly unchanged θ_{13} and θ_{23}. In contrast, if this sign condition is not fulfilled the θ_{13} and θ_{23} angles get large corrections. One sees therefore that if the sign condition is satisfied the atmospheric and solar neutrino features decouple: the atmospheric is mainly tree-level physics, while the solar neutrino anomaly is accounted for by genuine loop physics in a simple factorizable way. Thus the sign condition is helpful to get a better control on the parameters for the solar neutrino problem.

[3] The Λ parts lead only to a renormalization of the heaviest neutrino state whereas the ϵ part gives mass to the lighter neutrinos.

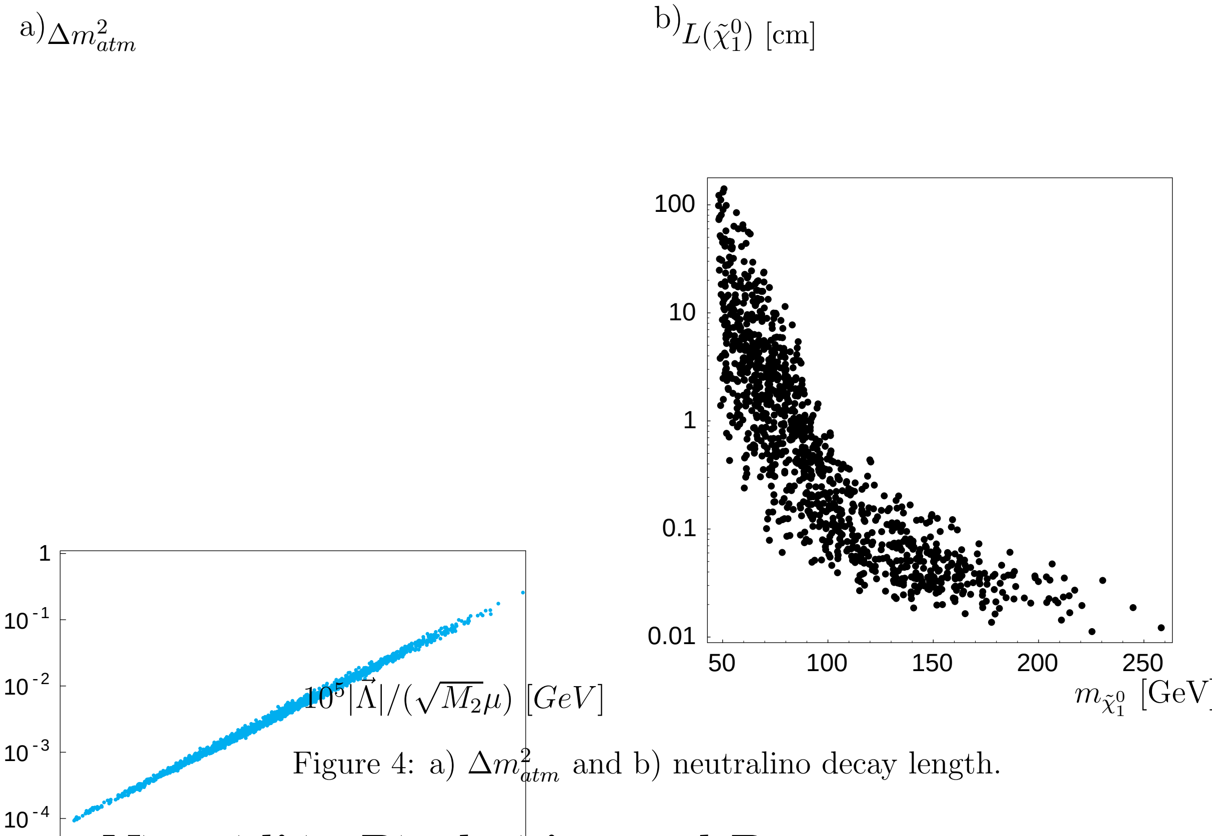


Figure 4: a) Δm^2_{atm} and b) neutralino decay length.

3 Neutralino Production and Decays

In this section we will discuss the production and the decay modes of the lightest neutralino $\tilde{\chi}^0_1$. In order to reduce the numbers of parameters we have performed the calculations in the framework of a minimal SUGRA version of bilinearly $\not{R}_p$SUSY model. Unless noted otherwise the parameters have been varied in the following ranges: M_2 and $|\mu|$ from 0 to 1 TeV, m_0 [0.2 TeV, 1.0 TeV], A_0/m_0 and B_0/m_0 [-3,3] and $\tan\beta$ [2.5,10], and for the $\not{R}_p$ parameters, $|\Lambda_\mu/\sqrt{\Lambda_e^2+\Lambda_\tau^2}| = 0.4-2$, $\epsilon_\mu/\epsilon_\tau = 0.8-1.25$, $|\Lambda_e/\Lambda_\tau| = 0.025-2$, $\epsilon_e/\epsilon_\tau = 0.015-2$ and $|\Lambda| = 0.05-0.2$ GeV^2. They were subsequently tested for consistency with the minimization (tadpole) conditions of the Higgs potential as well as for phenomenological constraints from supersymmetric particle searches. Moreover, they were checked to provide a solution to both solar and atmospheric neutrino problems. For the case of the solar neutrino anomaly we have accepted points which give either one of the large mixing angle solutions or the small mixing angle MSW solution.

We have seen in eq. (20) that the atmospheric scale is proportional $|\vec{\Lambda}|^2/\mathrm{Det}(m_{\tilde{\chi}^0})$. As has been shown in [6, 21] this statement remains valid after inclusion of 1-loop corrections provided that $|\vec{\epsilon}|^2/|\vec{\Lambda}| < 1$ implying that 1-loop corrections to the heaviest neutrino mass remain small. As we have seen in section (2.2), most of the couplings are proportional to $|\vec{\Lambda}|/\sqrt{\mathrm{Det}(m_{\tilde{\chi}^0})}$ and/or ϵ_i/μ. Although $|\vec{\Lambda}|/(\sqrt{M_2}\mu)$ has to be small in order to account for

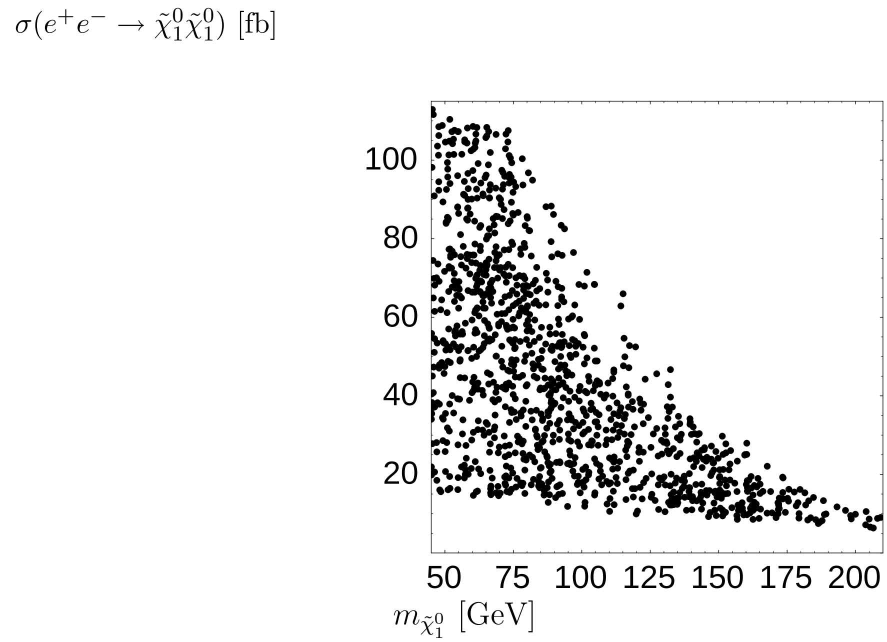


Figure 5: Production cross section for the process $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0)$ as a function of $m_{\tilde{\chi}_1^0}$ at a Linear Collider with 1 TeV c.m.s energy. ISR-corrections are included.

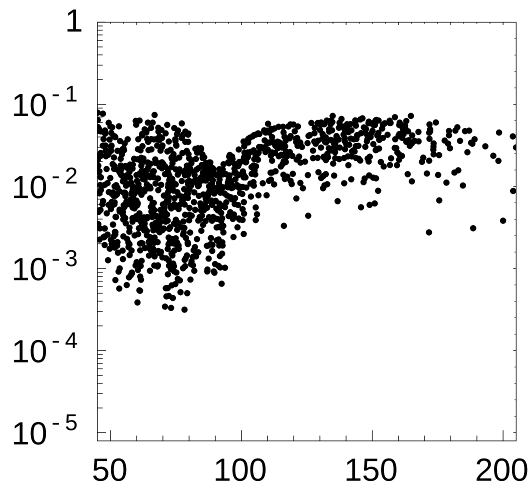
the atmospheric mass scale (see Fig. (4a)) the previously discussed couplings are still large enough so that the neutralino decays inside the detector, as can be seen in Fig. (4b).

