The interaction between center monopoles in $SU(2)$ Yang–Mills

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We study the potential between a static center monopole and antimonopole in $4d$ $SU(2)$ Yang–Mills theory. Using a new numerical method, we show that the 't Hooft loop is a dual order parameter with respect to the Wilson loop, for the deconfinement phase transition. We observe a $3d$ Ising–like critical behaviour for the dual string tension related to the spatial 't Hooft loop as a function of the temperature.

1. Introduction

In the $4d$ $SU(2)$ Yang–Mills theory a second order phase transition occurs. A critical temperature T_c separates a cold confined phase and a hot deconfined one. More than two decades ago, 't Hooft proposed[1] that the center degrees of freedom might play an important role in the confinement mechanism. Along with this approach, he introduced an operator – named 't Hooft loop – with a non trivial structure with respect to the center subgroup, and argued that it could be used as a dual order parameter for the deconfinement phase transition. The 't Hooft loop, $\tilde{W}(C)$, is associated with a given closed contour C, and is defined in the continuum $SU(N)$ theory by the following equal–time commutation relations[1]

$$[W(C), W(C')] \;=\; \left[\tilde{W}(C), \tilde{W}(C')\right] \;=\; 0 \qquad (1)$$

$$\tilde{W}^{\dagger}(C) W(C') \tilde{W}(C) \;=\; e^{i\frac{2\pi}{N} n_{CC'}} W(C') \qquad (2)$$

where $W(C')$ is the Wilson loop associated with the closed contour C' and $n_{CC'}$ is the linking number of C and C'. Just like the Wilson loop creates an elementary electric flux along C', the 't Hooft loop creates an elementary magnetic flux along the path C affecting any Wilson loop "pierced" by C. In that sense, the two types of loop are dual to each other. At zero temperature, it has been shown[1–3] that also the 't Hooft loop behaviour is dual to that of the Wilson loop: in the absence of massless excitations, an area law for one implies a perimeter law for the other, and vice versa. Hence, at $T = 0$ the 't Hooft loop obeys a perimeter law.

Several analytical[4,5] and numerical[6–9] studies have been carried out in order to investigate this issue of duality at finite temperature. At $T > 0$, the Lorentz symmetry is broken, so spatial and temporal loops can have different behaviours. Because the spatial string tension persists also above T_c for the Wilson loop, temporal 't Hooft loops are expected to show a perimeter law in both phases; spatial 't Hooft loops, on the other hand, are expected to obey a perimeter law in the confined phase and an area law – defining a dual string tension (strictly speaking it is an action density) – in the deconfined phase.

Using a novel computational approach, we have performed[9] a numerical study of the 't Hooft loop, showing that it has a dual behaviour with respect to the Wilson loop both at $T = 0$ and $T > 0$. Our main result is that the 't Hooft loop is indeed a dual order parameter for the deconfinement phase transition.

2. The lattice formulation

The 't Hooft loop characterizes the static potential between a center monopole and anti-

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monopole, just like the Wilson loop does for two electric charges. Thus, to obtain a 't Hooft loop, one creates a static monopole – antimonopole pair, then the two charges propagate and, at the end, annihilate. This is the standard definition[10,11] of the 't Hooft loop on the lattice. Specifically, consider the $SU(2)$ lattice gauge theory with the usual Wilson plaquette action,

$$S(\beta) = \beta \sum_P (1 - \frac{1}{2}\mathrm{Tr}(U_P)) \tag{3}$$

where the sum extends over all the plaquettes P and U_P is the path–ordered product of the links around P. Starting from $S(\beta)$, one defines the partition function

