OBSERVATION OF QUADRUPOLE MODE FREQUENCY AND ITS CONNECTION WITH BEAM LOSS

T.Uesugi*, S.Machida, Y.Mori, KEK-Tanashi, Midori-cho, Tanashi-shi, Tokyo 188-8501, Japan
M.Ikegami, JAERI, Tokai-mura, Naka-gun, Ibaraki-ken 319-1195, Japan
S.Ninomiya, A.Mochihashi, T.Oki, S.Hidaka, RCNP, Ibaraki, Osaka 567-0047, Japan
D.Arakawa, T.Toyama, KEK 1-1 Oho, Tsukuba-shi, Ibaraki-ken 305-0801, Japan
and K.Noda, NIRS, Anagawa, Inage-ku, Chiba-ken 263-8555, Japan

Abstract

Recent Simulation results imply that the number of particles in a high intensity synchrotron is limited by the resonance crossing of coherent mode oscillations, not by the incoherent one. In order to verify it, we measured the tune shift of quadrupole mode oscillations in HIMAC at National Institute of Radiological Sciences (NIRS) to show the connection with beam loss. In this paper, we discuss signals observed in a coasting beam. We also presents the numerical calculation of envelope motion for a bunched beam.

1 INTRODUCTION

It is believed that space charge effects limit the number of particles in a high intensity synchrotron. However, the detailed mechanism of beam loss is not clear. One model tells that the incoherent tune of an individual partcle is reduced by the space charge field to a major resonance line, and resonate with lattice field errors. Another model, proposed by Sacherer in 1960's[1], is that the coherent mode oscillations of a beam resonate with lattice field errors. The incoherent model is not self-consistent because it neglects the time evolution of distribution due to the space charge force.

Recent simulation results seem to support the latter model[2]. In order to verify this, we planned to measure the tune shifts of coherent mode oscillations and their connection with beam loss. We observed coherent mode signals in a coasting beam in HIMAC synchrotron. To start with, we discuss the three dimensional envelope equations, where it is shown that the envelope oscillation frequencies in a bunched beam do not differ much from that in a coasting beam. Then, we report our measurements.

2 ENVELOPE OSCILLATIONS IN A BUNCHED BEAM

2.1 3D envelope equations

For a bunched beam, envelope oscillations occur in three dimensions. Sacherer studied three dimensional envelope equations and found that they depend only on rms beam size, as far as the distribution has ellipsoidal symmetry [3]. They are

$$\begin{aligned}
\frac{d^2\tilde{x}}{d\phi^2} + \left(\nu_{0x}^2 - \frac{Nr_0R^2}{\beta^2\gamma^2}\lambda_3 g(\tilde{x},\tilde{y},\gamma\tilde{z})\right)\tilde{x} &= \frac{(R\epsilon_x)^2}{\tilde{x}^3} \\
\frac{d^2\tilde{y}}{d\phi^2} + \left(\nu_{0y}^2 - \frac{Nr_0R^2}{\beta^2\gamma^2}\lambda_3 g(\tilde{y},\gamma\tilde{z},\tilde{x})\right)\tilde{y} &= \frac{(R\epsilon_y)^2}{\tilde{y}^3} \\
\frac{d^2\gamma\tilde{z}}{d\phi^2} + \left(\nu_{0z}^2 - \frac{Nr_0R^2}{\beta^2\gamma^2}\lambda_3 g(\gamma\tilde{z},\tilde{x},\tilde{y})\right)\gamma\tilde{z} &= \frac{(R\epsilon_z)^2}{\tilde{z}^3}
\end{aligned} \tag{1}$$

where ϕ is the azimuth along the circumference of a synchrotron, R is the ring average radius, ν_{0x} is bare tune, and the geometric factor g is elliptic integral,

$$g(a,b,c) = \frac{3}{2}\int_0^\infty \frac{ds}{(a^2+s)^{3/2}(b^2+s)^{1/2}(c^2+s)^{1/2}} \tag{2}$$