In Fig. 5 we show the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0)$ in fb for $\sqrt{s} = 1$ TeV. Assuming now that an integrated luminosity of 1000 fb^{-1} per year can be achieved at a future linear collider (see [35, 36] and references therein) this implies that between 10^4 to 10^5 neutralino pairs can be directly produced per year. Due to the smallness of the R-parity violating couplings, most of the SUSY particles will decay according to the MSSM scheme implying that there will be many more neutralinos to study, namely from direct production as well as resulting from cascade decays of heavier SUSY particles. From this point of view the measurement of branching ratios as small as 10^{-5} should be feasible. As we will see in what follows this might be required in order to establish some of the correlations between neutrino mixing angles and the resulting neutralino decay observables, which is a characteristic feature of this class of models.

In this model the neutralino can decay in the following ways

$$\begin{aligned} \tilde{\chi}_1^0 &\to \nu_i\,\nu_j\,\nu_k && (56)\\ &\to \nu_i\,q\,\bar{q} && (57)\\ &\to \nu_i\,l_j^+\,l_k^- && (58) \end{aligned}$$

$\mathrm{Br}(\tilde{\chi}^0_1 \to \sum_{i,j,k} \nu_i \nu_j \nu_k)$



$m_{\tilde{\chi}^0_1}$ [GeV]

Figure 6: Invisible neutralino branching ratio summing over all neutrinos.

$$\to \quad l_i^\pm \, q \, \bar{q}' \tag{59}$$

$$\to \quad \nu_i \, \gamma \tag{60}$$

In the following we will discuss these possibilities in detail except $\tilde{\chi}^0_1 \to \nu_i \, \gamma$ because its branching ratio is always below 10^{-7}.

In the following discussion we have always computed the complete three-body decay widths even in cases where $m_{\tilde{\chi}^0_1}$ has been larger than one of the exchanged particle masses, so that two-body channels are open. This has turned out to be necessary because there are parameter combinations where the couplings to the lightest exchanged particle are $O(10)$ smaller than the coupling to one of the heavier particles, implying that the graph containing the heavy particle cannot be neglected with respect to the lighter particle contribution. An example is the case of Z-boson and S^0_1-mediated gaugino-like neutralino decays discussed later on. Here S^0_1 denotes the lightest neutral scalar. In addition we want to be sure not to miss possibly important interference effects as there are several graphs which contribute to a given process. A typical example is the process $\tilde{\chi}^0_1 \to \nu_i \, l_j^- \, l_k^+$ where 26 contributions exist, as can be seen from the generic diagrams shown in Fig. 1.

The first important question to be answered is how large the invisible neutralino decay modes to neutrinos can be. This is important to ensure that sufficient many neutralino decays can be observed. As can be seen from Fig. 6 the invisible branching ratio never exceeds 10%. The main reason for this behaviour can be found in the fact that for the

a) $\mathrm{Br}(\tilde{\chi}^0_1 \to b\bar{b} \sum_i \nu_i)$ b) $\mathrm{Br}(\tilde{\chi}^0_1 \to \sum_{q=u,d,s} q\bar{q} \sum_i \nu_i)$ c) $\mathrm{Br}(\tilde{\chi}^0_1 \to c\bar{c} \sum_i \nu_i)$

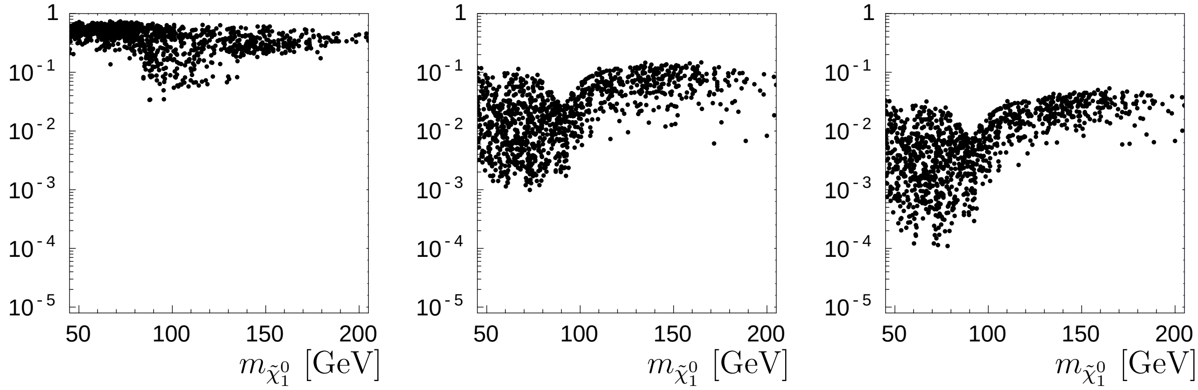


Figure 7: Neutralino branching ratios for the decays into $q\bar{q}\nu_i$ final states summing over all neutrinos.

SUGRA motivated scenario under consideration the couplings of the lightest neutralino to the Z-boson are suppressed. This and the comparison with other couplings will be discussed in some detail later on.

The mainly "visible" nature of the lightest neutralino decay, together with the short neutralino decay path discussed above, suggests the observability of neutralino-decay-induced events at collider experiments and this should stimulate dedicated detector studies.

In Fig. 7 we show the branching ratios for the decays into $q\bar{q}\nu_i$. Here we single out the b-quark (Fig. (7a)) and the c-quark (Fig. (7c)) because in these cases flavour detection is possible. One can clearly see that for $m_{\tilde{\chi}^0_1} \lesssim 1.1\, m_W$ the decay into $b\bar{b}\nu_i$ can be the dominant one. The reason is that the scalar contributions stemming from S^0_j, P^0_j and/or $\tilde{b}_k$ can be rather large. This can be understood with the help of eq. (34)-(39) where terms proportional to $h_D \epsilon_i/\mu$ appear. This kind of terms is absent in the corresponding couplings for the u-type squarks implying that the branching ratio for $c\bar{c}\nu_i$ is rather small as can be seen in Fig. (7c). One can see in Fig. (7b) and Fig. (7c) a pronounced 'hole' around 80-100 GeV. It occurs because for $m_{\tilde{\chi}^0_1} > m_W$ the W becomes on-shell implying a reduction for these decays. This is compensated as the Z becomes on-shell.

The semi-leptonic branching ratios into charged leptons are shown in Fig. 8. The decays into μ and τ are particularly important because, as we will see in Sec. 4, they will give a measure of the atmospheric neutrino angle. Note that these branching ratios are larger than 10^{-4} and in most cases larger than 10^{-3}, implying that there should be sufficient statistics

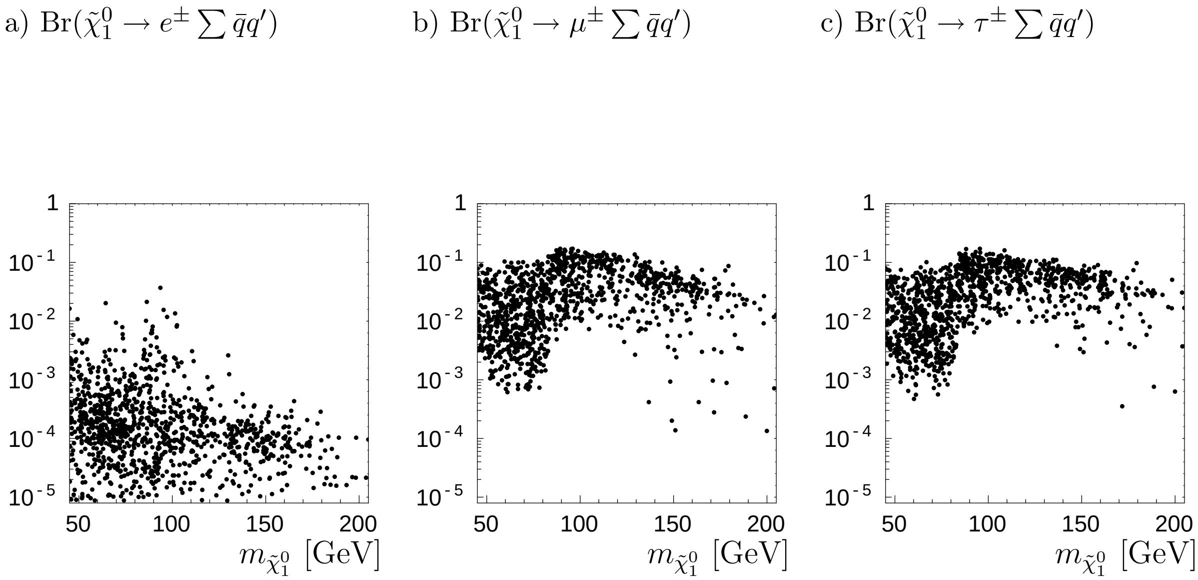


Figure 8: Neutralino branching ratios for the decays into $l^\pm q'\bar{q}$ final states summing over all $q'\bar{q}$ combinations.

for investigations. In case of the e final state it might happen that one can only give an upper bound on this branching ratio. This is just a result of the reactor neutrino bound [33]. Note that due to the Majorana nature of the neutralino one expects in large regions of the parameter space several events with same sign di-leptons and four jets.