$$Z(\beta) = \int [dU] \exp(-S(\beta)) \tag{4}$$

To insert a monopole – antimonopole pair, one must create a flux tube with non trivial value with respect to the center degrees of freedom: the center monopoles lie at the two ends of the tube. The flux is switched on by multiplying by a non trivial center element z the plaquettes along a path, in the dual lattice, joining the two monopoles. For $SU(2)$, the only non trivial center element is -1 and so multiplying a plaquette by z is equivalent to flipping its coupling. Replicating this construction at successive time–slices, we create an elementary magnetic flux along a closed contour C in the dual lattice, extending in one space- and in the time-direction (temporal 't Hooft loop). In a similar way, we can make the closed contour C extend only in spatial directions (spatial 't Hooft loop). The set $\mathcal{P}(\mathcal{S})$ of the plaquettes whose coupling is flipped, $\beta \to -\beta$, is dual to a surface $\mathcal{S}$ supported by C. The action $S_{\mathcal{S}}$ of the system where an elementary flux along a closed contour C has been switched on is then given, up to an additive constant, by

$$S_{\mathcal{S}}(\beta) = -\frac{\beta}{2} \left(\sum_{P \notin \mathcal{P}(\mathcal{S})} \mathrm{Tr}(U_P) - \sum_{P \in \mathcal{P}(\mathcal{S})} \mathrm{Tr}(U_P) \right) \tag{5}$$

and the partition function is

$$Z_C(\beta) = \int [dU] \exp(-S_{\mathcal{S}}(\beta)) \tag{6}$$

$Z_C(\beta)$ does not depend on the particular chosen surface $\mathcal{S}$, since different choices are related by a change of integration variables.

The expectation value of the 't Hooft loop gives the free energy cost to create the flux loop. It is obtained by comparison between the states with the loop switched on and the states where it is off:

$$\langle \tilde{W}(C) \rangle = Z_C(\beta)/Z(\beta) \tag{7}$$

This expression can be rewritten in the form

$$\langle \tilde{W}(C) \rangle = \langle \exp\big(-\beta \sum_{P \in \mathcal{P}(\mathcal{S})} \mathrm{Tr}(U_P) \big) \rangle \tag{8}$$

with the average taken with the standard Wilson action. The numerical computation of the ratio (7), (8) is a very hard numerical task due to the very poor overlap between the relevant phase space of the numerator and the denominator.

Recently, the 't Hooft loop, or special cases of it, have been studied numerically on the lattice. In [6,7] the sampling problem was solved by using a multihistogram method. In our study, we adopt a new approach, where the ratio $Z_C(\beta)/Z(\beta)$ is rewritten as a product of intermediate ratios, each easily measurable.

3. The numerical method

In this section we present the new numerical technique that has been developed to measure the expectation value of the 't Hooft loop operator. The direct evaluation of $\langle \tilde{W}(C) \rangle$ with (7), (8) by a single Monte Carlo simulation is not reliable. Importance sampling of $Z(\beta)$ leads to generating field configurations whose contribution to $Z_C(\beta)$ is negligible and vice versa. The usual approach to overcome this difficulty is the multihistogram method[6,7], where one performs several different simulations in which the coupling of the stack of plaquettes in $\mathcal{P}(\mathcal{S})$ is gradually changed from β to $-\beta$. Instead, we interpolate in the number of plaquettes in $\mathcal{P}(\mathcal{S})$ with flipped coupling. We make use of the following identity, where N is the total number of plaquettes belonging to $\mathcal{P}(\mathcal{S})$

$$\frac{Z_C(\beta)}{Z(\beta)} = \frac{Z_N(\beta)}{Z_{N-1}(\beta)} \cdot \frac{Z_{N-1}(\beta)}{Z_{N-2}(\beta)} \cdot \ldots \cdot \frac{Z_1(\beta)}{Z_0(\beta)} \tag{9}$$

where $Z_k(\beta)$, $k = 0, \ldots, N$ ($Z_N \equiv Z_C$ and $Z_0 \equiv Z$) is the partition function of the system

where only the first k plaquettes in $\mathcal{P}(\mathcal{S})$ have flipped coupling. Consider the following figure where $\mathcal{P}(\mathcal{S})$ is the set of bold and thin plaquettes, whose coupling has to be flipped to create the loop. We interpolate between 0 and N flipped plaquettes by considering a snake–like movement where the coupling is progressively flipped.