and λ_3 is about $1/5\sqrt{5}$, which very weakly depends on details of distribution. In the parenthesis of the left hand side of eq.1 corresponds to a square of depressed tune ν_x^2 when ($\tilde{x}$,$\tilde{y}$,$\gamma\tilde{z}$) is the matched beam size. Differentiating eq.1 and use $\epsilon_x = \nu_x\tilde{x}_0^2/R$, we have

$$\begin{aligned}
\frac{d^2}{d\phi^2}\Delta\tilde{x} + 4\nu_x^2\Delta\tilde{x} &= \frac{\lambda_3 Nr_0R^2}{\beta^2\gamma^2}\tilde{x}_0\Delta g|_{\tilde{x}_0,\tilde{y}_0,\gamma\tilde{z}_0} \\
\frac{d^2}{d\phi^2}\Delta\tilde{y} + 4\nu_y^2\Delta\tilde{y} &= \frac{\lambda_3 Nr_0R^2}{\beta^2\gamma^2}\tilde{y}_0\Delta g|_{\tilde{y}_0,\gamma\tilde{z}_0,\tilde{x}_0} \\
\frac{d^2}{d\phi^2}\gamma\Delta\tilde{z} + 4\nu_z^2\gamma\Delta\tilde{z} &= \frac{\lambda_3 Nr_0R^2}{\beta^2\gamma^2}\gamma\tilde{z}_0\Delta g|_{\gamma\tilde{z}_0,\tilde{x}_0,\tilde{y}_0}
\end{aligned} \tag{3}$$

where the subscript “0” denotes matched solutions. The equations above can not be solved analytically because of elliptic integral. One way to solve them is to take some approximations[4] and substitute arithmetic function for g. Here, we employ numerical integration.

2.2 Numerical solution

We solved eigen equations of eq.3 numerically, for a given focusing force, emittance and beam intensity. It is expected that, in the limit of long bunch and low synchrotron frequency, the behavior of envelope motion will be similar to that of a coasting beam.

In solving eq.3, matched beam size $(\tilde{x}_0, \tilde{y}_0, \gamma\tilde{z}_0)$ was calculated by eq.1 with $d^2/d\phi^2 = 0$ and elliptic integral

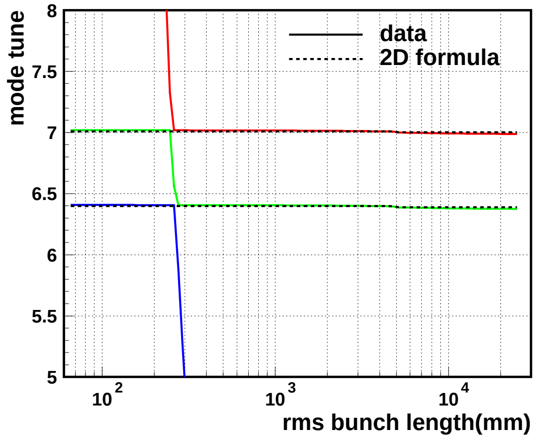

Figure 1: Bunch length dependence of eigen mode frequencies with the parameters of table1.

is evaluated with a numerical package available [5]. We chose parameters in Table1. In order to study bunch length dependence, we varied longitudinal focusing force, and adjusted beam intensity simultaneously so that transverse incoherent tune remains constant.

At first, we show in Fig.1 the eigen frequencies of envelope oscillations, together with the one calculated by a 2D formula.

$$\begin{aligned}\nu_{\pm}^2 &= \frac{\tilde{x}_0+\tilde{y}_0}{2\tilde{y}_0}\frac{\nu_x}{\nu_{0x}} \pm \sqrt{(2\nu_{0x}^2-2\nu_{0y}^2)^2-(\nu_{0x}\Delta\nu_x)^2} \\ &= 7.01^2, 6.39^2. \qquad (4)\end{aligned}$$

In each region except $\gamma\tilde{z}_0 \sim 250$mm, two of calculated modes agree with the values estimated by eq.4. Eigen vectors, shown in Fig.2, prove that these modes are pure horizontal, vertical and longitudinal one. At around $\nu_{0z} \sim \nu_{0x}, \nu_{0y}$, the oscillations form two coupled modes, ie. breathing(even) mode with higher frequency and quadrupole(odd) mode with lower frequency. This behavior is similar to the coupled phenomena of a single particle.