In Fig. 9 the fully leptonic branching ratios are shown. One can clearly see a difference between the branching ratios into channels containing different charged leptons of the same flavour, i.e. $\tau^-\tau^+$ versus $\mu^-\mu^+$ and e^-e^+. This difference is due to the importance of the S^0_1 state which corresponds mainly to the lightest Higgs boson h^0 of the MSSM. We have found that for gaugino-like $\tilde{\chi}^0_1$ the $R\!\!\!/_p$ couplings $S^0_1 - \tilde{\chi}^0_1 - \nu_i$ are in general larger than the corresponding $Z^0 - \tilde{\chi}^0_1 - \nu_i$ couplings. This can be understood by inspecting the formulas given in eq. (26) - (30) in Sec. 2.2, in particular the parts proportional to ϵ_k in eq. (28) – (30). Other reasons for having "non-universal" $\tau^-\tau^+$, $\mu^-\mu^+$ and e^-e^+ couplings are the graphs containing W or charged sleptons as exchanged particle (see Fig. 1). From eq. (24) one can see that the coupling O^{cnw}_{Ri1} is proportional to h^{ii}_E implying that they only play a role if a τ is present in the final state.

Notice also that the largeness of the branching ratios for neutralino decays into lepton-flavour-violating channels can be simply understood from the importance of $W^\pm$ and $S^\pm_n$ contributions present in Fig. 1 [4].

[4]The charged scalars are a mixture of the charged Higgs bosons and the charged sleptons, and in particular the later are the important ones

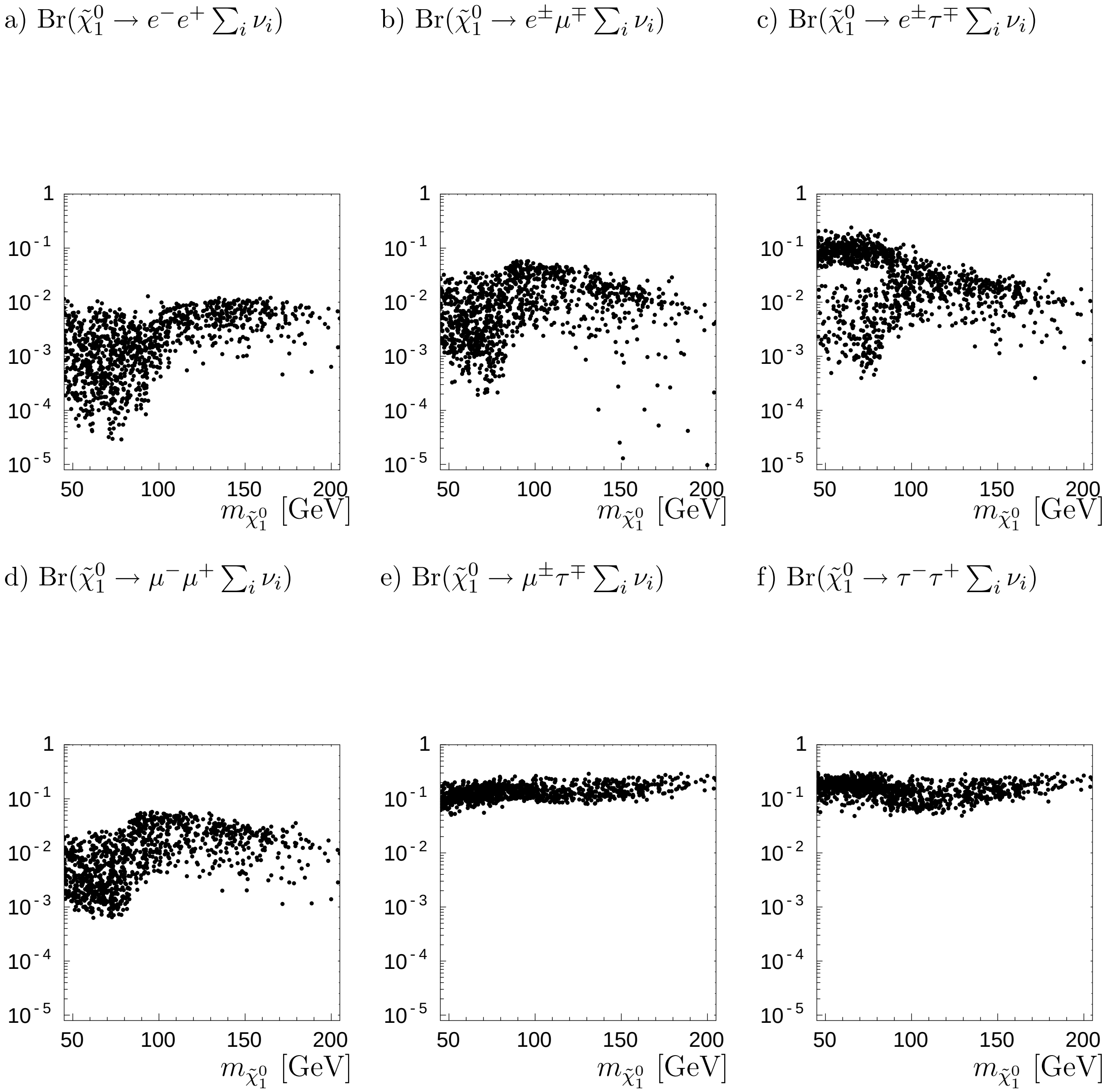


Figure 9: Neutralino branching ratios for the decays into various lepton final states summing over all neutrinos.

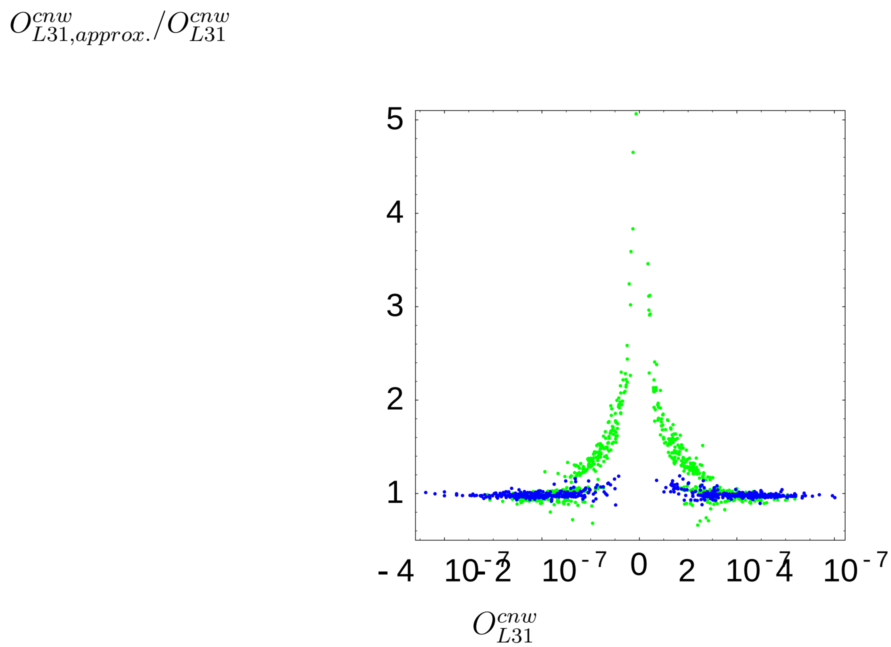


Figure 10: Approximated coupling $O^{cnw}_{L31,approx.}$ using formula eq. (25) divided by the exact calculated coupling as a function of the exact calculated coupling. The bright (dark) points are for $\mu > (<) 0$.

4 Probing Neutrino mixing via Neutralino Decays

In this section we will demonstrate that neutralino decay branching ratios are strongly correlated with neutrino mixing angles. We will consider two cases: 1) The situation before supersymmetry is discovered. In this case we demonstrate that neutrino physics implies predictions for neutralino decays which will be tested at future colliders. 2) The situation when the spectrum is known to the 1% level or better as could, for example, be achieved at a future linear collider [35, 37]. In this case our model allows for several consistency checks between neutrino physics (probed by underground and reactor experiments [3, 2, 33]) and neutralino physics. Moreover, some neutralino decay observables are sensitive to which of the possible solutions to the solar neutrino problem is the one realized, i.e. they can discriminate large angles solutions from the small angle MSW solution.