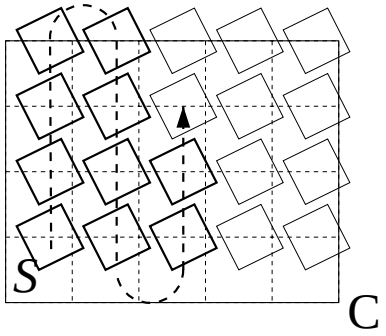


Figure 1 shows the snake–like flipping movement: bold plaquettes have coupling $-\beta$ and thin ones $+\beta$. Thus, performing N independent Monte Carlo simulations – each corresponding to a different number of plaquettes in $\mathcal{P}(\mathcal{S})$ with flipped coupling – one can have a reliable estimation of $\langle \tilde{W}(C) \rangle$. The efficiency of the method can be further enhanced through variance reduction tricks, described in detail in [9]. An advantage of our method over the multihistogram technique is that the products in (9) give information on smaller 't Hooft loops for free; moreover, the error analysis is simpler and less delicate than in a multihistogram analysis.

4. Results

We focus on the free energy $F(R)$ of a pair of static center monopoles as a function of their separation R. It can be obtained as $\lim_{R_t \to \infty} - \mathrm{Ln}[\langle \tilde{W}(R, R_t) \rangle]/R_t$ by taking elongated $R \times R_t$ rectangular loops, in the same way as one extracts the static potential between two chromoelectric charges. We take R_t as large as possible, i.e. equal to the lattice size L. This is analogous to measuring the correlation of two Polyakov loops, and is the correct approach at finite temperature. Therefore we must flip the coupling of $R \times L$ plaquettes. We do this with the snake–like movement: we scan first the R_t- and then the $R-$direction. With this ordering, the intermediate partition functions $Z_L, Z_{2L}, .., Z_{R \times L}$ in (9) provide us with the free energy at separations $1, 2.., R$ respectively. The final ratio $Z_{L \times L}/Z_0$ gives the free energy of a center vortex as computed in [6].

At zero temperature the 't Hooft loop is expected to obey a perimeter law: $Z_k/Z_0 \propto e^{-c\tilde{P}_k}$, where $\tilde{P}_k$ is the length of the contour C_k. For the sequence of 'snake' flipped plaquettes – up to boundary effects occurring for very small or very large contours – $\tilde{P}_k$ assumes only two different values[9]. Indeed numerical simulations show that Z_k/Z_0 centers around two values only, confirming the perimeter law. Furthermore, a direct measurement of Creutz ratios $\chi(R, R)$ shows a quick drop to zero with the distance R.

The free energy $F(R)$ can be fitted by a Yukawa form $\frac{e^{-mR}}{R}$, up to an irrelevant additive constant. However the screening mass m is rather large, so that the signal quickly dies out. The measured value of ca. 2 Gev for m is close to the lightest gluonic excitation, the scalar glueball (~ 1.65 GeV), as observed in [7,8]. More precise statements would require a much more ambitious numerical study.

At finite temperature, the spatial and the temporal loops can have different behaviours. This difference can be understood by the following argument. Consider a spatial 't Hooft loop, that is a loop in the (x, y) plane obtained by flipping the coupling of plaquettes in the (z, t) plane. The magnetic flux could diminish its free energy by spreading out, but this is limited by the finiteness of the t–direction. Conversely a temporal loop can spread its flux without limitation. Thus we expect higher free energies for spatial loops over temporal ones. Moreover, a dual behaviour of the 't Hooft loop with respect to the Wilson loop leads to the following scenario. The spatial Wilson loop obeys an area law below and above T_c and so we expect a perimeter law behaviour, with a Yukawa term, for the temporal 't Hooft loop. The temporal Wilson loop obeys a perimeter law above T_c and so the spatial 't Hooft loop should obey a perimeter law, with a Yukawa term,

below T_c and an area law above T_c, defining a dual string tension. This is precisely what we observe.