In conclusion, the coupling of transverse and longitudinal envelope oscillations due to space charge force seems negligible for a synchrotron where the synchrotron oscillation frequency is much less than betatron oscillation. More detailed discussion could be found in Ref.[6]

Table 1: Parameters used in envelope mode calculation.

particle	6MeV/u He^{2+} ($\gamma = 1.006$)
bare tune	(ν_{0x}, ν_{0y})=(3.6,3.3)
	ν_{0z}=0.00978 @$\gamma\tilde{z}$=5000mm
100% emittance	ϵ=(10,10,55000)πmm-mrad
beam intensity	1×10^{10}ppp @$\gamma\tilde{z}$=5000mm

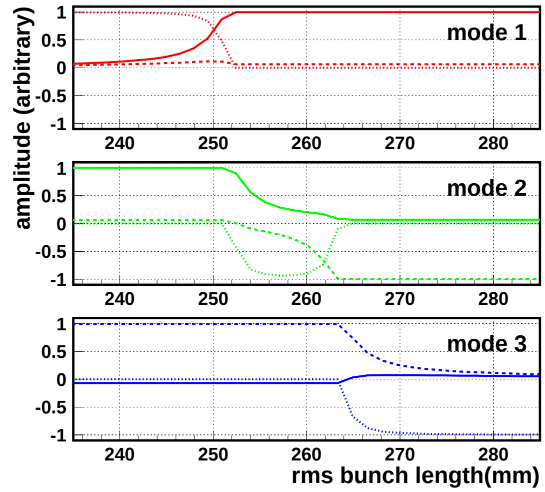

Figure 2: Amplitude ratio of eigen modes in x(solid line), y(broken line) and z(dot line). Mode 1, 2 and 3 is of the highest, middle and the lowest frequency.

3 MEASUREMENTS IN HIMAC

3.1 The HIMAC tune shifts

We chose a flatbase operation (no acceleration) with a He^{2+} beam to obtain the largest tune shift. Machine parameters are same as Table 1 except horizontal emittance of 264πmm-mrad and maximum beam intensity of 1×10^{11}ppp. That is due to multi-turn injection. The incoherent tune shift of vertical oscillation is calculated as -0.0446/10^{11}ppp. The coherent ones are -0.00216/10^{11}ppp for dipole mode and -0.0590/10^{11}ppp for quadrupole mode.

3.2 Quadrupole monitor

Among coherent mode oscillations, the tune shifts of quadrupole mode is dominant. Therefore, we installed a quadrupole pickup monitor in HIMAC in the summer of 1998. It is an electrostatic type pickup with four electrodes(Fig.3). The amplitude of quadrupole mode can be seen in a quadrupole channel output, which is subtracting the sum of vertical two electrodes signal from that of horizontal. The difference between opposite two electrodes are used to observe dipole mode oscillations. The estimated magnitude of output signal for a 6MeV/u He^{2+} beam is around 15mV/10^{11}ppp in each channel, and a quadrupole component is about the order of $(x/a)^2$ lower than that, where a is the radius of pickup electrodes.

3.3 Spectrum and tune shifts

We observed beam spectrum of a coasting beam of 3×10^{10}ppp with a real time spectrum analyzer. We used RFQ[7] in order to excite quadrupole mode oscillations. Fig.4 shows the beam spectrogram around 4 times of revo-

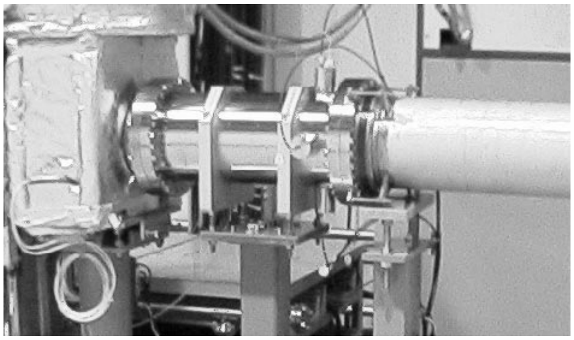

Figure 3: Quadrupole monitor in HIMAC.