4.1 Before SUSY is discovered

Let us first consider the situation before SUSY is discovered. Before working out the predictions for neutralino decays we would like to point out a fact concerning the 1-loop corrected neutrino/neutralino mass matrix. It has been noticed in [21] that the sign of the μ param-

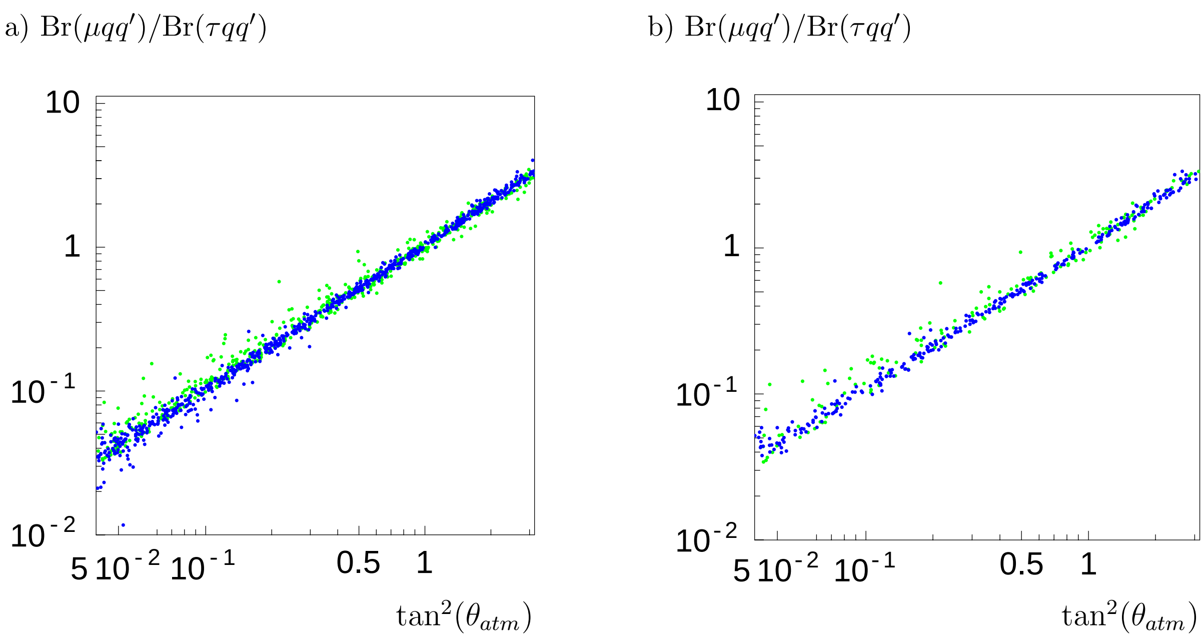


Figure 11: Testing the atmospheric angle. In case of the dark (bright) points $\mu < (>) 0$. In the second figure we have taken only those points with $|\sin 2\theta_{\tilde{b}}| > 0.1$.

eter determines to some extent how large the absolute radiative corrections are (see Fig. 5 of [21]) [5]. The reason is that depending on this sign the interference between the 1-loop graphs containing gauginos and the 1-loop graphs containing Higgsinos is constructive or destructive.

This fact has of course implications on whether the approximate couplings presented in Sec. 2.2 remain valid after the 1-loop corrections are taken into account [6]. A typical example is shown in Fig. 10 where the approximated coupling $O^{cnw}_{L31,approx.}$ divided by the coupling O^{cnw}_{L31} as a function of O^{cnw}_{L31}. One clearly sees that for $\mu < 0$ the tree-level formula is a good approximation within 20% whereas for $\mu > 0$ it can be off by a factor up to 5 in some extreme cases where a constructive interference between gaugino and Higgsino loops takes place. We have checked that the same is true for the other couplings involving either the charged leptons and/or ν_3.

As can be seen from the discussion in Sec. 2.2 the approximate formulas depend on the SUSY parameters, in particular on the parameters of the MSSM chargino/neutralino sector. However, one can see that the ratios of neutralino partial decay widths or of its branching

[5]The important information is the relative sign between μ and the gaugino mass parameters $M_{1,2}$. Since in [21] as well as here we assume that $M_{1,2} > 0$ then the absolute sign of μ becomes relevant.

[6]Of course the couplings involving ν_1 and/or ν_2 are exceptional ones, as the angle between these states is only meaningful after performing 1-loop corrections are included.

a) $\mathrm{Br}(e^{\pm}\mu^{\mp}\sum_i \nu_i)/\mathrm{Br}(\tau q q')$ b) $\mathrm{Br}(e^{\pm}\mu^{\mp}\sum_i \nu_i)/\mathrm{Br}(\tau q q')$

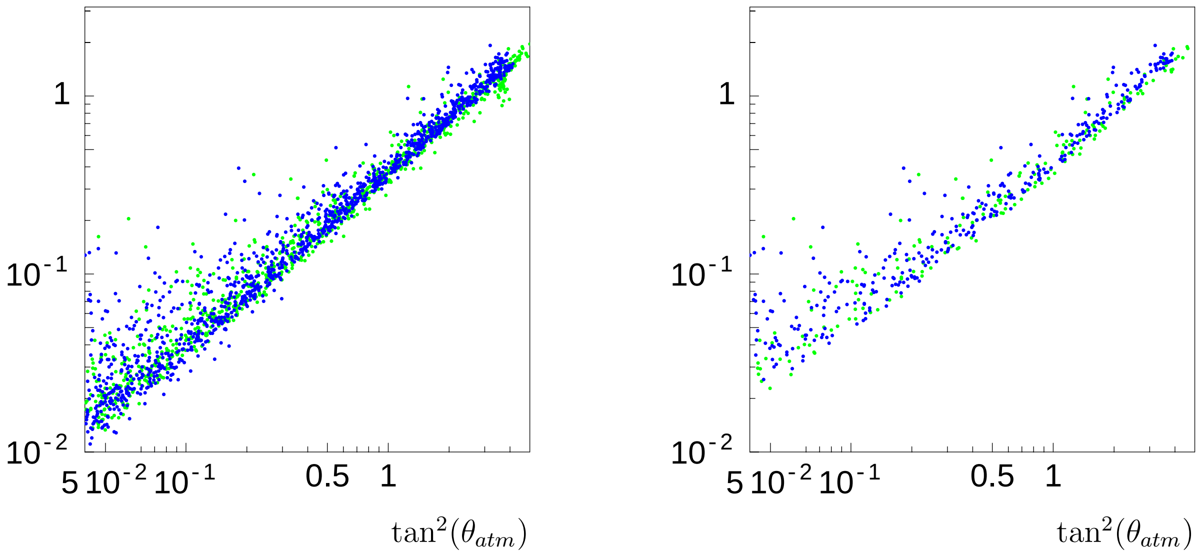


Figure 12: Testing the atmospheric angle. In case of the dark (bright) points $\mu < (>)0$. In the second figure we have taken only those points with $|\sin 2\theta_{\tilde{b}}| > 0.1$.

a) $\mathrm{Br}(\mu^{-}\mu^{+}\sum_i \nu_i)/\mathrm{Br}(\tau q q')$ b) $\mathrm{Br}(\mu^{-}\mu^{+}\sum_i \nu_i)/\mathrm{Br}(\tau q q')$

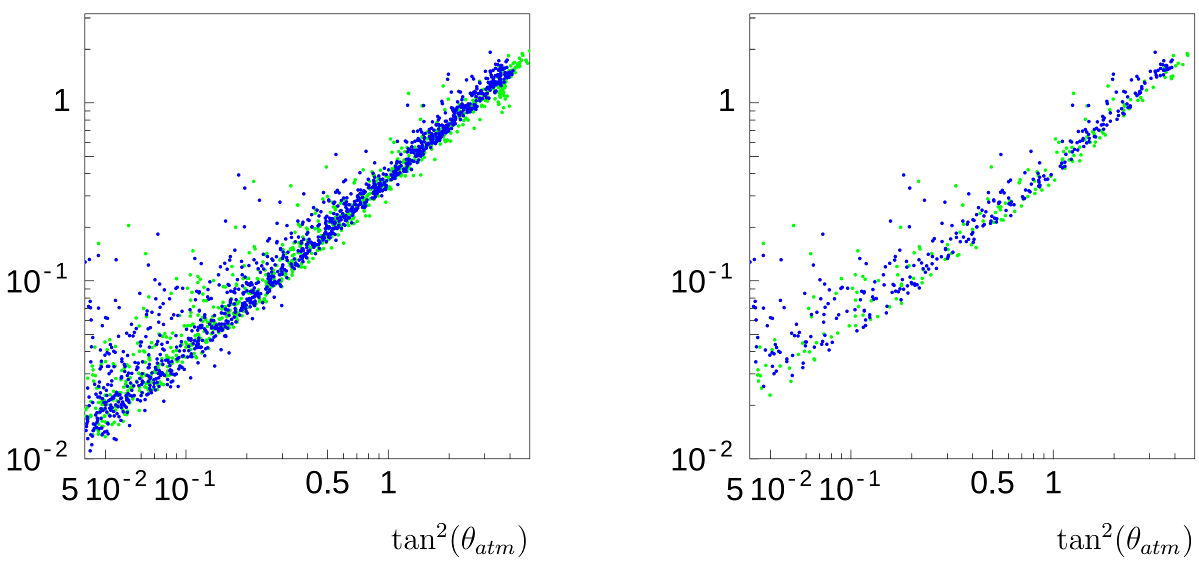


Figure 13: Testing the atmospheric angle. In case of the dark (bright) points $\mu < (>)0$. In the second figure we have taken only those points with $|\sin 2\theta_{\tilde{b}}| > 0.1$.

ratios is rather insensitive to the MSSM parameters. As has been pointed out in [21] the atmospheric angle depends on the ratio of Λ_μ/Λ_τ. This ratio (at tree level) can be obtained by taking the ratio $O^{cnw}_{L21}/O^{cnw}_{L31}$. This leads immediately to the idea that the semi-leptonic branching ratios into $\mu^\pm q\bar{q}'$ and $\tau^\pm q\bar{q}'$ should be related to the atmospheric angle. This is clearly demonstrated in Fig. 11 where we show the ratio of the corresponding branching ratios as a function of $\tan^2(\theta_{atm})$. One sees that present data imply that this ratio should be $O(1)$. In particular, the relative yield of muons and taus will specify whether or not the solution to the atmospheric neutrino anomaly occurs for parameter choices in the "normal" range or in the "dark-side", i.e. $\tan^2(\theta_{atm}) < 1$ or $\tan^2(\theta_{atm}) > 1$ [38].