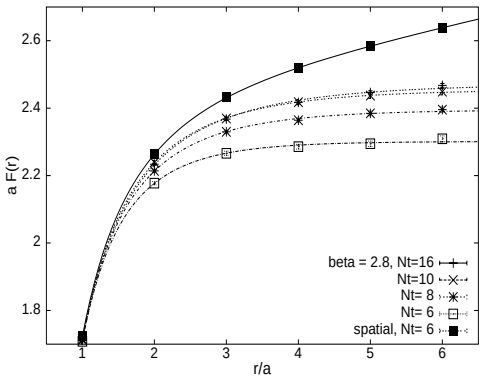
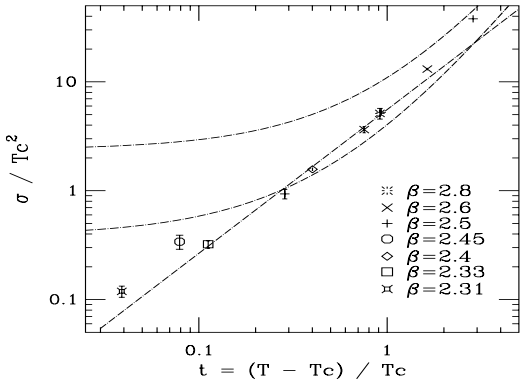


Figure 2 displays our results for the free energy vs. monopole separation at $T > T_c$. The spatial 't Hooft loop (full squares) shows a dual string tension; the temporal 't Hooft loop (other data) shows a screening mass increasing with T. An attempt to fit the data for the temporal loop with the ansatz $F_0 + c\frac{e^{-mR}}{R} + \sigma R$, including a linear term, gives a dual string tension σ consistent with zero. In contrast, this linear term is required to obtain an acceptable fit above T_c for the spatial loop: a dual string tension appears. We have measured the screening mass m at various temperatures both for the spatial and the temporal 't Hooft loops. In the former, it seems to be little affected by temperature, while in the latter we observe a linear dependence in T, much like for the glueball excitation which it presumably represents[12]. As for $T = 0$, an accurate numerical study is in order to make precise quantitative statements.



The dual string tension σ depends on temperature and must vanish at T_c. Figure 3 shows that it does so as $\sigma \propto t^{2\nu}$, where $t = \frac{T-T_c}{T_c}$ is the reduced

temperature. The straight line is a power law fit to the $t < 1$ data, and the curves are the perturbative result, to leading (upper) and next (lower) order. The fitted critical exponent ν, associated with the correlation length $\xi = \sigma^{-1/2}$, comes out very close to that of the $3d$ Ising model: 0.66(3) vs. ≈ 0.63. This should be expected since both models are in the same universality class. This dual string tension can then be taken as order parameter for the restoration of the (magnetic) Z_N symmetry, corresponding to *de*confinement [4,13].

5. Conclusions

Using a new numerical method, we have shown that the 't Hooft loop is a dual order parameter for the deconfinement phase transition in $SU(2)$. We have observed a dual string tension in the deconfined phase with $3d$ Ising–like critical exponent. We have measured the screening masses and studied their dependence on the temperature; more accurate investigations are required for more quantitative results on this point.

REFERENCES

1. G. 't Hooft, Nucl. Phys. B138 (1978) 1.
2. T. Tomboulis, Phys. Rev. D23 (1981) 2371.
3. S. Samuel, Nucl. Phys. B214 (1983) 532.
4. C. Korthals–Altes, A. Kovner and M. Stephanov, Phys. Lett. B469 (1999) 205.
5. T. Bhattacharya et al., Nucl. Phys. B383 (1992) 497.
6. T. G. Kovács, E. T. Tomboulis, hep-lat/0002004.
7. C. Hoelbling, C. Rebbi, V.A. Rubakov, hep-lat/0003010.
8. L. Del Debbio, A. Di Giacomo and B. Lucini, hep-lat/0006028.
9. Ph. de Forcrand, M. D'Elia and M. Pepe, hep-lat/0007034.
10. A. Ukawa, P. Windey and A. H. Guth, Phys. Rev. D21 (1980) 1013.
11. M. Srednicki, L. Susskind, Nucl. Phys. B179 (1981) 239.
12. S. Datta and S. Gupta, hep-lat/9906023.
13. C. Korthals–Altes, A. Kovner, hep-ph/0004052.