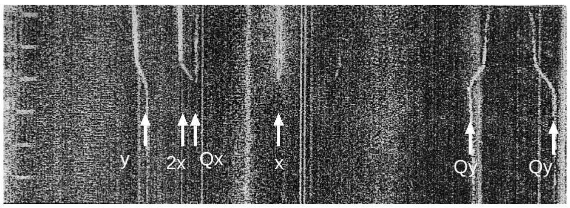

Figure 4: Spectrogram of coasting beam.

lution frequency. Quadrupole mode and dipole mode oscillations in horizontal and vertical space can be seen. Some of them are shifting in frequency as the beam current decreases at ∼1000msec after injection.

We estimated the space charge tune shift of each mode by plotting frequency versus beam current seen by a slow current transformer. Fitted value of tune shifts for quadrupole mode is about -0.082/10^{11}ppp, which is not in good agreement with theoretical value. A part of the discrepancy can be attributed to uncertainty of emittance. We are continuing measurements at other operationg points.

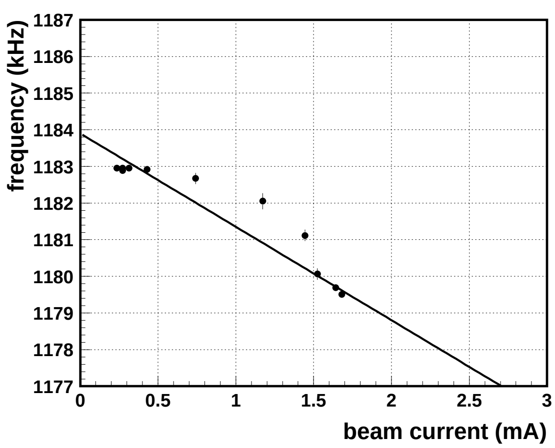

Figure 5: Tune shift vs circulating current.

3.4 Resonance crossing

Finally we show in Fig.6, where the operating poiont is moving gradually by varying defocusing field. As the current of defocusing magnet goes down, the quadrupole mode frequency approaches to revolution frequency. At 2500msec after beam injection, it finaly meets $2\nu_y$=7.0 resonance line and beam disappears rapidly. The tune shift of coherent quadrupole mode oscillations at the beam current just before the resonance is so small that it is difficult to determine if the resonance occurs by coherent mode oscillation or incoherent one.

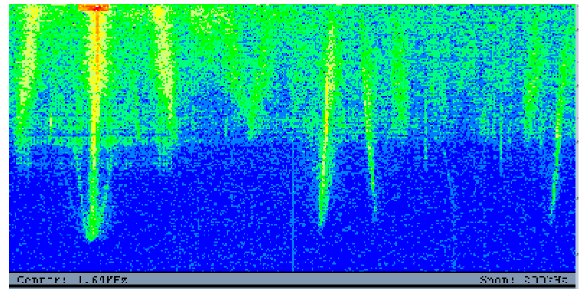

Figure 6: Half integer resonance. Frequency span is 200kHz centered at 1.64MHz. Time scale is 3000msec from top (beam injection) to bottom.

4 SUMMARY

First, we solved 3D envelope equations numerically, and found that the coupling between transverse and longitudinal motion due to space charge force is negligible in synchrotrons. Therefore, the mode frequencies in a coasting beam are fairly good approximations of that in a bunched one except one longitudinal mode.

Secondly, we presented our experiment in HIMAC synchrotron. We identified the coherent mode signals. The tune shift is not so good agreement with the expected value, at this moment. However, a part of the discrepancy can be attributed to uncertainty of emittance. We are carrying on the experiments to reveal the connection between tune shifts and beam loss.

5 ACKNOWLEDGEMENT

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