The observed width of the band simply expresses the residual SUSY parameter dependence, which comes partly from the 1-loop calculated mass matrix and partly from the different contributions to these decays. If for some reason $|\sin 2\theta_{\tilde{b}}| > 0.1$ the dependence on the parameters in the 1-loop calculation is considerably reduced because the sbottom/bottom loop dominates. This leads to a stronger correlation as seen in Fig. (11b). The fact that for $\mu > 0$ the band is wider is a consequence of the discussion in the previous paragraph.

In Fig. 12 and Fig. 13 we show two additional ratios which exhibit also a correlation with $\tan^2\theta_{atm}$: $\mathrm{Br}(\tilde{\chi}^0_1 \to e^\pm\mu^\mp \sum_i \nu_i)$ / $\mathrm{Br}(\tilde{\chi}^0_1 \to \tau^\pm q\bar{q}')$ and $\mathrm{Br}(\tilde{\chi}^0_1 \to \mu^-\mu^+ \sum_i \nu_i)$ / $\mathrm{Br}(\tilde{\chi}^0_1 \to \tau^\pm q\bar{q}')$. The (nearly) maximal mixing of atmospheric neutrinos implies that several other ratios of branching ratios are also fixed to within one order of magnitude, see Table 1 and Fig. 17.

In this model the so-called Chooz-angle is given by $|\Lambda_e/\Lambda_\tau|$ [21] where we already have used the fact that the atmospheric data implies $|\Lambda_\mu| \simeq |\Lambda_\tau|$. The same discussion as in the previous paragraph is valid. This leads automatically to the correlation between $\mathrm{Br}(\tilde{\chi}^0_1 \to e^\pm qq')/\mathrm{Br}(\tilde{\chi}^0_1 \to \tau^\pm qq')$ and U^2_{e3} which is shown in Fig. 14. For $U^2_{e3} < 0.01$ the correlation is less stringent because it implies that the tree level couplings have to be rather small and therefore loop corrections are more important. Note that existing reactor data [33] give the constraint on $U^2_{e3} \lesssim 0.05$ at 90% CL [1]. This in turn implies an upper bound of ~ 0.2 on this ratio of branching ratios.

The discussion of the solar angle is more involved. As illustrated in Fig. 15 this angle is strongly correlated with ϵ_e/ϵ_μ ratio. In order to get information on the ϵ_i from neutralino decays one must take into account that, as already mentioned, the solar angle acquires a meaning only once the complete 1-loop corrections to the mass matrix have been included. For an easier understanding we focus on leptonic decays of the type $\tilde{\chi}^0_1 \to l^+_i l^-_j \nu_k$ with $i \neq j$ which depend on the $\tilde{\chi}^0_1$-W-$l_{j,i}$ and W-$l_{i,j}$-ν_k couplings. The way a correlation appears

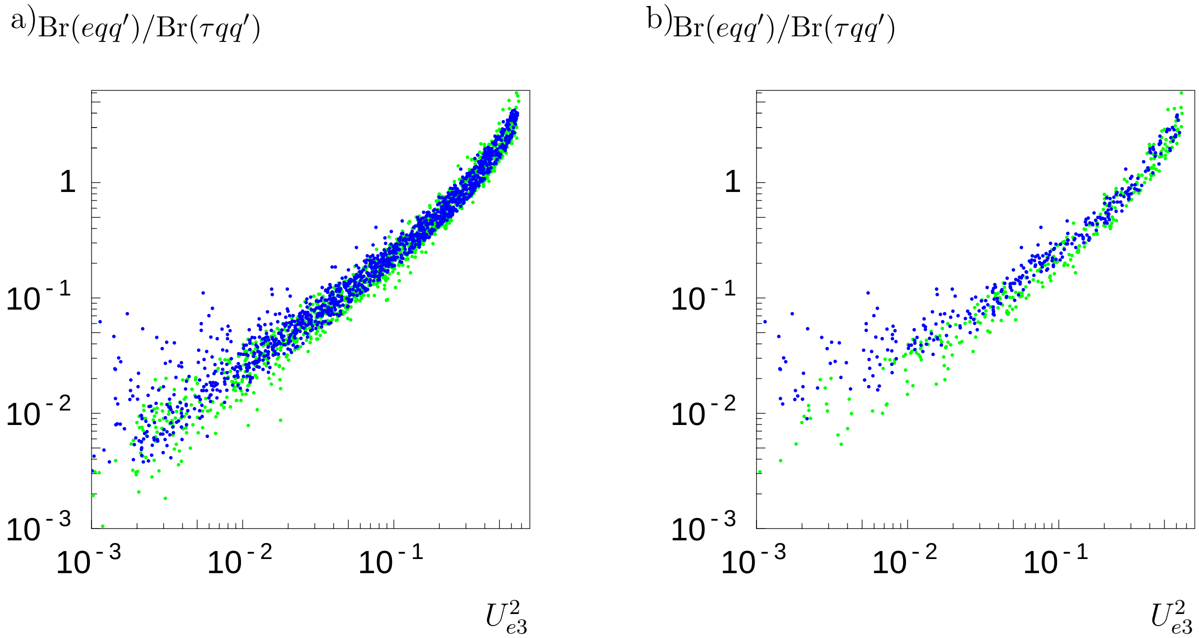


Figure 14: Testing the Chooz angle. In case of the dark (bright) points $\mu < (>)0$. In b) we have taken only those points with $|\sin 2\theta_{\tilde{b}}| > 0.1$.

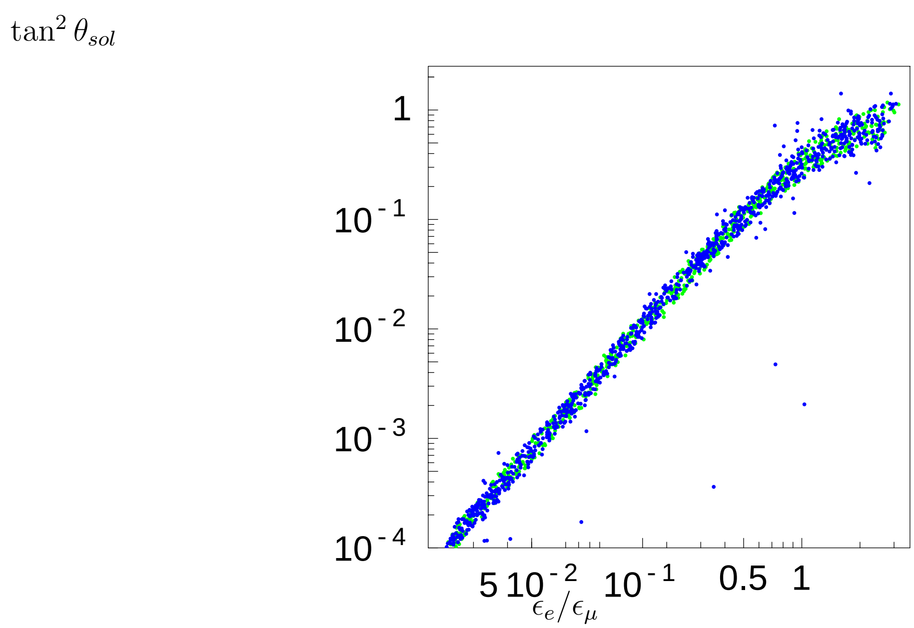


Figure 15: The solar mixing angle as a function of ϵ_e/ϵ_μ.

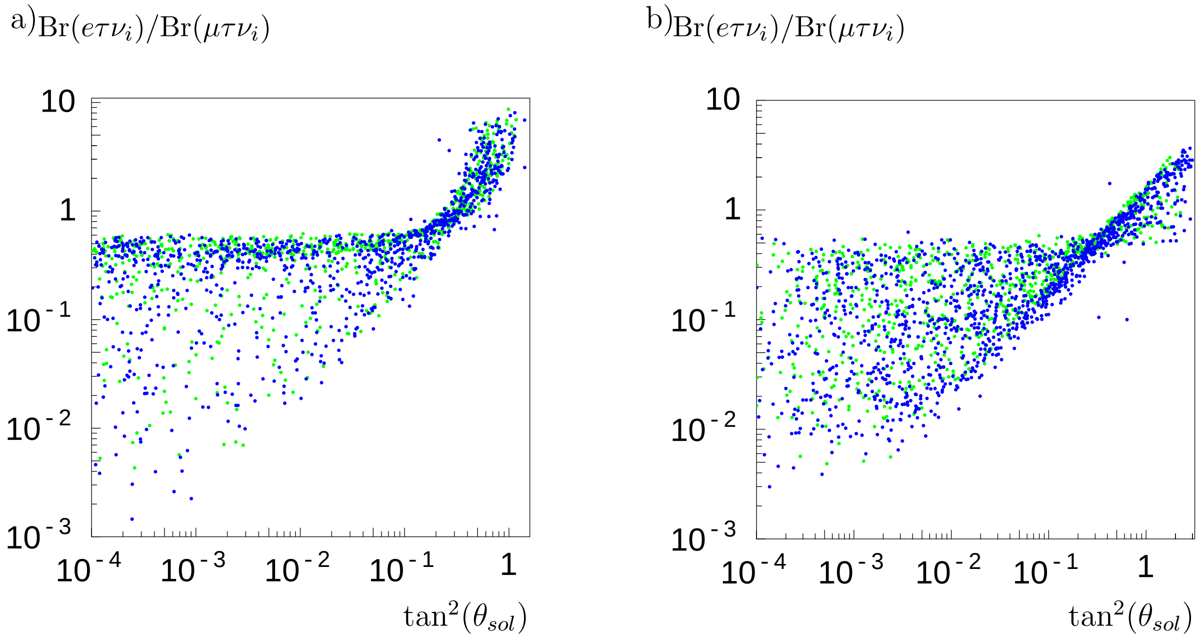


Figure 16: Testing the solar angle. In case of the dark (bright) points $\mu < (>)0$. In a) we have taken $\epsilon_\mu\Lambda_\mu/(\epsilon_\tau\Lambda_\tau) > 0$. and in b) $\epsilon_\mu\Lambda_\mu/(\epsilon_\tau\Lambda_\tau) < 0$

is non-trivial. To understand it note that the couplings W-l_i-ν_j depend on the neutrino mixing, since one must use the *mass eigenstates* for the calculation of the partial decay widths and not the electroweak eigenstates. In addition the ϵ_i enter via the ν_j-$S_k^\pm$-l_i and $\tilde{\chi}_1^0$-$S_k^\pm$-l_i couplings. Remarkably, despite the non-trivial way the ϵ_i parameters enter here, one still has some residual correlation with ϵ_i ratios, as displayed in Fig. 16 This figure shows that, although one does not get a strong correlation in this case, one can still derive lower and upper bounds depending on $\tan^2(\theta_{sol})$. For the favored case [1] of the large mixing angle solution one finds that $\mathrm{Br}(\tilde{\chi}_1^0 \to e\tau\nu_i)/\mathrm{Br}(\tilde{\chi}_1^0 \to \mu\tau\nu_i)$ is determined to be one to within an order of magnitude. For the general bilinear $R\!\!\!/_p$ model the spread in Fig. 16 is due to the lack of knowledge of SUSY parameters. As will be shown in the next subsection, a much stronger correlation appears once the SUSY parameters get determined.

In Table 4.1 we list upper and lower bounds on several ratios of branching ratios which are required by the consistency of the model. The values in the table are hardly dependent on the solution for the solar neutrino problem.

The values in table 4.1 can be viewed as important consistency checks of our model. However, one can also have observables which are able to discriminate between large and small angle solution of the solar neutrino problem. In Fig. 17 we show how several ratios of neutralino decay branching ratios can be used to discriminate between large and small angle solution of the solar neutrino problem. The numbers in Fig. 17 correspond to the following

Table 1: Ratio of branching ratios as required by the consistency of the model. In $\mathrm{Br}(q\bar{q}\sum_i \nu_i)$ we have summed over u, d, and s. Also in case of ν_i we have summed over all neutrinos.

Ratio	lower bound	upper bound
$\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(c\bar{c}\nu_i)$	2.5	6
$\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(\mu^\pm q\bar{q}')$	0.1	3.5
$\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(\tau^\pm q\bar{q}')$	0.1	3.5
$\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(e^+e^-\nu_i)$	5	35
$\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(e^\pm\mu^\mp\nu_i)$	0.3	9.5
$\mathrm{Br}(q\bar{q}\nu_i)$/ $\mathrm{Br}(\mu^+\mu^-\nu_i)$	0.3	9
$\mathrm{Br}(\mu^\pm q\bar{q}')$ / $\mathrm{Br}(\tau^\pm q\bar{q}')$	0.5	3
$\mathrm{Br}(\mu^\pm q\bar{q}')$ / $\mathrm{Br}(\mu^+\mu^-\nu_i)$	1	5
$\mathrm{Br}(\tau^\pm q\bar{q}')$ / $\mathrm{Br}(\mu^+\mu^-\nu_i)$	0.5	6.5
$\mathrm{Br}(e^\pm\mu^\mp\nu_i)$ / $\mathrm{Br}(\mu^+\mu^-\nu_i)$	0.4	1.6

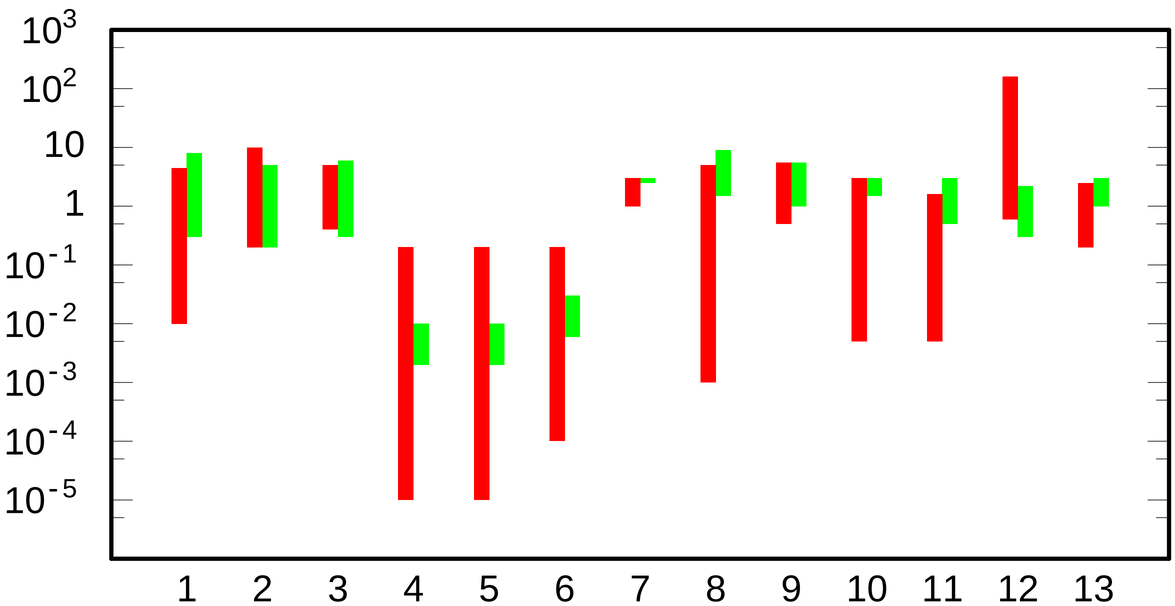


Figure 17: Predicted ranges for the ratios of various branching ratios. The dark stripes are the ranges if one of the large mixing solutions (LMA, LOW or just-so) is realized in nature, the bright stripes are if SMA is realized in nature. The various ratios are given in the text.

a) $\mathrm{Br}(\mu^\pm q\bar{q}')/\mathrm{Br}(\tau^\pm q\bar{q}')$ b) $\mathrm{Br}(e^\pm\mu^\mp\sum_i\nu_i)/\mathrm{Br}(\tau^\pm q\bar{q}')$ c) $\mathrm{Br}(\mu^+\mu^-\sum_i\nu_i)/\mathrm{Br}(\tau^\pm q\bar{q}')$

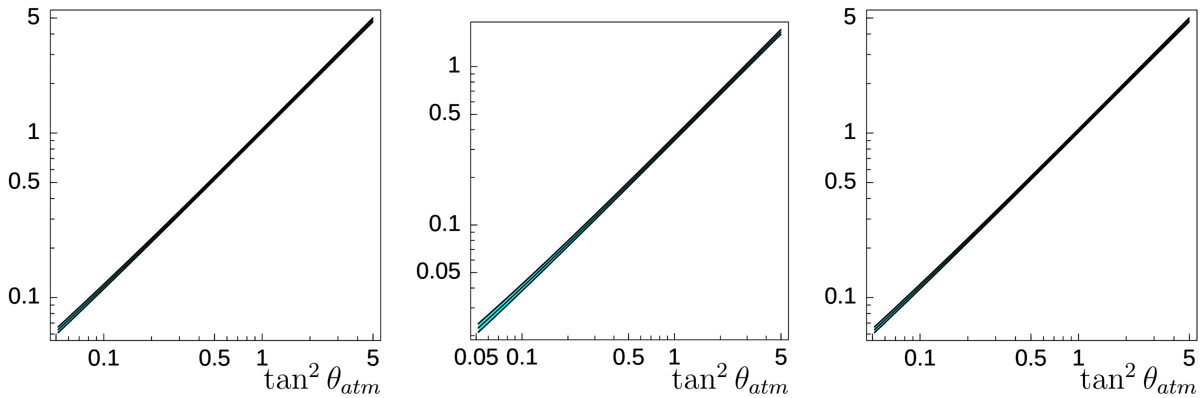


Figure 18: Correlations between $\tan^2\theta_{atm}$ and ratios of branching ratio for the parameter point specified in the text assuming that 10^5 neutralino decays have been measured. The bands correspond to an 1-σ error.

branching ratios: 1 ... $\mathrm{Br}(q\bar{q}\nu_i)$ / $\mathrm{Br}(e^\pm\tau^\mp\nu_i)$, 2 ... $\mathrm{Br}(bb\nu_i)$ / $\mathrm{Br}(\mu^\pm\tau^\mp\nu_i)$, 3 ... $\mathrm{Br}(bb\nu_i)$ / $\mathrm{Br}(\tau^-\tau^+\nu_i)$, 4 ... $\mathrm{Br}(e^\pm q\bar{q}')$ / $\mathrm{Br}(\mu^\pm q\bar{q}')$, 5 ... $\mathrm{Br}(e^\pm q\bar{q}')$ / $\mathrm{Br}(\tau^\pm q\bar{q}')$, 6 ... $\mathrm{Br}(e^\pm q\bar{q}')$ / $\mathrm{Br}(e^\pm\mu^\mp\nu_i)$, 7 ... $\mathrm{Br}(\mu^\pm q\bar{q}')$ / $\mathrm{Br}(e^\pm\mu^\mp\nu_i)$, 8 ... $\mathrm{Br}(\mu^\pm q\bar{q}')$ / $\mathrm{Br}(e^\pm\tau^\mp\nu_i)$, 9 ... $\mathrm{Br}(\tau^\pm q\bar{q}')$ / $\mathrm{Br}(e^\pm\mu^\mp\nu_i)$, 10 ... $\mathrm{Br}(\tau^\pm q\bar{q}')$ / $\mathrm{Br}(e^\pm\tau^\mp\nu_i)$, 11 ... $\mathrm{Br}(e^\pm\mu^\mp\nu_i)$ / $\mathrm{Br}(e^\pm\tau^\mp\nu_i)$, 12 ... $\mathrm{Br}(e^\pm\tau^\mp\nu_i)$ / $\mathrm{Br}(\mu^+\mu^-\nu_i)$, and 13 ... $\mathrm{Br}(\mu^\pm\tau^\mp\nu_i)$ / $\mathrm{Br}(\tau^+\tau^-\nu_i)$. In $\mathrm{Br}(q\bar{q}\sum_i\nu_i)$ we have summed over u, d, and s. Also for the case of ν_i we have summed over all neutrinos.

4.2 After the SUSY spectrum is measured

In the previous section we have discussed the predictions which can be established between neutralino decay branching ratios and neutrino mixing angles before the first SUSY particle is discovered. Let us assume now that the entire spectrum has been measured with some precision, e.g. at a future Linear Collider [35, 37]. As a typical example we discuss the point $M_2 = 120$ GeV, $\mu = 500$ GeV, $\tan\beta = 5$, setting all scalar mass parameters to 500 GeV, and also the A-parameter is assumed to be equal for all sfermions $A = -500$ GeV. Note, that we have taken μ positive to be conservative, as this corresponds to a 'worst-case' scenario." There are at least two parameters which need to be measured precisely: $\tan\beta$ and $|\sin 2\theta_{\tilde{b}}|$ because the 1-loop mass matrix is dominated by the sbottom/bottom loop if at least one of these parameters is large.

$\mathrm{Br}(e^{\pm}q\bar{q}')/\mathrm{Br}(\tau^{\pm}q\bar{q}')$

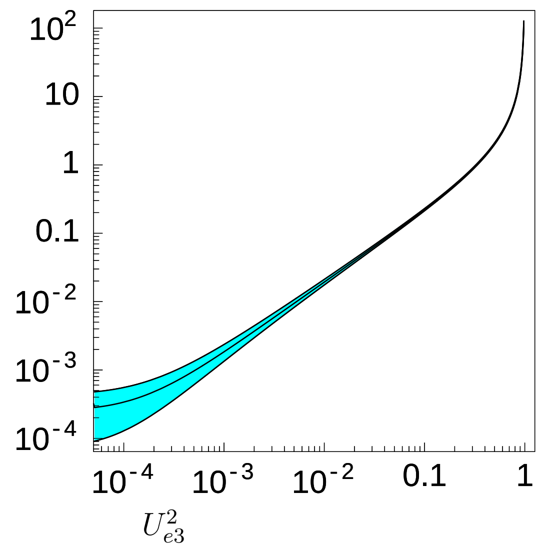


Figure 19: Correlation between U_{e3}^2 and the ratio $\mathrm{Br}(e^{\pm}q\bar{q}')/\mathrm{Br}(\tau^{\pm}q\bar{q}')$ for the parameter point specified in the text assuming that 10^5 neutralino decays have been measured. The band corresponds to an 1-σ error.

$\mathrm{Br}(e^{\pm}\tau^{\mp}\sum_i \nu_i)/\mathrm{Br}(\mu^{\pm}\tau^{\mp}\sum_i \nu_i)$

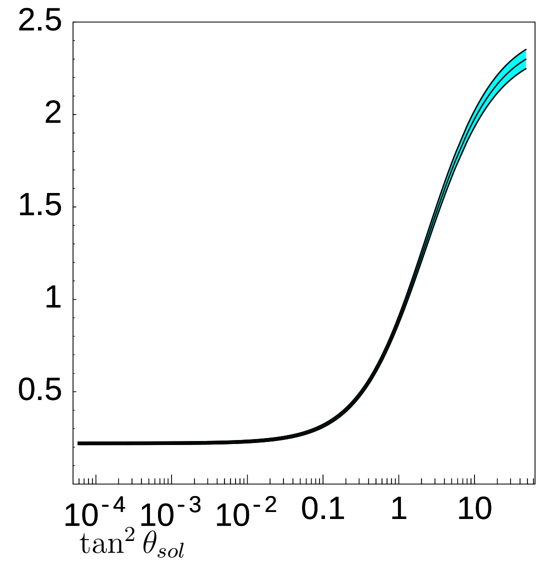


Figure 20: Correlation between $\tan^2\theta_{sol}$ and the ratio $\mathrm{Br}(e^{\pm}\tau^{\mp}\sum_i \nu_i)/\mathrm{Br}(\mu^{\pm}\tau^{\mp}\sum_i \nu_i)$ for the parameter point specified in the text assuming that 10^5 neutralino decays have been measured. The band corresponds to an 1-σ error.

In Fig. 18 — Fig. 20 the same relationships as discussed above are displayed assuming that the particle spectrum and the corresponding mixing angles are known to the 1% level or better. In addition we have assumed that 10^5 neutralino decays have been identified and measured. Taking at the moment only the statistical error this translates to a relative error on the branching ratio Br(X) of the form $1/\sqrt{10^5 \mathrm{Br}(X)}$. It is clear from these figures that there exist excellent correlations between the ratio of various branchings and $\tan^2\theta_{atm}$ as well as the parameter U_{e3}^2 probed in reactor experiments. For the solar angle we observe a strong dependence on $\tan^2\theta_{sol}$ for the case of large mixing angle solutions (LMA, LOW or vacuum) of the solar neutrino problem. For the small mixing angle MSW solution, even though the dependence on $\tan^2\theta_{sol}$ becomes, unfortunately, rather weak, the ratio of branching ratios for $\mathrm{Br}(e^\pm\tau^\mp\sum_i\nu_i)/\mathrm{Br}(\mu^\pm\tau^\mp\sum_i\nu_i)$ is predicted with good accuracy for any $\tan^2\theta_{sol} \lesssim 0.1$.

5 Conclusions

Supersymmetry with broken R Parity provides a predictive hierarchical pattern for neutrino masses and mixings determined in terms of just three independent parameters, assuming CP to be conserved in the lepton sector. This can solve the solar and atmospheric neutrino anomalies in a way that allows leptonic mixing angles to be probed at high energy colliders, providing an independent determination of neutrino mixing. Taking into account data from atmospheric neutrino experiments we have derived specific predictions for neutralino decays, illustrated in Fig. 11, Fig. 12 and Fig. 13. Probing the solar angle is more involved due to the intrinsic spread in SUSY parameters, see Fig. 16. However, we have demonstrated that, with about 10^5 neutralino decays, and a determination of the spectrum of the theory to within 1% or better, there are very stringent correlations between solar neutrino-physics and neutralino-physics (see Fig. 20). We showed that several ratios of neutralino decay branching ratios can be used to discriminate between large and small angle solutions of the solar neutrino problem. Therefore, the hypothesis that bilinear R-parity violation is the origin of neutrino mass and mixing can be easily ruled out or confirmed at future collider experiments. This statement is actually more general, to the extent that the bilinear model is an effective theory of a model where R-parity is violated spontaneously.

Acknowledgments

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A Approximate Diagonalization of scalar mass matrix

Let us assume that the following simplifying conditions hold

$$|\epsilon_i \epsilon_j| \ll |B_k \epsilon_k| \tag{61}$$

$$|v_i| \ll v_D, v_U \tag{62}$$

$$|\epsilon_i| \ll |\mu| \tag{63}$$

$$\begin{aligned}&2|\cos\alpha\,(B_i\epsilon_i - g_Z v_U v_i) + \sin\alpha\,(g_Z v_D v_i - \mu\epsilon_i)\,| \ll \\ &|\cos^2\alpha\Delta M_{rad} + g_Z\,(v_U\cos\alpha - v_D\sin\alpha)^2 + B_0\mu\,(\cot\beta\cos\alpha - \tan\beta\sin\alpha)^2 - \frac{\epsilon_e}{v_i}\,(v_D\mu - B_i v_U)\,|\end{aligned} \tag{64}$$

$$\begin{aligned}&2|-\sin\alpha\,(B_i\epsilon_i - g_Z v_U v_i) + \cos\alpha\,(g_Z v_D v_i - \mu\epsilon_i)\,| \ll \\ &|\sin^2\alpha\Delta M_{rad} + g_Z\,(v_U\sin\alpha + v_D\cos\alpha)^2 + B_0\mu\,(\cot\beta\sin\alpha + \tan\beta\cos\alpha)^2 - \frac{\epsilon_e}{v_i}\,(v_D\mu - B_i v_U)\,|\end{aligned} \tag{65}$$

where $g_Z = (g^2 + g'^2)/4$, $\Delta M_{rad} = 3g^2 m_t^4/(16\pi^2 m_W^2 \sin^2\beta \sin^2\theta)$ with $\sin^2\theta = (v_U^2 + v_D^2)/(v_U^2 + v_D^2 + v_1^2 + v_2^2 + v_3^2)$ and α is the mixing angle that diagonalizes the upper left 2×2 sub-matrix which corresponds to the usual Higgs mass matrix in the MSSM limit. Under these approximations the mixing matrix reads

$$R^{S^0} = \begin{pmatrix} c_2c_4c_6\sin\alpha & c_2c_4c_6\cos\alpha & s_2 & s_4 & s_6 \\ c_1c_3c_5\cos\alpha & -c_1c_3c_5\sin\alpha & s_1 & s_3 & s_5 \\ -s_1\cos\alpha - s_2\sin\alpha & s_1\sin\alpha - s_2\cos\alpha & c_1c_2 & 0 & 0 \\ -s_3\cos\alpha - s_4\sin\alpha & s_3\sin\alpha - s_4\cos\alpha & 0 & c_3c_4 & 0 \\ -s_5\cos\alpha - s_6\sin\alpha & s_5\sin\alpha - s_6\cos\alpha & 0 & 0 & c_5c_6 \end{pmatrix} \tag{66}$$

with

$$s_{2i} = \frac{\cos\alpha\,(B_i\epsilon_i - g_Z v_U v_i) + \sin\alpha\,(g_Z v_D v_i - \mu\epsilon_i)}{\cos^2\alpha\Delta M_{rad} + g_Z\,(v_U\cos\alpha - v_D\sin\alpha)^2 + B_0\mu\,(\cot\beta\cos\alpha - \tan\beta\sin\alpha)^2 - \frac{\epsilon_i}{v_i}\,(v_D\mu - B_i v_U)} \tag{67}$$

$$s_{2i-1} = \frac{-\sin\alpha\,(B_i\epsilon_i - g_Z v_U v_i) + \cos\alpha\,(g_Z v_D v_i - \mu\epsilon_i)}{\sin^2\alpha\Delta M_{rad} + g_Z\,(v_U\sin\alpha + v_D\cos\alpha)^2 + B_0\mu\,(\cot\beta\sin\alpha + \tan\beta\cos\alpha)^2 - \frac{\epsilon_i}{v_i}\,(v_D\mu - B_i v_U)} \tag{68}$$

where $i = 1, 2, 3$ and $c_i = 1 - s_i^2/2$. For the case where $\frac{\epsilon_i}{v_i}(v_D\mu - B_i v_U)$ are equal for all i, the $\epsilon_i\epsilon_j$ part in the mixing between the sneutrinos becomes important. Therefore

$$R^{P^0} \Rightarrow R^\epsilon R^{P^0} \tag{69}$$

$$R^{S^0} \Rightarrow R^\epsilon R^{S^0} \tag{70}$$

$$R^\epsilon = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\epsilon_\tau}{\sqrt{\epsilon_1^2+\epsilon_3^2}} & 0 & \frac{\epsilon_e}{\sqrt{\epsilon_1^2+\epsilon_3^2}} \\ 0 & 0 & 0 & \frac{-\epsilon_\mu}{\sqrt{\epsilon_1^2+\epsilon_2^2}} & \frac{\epsilon_e}{\sqrt{\epsilon_1^2+\epsilon_2^2}} \\ 0 & 0 & \frac{\epsilon_e}{|\epsilon|} & \frac{\epsilon_\mu}{|\epsilon|} & \frac{\epsilon_\tau}{|\epsilon|} \end{pmatrix} \tag{71}$$

On the other hand if $\frac{\epsilon_e}{v_1}(v_D\mu - B_1 v_U) = \frac{\epsilon_\mu}{v_2}(v_D\mu - B_2 v_U) \neq \frac{\epsilon_\tau}{v_3}(v_D\mu - B_3 v_U)$ and $|\epsilon_i\epsilon_j| \ll |\frac{\epsilon_\mu}{v_2}(v_D\mu - B_2 v_U) - \frac{\epsilon_\tau}{v_3}(v_D\mu - B_3 v_U)|$ one has

$$R^\epsilon = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{-\epsilon_\mu}{\sqrt{\epsilon_1^2+\epsilon_2^2}} & 0 & \frac{\epsilon_e}{\sqrt{\epsilon_1^2+\epsilon_2^2}} \\ 0 & 0 & 0 & \frac{-\epsilon_e}{\sqrt{\epsilon_1^2+\epsilon_2^2}} & \frac{-\epsilon_\mu}{\sqrt{\epsilon_1^2+\epsilon_2^2}} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c_7 & 0 & s_7 \\ 0 & 0 & 0 & c_8 & s_8 \\ 0 & 0 & -s_7 & -s_8 & c_7 c_8 \end{pmatrix} \tag{72}$$

$$s_7 = \frac{\epsilon_e\epsilon_\tau}{\frac{\epsilon_e}{v_1}(v_D\mu - B_1 v_U) - \frac{\epsilon_\tau}{v_3}(v_D\mu - B_3 v_U)} \tag{73}$$

$$s_8 = \frac{\epsilon_\mu\epsilon_\tau}{\frac{\epsilon_e}{v_1}(v_D\mu - B_1 v_U) - \frac{\epsilon_\tau}{v_3}(v_D\mu - B_3 v_U)} \tag{74}$$

$$c_{7,8} = 1 - \frac{s_{7,8}^2}{2} \tag{75}$$

We have checked, that the eigenvalues obtained with these mixing matrices agree with those obtained in [39] in lowest order of the R-parity breaking parameters.

B Couplings

Most of the couplings necessary for the calculation of the neutralino decays have already been given in [21] where also the definition of the mixing matrices are given. The remaining couplings involve $S^\pm_k$-u_i-d_i and $\tilde{q}_j$-q'_k-$l^\pm_i$. Neglecting the generation mixing among sfermions the corresponding Lagrangian is given by:

$$\begin{aligned}
\mathcal{L} &= \tilde{d}_j \overline{u}_k \left(C^{\tilde{d}}_{Lkl_i} P_L + C^{\tilde{d}}_{Rkl_i} P_R \right) l^+_i + \tilde{u}_j \overline{u}_k \left(C^{\tilde{u}}_{Lkl_i} P_L + C^{\tilde{u}}_{Rkl_i} P_R \right) l^-_i \\
&\quad + S^-_k \bar{d}_i \left(a_{S^-_k i} P_L + b_{S^-_k i} P_R \right) u_i + \text{h.c.} \qquad (76)
\end{aligned}$$

with

$$a_{S^-_k i} = h^{ii}_D R^{S^\pm}_{k1} \qquad (77)$$

$$b_{S^-_k i} = (h^{ii}_U)^* (R^{S^\pm}_{k2})^* \qquad (78)$$

$$C^{\tilde{d}}_{Lkl_i} = h^{kk}_U (R^{\tilde{d}}_{j1})^* V^*_{i2} \qquad (79)$$

$$C^{\tilde{d}}_{Rkl_i} = -g (R^{\tilde{d}}_{j1})^* U_{i1} + (h^{kk}_D)^* (R^{\tilde{d}}_{j2})^* U_{i2} \qquad (80)$$

$$C^{\tilde{u}}_{Lkl_i} = h^{kk}_D (R^{\tilde{u}}_{j1})^* U^*_{i2} \qquad (81)$$

$$C^{\tilde{u}}_{Rkl_i} = -g (R^{\tilde{u}}_{j1})^* V_{i1} + (h^{kk}_U)^* (R^{\tilde{u}}_{j1})^* V_{i2} \qquad (82)$$

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